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Effects of Mix Property On Time Dependent Behavior of Reinforced Concrete Slabs

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Abstract

A nonlinear, layered, finite element model for predicting the time dependent behavior of reinforced concrete slabs under sustained transverse loading is presented. The effects of biaxial creep and shrinkage are considered by using the provisions of ACI Committee 209. Both elastic- perfectly plastic and strain hardening plasticity approach have been employed to model the compressive behavior of the concrete. The yield condition is formulated in terms of two-stress invariants. The movement of the subsequent loading surfaces is controlled by the hardening rule, which is extrapolated from the uniaxial stress-strain relationship defined by a parabolic function. Concrete crushing is a strain controlled phenomenon, which is monitored by a fracture surface similar to the yield surface. A smeared fixed crack approach is used to model the behavior of the cracked concrete, coupled with a tensile strength criterion to predict crack initiation. An attention is given also to the post-cracking shear strength. The steel is considered either as an elastic perfectly plastic material or as an elastic-plastic material with linear strain hardening. Steel reinforcement is assumed to have similar tensile and compressive stress-strain relationship. A computer program coded in FORTRAN77 language is written to implement the present study. This program is arranged to give a complete listing of stresses and deformations in every concrete or steel layer. Several examples for which experimental results are available are analyzed, using the proposed model .The comparison showed very good agreement especially for the maximum deflection, the different about 1%.

تاثير خواص الخلطة على السلوك المعتمد على الزمن للالواح الكونكريتية المسلحة

الخلاصة

استعملت طريقة العناصر المحددة اللاخطية لدراسة اداء البلاطات الخرسانية المسلحة المتمد على الزمن سواء كانت هذه الاحمال ثابتة او متغيرة. مثل تاثير الزحف والانكماش اعتمادا على النموذج المقترحمن قبل لجنة ٢٠٩ لمدونة الخرسانة الامريكية مثل سلوك الخرسانة في حالة الانضغاط كمادة مرنة-تامة اللدونة ام كمادة مرنة مع تصلد انفعالي لدن. شروط الخضوع عبر عنها بدلالة متغيرين للاجهاد. حكمت حركة سطوح التحميل بقاعدة التصلب التي تتحدد من علاقة الاجهاد- الانفعال الاحادي المحور والمعبر عنها بدلالة القطع المكافئ. سحق الخرسانة هي ظاهرة محكومة بالانفعال ومنظمة بسطح يشبه سطح الخضوع.

استخدم اسلوب الشق الثابت لتمثيل الخرسانة المتشققة مع شرط مقاومة الشد للتنبؤ بحدوث الشق. كما اخذ بنظر الاعتبار تاثير تصلب الشد في الخرسانة المتشققة. اخذ بنظر الاعتبار ايضا مقاومة القص بعد التشقق. حديد التسليح اعتبر كمادة مرنة-تامة اللدونة مع تصلد انفعال خطب. اعتبر سلوك حديد التسليح متشابه تحت تاثير قوى الشد والانضغاط. حللت عدة امثلة ذات نتائج عملية متوفرة باستعمال الاسلوب المقترح. اظهرت المقارنة توافقا جيدا.

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NOTATION

Ae Element area
B Strain- nodal displacement matrix
B_b Bending strain-displacement matrix
B_s Transverse shear strain-displacement matrix
D Elasticity matrix
D_b Flexural rigidities
D_s Shear rigidities
E Young's modulus
F Percent fines by weight
H Ambient humidity or hardening parameter
t Time in days
K Stiffness matrix
K_b Bending stiffness matrix
K_s Transverse shear stiffness matrix
M_x,M_y,M_{xy} Generalized stress resultants (moments)
S slump
N Shape functions
Q_x,Q_y Generalized stress components (transverse shear forces)
w, d_i Displacements
γ_{xz}, γ_{yz} Transverse shear strains in Cartesian coordinate system.
ε_b Bending strain tensor
ε_s Transverse shear strain tensor
θ_{xi}, θ_{yi} Rotations
ν Poisson's ratio
f_c' Concrete cylinder compressive Strength.
$f'_{c\max}$ Maximum compressive strength in the direction parallel to the crack

INTRODUCTION

The failure of a high parabolic (HP) gable shell roof in Virginia,seven years after its construction[1], underlines the importance of the application of realistic and rational models, load conditions and methods of analysis and design of such structures. To guarantee the serviceability of any structure throughout its useful period, it may be important to perform an analysis to obtain the response history during that period incorporating inelastic and time-dependent effects. Moreover, for the correct estimation of safety against failure, an ultimate analysis becomes mandatory. The development of the finite element method with the simultaneous application of increasingly efficient and sophisticated digital computers in recent years has resulted a significant progress in the investigation of the behavior of reinforced concrete structures for time dependent effects such as creep, shrinkage, temperature changes and load history.

Jason et al [2] developed a test method to assess the potential of

shrinkage cracking and developed a theoretical model to predict cracking and a method for evaluation of a nonexpansive shrinkage reducing admixture. Geist [3] presented a technical note for the fundamental computation for the foundation of an algorithm used to predict the creep behavior of material under stress. The model class employs four components with matched derivatives to represent the traditional primary, secondary, and tertiary stages of creep.

Baweja et al [4] showed how the mechanism of elastic composite materials can be adopted to predict the basic creep of concrete with aging due to hydration. The prediction is made on the basis of the given composition of concrete on the elastic constants of the aggregate and the aging viscoelastic properties of the Portland cement by Dvoraks transformation field analysis.

Ghali and Azarnejad [5] presented a summary of methods of analysis to predict the immediate and time dependent strains in reinforced concrete sections with or without prestressing.

AL-Naimi [6] adopted three-dimensional computational models of eight and twenty-node elements for the idealization of the concrete brick element by assuming perfect bond with imbedded steel . Nonlinear material and time -dependent effects have been included in this analysis.

