Inverse Kinematics-Based Trajectory Generation For Robot-Assisted 3D Surface Machining

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Abstract

Machining and tracking of 3D surfaces using industrial robot is not a new method, but the new in this paper is the use of simple trigonometric relations in the calculations of robots joints variables by using the inverse kinematics approach rather than the previous conventional methods like forward kinematics, decoupling, and sensor based machining. Calculations of the joints variables are mainly based on knowing the robot reference point (origin point) and the coordinates of the tip of the end effectors which is the cutter contact point (CC-P) at the surface. The coordinate of the cutter location point (CL-P) is the coordinate of the normal vector that passing through the intended cutter contact point. The joints variables are calculated based on simple trigonometric relationships. The results of the proposed method are verified based on hand-made simulation programs organized for this purpose. The simulation results explore the high accuracy and efficiency of the proposed method and its high speed in prediction of joints variables.

Keywords 3D surface machining, Inverse Kinematics, Surface tracking, CL-Points, CC-Points, NURBS surface, Tool Path Generation.

توليد مسار الإنسان الآلي لغرض تشغيل الأسطح الثلاثية الأبعاد باستخدام الطريقة الكينماتيكية العكسية

الخلاصة

إن عملية استخدام الروبوت في تشغيل الأسطح الثلاثية الأبعاد وتعقب المسارات المعقدة هي ليست بالأمر الجديد ولكن الجديد في هذا البحث هو استخدام العلاقات المثلثية البسيطة لحساب متغيرات الروبوت (زوايا الأذرع وإحداثيات نهايات الأذرع) بالاعتماد على الطريقة الكيناماتيكية العكسية (inverse kinematics) بدلا من الطرق التقليدية مثل الطريقة الكيناماتيكية الأمامية (forward kinematics method) والطريقة المعتمدة على المتحسسات أو ما يسمى بالـــ (decoupling method). يعتمد البحث في اشتقاق متغيرات الروبوت لأي سطح ثلاثي الأبعاد على معلومية النقطة المرجعية للروبوت وهي نقطة الأصل وإحداثيات نقطة نهاية المؤثر والتي تعتبر نقطة تماس العدة مع السطح (CC-P). إن إحداثيات نقطة موقع العدة (CL-P) هي إحداثيات المتجه العمودي على السطح والمار بـــنقاط التماس (CC-P). ومن خلال تطبيق بعض العلاقات المثلثية البسيطة تم حساب بقية متغيرات الروبوت وحسب الشكل المطلوب. تم التحقق من النتائج المطلوبة باجراء عملية المحاكاة. أثبتت النتائج دقة الطرقة المستخدمة ونجاحها وسرعتها العالية في حساب المتغيرات المطلوبة.

1. Introduction

Using industrial robot in tracking a 3D contours is a widely used method in recent industry [1-4]. The machining of 3D surfaces is usually done on 3-axes or 5-axes

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CNC milling machines [5]. The main objective of using robot instead of CNC machines in machining or finishing of 3D surfaces is to increase the machining flexibility thought increasing the degree of freedom DOF. The previous works related to the use of robots in 3D contour tracking which utilizes the inverse kinematics method were based on converting the transformation matrices to a system of explicit equations or based on decoupling methods [6,7]. These methods are iterative and hence they are unstable from the numerical point of view and they are not easy to be solved especially for 3 or more than 3 DOF. Fortunately, the machining of 3D surfaces with the aid of robots not -in general- requires a force or moment sensors to In this paper a new method for using a 6 degree of freedom (6-DOF) articulated robot is used for the machining of 3D surfaces. The method is based on position control alone utilizing the cutter contact data CC-data of the surface to be machined and cutter location data CL-data of the end-effector. The proposed method uses the trigonometric relations to extract the joints variables. Therefore, a unique solution can be obtained for each CL-data rather than many solutions obtained by previous iterative methods.

