On Arps – Closed Sets in Topological Spaces

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ABSTRACT

In this paper, we introduce a new class of closed sets which is called αrps – closed sets in topological spaces and we given the relationships of these sets with some other sets. Also, we study some of their properties. Further, will be introduce and study new type of spaces namely $T_{\alpha rps} - \text{space}$ and new type of continuous functions which are (αrps-continuous functions , αrps- irresolute functions and strongly αrps-continuous functions) , we introduce several properties of these functions are proved.

حول ألمجموعات المغلقة – αrps في ألفضاءات التبولوجية

الخلاصة

في هذا البحث , قدمنا صنف جديد من ألمجموعات ألمغلقة تدعى المجموعات ألمغلقة- αrps وتم اعطاء العلاقات بين هذه المجموعات مع بعض انواع اخرى من المجموعات في ألفضاءات ألتبولوجية. وكذلك درسنا بعض من خواصها . بالأضافة الى ذلك, سوف نقدم وندرس نوع جديد من الفضاءات تسمى بالفضاءات – $T_{\alpha rps}$ و نوع جديد من الدوال المستمرة وهي (الدوال المستمرة-αrps , الدوال المحيرة – αrps والدوال الأقوى –αrps المستمرة) . قدمنا براهين لبعض خواص لهذه الدوال .

INTRODUCTION

Semi –open sets , regular open sets , α-open and preopen sets have been introduced and investigated be Levin [15],Stone[25], Njastad [22]and Mashhour [20] respectively. In 1970, Levin [16] introduced generalized closed sets and studied their basic properties. After the introduction of generalized closed sets there are many research papers which deal with different types of generalized closed sets. Arya[3] , Bhattachary and Iahiri [6], Maki et a [17, 18] introduced generalized semi – closed sets, semi- generalized closed sets, α-generalized closed sets and generalized α- closed sets respectively. Also Dontchev [10], Maki et a [19].Ganambal [13].Palaniappan and Rao[23]. Nagaven and Ganster[11],they also introduced and investigated generalized semi- preclosed sets ,generalized preclosed, gp- sets , generalized pre regular closed sets, weakly generalized closed, regular generalized- closed sets and generalized b- closed sets .

In [24] , (Shyla and Thangavelu , 2010) introduced and studied rps – closed and rps- open sets in topological spaces .

In this paper a new type of closed sets called αrps- closed sets are introduced and its properties are studied. Applying these sets, we obtain a new space which is namely $T_{\alpha rps}$ $-$ spaces . Further we study (αrps- continuous , αrps- irresolute and strongly αrps-continuous)functions and we proved some their properties Throughout this paper (X, τ), (Y, σ) and (Z ,μ) (or simply X,Y and Z) represent non – empty topological spaces .For a subsets A of a spaces X. cl(A), int (A) and A^c denote the closure of A ,the interior of A and the complement of A respectively.

PRELIMINARIES

Some definition and basic concepts have been given in this section.

Definition (2-1): A subset A of a space X is said to be:

1- semi- open [15] if $A \subseteq c\,l(\text{in t}(A))$ and semi- closed set if int(cL(A)) ⊆ A .

2-α-open set [22] if $A \subseteq \text{int}(\text{cl}(\text{in t}(A))$ and α-closed set if $\text{cl}(\text{int}(\text{cl}(A)) \subseteq A$.

3-Preopen set [20] if A⊆ int(cl(A)) and preclosed if cl(int(A) ⊆A.

4-semi-preopen set [1] if $A \subseteq \text{cl}(\text{int}\,(\text{cl}(A))$ and semi- preclosed if $\text{int}(\text{cL}(\text{in t}(A)) \subseteq A$.

5-b-open set [2] if A⊆ int(cl(A)) ∪ cl(int(A)) and b- closed set if int (cl(A)) ∩ cl(int(A)) ⊆ A.

6-regular open [25] if A = int(cl(A)) and regular closed if A = cl(int(A)) .

7-regular α-open [27] if there is a regular open set U such that U⊆ A⊆ αcl(U) .

The semi –closure (resp . α- closure , resp. pre- closure , resp . semi- pre closure , resp . b- closure) of a sub set A of X is the intersection of all semi-closed (resp α- closed , resp . pre- closed , resp. semi- pre closed , resp. b- closed) sets containing A and denoted by scl(A) (resp. αcl(A) , resp . pcl(A) , resp. spcl(A), resp. bcl(A) Clearly, $\text{bcl}(A) \subseteq \text{scl}(A) \subseteq \alpha\text{cl}(A) \subseteq \text{cl}(A)$ and $\text{spcl}(A) \subseteq \text{pcl}(A) \subseteq \alpha\text{cl}(A) \subseteq \text{cl}(A)$.

Remark (2-2): In [2] , [15], [22] , it has been proved that :

(i) Every open set in a space X is an α-open (resp. preopen , semi-preopen and b-open sets .Also every closed set is α-closed(resp. preclosed , semi-preclosed and b-clsed set)

(ii) Every α-open set in a space X is a preopen (resp . semi-preopen , semi- open and b-open) set. Also every α-closed set in space X is a preclosed (resp . semi-preclosed , semi-closed and b-closed) set .

(iii) The union of any family of α-open sets is α-open set and the intersection of any finite sets of α-closed sets is α-closed.

Definition (2-3): A sub set A of a space X is said to be a :

1- **Generalized closed set** (briefly, g- closed) [16] if cl(A) ⊆ U whenever A⊆ U and U is an open set in X .

2- **Generalized semi- closed set** (briefly, gs- closed) [3] if scl(A) ⊆ U whenever A⊆ U and U is an open set in X .

3- **semi- generalized closed set** (briefly, sg- closed) [6] if scl(A) ⊆ U whenever A⊆ U and U is a semi- open set in X .

4- **generalized α- closed set** (briefly , gα- closed) [18] if αcl(A) ⊆ U whenever A⊆ U and U is an α- open set in X .

5- **α- generalized closed set** (briefly, α g- closed) [17] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an open set in X .
6- **generalized pre closed set** (briefly , gp- closed) [19] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is an open set in X .
7- **generalized semi-pre closed set** (briefly , gsp- closed) [10] if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is an open set in X .
8- **generalized pre regular closed set** (briefly , gpr- closed) [13] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is a regular open set in X .
9- **regular generalized closed set** (briefly , rg- closed) [23] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is a regular open set X.
10- **regular weakly generalized closed set** ((briefly , rwg-closed) [21] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is a regular open set .
11- **regular generalized α- closed set** ((briefly , rgα-closed) [21] if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is a regular α- open set .
12- **generalized b- closed set** (briefly , gb- closed) [13] if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is an open set in X .
13-**Pre- semi closed** [29] if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is a g- open
14- **regular pre-semi closed** (briefly , rps-closed) [24] if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is an rg- open set in X.