AL-Timeemy [7] and AL Saaedi [8] presented a finite element model for creep behavior of reinforced concrete beams. Eight-node isoparametric element was used to represent the concrete with smeared layer of reinforcement.

They included the nonlinear effect in this analysis.

TIME – DEPENDENT ANALYSIS

Mechanisms of creep and shrinkage in concrete are not fully understood and the prediction of creep and shrinkage behavior in concrete is not precise at best. Creep and shrinkage are often treated as separate and independent phenomena. Actually, the effect of creep is significantly greater when accompanied by shrinkage e.g., drying creep is the additional creep resulting from drying of concrete. Over the last two decades, number of predictive models have been developed to take into account the interdependence of creep and shrinkage.

In the time dependent analysis of reinforced concrete, it is convenient to represent creep of concrete by using a creep coefficient, defined as the ratio of creep strain to elastic (instantaneous) strain. This presumes that the creep strain is linearly related to the applied stress through the elastic term. The assumption of linearity of creep must be made for the principle of superposition to be used in a time dependent analysis considering a multi- phase process. This condition is generally met when the applied stress is less than about 50 % of the concrete strength [9, 10].

The long term creep strains of concrete under service stress are typically 2 to 6 times larger than the short –term elastic strains, and even short term deformation of about 10 minute duration comprises 25-50 % of the creep. Thus, consideration of creep is important for realistic analysis of concrete structures [9, 11]. The ACI Committee 209 [12] proposed the following form of equations for predicting the compressive strength (f_c'), direct tensile strength (f_t) and

the modulus of elasticity (E_0) at any time (t).

$$f_c(t) = \frac{t}{4 + 0.85 \times t} . f_c(28) \ldots.(1)$$

$$ft = 0.007\sqrt{W . fc(t)} \ldots\ldots\ldots..(2)$$

$$E_0 = 0.043W^{1.5}\sqrt{fc(t)} \ldots\ldots..(3)$$

$$e_{cu}(t) = \frac{4\,fc(t)}{E_0(t)} \quad \ldots\ldots\ldots\ldots\ldots(4)$$

where *W is* the unit unite weight and f_c , *ft* and E_o are in (SI) unite.

BASIC THEORY

The variation of the displacement and rotation fields over a Mindlin plate element is given by the following expression [13]

$$\left[W, q_x, q_y\right]^T = \sum_{i=1}^{n} N_i \, d_i \quad \ldots\ldots\ldots(5)$$

The plate curvature-displacement and shear strain-displacement relations are then written as:

$$e_b = \sum_{i=1}^{n} B_{bi} \; di \qquad \ldots\ldots\ldots.(6)$$

$$e_s = \sum_{i=1}^{n} B_{si} \; di$$

The moment-curvature and shear force-shear strain relationships are given as:

$[M_x, M_y, M_{xy}]^T = D_b \, \varepsilon_b$, $[Q_x, Q_y]^T = D_s \, \varepsilon_s$ ……. (7)

The stiffness matrix (Kij) contributions from the bending and shear relations can be written as:

$$K^e_{bij} = \int_{Ae} B^T_{bi} D_b B_{bj} dA$$

$$K^e_{sij} = \int_{Ae} B^T_{si} D_s B_{sj} dA \ldots\ldots\ldots (8)$$

CREEP FORMULATION

Deflection due to creep depends on the creep characteristics of the concrete, the quantity of compression steel, extent of the concrete cracking, and the boundary conditions of the member.

Creep is usually defined as the part of deformation under sustained loading which is in excess of instantaneous elastic deformation .The ACI Committee209 [12] suggests the use of the following empirical equation to calculate creep for "standard condition" and the use of correction factors for condition other than the standard case:

$$c(t-t) = \frac{(t-t)^{0.6}}{10 + (t-t)^{0.6}} \times cu \ldots.(9)$$

where $c(t-t)$ is the creep coefficient defined as the ratio of creep strain at (t) day after loading to the initial instantaneous strain at loading (at time τ), *cu* is the ultimate creep coefficient to be determined from experimental data defined as ratio of the creep strain at infinite time after loading to the initial strain at time of loading .The ultimate creep coefficient is calculated as follows:

$$cu = 2.3 \times F_t^C \times F_H^C \times F_t^c \times F_S^C \times F_A^C \ldots\ldots\ldots(10)$$

where, $F_t^C, F_H^C, F_t^C, F_S^C$ and F_A^C are correction factors for loading age, humidity, minimum member thickness, slump, percent of fine and air content. All the above correction factors assume the value of one for the following "standard condition"102 mm or less slump, 40 percent ambient relative humidity, 152 mm or less minimum member thickness, and loading age of 7 days for moist cured

concrete and 1-3 days for steam cured concrete.

For conditions other than the standard cases, the correction factors are calculated as follows [12]:

$F_t^C = 1.25 \times t^{-0.118}$

for moist cured concrete

$F_t^C = 1.13 \times t^{-0.094}$

for moist cured concrete ……..(11)

$F_H^C = 1.27 - 0.0067 \times H$

$H \geq 40\%$ ……..…… (12)

$F_T^C = 1.14 - 0.0092 \times T$

for $\leq$ 1 year loading

$F_T^C = 1.1 - 0.00067 \times T$ ……(13)

for ultimate loading

$F_S^C = 0.82 + 0.00264 \times S$ ……(14)

$F_F^C = 0.88 + 0.0024 \times F$ ……...(15)

$F_A^C = 1.0$

for $A \leq 6\%$

$F_A^C = 0.46 + 0.09 \times A$

for $A \geq 6\%$ ……... (16)

where t is the loading age in days, H is the ambient relative humidity in percent, T is the minimum thickness in mm, S is the slump in mm, F is the percent of fine material by weight and A is the air content in percent.