2. The Proposed Setup and Machining Strategy

The proposed articulated machining robot is the same as the well known industrial articulated robots which is also known as anthropomorphic arm as it is closely resembles the human arm [10]. The proposed setup of the machining robot is shown in Fig.2. In this setup a toroidal cutter is attached to the end effector enabling it tracking the required contour and rotating though a suitable cutting speed. The robot reference point (RRP) is coincide with the origin of the global coordinate system (0,0,0) , while the work piece reference point (WRP) is located so that the end effector can reach any point in the designed surface. The zigzag motion is the trajectory fashion nominated in this paper due to its simplicity and widely used compared with the other tool motion strategies as shown in Fig.3. The machining tool firstly sinks at the WRP and directed to move along the forward motion (red paths) towards point P_2 (see Fig.2). The tool is then reverses its motion towards the point P_3 to follow the cross motion and so on till the cutting tool finishes its motion at point P_{22}.

3. The Proposed Inverse Kinematics Method

The proposed inverse kinematics method is purely based on trigonometric relationships of the given reference frame (robots arms). Therefore the geometrical approach is used for solving the positioning problem rather than inverse matrices transformations method which entails iterative search method which leads to multiple solutions. The proposed geometrical method has a closed form solution; hence, it gives a unique solution for a given CL-data. Therefore, the closed form solution is preferable in 3D curve/surface tracking [11]. In our inverse

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kinematics method the knowing variables are (see Fig.4):

$$\Re_k = [\,\mathbf{a}\ \ \mathbf{b}\ \ \mathbf{c}\ \ \mathbf{d}\ \ \mathbf{P_o}\ \ \mathbf{CC\text{-}P}\,] \quad \ldots(1)$$

Where: a,b, and, c are the lengths of the robots arms, d is the length of the end effector (containing the unclamped tool length), $\mathbf{P_o}$ and **CC-P** is the position vector of the robot reference point RRP and the given cutter contact point (CC-P) on the required surface.

The required four unknown variables are:

$$\Re_u = (\,\alpha\ \beta\ \gamma\ \mathbf{P_2}\,) \quad \ldots(2)$$

Where: α, β, γ are the shoulder, elbow and wrist angles joint respectively. The following steps are followed to find the unknown variables of the final configuration.

3.1 Surface Generation and Cutter Location Data CL-Data Extraction

The NURBS surface is used for the generation of the required 3D parametric surface due to its widely used in CAGD. The required 3D NURBS surface is constructed using MATLAB programming environment according to the following mathematical format [12] :

$$\mathbf{r}\ S(u,v) = \frac{\sum_{i=0}^{p}\sum_{j=0}^{q} B_{i,m}(u).B_{j,n}(v).P_{i,j}.w_{i,j}}{\sum_{i=0}^{p}\sum_{j=0}^{q} B_{i,m}(u).B_{j,n}(v).w_{i,j}} \quad \ldots\ldots\ldots(3)$$

Where p,q are the number of control points (4 in this study), $P_{i,j}$ is the control points, $w_{i,j}$ the weight being associated with them, and Bi,n is the basis function defined by:

$$B_{i,0}(t) = \begin{cases} 1 & if\ t_i \le t < t_{i+1} \\ 0 & else \end{cases}$$

$$\forall k > 0, B_{i,k}(t) = \frac{t - t_i}{t_{i+k} - t_i} B_{i,k-1}(t) + \frac{t_{i+k+1} - t}{t_{i+k+1} - t_{i+1}} B_{i+1,k-1}(t)$$

The cutter contact data CC-Data are generated based on isoperimetric decartelization approach [13]. The decartelization isoparametric increment are (Δu=Δv=0.05), accordingly the whole surface is descretized to 441 CC-point, 21 forward paths and 20 traverse paths as shown in Fig.5. To generate the cutter location data (CL-Data), the normal vector at each cutter contact point CC-point must be determined. The CC-Data is then translated along the unit normal vector at a distance equal to the length of the end effector (d) as follows (see Fig.4):

$$\vec{CLP} = \vec{S}(u,v) + d.\vec{N} \quad \ldots(4)$$

$$\mathbf{r}\ \vec{N} = \frac{\partial \vec{S}}{\partial u} \times \frac{\partial \vec{S}}{\partial v} \Big/ \left\| \frac{\partial \vec{S}}{\partial u} \right\| . \left\| \frac{\partial \vec{S}}{\partial v} \right\| \quad \ldots\ldots(5)$$

