

Optimal Subcarrier Allocation Algorithm for OFDMA Systems

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Received on: 3/9/2012 & Accepted on: 4/4/2013

ABSTRACT

An optimal subcarrier allocation algorithm is proposed for orthogonal frequency division multiple access systems over Rayleigh fading channels with channel state information feedback. The transmitted orthogonal frequency division multiple access frame is divided into groups called subchannels according to the detected number of users at the base station. The pilots are reduced for the stable subchannel and replaced with additional data subcarriers in order to increase the total system throughput. Simulation and semi-analytical results demonstrate that the throughput performance of the proposed method outperforms the conventional approaches.

Keywords: OFDMA, Optimized Methods, Subcarrier Allocation.

خوارزمية مثلى في تحديد الحوامل الفرعية لانظمة OFDMA

الخلاصة

خوارزمية امثلية لانظمة التشفير والتغيير باستخدام طريقة تحديد الحوامل الفرعية المتحركة لانظمة الاتصالات متعددة الحوامل التي ترسل فوق القنوات ذات الضمور العالي. المقطع المرسل باستخدام انظمة الاتصالات متعددة الحوامل يقسم الى قنوات فرعية بالاعتماد على عدد المستخدمين لشبكة الاتصالات للهواتف الخلوية، بينما حوامل المعلومات الفرعية الخاصة بقناة النقل تقلل في القنوات المستقرة وتستبدل بحوامل بيانات اضافية لزيادة كمية المعلومات المرسلة، النتائج المستخرجة من انظمة المحاكاة المصممة ونتائج التحليلات الرياضية تبين الزيادة الحاصلة بالمعلومات المرسلة بالنسبة للانظمة المستخدمة سابقا في انظمة التشفير والتغيير المتكيفة.

INTRODUCTION

Orthogonal frequency division multiple access (OFDMA) technology is expected to be widely used in many current and future wireless communication systems due to its ability to overcome, with low receiver complexity, many technical challenges, such as inter symbol interference (ISI) caused by multipath channels. In the Mobile WiMAX IEEE 802.16e standards, adaptive modulation and

coding (AMC) is adopted in conjunction with OFDMA systems to produce an efficient radio system that exploits the channel capacity in order to enhance the system throughput performance compared to fixed modulation and coding techniques [1].

Many issues have been published in the area of pilot allocation in terms of pilot pattern design. The authors of [2] produced an OFDMA system that adaptively selects distinct pilot patterns based on the feedback prediction of the channel estimation error and the quality of service (QoS). However, they have not considered the SNR fluctuation values and the coherence bandwidth as main constraints for pilot reduction, as well as the subchannelization and the multi-users issues, which are adopted in many standards such as Mobile WiMAX. In [3], the optimal number and pattern for the pilots in the OFDMA frame was presented. The selection of the suitable pattern and the size of pilots was based on the bit error rate (BER) achieved by both theoretical and simulated evaluations.

In [4], an optimal pilot pattern design based on time and frequency domain pilot density scattering was introduced in order to minimize the mean square error (MSE) of the channel estimation. Additionally, a best multiplexed pilot and data scheme was derived in [5] to degrade the interference between them using left and right null data insertion for each pilot tone.

In this paper, the transmitter of the BS divides the OFDMA frame into groups of symbols called subchannels, which employ independent MCSs, based on the corresponding users. The utilized data and pilots are divided uniformly across the considered users in order to guarantee fair service for them. Additionally, a pilot allocation algorithm is proposed to reduce the utilized pilots for channel estimation of each subchannel according the corresponding SNR value, evaluate the variance of the SNR fluctuation values, and compute the measured channel coherence bandwidth [6]. Furthermore, the unused pilots for each user are replaced by additional data subcarriers, in which the sum of data and pilot subcarriers in each band is kept the same. The number of required pilots and the optimal MCS level of each user is returned to the transmitter utilizing the feedback CSI. As a result, the system throughput is improved by increasing the data transmission for the users with stable profile. The proposed AMC strategy employs nine different MCS levels that include Quadrature Phase Shift Keying (QPSK), 16 Quadrature Amplitude Modulation (16-QAM), and 64-QAM combined with three convolutional coding rates (1/2, 2/3 or 3/4).