SHRINKAGE FORMULATION

Shrinkage results in a shortening but not curvature in plain concrete. However, for singly reinforced sections or sections containing unequal amounts of tensile and compressive reinforcement, the top fibers shorten due to shrinkage (for positive moment region) and the shortening of the bottom fibers are resisted or prevented by the reinforcement. These unsymmetrical strains cause warping in the section and, hence, shrinkage deflection is appears. For thin slabs, since compression reinforcement is not usually provided, these additional deflections due to shrinkage -induced warping can be assumed considerable and will result in warping deflection. ACI Committee209 suggested the use of the following empirical equation to calculate shrinkage strain:

$$e_{sh}(t - t_0) = \frac{(t - t_0)}{35 + (t - t_0)} . (e_{sh})_u \quad \text{….(17)}$$

where $e_{Sh}(t - t_0)$ is the shrinkage strain after (t-t_0) days from the completion of curing $(e_{sh})_u$ is the ultimate shrinkage strain, t is the time in days since concrete casting and t_0 is the curing time. ACI Committee 209 recommends the following value for the ultimate shrinkage strain

$$(e_{sh})_u = 780 \times 10^{-6} \times CF^s \quad \text{……..(18)}$$

where CF^S is the shrinkage correction factor given as:

$$CF^S = F_H^S \times F_T^S \times F_B^S \times F_S^S \times F_F^S \times F_A^S \quad \text{…….. (19)}$$

where $F_H^S, F_T^S, F_B^S, F_S^S, F_F^S$ and F_A^S are the shrinkage correction factor for humidity, minimum thickness, slump, cement content, percent fines, and air content respectively.

All the above correction factors assume the value of unity for the following "standard condition": 40 percent ambient relative humidity, 152 mm or less member thickness, and 102mm or less slump. Other than the standard cases, the correction factors are calculated as follows:

$F_H^S = 1.4 - 0.01 \times H$

$40 \le H \le 80\%$

$F_H^S = 3.0 - .03 \times H$

$80 \le H \le 100\%$ ……...(20)

$F_T^S = 1.23 - 0.0015 \times T$

for $\le$ 1 year loading

$F_T^S = 1.17 - 0.00114$

for ultimate loading . ……. (21)

$F_B^S = 0.75 + 0.00061 \times B$ ……(22)

$F_F^S = 0.3 + 0.014 \times F$

for F $\le 50\%$

$F_F^S = 0.9 + 0.002 \times F$

for F $\ge 50\%$ ……. (23)

$F_A^S = 0.95 + 0.008 \times A$ …… ..(24)

where H is the ambient relative humidity in percent, T is the minimum member thickness in mm, S is the slump in mm, B is the number of 50kg sacks of cement per cubic meter of concrete, F is the percent of fine aggregate by weight, and A is the air content in percent.

MATERIAL MODELLING

Based on the flow theory of plasticity the nonlinear compressive behaviour of concrete is modelled. Adopting Kupfer's results[14], the yield condition for the slabs can be written in terms of the stress components as:

$$f(\sigma)=\{1.355[(\sigma_x^2+\sigma_y^2-\sigma_x\sigma_y)+3(\tau_{xy}^2+\tau_{xz}^2+\tau_{yz}^2)]+0.355\sigma_o(\sigma_x+\sigma_y)\}^{1/2}=\sigma o \quad \text{……..(25)}$$

where (σ_o) is the equivalent effective stress taken as the compressive strength ($f_c^{'}$) obtained from uniaxial test.

Both perfect plastic and strain hardening plasticity approaches are employed which are illustrated, for one dimension, in Fig.(1).

The crushing type of concrete is a strain-controlled phenomenon. A simple way is used by converting the yield criterion in stresses into yield criterion directly in terms of strains, thus crushing condition can be expressed in terms of the total strain components as:

$$1.355\ [(\varepsilon_x^2+\varepsilon_y^2-\varepsilon_x\varepsilon_y)+0.75(\gamma_{xy}^2+\gamma_{xz}^2+\gamma_{yz}^2)]+0.355\varepsilon_u(\varepsilon_x+\varepsilon_y)=\varepsilon_u^2 \quad \text{……(26)}$$

when equation (26) is satisfied, the strain (ε_u) reaches the crushing surface, and the concrete is assumed to lose all its characteristics of strength and stiffness.

The response of concrete in tension is assumed to be linearly elastic until the fracture surface is reached. Cracks are assumed to form in planes perpendicular to the direction of maximum principal tensile stress if this maximum stress reaches the specified concrete tensile strength. After cracking, a gradual release of the concrete stress component normal to the cracked plane is adopted according to a tension stiffening diagram illustrated in Fig.(2). The process of loading and unloading of cracked concrete is also shown in Fig.(2). A reduced shear modulus taken as a function of the current tensile strain is used to simulate aggregate interlock and dowel action.

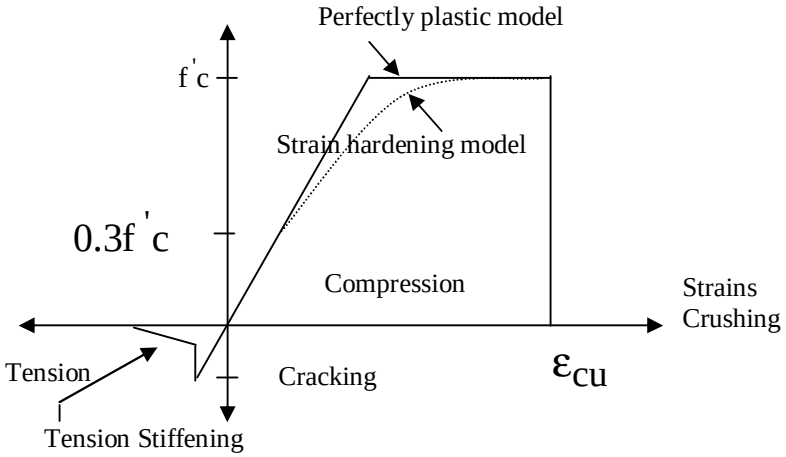


Fig.(1) Uniaxial representation of the concrete constitutive model.