The complement of a g-closed (resp. gs-closed ,sg- closed ,gα-closed ,αg-closed , gp-closed, gsp-closed, gpr-closed ,rg-closed, rwg-closed, rgα-closed and gb –closed) sets is called a g-open (resp. gs-open ,sg- open ,gα-open ,αg-open , gp-open, gsp-open, gpr-open ,rg-open, rwg-open, rgα-open and gb –open)sets , also the complement of rps – closed is called *rps- open set.*

Remark (2-4) *:* In [4],[12],[14] and [27] it has been proved that :
1-Every sg-closed set is a(gs-closed , gsp-closed and gb-closed) set respectively.
2-Every gs-closed set is a gsp-closed set.
3-Every gb-closed set is (sg-closed set, gs-closed set and gsp-closed set) Respectively.
4-Every gα-closed set is a (αg-closed , gp-closed, gpr-closed and rgα-closed) set .
5-Every αg-closed set is a gp-closed set and gpr-closed set.
6-Every gp-closed set is a gpr-closed set .
7-Every rgα-closed set is an rg-closed set and rwg-closed set.

Remark (2-5): In [24] , it has been proved that :
(i) Every open set is rps-open. Also every closed set in X is rps-closed.
(ii) Every semi- open set is rps-open. Also every semi- closed set in X is rps-closed.
(iii) Every α- open set is an rps-open . Also every α- closed set in X is an rps-closed.
(iv) Every semi-pre open set is an rps-open . Also every semi-pre closed set in X is an rps-closed. But the converse of remark(2-5) need not be true in general .

Example(2-1) *:* Let X ={a,b,c,d} , τ ={X,Φ,{a},{b},{a, b},{b ,c},{a ,b ,c}} . Then
(i) {b c} is an rps-closed set but is not closed , α-closed and preclosed set in (X ,τ) . Also, $\{b,c\}^c = \{a,d\}$ is not open , α-open and preopen set in (X ,τ) .
(ii){a ,b, d} is an rps closed set in (X ,τ) but not semi-closed and semi-preclosed ,and $\{a,b,d\}^c = \{c\}$ is an rps open set but is not semi-open and semi-preopen.

Definition (2-6): A topological space X is said to be:

1-$T*_{1/2}$ – spaces [23] if every rg- closed sets is closed
2-A $T_b -$ space [8] if every gs- closed sets is closed.
3-**An αT_b-spaces** [7] if every αg- closed sets is closed.
4-**A $T_{1/2}$ - space** [16] if every g- closed sets is closed.

Definition (2-7),[15]: A function $f : (X, \tau) \to (Y, \sigma)$ is said to be continuous function if the inverse image of each open(closed) set in Y is an open(closed) set in X

Definition (2-8): A function *f:* $(X, \tau) \to (Y, \sigma)$ from topological space X in to topological space Y is said to be:

1-**generalized continuous** (briefly, g- continuous) [5] if $f^{-1}(A)$ is a g- closed set in X for every closed set A in Y.

2-**generalized semi continuous** (briefly, gs- continuous) [9] if $f^{-1}(A)$ is a gs-closed set in X for every closed set An in Y.

3- **semi- generalized continuous** (briefly, sg- continuous) [26] if $f^{-1}(A)$ is a sg-closed set in X for every closed set A in Y.

4- **α- generalized continuous** (briefly, αg- continuous) [17] if $f^{-1}(A)$ is an αg-closed set in X for every closed set A in Y.

5- **Generalized α- continuous** (briefly, gα- continuous) [7] if $f^{-1}(A)$ is a gα-closed set in X for every closed set An in Y.

6- **Generalized pre- continuous** (briefly, gp- continuous) [14] if $f^{-1}(A)$ is a gp-closed set in X for every closed set A in Y .

7- **Generalized semi pre- continuous** (briefly, gsp- continuous) [16] if $f^{-1}(A)$ is a gsp- closed set in X for every closed set A in Y .

8- **Generalized preregular- continuous** (briefly , gpr- continuous) [12] if $f^{-1}(A)$ is a gpr- closed set in X for every closed set A in Y .

9- **Regular generalized- continuous** (briefly , rg- continuous) [23] if $f^{-1}(A)$ is an rg- closed set in X for every closed set A in Y .

10- **Regular weakly generalized- continuous** (briefly , rwg- continuous) [21] if $f^{-1}(A)$ is an rwg- closed set in X for every closed set A in Y .

11- **Regular generalizedα- continuous** (briefly , rgα- continuous) [28] if $f^{-1}(A)$ is a rgα- closed set in X for every closed set A in Y .

12- **Generalized b- continuous** (briefly ,gb- continuous) [4] if $f^{-1}(A)$ is a gb-closed set in X for every closed set A in Y .

αRPS – CLOSED SETS IN TOPOLOGICAL SPACES :

In this section, we introduce a new type of closed sets namely αrps-closed sets in topological spaces and study some of their properties.

Definition (3-1)*:* A subset A of a topological spaces X is said to be αrps-closed set if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an rps-open set in X . The complement of an αrps- closed set is called αrps- open and the class of all αrps- closed (resp. αrps-open) subset of X is denoted by αRPSC(X) (resp. αRPSO(X)).

Example(3-1): Let X={a,b,c} with the topology τ={X, Φ,{a}}on X , then αRPSC(X)= {X, Φ, {b},{c},{b,c}} and αRPSO(X) = { X, Φ,{a},{a,b},{a,c}} .

Proposition(3-2) : Let (X,τ) be a topological space . Then

(i) Every α-closed set in X is an αrps-closed.

(ii) Every closed set in X is an αrps-closed.

(iii) Every regular closed set in X is a αrps-closed.

Proof :(i) Let A be an α-closed set in X and let $A \subseteq U$ where U is a rps-open set in X. Since A is an α-closed in X .Thus, we have $\alpha cl(A)=A\subseteq U$. Therefore, $\alpha cl\ (A)\subseteq U$ whenever $A\subseteq U$ and U is a rps-open set . Hence, A is a αrps-closed set in X.