Where $\vec{S}(u,v)$ is the instantaneous cutter contact point, $\vec{N}$ is the unit normal vector and $\frac{\partial \vec{S}}{\partial u}, \frac{\partial \vec{S}}{\partial v}$ is first partial derivative of the NURBS

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surface with respect to the parameter u and v respectively. It is worth pointing that translating the CC-point along with the normal vector guarantee the perpendicularly of the tool over the surface.

3.2 Calculation of the Robot's Arms Orientations

The joints variables $\Re_u$ = [α β γ $\mathbf{P_2}$] of the general situation o the robot's arms (see Fig.6) can be calculated according the following trigonometric formulation (see Fig.6):

$$a = 90 + f + \Omega \quad \text{.....(6)}$$

$$b = 180 - h \quad \text{.....(7)}$$

$$g = arcCos\ (\frac{V_1.V_2}{\|V_1\|.\|V_2\|}) \quad \text{.....(8)}$$

Now we are about finding φ, Ω, η to find the required joints variables $\Re_u$. Applying Cosine's law to the triangle P_1P_2 CL-P gives:

$$L^2 = b^2 + c^2 - 2.b.c.Cos(b)$$

.....(9.a)

$$Or\ \ L^2 = b^2 + c^2 + 2.b.c.Cos(h)$$

.....(9.b)

$$\therefore h = arcCos(\frac{L^2 - b^2 - c^2}{2.b.c})$$

.....(9.c)

Substituting Eq.9.c in Eq.7 yields:

$$b = 180 - arcCos(\frac{L^2 - b^2 - c^2}{2.b.c})$$

.....(10)

Applying the Sine law to the same triangle gives:

$$\frac{Sin(\Omega)}{c} = \frac{Sin(b)}{L} \quad \text{.....(11.a)}$$

$$\therefore \Omega = arcSin(\frac{c}{L}Sin(b)) \text{ ..... (11.b)}$$

Substituting Eq.10 in Eq.11.b yields:

$$\therefore \Omega = ArcSin[\frac{c}{L}Sin\{180 - arcCos(\frac{L^2 - b^2 - c^2}{2.b.c})\}]$$

..... (11.c)

$$f = \arctan(\frac{L_1}{L_2}) \quad \text{.....(11.d)}$$

$$L_1 = \left|Z_{CLP} - Z_{P_1}\right| \quad \text{..... (11.e)}$$

$$L_2 = \left|X_{CLP} - X_{P_1}\right| \quad \text{..... (11.f)}$$

Substituting Eq.11.f and Eq.11.e in Eq.11.d gives:

$$a = 90 + \arctan(\frac{\left|Z_{CLP} - Z_{P_1}\right|}{\left|X_{CLP} - X_{P_1}\right|}) + arcSin[\frac{c}{L}Sin\{180 - arcCos(\frac{L^2 - b^2 - c^2}{2.b.c})\}]$$

..... (12)

To find the elbow joint variable (γ) it is necessary to find the coordinates of the elbow joint as follows:

$$X_{P_2} = b.Cos(f + \Omega) \quad \text{.... (13.a)}$$

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$$Y_{P_2} = b.Sin(f + \Omega) \quad \ldots(13.b)$$

$$\vec{V}_1 = CL\vec{P} - \vec{P}_2 \quad \ldots(13.c)$$

$$\vec{V}_2 = \vec{P}_1 - \vec{P}_2 \quad \ldots(13.d)$$

Equation 13.c and 13.d can be used with the aid of Eq.8 to fid the elbow orientation (γ). It is worth pointing out that our mathematical formulation is a function of two variables namely: the cutter location points CL-P and the length between the shoulder joint and wrist joint (L) (see Fig.6). Accordingly the experimental runs show that thee the joint variables $\Re_u$ can be updated very fast within 0.187 second which is a very important task in contour tracking using robot.