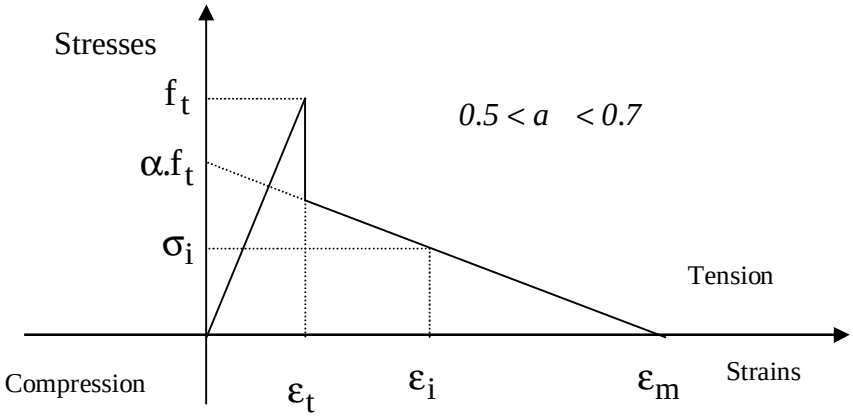


Fig.(2) Tension stiffening in concrete after cracking.

The tensile cracks produce damage to concrete with the transverse strain having a degrading effect not only on the compressive strength but also in the compressive stiffness, so that the concrete in this case becomes softer and weaker than that in a standard cylinder test. In the present study the relationship suggested by Belarbi and Hsu [17] is adopted,

$$f_{c\max} = \frac{0.9 f_c^{'}}{\sqrt{1+400e_1}} \quad \text{.....(27)}$$

where $(f_c^{'})$ is the concrete cylinder compressive strength and (e_1) is the

average principal tensile strain of concrete in direction (1).

Steel reinforcement is modeled by considering the steel bars as layers of equivalent thickness. Each steel layer exhibits uniaxial response, having strength and stiffness characteristics in the bar direction. A bilinear idealization is adopted in order to model the elasto-plastic stress-strain relationships.

An incremental /iterative Newton-Raphson method is employed in order to trace the response of the structure through the loading history.

NUMERICAL EXAMPLES

EXAMPLE 1

A one-way slab under uniform load given in reference [15] is shown in Fig. (3). The material properties are given in Table (1). Taking advantage of symmetry only one quarter of the slab is considered and idealized by 16 slab elements. Results of load deflection curves are shown in Fig.(4). Very good agreement is obtained by the proposed elements compared with the experimental results, especially at the maximum deflection.

Table.1 Material properties for slab of example (1)

Slab No	E_c MPa	f_c' MPa	n_c	f_t' MPa	a	e_m	f_y MPa	E_s' MPa	E_s MPa	H	S mm	F
1	25097	32.26	0.15	3.2	0.6	0.002	339	0.0	200000	70 %	190	36.9 %

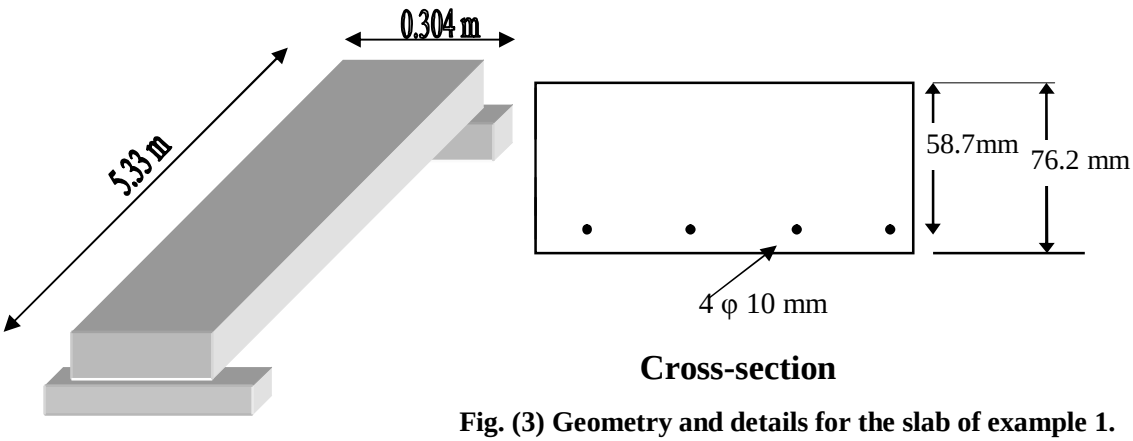


Fig. (3) Geometry and details for the slab of example 1.