(ii) Let A be a closed set in X and let $A \subseteq U$ where U is a rps-open set in X. Since A is a closed set in X .Then $cl(A)=A$. But $\alpha cl(A)\ \ cl(A)=A \subseteq U$.Thus ,we have $\alpha cl(A) \subseteq U$ whenever $A\subseteq U$ and U is a rps-open set . Hence, A is a αrps-closed set in X.

(iii) It follows from the fact every regular closed set is closed and by, (ii) we have every regular set is a αrps-closed.

Corollary (3-3): Let (X, τ) be a topological space. Then

(i) Every open (α-open) set in X is an αrps-open.

(ii) Every regular open set in X is an αrps-open .

Proof : (i) Let A be an open (α-open) set in X .Then A^c is a closed (α-closed) set in X and by proposition(3-2),(i)and (ii) we get A^c is an αrps- closed in X. Then A is an αrps-open set in X .

(ii) Let A be a regular open set in X .Then A^c is a regular closed set in X and by proposition(3-2),(iii) we get A^c is an αrps- closed in X .Then, A is an αrps-open in X.

Remark (3-4): The converse of the proposition (3-2) and corollary (3-3)are not true in general, as the following show .

Example(3-2): Let X={a,b,c,d} and Y={ a,b,c} be two topological spaces

- **(i)** Consider the topology τ = {X, Φ, {a}, {b}, {a,b}, {a,b,c} }. Then the set {c} is an αrps-closed set in (X, τ), but is not closed set in (X, τ). Also, $\{c\}^c$ ={a,b,d}} is an αrps-open set in (X ,τ), but is not open set in (X ,τ).
- **(ii)** Consider the topology σ={Y, Φ, {a}, {b}, {a, b} }. Then the set {c} is an αrps-closed set in (Y, σ) but is not regular closed in (Y, σ). Also, $\{c\}^c$ ={a,b} is an αrps-open set in (Y ,σ), but is not regular open set in (Y ,σ) .

Proposition (3-5) : Let (X ,τ) be a topological space . Then

(i) Every αrps-closed set in X is a sg-closed.

(ii) Every αrps-closed set in X is a gs-closed.

(iii) Every αrps-closed set in X is a gsp-closed.

(iv) Every αrps-closed set in X is a gb-closed.

Proof: (i) Let A be an αrps- closed set in X and $A \subseteq U$, where U is a semi- open set in X. By remark (2-5) (Every semi-open set is a rps- open) and since A is an αrps-closed set. Then $\alpha cl(A)\subseteq U$. But $scl\ (A) \subseteq \alpha cl\ (A)\subseteq U$. Thus, we have $scl\ (A) \subseteq U$ whenever $A\subseteq U$ and Uis a semi-open set in X. Therefore, A is a sg-closed set in X.

(ii) It follows from (i) and by remark (2-4) (Every sg-closed set is gs-closed). Therefore, every αrps-closed set is a gs-closed.

(iii) It follows from (ii) and by remark (2-4) (Every gs-closed set is gsp-closed). Therefore, every αrps-closed set is a gsp-closed.

(iv) It follows from (ii) and by remark (2-4) (Every gs-closed set is gb-closed). Therefore, every αrps-closed set is a gb-closed.

Corollary (3-6): Let (X, τ) be a topological space. Then

(i) Every αrps-open set in X is a sg-open.

(ii) Every αrps-open set in X is a gs-open.

(iii) Every αrps-open set in X is a gsp-open .
(iv) Every αrps-open set in X is a gb-open .
Proof: (i) Let A be an αrps-open set in X . Then A^c is an αrps-closed set in X and by proposition (3-5),(i) we get A^c is an sg-closed set in X . Thus, A is a sg-open set in X.
(ii) Let A be a αrps-open set in X. Then A^c is a αrps-closed set in X and by proposition (3-5), (ii) we get A^c is a gs-closed set in X Thus, A is a gs-open set in X.
(iii) Let A be a αrps-open set in X. Then A^c is an αrps-closed set in X and by proposition (3-5), (iii) A^c is a gsp-closed set in X. Thus, A is a gsp-open set in X.
(iv) Let A be a αrps-open set in X. Then A^c is a αrps-closed set in X and by proposition (3-5), (iv) we get A^c is a gb-closed in X. Hence, A is a gb-open set in X.
Remark (3-7): The converse of the proposition(3-5) and corollary (3-6)are not true in general, as the following show :
Example(3-3): Let X={a,b,c,d } wih the topology τ={ X,Φ,{a},{b}, {a,b}, {a,b,c}} . Then the set {a,d} is an sg-closed (resp . gs-closed , gsp-closed and gb-closed)set in (X,τ), but is not αrps-closed set in (X,τ). Also, $\{a,d\}^c$ ={b,c} is a sg-open (resp . gs-open , gsp-open and gb-open)set in (X,τ),but is not αrps open set in (X,τ) .
Proposition (3-8): Let (X, τ) be a topological space .Then
(i) Every αrps-closed set in X is gα-closed.
(ii) Every αrps-closed set in X is αg-closed.
(iii) Every αrps-closed set in X is gp-closed.
(iv) Every αrps-closed set in X is gpr –closed .
Proof: (i) Let A be an αrps-closed set in X. Let A ⊆ U , where U is an α-open set in X . By Remark (2-5) and since A is an αrps-closed set in X . Then αcl(A)⊆ U whenever A ⊆ U when U is an α-open set in X . Hence, A is a gα-closed .
(ii) It follows from(i) and by remark(2-4) (Every gα-closed set is an αg-closed) . Hence, every αrps-closed set is an αg-closed.
(iii) It follows from(ii) and by remark(2-4) (Eevery αg-closed set is a gp-closed) . Hence, every αrps-closed set is a gp-closed.
(iv) It follows from(iii) and by remark(2-4) (Every gp-closed set is a gpr-closed). Hence, every αrps-closed set is a gpr-closed.
Corollary(3-9): Let (X,τ)be a topological space . Then
(i) Every αrps-closed set is rgα-closed .
(ii) Every αrps-closed set is rg-closed .
(iii) Every αrps-closed set is rwg-closed .
Proof :(i)Follows from the proposition(3-8) ,(i) and Remark(2-4) Therefore, every αrps- closed set is an rgα- closed .
(ii) Follows from, (i) and remark (2-4) (Every rgα-closed set is an rg-closed set). Therefore, every αrps- closed set is an rg- closed (Every rgα-closed set is an rwg-closed set). Therefore, every αrps- closed set is an rwg- closed set.
(iii) Follows from (i) and remark(2-4) .
Corollary (3-10): Let (X,τ) be a topological space . Then
(i) Every αrps-open set in X is a gα-open set.
(ii) Every αrps-open set in X is an αg-open set.