It is important to note that the OFDMA system model of [6] has been adopted in this paper. This paper motivates from [6] by proposing a novel algorithm that optimally choose the perfect number of the required pilots. In addition, major modifications in terms of mathematical model, number of MCSs and system design have been considered.

The remainder of the paper is organized as follows. In Section II, a description of the proposed system model is presented. Section III outlines the proposed transmission technique. Section IV demonstrates the performance of the proposed system via simulation and semi-analytical results. Finally, conclusions are drawn in Section V.

PROPOSED SYSTEM MODEL

Figure (1) shows the block diagram of the AMC based OFDMA system. The transmitter of the assigned BS exploits the number of users, K , to divide the OFDMA frame into, N_{SC} , subchannels.

The user random fairness distribution, i.e. allocating one user for each subchannel, is considered in this paper [7]. The subchannels are employed to send the N_p pilots and N_d data symbols in different MCS groups for the assigned users. Thus, for full pilot use, each user has $\delta_{k,p} = N_p / N_{SC}$ pilots and $\delta_{k,d} = N_d / N_{SC}$ data, where $k = \{1, \dots, K\}$ is the user's index. In this paper, $\delta_{k,p}$ and $\delta_{k,d}$ can be varied according to the assigned CSI, which contains the optimal MCS number and the pilots size for each user. This proposed pilot allocation will remove the requirement to send any side information, which is costly in terms of the effective bandwidth. The pilots are distributed over the data subcarriers by utilizing a comb-pilot approach that allocates a pilot for every $\delta_{k,d} / \delta_{k,p}$ data.

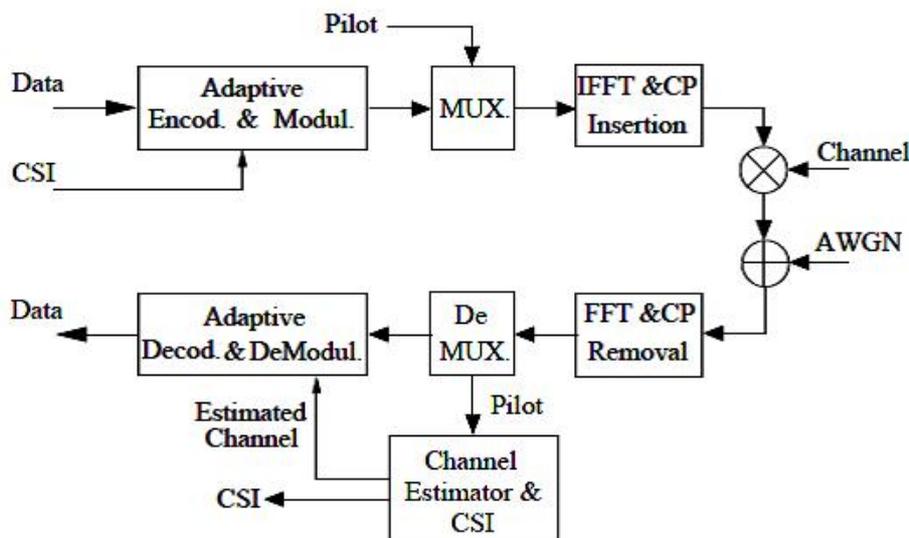


Figure (1) the system block diagram.

To check the validity of the proposed technique, the system is tested using Rayleigh fading channel. This channel contains six paths, $l = \{1, \dots, 6\}$, with distinct signal delays, τ_l , and power values, P_l , [8]. Furthermore, the channel paths' average power values are assumed to be constant over the duration of the transmitted OFDMA frame. Additionally, they are normalized so that, $E\{|H(n)|^2\} = \sum_{l=1}^L P_l = 1$, where $H(n)$ is the frequency response of the simulated channel with index $n = \{1, \dots, NFFT\}$. $NFFT$ is the fast Fourier transform (FFT) size. Different time-varying channels are

generated for the various users to simulate separate physical positions of the user around the BS.

At the receivers of the corresponding users, the pilots are extracted from the assigned subcarriers of the OFDMA frame and fed into the channel estimator. The pilot coefficients \hat{H}_p for each user are estimated by employing a Least Square (LS) method as in [9] and [10]:

$$\hat{H}_p = \frac{Y_p}{X_p} \quad \dots (1)$$

Where X_p and Y_p represent the transmitted and received pilot symbols for each user, respectively. In addition, this method is efficient and results in a low channel estimation error for practical SNR values. The data coefficients \hat{H}_d are obtained by implementing 1st order linear interpolation.