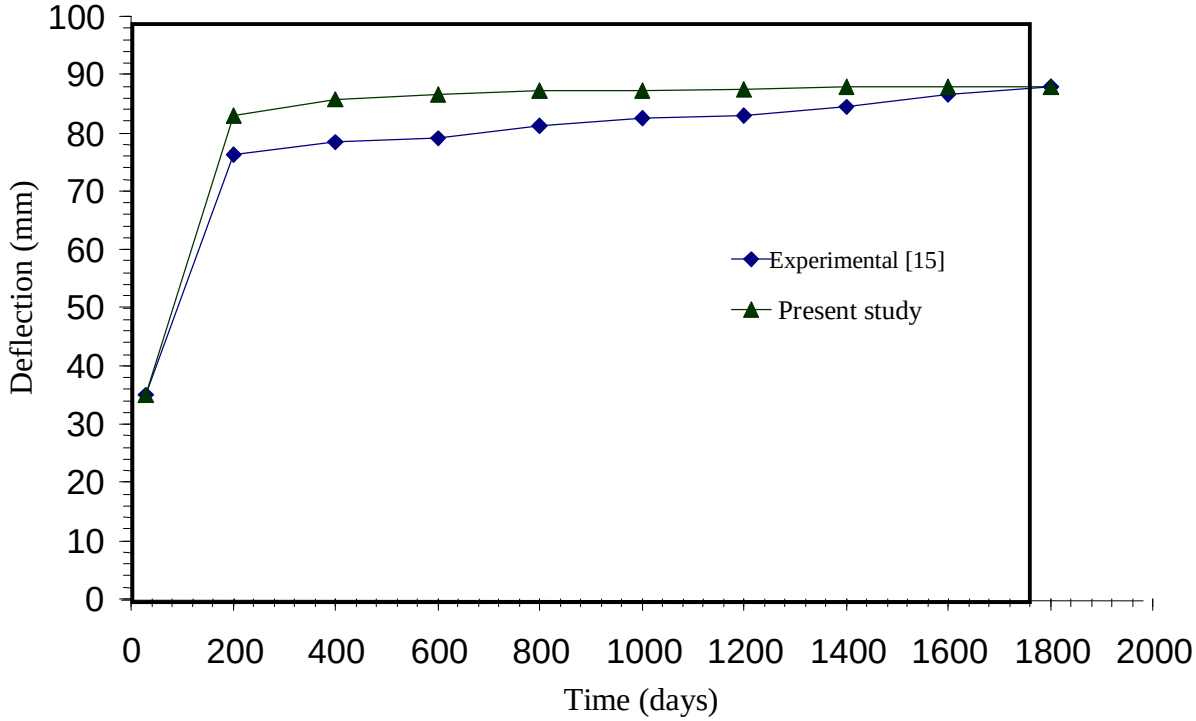


Fig.(4) Variation of deflection with time for the slab of example 1.