(iii) Every αrps-open set in X is a gp-open set.
(iv) Every αrps-open set in X is a gpr-open set.
Proof *:* (i)Let A be an αrps-open set in X .Then A^c is an αrps-closed set in X and by proposition(3-8),(i) we get A^c is a gα-closed in X . Hence, A is a gα -open set in X.
(ii) Let A be a αrps-open set in X. Then A^c is a αrps-closed set in X and by proposition (3-8), (ii) we get A^c is an αg-closed in X. Hence, A is an αg -open set in X.
(iii) Let A be an αrps-open set in X .Thus, A^c is an αrps-closed set in X and by using proposition (3-8),(iii) we get A^c is a gp-closed set in X . Then, A is a gp -open set in X.
(iv) Let A be a αrps-open set in X .Then A^c is a αrps-closed in X and by proposition (3-8), (IV) we get A^c is a gpr-closed set in X. Hence, A is a gpr –open set in X.

The proof of the following corollary it is easy. Hence, it is omitted.

Corollary (3-11): Let (X, τ) be a topological space. Then
(i) Every αrps- open set is rgα-open. (ii)Every αrps- open set is rg-open.
(iii) Every αrps- open set is rwg-open.

Remark (3-12): The converse of the proposition (3-8) and corollary (3-9),(3-10)and corollary(3-11) need not be true in general, as seen from the following example:

Example (3-4): Let X={a,b,c,d} and Y={a,b,c}be two topological spaces
(i) Consider the topology τ = {X,Φ,{a}, {b},{a,b},{a,b, c}}.Then the set {b,d} is an αg-closed(resp. gp-closed and gpr-closed) set in (X,τ), but is not αrps- closed set in (X τ) . Also, $\{b,d\}^c$ ={a,,c} is an αg-open (resp. gp-open and gpr-open) set in (X ,τ), but is not αrps- open set in(X ,τ) .
(ii) Consider the topology σ={Y, Φ, {a}, {b,c} }. Then the set {c} is a gα-closed set (resp. rgα-closed, rg-closed and rwg-closed) in (Y,σ)but is not αrps-closed set in (Y ,σ). Also, $\{c\}^c$ ={a,,b} is an gα-open (resp.rgα-open, rg-open and rwg-open) set in (Y ,σ) but is not αrps- open set in(Y,σ) .

Remark (3-13): The concept of g-closed set and αrps-closed set are independent.

Example(3-5): Let X={a,b,c,d} and τ ={X,Φ,{a}, {b}, {a,b}, {a,b,c} }.Then {a,d} is a g-closed set in (X ,τ). But is not αrps-closed set in (X ,τ) and {c} is an αrps-closed set in(X ,τ) but is not g-closed set in (X, τ) .

The following propositions given the condition to make g- closed sets and αrps-closed sets are equivalent.

Proposition (3-14): If X is a $T_{1/2}$ - space, and then every g-closed set in X is αrps-closed.

Proof: Let A be a g-closed set in X .Since X is a $T_{1/2}$ - space and by definition (2-6) we get A is a closed in X and by proposition (3-2), (ii), A is a αrps-closed set in X.

Proposition (3-15): If X is a T_b –space, and then every αrps-closed set in X is g-closed.

Proof*:* Let A be a αrps-closed set in X and by corollary (3-9), (ii) we get A is a gs-closed in X. Since X is a T_b -space and by definition (2-6), (2) we get A is a closed set in X and since every closed set is a g-closed set Therefore, A is a g-closed set in X.

In following proposition and next results, we introduce some properties of αrps-closed sets:

Proposition (3-16): A subset A of a space X is an αrps-closed set if and only if $\alpha cl(A) - A$ does not contain any non- empty rps-closed set in X .

Proof: Let A be a αrps-closed set in X. we prove the result by contradiction. Let $G \neq \Phi$ be an rps-closed set in X such that $G \subseteq \alpha cl(A) - A$, since $\alpha cl(A) - A = \alpha cl(A) \cap A^c$, then $G \subseteq \alpha cl(A) \cap A^c$. Therefore , $G \subseteq \alpha cl(A)$ and $G \subseteq A^c$ .Since X-G is an rps-open set in X and A is an αrps-closed set ,thus $\alpha cl(A) \subseteq X-G$ .That is $G \subseteq [\alpha cl(A)]^c$ .Hence,$G \subseteq \alpha cl(A) \cap [\alpha cl(A)]^c = \Phi$. That is $G=\Phi$(which is contradiction). Since $G \neq \Phi$. Thus, αcl(A)-A does not contain any non-empty rps-closed set in X. Conversely , supposes αcl(A)-A does not contain any non-empty a rps-closed set in X and let $A \subseteq G$,G be an rps-open , suppose that αcl(A) is not contained in G , then $\alpha cl(A) \cap G^c$ is a non-empty rps-closed set of $\alpha cl(A) - A$ (which is contradiction) . Therefore, $\alpha cl(A) \subseteq G$ and hence A is an αrps-closed set in X .

Corollary (3-17): If A is an αrps-closed set in X .Then A is an α-closed set if and only if αcl(A)-A is closed .

Proof: Let A be an αrps-closed set in X, if A is an α-closed set, then we get αcl(A)=A. Thus, αcl(A)-A which is closed set . Conversely, let αcl (A)-A be a closed set .Then by proposition (3-16) we have αcl(A)-A does not contain any non-empty rps-closed set . Since αcl(A)-A is a closed subset, then $\alpha cl(A)-A = \Phi$ Thus, αcl(A)=A and so A is an α-closed set.

Proposition (3-18)*:* The union of two αrps-closed subsets of X is also αrps-closed set in X.

Proof : Let A and B be two αrps-closed set in X .Let U be an rps-open set in X , such that $A \cup B \subseteq U$.Then $A \subseteq U$ and $B \subseteq U$.Since A and B are αrps-closed set in X. Hence, $\alpha cl(A) \subseteq U$ and $\alpha cl(B) \subseteq U$.Hence, $\alpha cl(A \cup B) = (\alpha cl(A) \cup \alpha cl(B)) \subseteq U$.That is $\alpha cl(A \cup B) \subseteq U$ whenever U is a rps-open in X . Then $A \cup B$ is an αrps-closed set in X

Proposition (3-19): If A is an αrps-closed subset in X and $A \subset B \subset \alpha cl(A)$ Then B is an αrps-closed set in X .