3.3 Singularity

It is worth noting that from previous mathematical formulation and some other equations that the following deliverance condition should be satisfied to avoid singularity:

$$\frac{L^2 - b^2 - c^2}{2.b.c} \geq 1 \quad \ldots. \ (14.a)$$

$$\therefore 2.b.c \geq L^2 - b^2 - c^2 \quad \ldots. \ (14.b)$$

$$let \ b = l_1 L; c = l_2 L \quad \ldots. (14.c)$$

$$\therefore 2.b.c = 2 l_1 l_2 L^2; \quad \&$$

$$L^2 - b^2 - c^2 = L^2 - l_1{}^2 L^2 - l_2{}^2 L^2$$

$\ldots.(14.d)$

Substituting Eq.14.d in Eq.14.b gives:

$$2 l_1 l_2 \geq 1 - l_1^2 - l_2^2 \quad \ldots..(14.e)$$

Let $\lambda_1 = \lambda_2 = \lambda$ so that a=b, this will lead to:

$$l \geq 0.5 \Rightarrow b = c = 0.5L$$

$\ldots\ldots(14.f)$

Since that L is variable and depend upon the joints variables (see Eq.9) and to ensure machining the whole surface, the length of the arms a and b should be related to the longest L. Therefore the arm's length is selected to be 725 mm as the maximum length (L=1400 mm) according to our 3D surface.

4. Implementation and Results

In order to explore and test the validity and efficiency of the proposed inverse kinematics method an experimental example has been presented as follows:

4.1 Surface and Tool Path Generation

The proposed inverse kinematics approach is tested and applied to a 3D NURBS surface shown in Fig.5. The required surface is generated using MATLAB programming software while the required zigzag tool path using linear interpolation is generated using the well known professional CAD/CAM software UGS NX 5.0. The selected scallop height is 0.254 mm to attain 0.05 mm chordal deviation.

Table 1 shows the technical parameter of the robot used to machine the intended surface (for the simulation purposes). The cutting tool diameter (D=10 mm) is selected to avoid gauging so that its diameter is smaller than the

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minimum Gaussian radius of curvature over the whole surface.

4.2 Results Verification

In order to verify the results of our inverse kinematics method, the forward kinematics method is used. To insure that the tip of the end effector (milling cutter) touch the surface at each given CC-P for a given joint variable, the well known forward kinematics procedure is used. The results of the joints variables of the designed surface based on the aforementioned descretization approach (section 2.1) are listed in Table 2. There is no error measured between the calculated position of the end effector and the verified position.

4.3 Performance Simulation

The purpose of the robot simulation is to evaluate how the robot will react to the calculated joints variables. The proposed simulation method is realized utilizing MATALB software. The cutter contact file, cutter location file, and the joints variables are processed and the Figure 7 shows the output of the UG software and the selected machining parameter. The coordinates of the five positions shown above in the previous Fig.6 are shown in previous Table 2. This table also includes the joint variables calculated through applying the proposed inverse kinematics approach (Eqs.6-13). In order to verify these results, the system of verification equations (see Appendix A) is used and the coordinates are listed in the same table. The verification results reflect the robustness of our inverse kinematics approach in the extraction of joints variables from the given robot's tip position.

simulation results (sample of result to avoid ambiguity of the simulation) are shown in Fig.8.

5. Conclusions

An inverse kinematics approach was proposed in this paper in order for machining of 3D surfaces rather than the use of CNC milling machine. From the experimental runs it could be concluded that very small time is needed for the calculations of the joints variable (0.187s). The gathered results show the accuracy of the proposed method where the error between the required and verified CC-P is zero for all the end effector positions. The use of industrial robot in the machining of 3D surfaces facilitates the machining process compared with the ordinary CNC milling machine. The verification results exhibit the high precision in the calculations of the joints variable from the given CL-data. The proposed method establishes for the concept of portable milling machine which very important task for many industries such as marine applications.