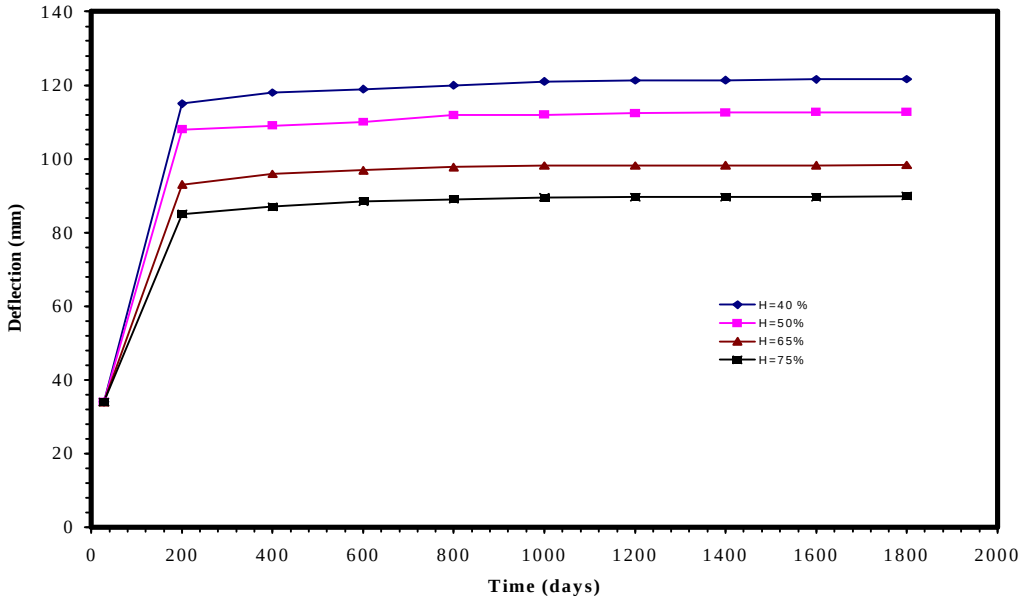


Fig.(5)Effects of variation of humidity on deflection of slab No.1

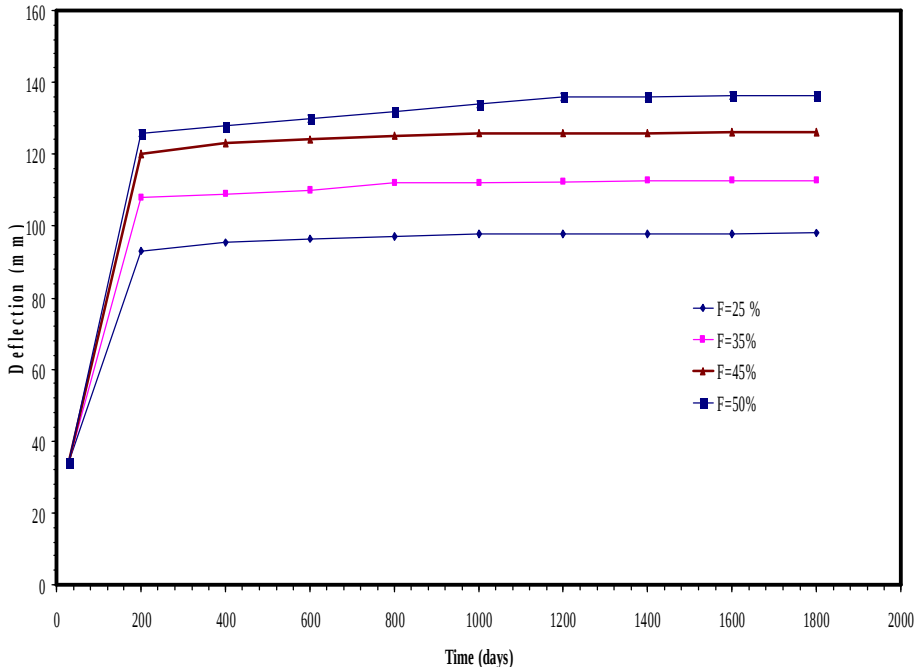
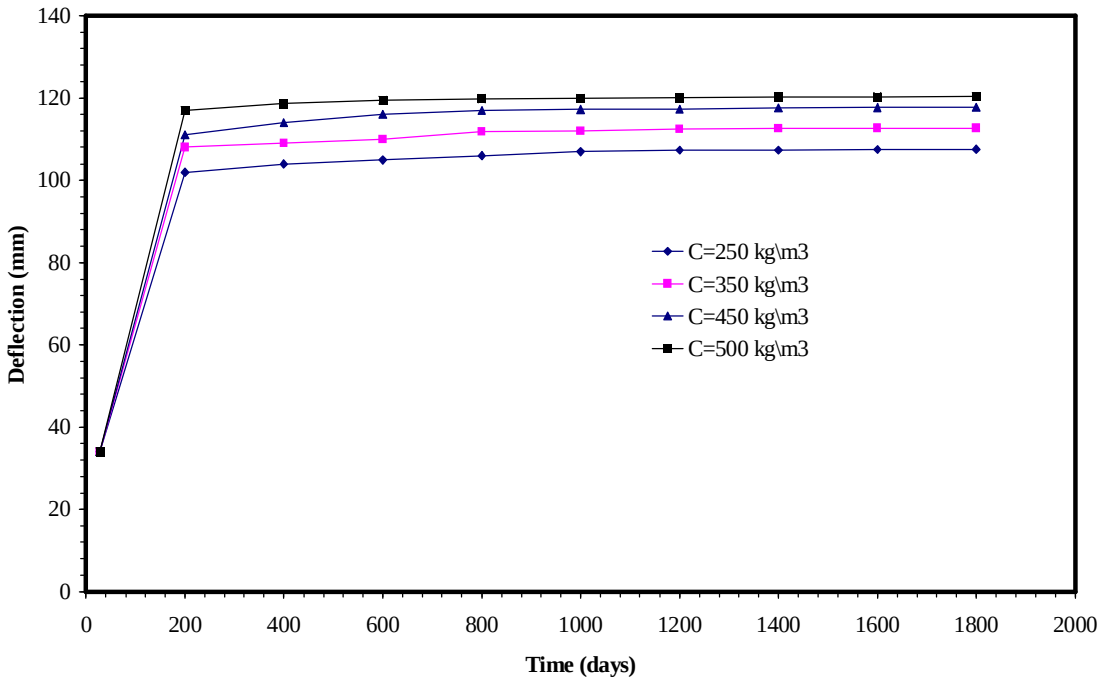


Fig.(6)Effects variation of fine percent on deflection of slab No.1



Fig(7)Effects variation of slump on deflection of slab No 1.

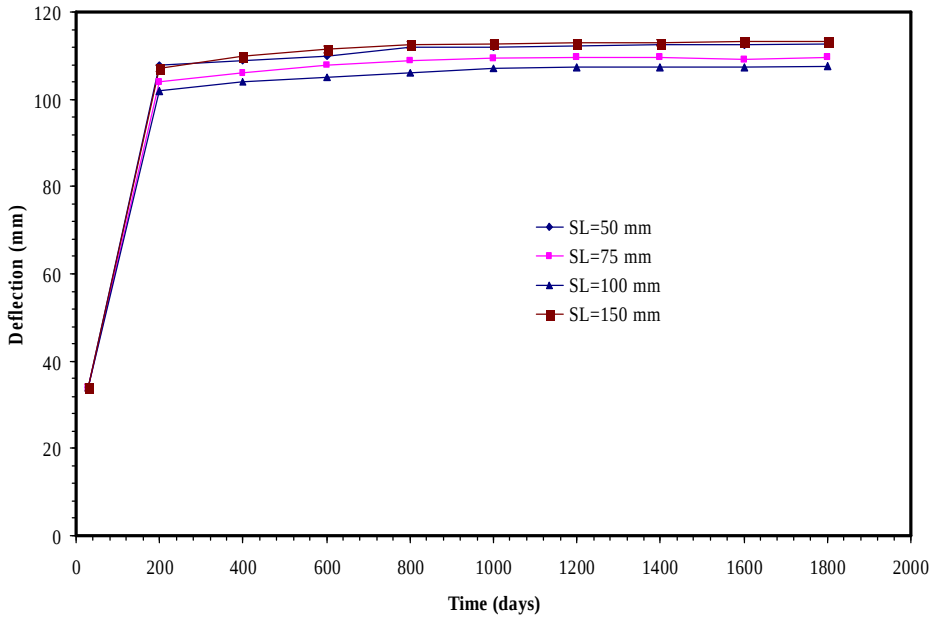


Fig (8)Effects of variation cement content on deflection of slab No.1

From the results given in Fig. (5), the relative humidity affects the deflection in such a manner that a reduction in relative humidity results in an increase in deflection.

Figs (6-8) showed that an increase in the percentage of fines and cement content of the mix results in an increase in deflection. This is also true for the effect of the slump of fresh concrete.

EXAMPLE 2

A simply supporting square slab under a central point load was tested by McNiece and Jofriet[18]. The test specimen has dimensions of (914.4*914.4) mm with (44) mm thickness and an effective depth of (33.7) mm to steel reinforcement. The geometry and details are shown in Fig. (9)

The material properties of the tested slab are summarized in Table (3). The slab is modeled by nine elements. Due to the symmetry of the slab, only one quarter of the slab is analyzed. Two steel layers are used to represent the reinforcement and eight concrete layers are found to be enough for the analysis.

From the results given in Fig. (10), the relative humidity affects the deflection in such a manner that a reduction in relative humidity results in an increase in deflection.

Figs (11-13) showed that an increase in the percentage of fines and cement content of the mix results in an increase in deflection. This is also true for the effect of the slump of fresh concrete.