Proof : Let A be an αrps-closed subset in X, such that $A \subset B \subset \alpha cl(A)$ and G be an rps-open set of X , such that $B \subseteq G$ .Since A is an αrps-closed set in X, we get $\alpha cl(A) \subseteq G$. Now, $\alpha cl(A) \subseteq \alpha cl(B) \subseteq \alpha cl(\ \alpha cl(A)\ ) = \alpha cl A \subseteq G$ Thus, $\alpha cl(B) \subseteq G$. Whenever, G is an rps-open set in X . Hence, B is an αrps-closed set in X. The converse need not be true in general. As seen from the following example:

Example (3-6) : Let X={a,b,c} and τ={X,Φ,{a},{a,c}}on X . Let A={b} and B={b,c} be two αrps-closed set in (X,τ) such that $A \subset B$, But $B \subseteq \alpha cl(A)$.

Proposition (3-20): Let A and B be two αrps-open subsets of X .Then $A \cap B$ is an αrps-open set in X.

Proof: If A and B be two αrps-open subsets of X. Then $A^c$and $B^c$are both αrps-closed set in X .By proposition (3-18) we get $A^c \cup B^c$ is also αrps-closed set in X. But $A^c \cup B^c = (A \cap B)^c$ is αrps-closed set in X .Thus, $A \cap B$ is αrps-closed in X.

Proposition (3-21)*:* Let A be any subset of a topological space (X,τ) , if A is an rps-open and αrps-closed set in X . Then A is α-closed set.

Proof: Suppose that a subset A of a space X is an rps-open and αrps-closed set in X Thus, A ⊆αcl (A) and αcl (A) ⊆A. Then, A=αcl(A) . Hence,A is an α-closed set in X .

Corollary (3-22) : If A is an rps-open and αrps-closed subset in X and F is an α-closed set in X .Then A∩F is an αrps-closed subset in X .

Proof: Suppose that A be an rps-open and αrps-closed subset in X . Then by proposition (3-21) we get A is an α-closed set in X .Since F is an α-closed in X then by Remark(2-2) ,(iii) we have A∩F is an α-closed set in X .Also ,by proposition (3-2),(i)we obtain A∩F is an αrps-closed set in X .

Proposition (3-23): For an element x ∈X, the set X-{x} is a αrps-closed set or rps-open set.

Proof: let X-{x} is not rps-open set .Then X is the only rps-open set containing X-{x}.This implies αcl(X-{x}) ⊆ X .Therefore, X-{x} is αrps-closed set in X .

Next, we introduce $T_{\alpha rps} - \text{space}$ as an application of αrps-closed sets in topological spaces and we study some of it is properties.

Definition (3-24): A topological space (X,τ) is said to be $T_{\alpha rps} - \text{space}$ if every αrps-closed set in X is an α-closed .

Proposition (3-25): A space (X,τ) is a $T_{\alpha rps} - \text{space}$ if and only if every singleton set in X is rps-closed or α-open .

Proof*:* Let X be $T_{\alpha rps} - \text{space}$. To prove every singleton set in X is rps-closed or α-open .Suppose {x} is not rps-closed set. Then X-{x} is not rps-open set .Thus, X is the only rps-open set containing X-{x} and hence αcl(X-{x}) ⊆ X. Then X-{x} is αrps-closed in X. Since X is $T_{\alpha rps} - \text{space}$ and by definition (3-24) we have X-{x} is a α-closed set in X. Therefore, {x} is an α-open set. Conversely, Let A is a αrps-closed set in X. Since A is an αrps-closed set in X and by proposition (3-16) we get αcl(A)-A does not contain any non-empty a rps-closed set in X .Let x∈ αcl(A). By our assumption{x} is either rps-closed set or α-open set. Case (i): Let x∈ αcl (A) such that {x} is an rps-closed. Since {x} is an rps-closed set and by proposition (3-16) we get x∉αcl (A)-A. Thus, x∈ A Therefore, A=αcl(A).Hence, A is an α-closed set. Case (ii): Let {x} be not rps-closed for each x ∈ αcl (A). Now if x ∈ αcl (A). Then {x} is an α-open set and {x}∩A≠Φ that implies x∈A.Therefore, A=αcl (A). Hence, A is an α-closed set. From case (i) and case (ii) it follows from definition(3-24) we obtain X is $T_{\alpha rps} - \text{space}$.

Proposition (3-26): If (X,τ) is a $T_{\alpha rps} - \text{space}$ Then every αrps-open set in X is an α-open set.

Proof: Let A be an αrps-open set in X. Then A^c is an αrps-closed in X. Since X is a $T_{\alpha rps} - \text{space}$ and by definition (3-24) we get A^c is an α-closed set in X Hence, A is an α-open set in X.

Proposition (3-27): Every $T*_{1/2}$- space is a $T_{\alpha rps} - \text{space}$.

Proof: Let (X,τ) be a $T*_{1/2}$- space and let A be an αrps-closed set in X . Then by corollary (3-9), (ii) we get A is an rg-closed set in X .Since X is a $T*_{1/2}$- space and by definition (2-6),-1-we have A is a closed set in X and by remark(2-2), (ii) we obtain A is an α-closed set in X .Therefore, X is a $T_{\alpha rps}$ – spaces .

The converse of proposition (3-27) need not be true in general . As seen from the following example:

Example (3-7): Let X= {a,b,c} with the topology τ={X,Φ,{a},{a,c}} . Since αRPSC(X)= { X,Φ,{b},{c},{b,c} } are α-closed sets in (X,τ) . Then X is a $T_{\alpha rps} - \text{space}$. But is not $T*_{1/2}$- space .Since, the set of all rg-closed in (X,τ) are { X,Φ, {a},{b},{c},{a,b},{a,c},{b,c} }≠ α-closed sets in (X,τ) .

Proposition (3-28): Every T_b- space is a $T_{\alpha rps} - \text{space}$.

Proof: Let (X,τ) be a T_b- space and let A be an αrps-closed set in X .By proposition(3-5),(ii) we get A is a gs-closed set in X . Since X is a T_{b-} space and by definition (2-6),-2-we have A is a closed set in X and also by Remark (2-2),(ii). We get A is an α-closed set in X. Hence, X is a $T_{\alpha rps} - \text{space}$.