Assuming perfect knowledge of the simulated channel, the received data signal in frequency domain, $Y_d = X_d H_d + W_d$, is written as a function of the transmitted data signal, X_d , the corresponding channel values, H_d , and the additive white Gaussian noise (AWGN) samples, W_d . The SNR value of each data subcarrier for the distinct users in terms of the channel estimation error is evaluated as

$$\gamma_k = P_{k,s} \frac{|\hat{H}_k|^2}{MSE_k P_{k,s} + \sigma_{w_k}^2} \dots (2)$$

Where $\sigma_{w_k}^2$ is the variance of AWGN samples, $P_{k,s}$ is the average symbol power, and MSE_k is the mean square error of the related estimated channel. The minimum SNR of each user is selected to guarantee that the required performance is maintained for each user as

$$\gamma_{min} = \min\{\gamma_k\} \quad \dots (3)$$

Based on (3), the suitable MCS number of each individual user in the next OFDMA symbol is selected at the assigned receiver according to the Table (1). The SNR threshold values of Table I are achieved from the simulation results of the utilized MCS levels over Rayleigh fading channel at BER level of 10^{-5} following [11], [12].

Table (I) SNR threshold values for distinct MCSs.

Modulation type	Code rate	Threshold SNR (dB)
QPSK	1/2	$\gamma_{min} < 10.25$
QPSK	2/3	$10.25 < \gamma_{min} < 13.25$
QPSK	3/4	$13.25 < \gamma_{min} < 14.52$
16-QAM	1/2	$14.52 < \gamma_{min} < 16.1$
16-QAM	2/3	$16.1 < \gamma_{min} < 20.25$
16-QAM	3/4	$20.25 < \gamma_{min} < 22.5$
64-QAM	1/2	$22.5 < \gamma_{min} < 28.5$
64-QAM	2/3	$28.5 < \gamma_{min} < 39.1$
64-QAM	3/4	$\gamma_{min} > 39.1$

The CSI required at the transmitter for MCS selection is feedback from the receiver unit. It includes the number of the optimal MCS and the optimized pilots size for each user. It is assumed to be returned using a time division duplex (TDD) link. In this paper, the distances between the assigned BS and the users are assumed to be short and hence the feedback information delay is neglected due to the small propagation delay.

PROPOSED TRANSMISSION TECHNIQUE

The proposed criterion, which is based on the optimal pilot allocation, can be divided into two parts as follows:

A. Problem formulation

In this paper, the proposed technique is based on the scalability of OFDMA systems, such as Mobile WiMAX, which endorses the reduction in pilot size [10]. The proposed system throughput is improved by replacing unnecessary pilots within a user with information data streams, and hence, increasing the total transmitted data size. This reduction in the number of pilots for each user, which is assigned to each user individually, depends on the measured coherence bandwidth of each user, $BW_{k,coh}$, the variance of the SNR fluctuations, σ_k^2 , and the minimum SNR, γ_{min} , of the frequency bands. The range of the pilot reduction, λ_k , for each user can be varied between zero and $\delta_{k,p} - 2$.

The proposed pilot allocation technology can be formulated as an optimization problem that aims to maximize the system throughput as a function of the adjusted number of data subcarriers, $\xi_{k,d}$, for p-th OFDMA transmitted frame as follows:

Maximize:

$$\varphi_p = \sum_{k=1}^K \eta(k) \xi_{k,d} \rho_k [1 - P_{e_k}], \dots (4)$$

Subjected to:

$$BW_{k,coh} \geq \frac{BW_{k,eff}}{2} \quad \dots (5)$$

$$\gamma_{min} \geq 10dB \quad \dots (6)$$

$$\sigma_{k \leq 0.2}^2, \quad \dots (7)$$

$$Y_k \leq 0.001, \quad \dots (8)$$

Where $\eta(k)$ refers to the user allocation vector that assigns different subcarrier to distinct users, P_{ek} is the BER and $BW_{eff} = N_d \frac{BW_{sys}}{KN_{FFT}}$, denotes the effective channel bandwidth assigned to each user. It worthpointing out that $\xi_{k,d}$ can be evaluated as a function of $\delta_{k,d}$ and λ_k as follows [6]:

$$\xi_{k,d} = \delta_{k,d} + \lambda_k, \quad \dots (9)$$

Where λ_k is the pilot reduction value that can be computed for each user as a trade-off criterion between $BW_{k,coh}$, λ_{min} and σ_k^2 as shown below:

$$\lambda_k = \zeta_k \frac{BW_{k,coh} \lambda_{min}}{\sigma_k^2}, \quad \dots (10)$$

Where ζ_k is a variable selected to satisfy the condition, $mod [\xi_{k,d}, \xi_{k,p}] = 0$, that guarantees uniform distribution of pilots and data within each user.