Appendix A

The forward kinematics equations used to verify the results of the proposed inverse kinematics method.

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$$X_{CCP} = [b.Cos(f+\Omega) + c.Cos(f+\Omega+h) + d.Cos(180+f+\Omega+h-g)].Cos(z)$$
$$Y_{CCP} = [b.Cos(f+\Omega) + c.Cos(f+\Omega+h) + d.Cos(180+f+\Omega+h-g)].Sin(z)$$
$$Z_{CCP} = [b.Sin(f+\Omega) + c.Sin(f+\Omega+h) + d.Sin(180+f+\Omega+h-g)]$$

References

[1] F. Nagata, K. Watanabe, et al., "Polishing Robot Using a JoystickControlledTeaching," Journal of Robotics and Mechatronics – Special Issue on Recent Advances in Robot Control–, Vol. 13, No. 5, pp. 517-525, 2002.

[2] F. Nagata, K. Watanabe and K. Izumi, "Furniture Polishing Robot Using a Trajectory Generator Based on Cutter Location Data," Procs. of 2001 IEEE Int. Conf. on Robotics and Automation, pp. 319-324, 2001.

[3] F. Nagata, K. Watanabe, et al., "3D Machining and Finishing System for New Designed Furniture," Procs. of 2002 Japan USA Symposium on Flexible Automation, Vol. 3, pp. 1239-1245, 2002.

[4] Fukuda, T. and Nakagawa, S., "Dynamically Reconfigurable Robotic System," Proc. of the 1988 IEEE Int. Conf. Robotics and Automation, pp. 1581-1586.

[5] D.C.H. Yang, J.J. Chuang, and T.H. OuLee "Boundary Conformed Tool Path Generation of Trimmed Free-Form Surfaces". Computer Aided Design. 35 (2003), 127-139.

[6] Mark W.Spong, Seth Hutchinson, and M.Vidyasagar "Robot Dynamic and Control" Prentice Hall Inc., 2nd Ed.2004.

[7] Xin-yun Wang, Nobuki Yukawa, Yoshinori Yoshita, Takuya Sukeda and Takashi Ishikawa "Research on some basic deformations in free forging with robot and servo-press". Journal of Material processing Technology, 209, 6, (2009), 3030-3038.

[8] E. Abele, M. Weigold and S. Rothenbücher, "Modeling and Identification of an Industrial Robot for Machining Applications" CIRP Annals - Manufacturing Technology Volume 56, Issue 1, 2007, 387-390 .

[9] H. Chanal, E. Duc, J.Y. Hascoët and P. Ray "Reduction of a parallel kinematics machine tool inverse kinematics model with regard to machining behaviour"

[10] Tien-Chien Chang, Richard A. Wysk, Hsu-Pin Wang "Computer Aided Manufacturing" Prentice Hall Inc., 2nd Ed.1998.

[11] Charles M. Bergren "Anatomy of a Robot" McGraw-Hill, 2003.

[12] Robert M.Cheatham, W.Edward Red, and C.Greg Jensen "Direct Process Control Using n-Dimensional NURBS Curves". Computer Aided Design & Applications, 2, 6, 2005, 825-834.

[13] Sang C.Park "Sculptured surface machining using triangular mesh slicing" Computer-Aided Design 36 (2004) 279-288.

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Table (1) The Experimental Technical Parameters of the Proposed Robot.

a (mm)	b (mm)	c (mm)	d (mm)	D (mm)
250	700	700	100	10

Table (2) The Results and Verifications of the Joints Variables of the Five Positions Shown in Fig.6.