The converse of proposition (3-28) need not be true in general. As seen from the following example:

Example (3-8): Let X= {a,b,c} , τ={ X,Φ,{a}{b},,{a,b}} . Since αRPSC(X)= { X,Φ,{c},{a,c},{b,c} }=α-closed sets in (X,τ) . Then X is a $T_{\alpha rps} - \text{space}$. But is not T_b- space ,since the set of all gs-closed sets in (X,τ) are { X,Φ, {a},{b},{c},{a,b},{a,c},{b,c} }≠ α-closed sets in (X,τ) .

Proposition (3-29): Every αT_b - space is a $T_{\alpha rps} - \text{space}$.

Proof: Let (X,τ) be an αT_b - space and let A be an αrps-closed set in X .By using proposition(3-8) ,(ii) we get A is an αg-closed set in X . Since X is an αT_b - space and by definition (2-6)-we have A is a closed in X and also by Remark (2-2),(ii) Thus, A is an α-closed in X .Hence, X is a $T_{\alpha rps}$ –space .

The converse of proposition (3-29) need not be true in general. As seen from the following example:

Example (3-9): Let X= {a,b,c} with the topology τ={ X,Φ,{a},{b,c} } . Since αRPSC(X) ={X, Φ, {a},{b,c},}}=α-closed sets in (X,τ) . Then X is a $T_{\alpha rps} - \text{space}$. But is not αT_b - space , since the set of all αg-closed sets in (X,τ) are { X,Φ, {a},{b},{c},{a,b},{a,c},{b,c} }≠ α-closed sets in (X,τ) .

Proposition (3-30): If X is a $T*_{1/2}$- space , then every αrps-closed set in X is closed .

Proof*:* Let A be an αrps-closed set in X .By corollary (3-9) ,(ii) we get A is an rg-closed in X . Since X is a $T*_{1/2}$- space , then, A is a closed set in X .

The following proposition and corollary it is easy .Thus, we omitted the proofs :

Proposition (3-31): Let (X,τ) be a topological space . Then every αrps-closed set in X is closed if

(i)X is T_b- space

(ii)X is αT_b – space.

Corollary (3-32): Let (X,τ) be a topological space . Then every αrps-open set in X is open if X is

(i) $T*_{1/2}$- space

(ii) αT_b – Space, (iii) T_b- space.

Remark(3-33): The converse of proposition(3-30) ,(3-31) and corollary (3-32) need not be true in general .It is easy seen that in example(3-9) (Every αrps-closed set in (X ,τ) is closed and also every αrps-open set in (X ,τ) is open . But X is not T_b- space, not αT_b – space and not $T*_{1/2}$- space .

ARPS-CONTINUOUS FUNCTION, αRPS-IRRESOLUTE FUNCTION and STRONGLY αRPS-CONTINUOUS FUNCTION.

In this section, we introduce and study new types of continuous functions and will be study some of their properties.

Definition (4-1): A function *f*: (X,τ) → (Y,σ) is said to be αrps-continuous continuous if f^{-1} (A) is an αrps-closed set in X for every closed set A in Y.

Proposition (4-2) : A function f : (X, τ) → (Y, σ) is αrps- continuous if and only if f^{-1} (A) is an αrps-open set in X for every open set A in Y .

Proof: Let *f* be an αrps-continuous function and A be an open set in Y .Then A^c is a closed set in Y.Thus, f^{-1} (A^c) is an αrps-closed set in X . But f^{-1} (A^c) =X-f^{-1} (A) $=(f^{-1}(A))^c$.Hence, f^{-1} (A) is an αrps-open set in X .Conversely, let A be a closed in *Y*. Then A^c is an open set in X .By assumption f^{-1} (A^c) is an αrps-open set in X . But f^{-1} (A^c) =X-f^{-1} (A) $=(f^{-1}(A))^c$.Hence, f^{-1} (A) is an αrps-closed set in X .

Proposition (4-3) : Every continuous function is αrps-continuous .

Proof : Follows from the definition (4-1) and the fact every closed set is an αrps-closed . The converse need not be true in general. As see from the following example:

Example(4-1): Let X=Y={a,b,c}, τ={X,Φ,{a}} and σ={Y,Φ,{a,b}} on X,Y respectively. Let *f:* (X,τ)→ (Y,σ) be the identity function . Then f is an αrps-continuous, but is not continuous function .Since for the closed set {c} in Y , f^{-1}({c})={c} is not closed set in X .

Proposition (4-4): Let *f*:(X,τ)→(Y,σ) be an αrps-continuous function from topological space X in to topological space Y. If X is a T_b- space or αT_b – space, then *f* is continuous.

Proof: Let A be an open set in Y .Thus, f^{-1} (A) is an αrps-open set in X. Since X is a T_b- space or αT_b – space and by corollary (3-32) we get f^{-1} (A) is an open set in X . Hence, *f* is a continuous function.

Proposition (4-5): Let *f*: (X, τ) → (Y, σ) be a function from topological space (X,τ) in to topological space (Y,σ). If f is an αrps-continuous function , then f is a :

(i)g-continuous function.

(ii)gs-continuous function.

(iii)gsp-continuous function.

(iv)gb-continuous function..

Proof :(i)Let A be closed set in Y .Thus, f^{-1} (A) is αrps-closed set in X and by proposition(3-5) ,(i) we have A is sg-closed set. Then, *f* is sg-continuous.

(ii) Let A be a closed set in Y .Thus, f^{-1} (A) is an αrps-closed set in X and by proposition (3-5), (ii) we have A is a gs-closed set. Then, *f* is gs-continuous .

(iii) Let A be a closed set in Y .Thus, f^{-1} (A) is αrps-closed set in X and by proposition (3-5),(iii) we have A is gsp-closed Therefore, *f* is gsp-continuous .

(IV) Let A be a closed set in Y .Thus, f^{-1} (A) is an αrps-closed set in X and by proposition (3-5),(iv) we have A is a gb-closed .Therefore, *f* is gb-continuous

From Definition (4-1), Proposition (3-8) and corollary (3-9) we get the following proposition and it is prove easy. Thus, we omitted it is.

Proposition (4-6): Every αrps-continuous function, f :(X, τ) → (Y, σ) is a

(i)gα-continuous . **(ii)**αg-continuous . **(iii)**gp-continuous .

(iv)gpr-continuous . **(v)**rgα-continuous . **(vi)**rg-continuous .

(vii)rwg-continuous .

The converse of a proposition (4-5) and (4-6)may not be true in general, as shown in the following example.