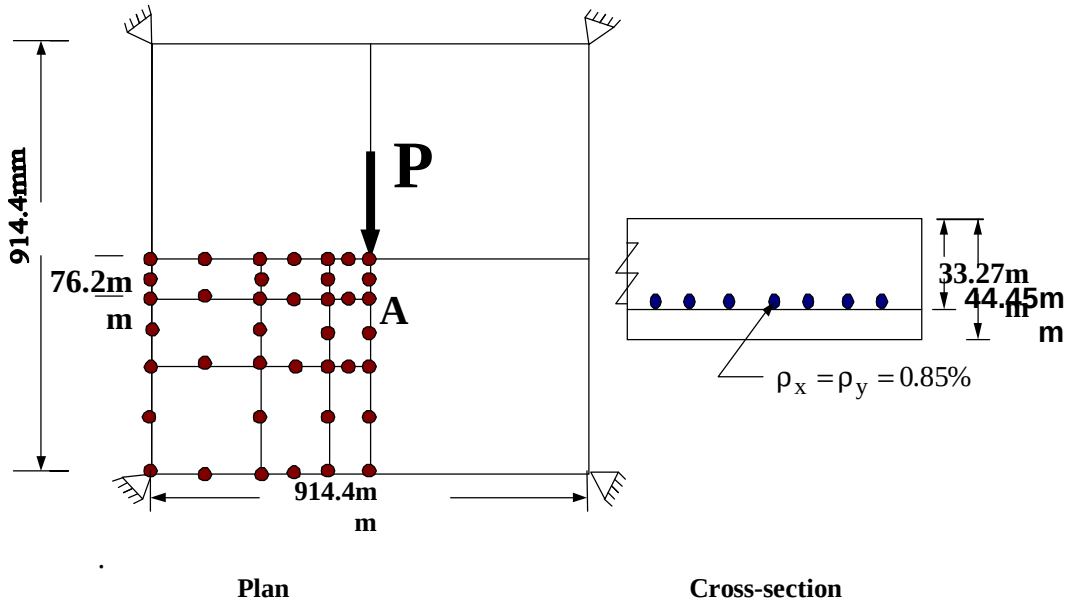


Fig.(9) Geometry and details for slab of example 2.

Table.2 Material properties for slab of example (2)

Slab No.	Ec (MPa)	fc' (MPa)	ft' (MPa)	v_c	Es (MPa)	f_y (MPa)	ε_u
5	28620	37.93	3.38	0.15	200000	414	0.003

Table.3 Assumed mix properties for slab of example (2)