	Required End Effector CC-Points			The Five Joints Variables				Verified End Effector CC-Points		
#	X	Y	Z	$f+\Omega$	h	g	z	X	Y	Z
1	945.15	950	300	26	17	87	45.14	945.15	950	300
2	960.49	600	350	54	81	55	31.99	960.49	600	350
3	773.8	756.3	290	63	100	39	44.34	773.8	756.3	290
4	601.96	600	200	71.97	125.65	26.28	44.90	601.96	600	200
5	585.43	950	300	70	113.5	32.76	58.35	585.43	950	300

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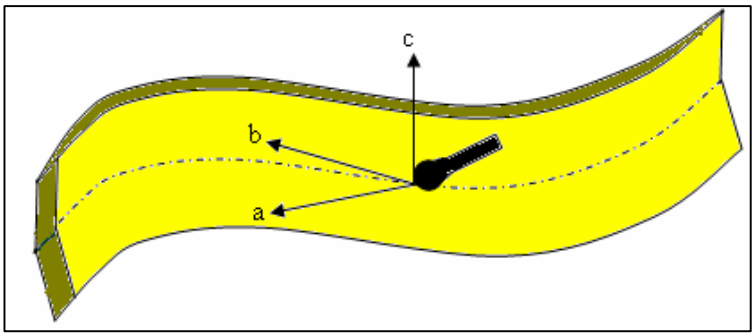

Figure (1) Deburring of 3D surfaces.

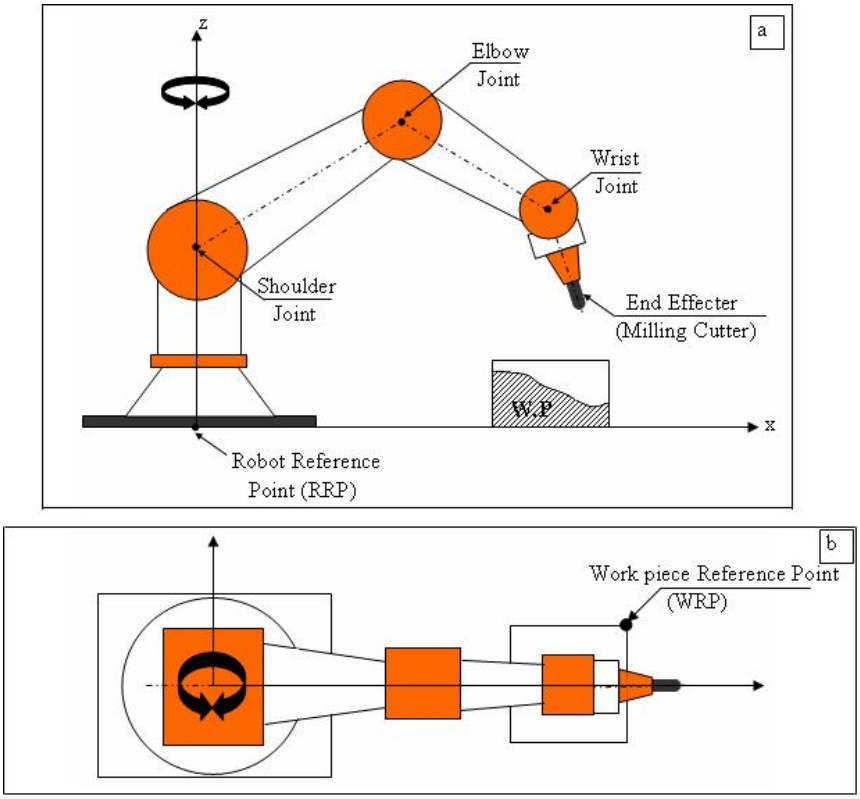

Figure (2) The Proposed Setup of the Machining Robot,
(a) Front View, (b) Top View.