Example(4-2):et X=Y={a,b,c} with the topologies τ={X,Φ,{a},{b},{a,b}} and σ={Y,Φ,{a},{b,c}}on X and Y respectively. Let f : (X,τ)→ (Y,σ) be the identity function . Then it is observe that f is not αrps-continuous function since for the closed set {a} in (Y,σ) , f^{-1}({a}) ={a} is not αrps-closed set in (X,τ). However, f is sg-continuous (resp. gs-continuous, gsp-continuous, gb-continuous) functions.

Example (4-3): Let X=Y={a,b,c} with the topologies τ={X,Φ,{a},{b ,c}} and σ={Y,Φ,{a},{a,c}}on X and Y respectively .Let *f:*(X,τ)→ (Y,σ) be the identity function .Then f is not αrps-continuous .Since for the closed set {b} in (Y,σ),f^{-1}({b}) ={b} is not αrps-closed set in (X,τ). However, f is a gα-continuous (αg-continuous, gp-continuous, gpr-continuous, rgα-continuous, rg- continuous and rwg-continuous) functions.

Proposition (4-7): If f : (X, τ) → (Y, σ) is any function from topological space X in to topological space Y and X is a T_b- space

(i) If f is a sg-continuous function , then it is αrps-continuous.

(ii) If f is a gs-continuous function ,then it is αrps-continuous.

Proof *:(i)* Let f :(X,τ)→(Y,σ) be a sg-continuous function and let A be a closed set in Y. Thus, f^{-1} (A) is a sg-closed set in X , since (Every sg-closed set is gs-closed) . Then f^{-1} (A) is gs-closed in X and since X is a T_b- space , Then f^{-1} (A) is a closed set in X ,and by proposition(3-2),(ii) we get f^{-1} (A) is an αrps-closed set in X . Therefore, f is an αrps-continuous.

(ii) Let f :(X,τ)→(Y, σ) be a gs-continuous function and A be a closed set in Y Thus, f^{-1} (A) is an gs-closed set in X and since X is a T_b- space. Thus, f^{-1} (A) is a closed set in X ,and by using proposition(3-2) , (ii)we get f^{-1} (A) is an αrps-closed set in X. Therefore, f is a αrps-continuous.

Similarly, we prove the following proposition:

Proposition (4-8): If *f*: (X, τ) → (Y, σ) is any function and let X be an αT_b – space and $T*_{1/2}$- space.

(i) If f is αg-continuous function , then it is αrps-continuous.

(ii) If f is gα-continuous function, then it is αrps-continuous .

(iii) If f is rg-continuous function, then it is αrps-continuous.

(iv) If f is rgα-continuous function, then it is αrps-continuous .

Now, we given other type of αrps-continuous function is called αrps-irresolute:

Definition (4-9): A function *f*: (X, τ) → (Y, σ) is said to be αrps-irresolute continuous if f^{-1} (A) is an αrps-closed set in X for every αrps- closed set A in Y .

Proposition (4-10) : A function *f :* (X, τ) → (Y, σ) is αrps- irresolute continuous if and only if f^{-1} (A) is an αrps-open set in X for every αrps- open set A in Y .

Proof *:* This proof is similar to that of proposition (4-2) .

Proposition (4-11): Every αrps-irresolute function is αrps-continuous.

Proof : Let *f*:(X, τ)→ (Y, σ)be an αrps-irresolute function and A be a closed set in Y. By proposition(3-2),(ii) we have A is an αrps-closed set in Y .Thus, f^{-1} (A) is an αrps-closed in X . Therefore, f is an αrps- irresolute . The converse need not be true

Example(4-4): LetX=Y={a,b,c,d}with the topologies τ = { X, Φ, {a} ,{b}, {a,b} , {b,c},{a,b,c}},σ={ Y,Φ,{d},{c,d},{a,c,d},{b,c,d}} on X and Y respectively .Define f :(X,τ) →(Y,σ) by f (a)=c, f (b)=f (c)=b and f (d)=d . It is observe that f is an αrps-continuous, but is not αrps-irresolute. Since for the αrps-closed set {c} in (Y, σ), $f^{-1}(\{c\}) = \{a\}$ is not an αrps-closed set in (X ,τ) .

Remark (4-12): The notions αrps-irresolute function and continuous function are independent .It is observe seen that in Example (4-1), such that f is an αrps- αrps- but is not a continuous function (Since for the closed set {c} in (Y,σ), $f^{-1}(\{c\})=\{c\}$ is not a closed set in (X,τ)) . Also in Example (4-4) it is easy seen that f is a continuous but is not an αrps- -irresolute function .

Proposition (4-13) : If f : (X,τ)→(Y,σ) is an αrps- continuous function and Y is a T_b- space. Then f is an αrps – irresolute function.

Proof : Let A be an αrps –closed set in Y . By proposition (3-5) , (ii) we get A is a gs-closed set in Y . Since Y is a T_b- space, then A is a closed set in Y, and so, f^{-1} (A) is an αrps-closed set in X.Hence, f is an αrps – irresolute function.

The following proposition it is easy. Thus, we omitted the proof:

Proposition (4-14):

1- If f : (X ,τ)→(Y,σ) is an αrps irresolute function and X is a T_b- space, then f is a continuous function.

2- If f : (X ,τ)→(Y,σ) is continuous function .and Y is a T_b- space ,then f is an αrps – irresolute function.

Proposition (4-15) : If $f:(X,\tau) \to (Y,\sigma)$ and $g:(Y,\sigma) \to (Z,\mu)$ are both αrps-irresolute. Then $g0f:(X,\tau) \to (Z,\mu)$ is an αrps-irresolute. .

Proof: Let A be an αrps-closed set in Z. Thus $g^{-1}(A)$ is an αrps-closed set in Y. Since f is an αrps – irresolute ,then $f^{-1}(g^{-1}(A))$ is an αrps-closed set in X . But $f^{-1}(g^{-1}(A)) = (g0f)^{-1}(A)$ .Hence, $g0f:(X,\tau) \to (Z,\mu)$ is αrps-irresolute function.

Similarly, we prove the following proposition:

Proposition (4-16) *:* If $f:(X,\tau) \to (Y,\sigma)$ and $g:(Y,\sigma) \to (Z,\mu)$are any two functions . Then $g0f:(X,\tau) \to (Z,\mu)$ is an αrps-continuous function if:

(i)f is αrps-irresolute and g is αrps-continuous .