Slab No.	T_0 Days	H %	S Mm	B kg/m^3	A %	T mm	F %
5	15	50	100	350	4	44.5	35

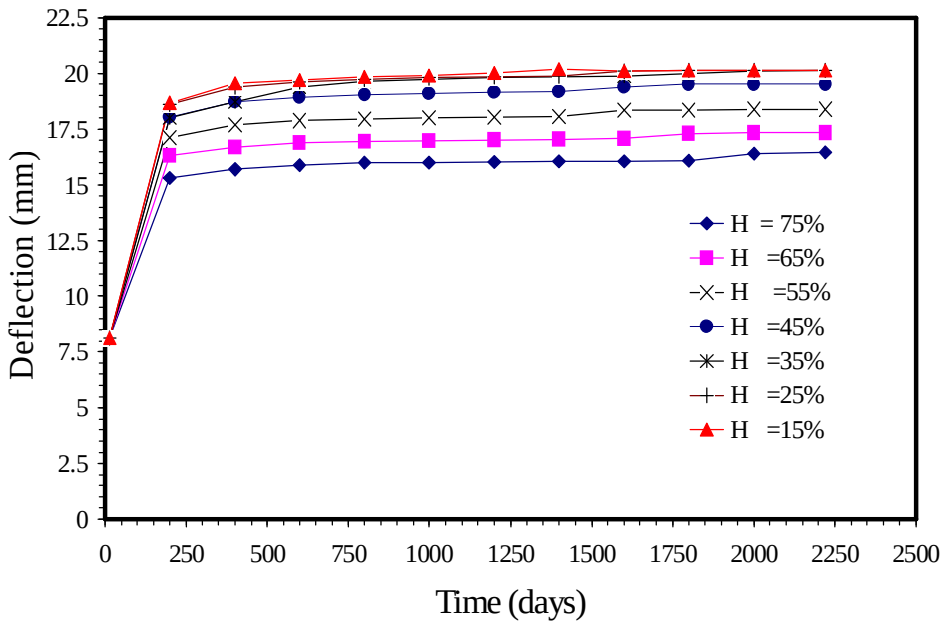


Fig (10) Effects of variation humidity on deflection of slab No.2

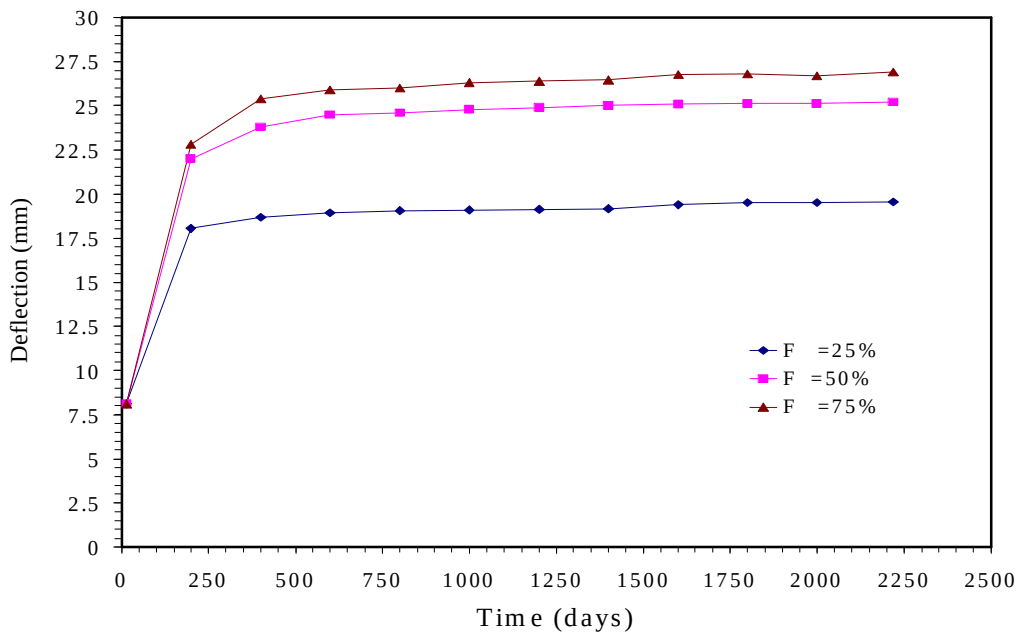


Fig (11)Effects variation fine percent on deflection of slab No.2

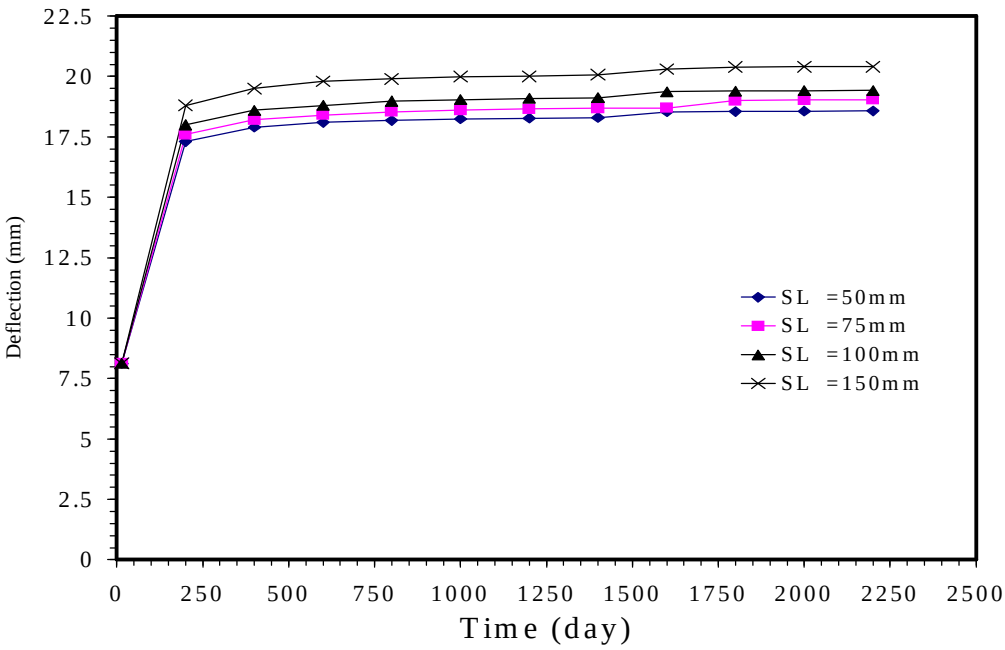


Fig (12) Effects of variation slump on deflection of slab No.2

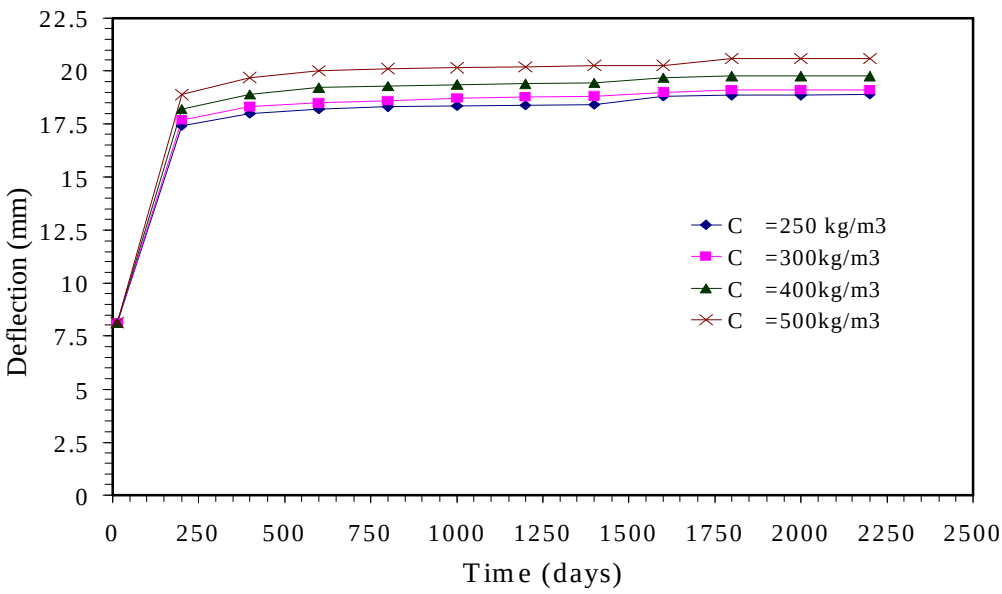


Fig (13) Effects of variation cement content on deflection of slab No.2

CONCLUSIONS

The developed models concerning the plasticity theory proved to give satisfactory results for time dependent analysis of reinforced concrete slabs.

The variation of deflections with time obtained by the nonlinear finite element analysis is in very good agreement with the experimental results especially for the maximum deflection, the difference about 1%.

Long time response due to creep and shrinkage effects are important for slab structures. Design procedures for such structures should incorporate the effects of long time behavior of concrete to determine both the serviceability and the ultimate safety criteria during the design life of such structures.

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