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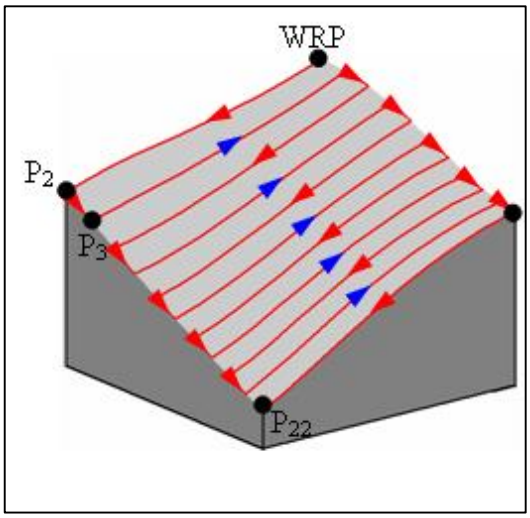

Figure (3) The Proposed Zig-Zag Tool Path Method.

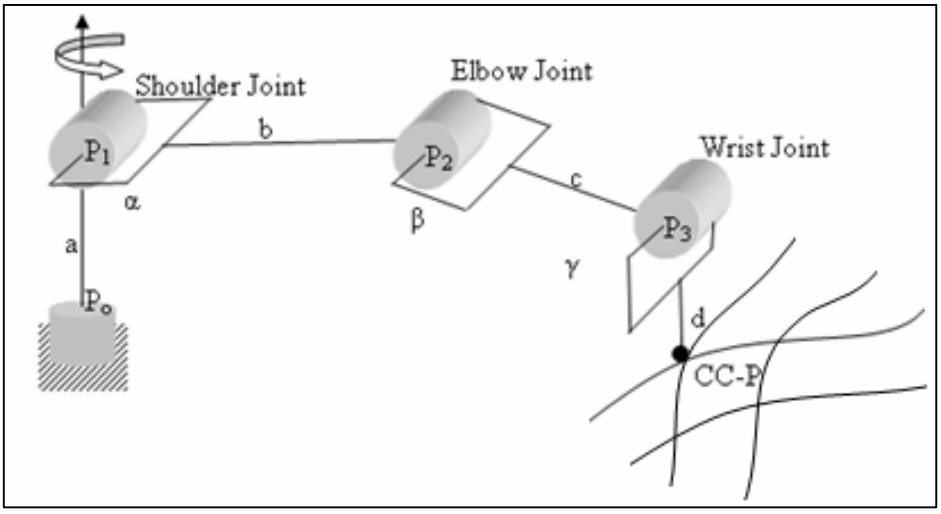

Figure (4) Schematic Representation of the Proposed Robot Showing the Known and

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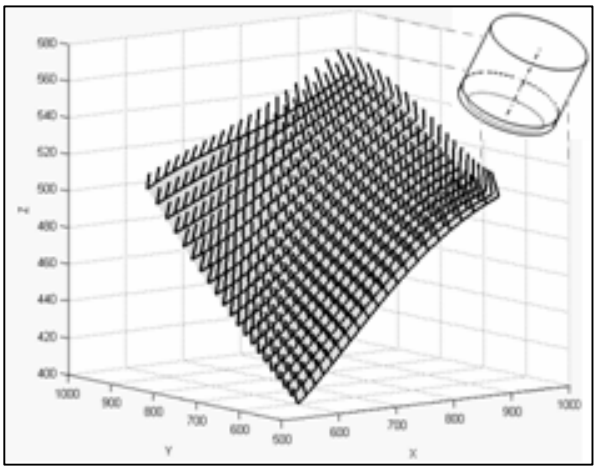

Figure (5) The Descretization method o the 3D Surface Showing the CL-Points and Surface Normal.