(ii)f is αrps- continuous and g is continuous .

Remark (4-17): If $f:(X,\tau) \to (Y,\sigma)$ and $g:(Y,\sigma) \to (Z,\mu)$ are both αrps-continuous function. Then $g0f:(X,\tau) \to (Z,\mu)$ is not necessarily αrps- continuous function .It is easy seen that from the following example:

Example (4-5): Let X=Y=Z={a,b,c} with topologies τ={X,Φ,{a},{b,c}}, σ={ Y,Φ,{a}, {a,c}} and μ={Z,Φ,{a}, {a,c}}on X,Y Z respectively . Let *f:* (X ,τ)→ (Y,σ) be the identity function and defined $g:(Y,\sigma) \to (Z,\mu)$ by g(a)=a,g(b)=c and$g$(c)=b, then f and g are αrps-continuous .But gof is not αrps- continuous function. Since for the a closed set {b} in (Z,μ) . $(gof)^{-1}(\{b\})=\{c\}$ is not αrps-closed set in (X ,τ) .

Proposition (4-18): If $f:(X,\tau) \to (Y,\sigma)$ and $g:(Y,\sigma) \to (Z,\mu)$ are both αrps-continuous function and Y is a $T*_{1/2}$- space. Then their composition $g0f:(X,\tau) \to (Z,\mu)$ is also αrps- continuous function.

Proof: It is clear.

In the following definition, we introduce another type of αrps-continuous function.

Definition (4-19): A function $f:(X,\tau) \rightarrow (Y,\sigma)$ is called strongly αrps-continuous continuous if f^{-1} (A) is closed set in X for every αrps- closed set A in Y .

Proposition (4-20): *If f:* (X, τ) → (Y, σ) is a strongly αrps-continuous if and only if f^{-1} (A) is open set in X for every αrps- open set A in Y.

Proof*:* This proof is similar to that of proposition (4-2).

Proposition (4-21): Every strongly αrp-continuous function is continuous.

Proof: Let *f*:(X,τ)→(Y,σ) be a strongly αrp-continuous function and A be a closed set in Y. By proposition (3-2) ,(ii) we have A is an αrps-closed set in Y .Thus, f^{-1} (A) is a closed set in X. Therefore, *f* is continuous .The converse need not be true in general . As seen from the following example:

Example(4-6): Let X={a,b,c,d} and τ={X,Φ,{a},{a,c}}on X. Define f :(X,τ) →(X,τ) by *f*(a)=a, *f*(b)=b and *f*(c)=c. Then *f* is continuous ,but is not strongly αrps-continuous. Since for the closed set {c} in (X,τ) , f^{-1}({c})={a} is not a closed in (X,τ) .

Proposition (4-22): Let (X,τ) be any topological space, Y be a$T*_{1/2}$- space and *f*:(X,τ)→(Y,σ)is a any function. Then the following are equivalent:

(i)*f* is a strongly αrp-continuous function .

(ii)*f* is a continuous function .

Proof *:(i)*→ (ii)Follows from proposition(4-21) .

(ii)→ *(i)* Let A be an αrps –closed set in Y .Since Y is a $T*_{1/2}$- space and by proposition (3-30) we get A is a closed set in Y, then f^{-1} (A) is a closed set in X . Therefore, *f* is an αrps -continuous function.

Corollary (4-23): Every strongly αrp-continuous function is an αrps-continuous.

Proof*:* Follows from the proposition (4-21) and a proposition (4-3) . The converse need not be true in general. As seen from the following example:

Example(4-7): Let X={a,b,c}and Y={a,b}, τ={ X,Φ,{a},{a,c}}and σ={Y,Φ,{a,}}on X and Y respectively. Define f : (X,τ)→ (Y,σ) by*f*(a)=a,*f*(b)=c and *f*(c)=b. Then is f an αrps-continuous function but is not strongly αrp-continuous . Since for the closed set {b} in (Y,σ), f^{-1}({b})={c} is not a closed set in (X,τ) .

Corollary(4-24): Every strongly αrp-continuous function is an αrps-irresolute .

Proof : Follows from the corollary(4-23) and a proposition(4-11) .

Remark(4-25) : The converse of a corollary (4-24) need not be true in general. In example(4-7), *f* is a αrps-irresolute, but is not strongly αrps-continuous function

Proposition (4-26): Let *f*: (X,τ) → (Y,σ) be any function and X,Y are both$T*_{1/2}$-spaces . Then the following are equivalent :

(i)*f* is a strongly αrps-continuous function.

(ii)*f* is a continuous function.

(iii)*f* is an αrps-irresolute function.

(iv)*f* is an αrps-continuous function.

Proof: Follows from a propositions(4-21),(4-22) and corollaries(4-23),(4-24).

Proposition (4-27) : If $f:(X,\tau) \rightarrow (Y,\sigma)$ and $g:(Y,\sigma) \rightarrow (Z,\mu)$ are both strongly αrps-continuous function, then g0f: $(X,\tau) \rightarrow (Z,\mu)$is also strongly αrp-continuous.

Proof*:* Let A be an αrps-closed set in Z. Thus, $g^{-1}(\mathrm{A})$ is a closed set in Y By proposition (3-2),-ii-we get $g^{-1}(\mathrm{A})$ is an αrps- closed set in Y, since f is a strongly αrps-continuous ,then $f^{-1}(g^{-1}(\mathrm{A})$) is a closed set in X . But $f^{-1}(g^{-1}(\mathrm{A})$) $=(g0f)^{-1}(\mathrm{A})$. Therefore, $g0f:(\mathrm{X},\tau)\to(\mathrm{Z},\mu)$is a strongly αrps-continuous function?

Similarly, we prove the following propositions.

Proposition (4-28) *:* If $f:(\mathrm{X},\tau)\to(\mathrm{Y},\sigma)$ is a continuous function and $g:(\mathrm{Y},\sigma)\to(\mathrm{Z},\mu)$ is strongly αrps-continuous. $g0f:(\mathrm{X},\tau)\to(\mathrm{Z},\mu)$ is strongly αrps-continuous .

Proposition (4-29) : If $f:(\mathrm{X},\tau)\to(\mathrm{Y},\sigma)$is a αrps- continuous function(or αrps-irresolute) and $g:(\mathrm{Y},\sigma)\to(\mathrm{Z},\mu)$ is strongly αrps-continuous function Then $g0f:(\mathrm{X},\tau)\to(\mathrm{Z},\mu)$is an αrps-irresolute function.

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