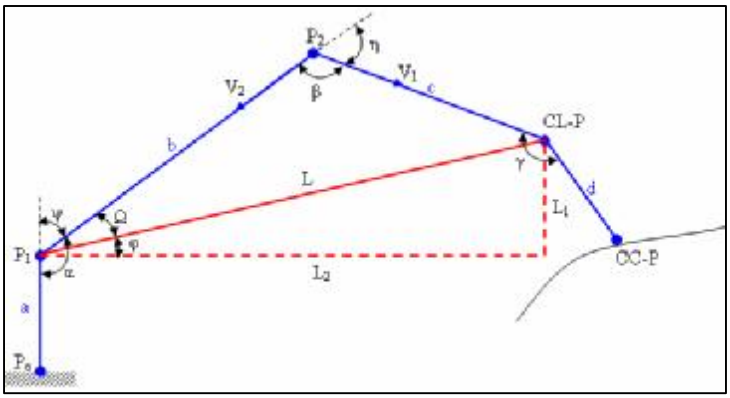

Figure (6) The Joint's Variables in General Situation and their Notations

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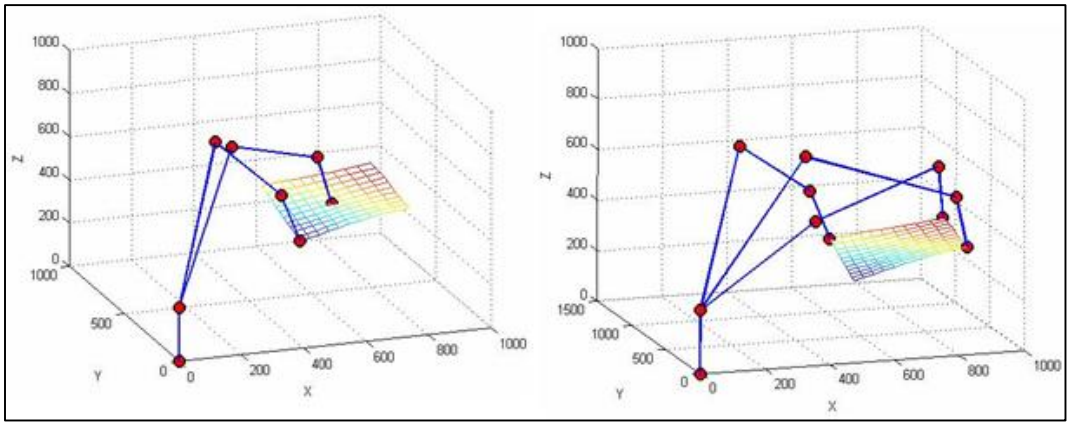

Figure (7) Simulation the Results of the Robot's Arms Positions at Different Locations.

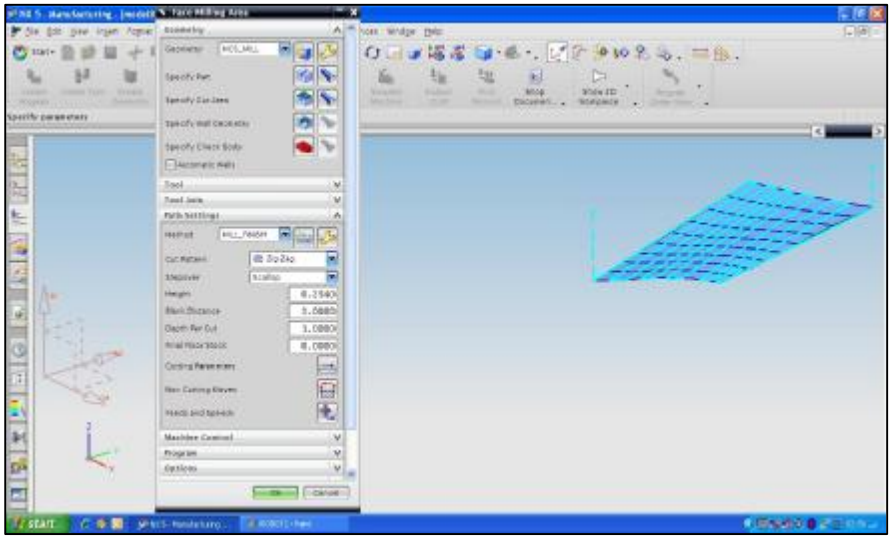

Figure (8) The tool path and the machining parameter using UG system

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