Additionally, the transmitter adjusts the pilots, $\xi_{k,p} = \xi_{k,p} - \lambda_k$, which are distributed uniformly among the designated data, $\xi_{k,d}$, based on a pilot-comb method. Since $\xi_{k,d}$ is locally computed and fed back to the transmitter as CSI, the receiver of each user is aware of the structure for the next transmission interms of the data and pilot subcarrier indices for each user.

Furthermore, (1) equations (5)-(8) refer to the detected coherence bandwidth, minimum SNR value, variance of the SNR fluctuation and MSE of the channel estimation error constraints, respectively. These constraints control the pilot reduction strategy. Moreover, Y_k represents the evaluated error between the channel estimation MSE of full use of pilots and the proposed approaches. Y_k can be mathematically formulated as:

$$Y_k = MSE_{k,d}^{Full} - MSE_{k,d}, \dots (11)$$

Where $MSE_{k,d}^{Full}$ is the channel estimation mean square error for the system with full-use of available pilots. Additionally, it is important to note that constraint (6)

prevents pilot reduction for SNR values below 10 dB due to the expected high channel estimation error caused by the utilized LS method.

To measure the coherence bandwidth, a correlation approach between the channel coefficients of each user is adopted in the frequency domain as expressed in [6]. On the other hand, the SNR fluctuation, ϑ_k , of the channel coefficients within each subchannel that is assigned to a user is evaluated depending on the estimated SNR values, λ_k . Furthermore, the channel estimation error is considered in the evaluation of λ_k as explained mathematically in (2). Hence, ϑ_k is calculated as [6]:

$$\vartheta_k = \lambda_k - \Gamma_{k,av}, \quad \dots (12)$$

where $\Gamma_{k,av} = E\{\lambda_k\}$ is the average SNR value of such user. The variance of the corresponding SNR fluctuations, σ_k^2 , can be calculated as [6]:

$$\sigma_k^2 = E\{\vartheta_k^2\}, \quad \dots (13)$$

The problem mathematically described by (4)-(8) can be solved using common optimization methods, such as Lagrange multipliers. However, a suboptimal ad hoc pilot adjusting algorithm is proposed to tackle the investigated problem efficiently as explained in the next subsection.

B. Pilot allocation algorithm

As mentioned before, the number of the required pilots for the channel estimation of each user depends on the measured coherence bandwidth, and the variance of the SNR fluctuations, particularly between the neighboring pilots. The minimum SNR value of each user is used to measure the quality of the channel state due to background noise.

Figure (2) demonstrates the proposed pilot adjusting algorithm in flow chart form. At the receiver of each user, the coherence bandwidth is measured. At the same time, the minimum SNR value, λ_{min} , and the variance of the SNR fluctuations, σ_k^2 , for each user are computed according to equation (3) and equation (13), respectively. These metrics are used as first level constraints in the proposed algorithm. The constraints expressed in equation (5)-(7) control the adjustment of the pilots and the data at low SNR levels, where the channel estimation error is high due to noise. Additionally, at low SNR values, the system preferably uses the full number of available pilots to mitigate against the expected high channel estimation error. This error influences the computational accuracy of the variance of the SNR fluctuations and the correlation operation of the coherence bandwidth measurements. If the first level constraints are satisfied, the proposed pilot allocation algorithm continues to decrease or increase the number of allocated pilots. At this stage, the pilot adjustment value, λ_k , as well as the optimized new number of data for each user can be obtained following equation (9) and equation (10). As mentioned earlier, the value of λ_k for each user assigned to a user is controlled by the constant ζ_k , which keeps the condition, $mod[\xi_{k,d}, \xi_{k,p}] = 0$, active in order to achieve a uniform distribution of adjusted pilots over data subcarriers.

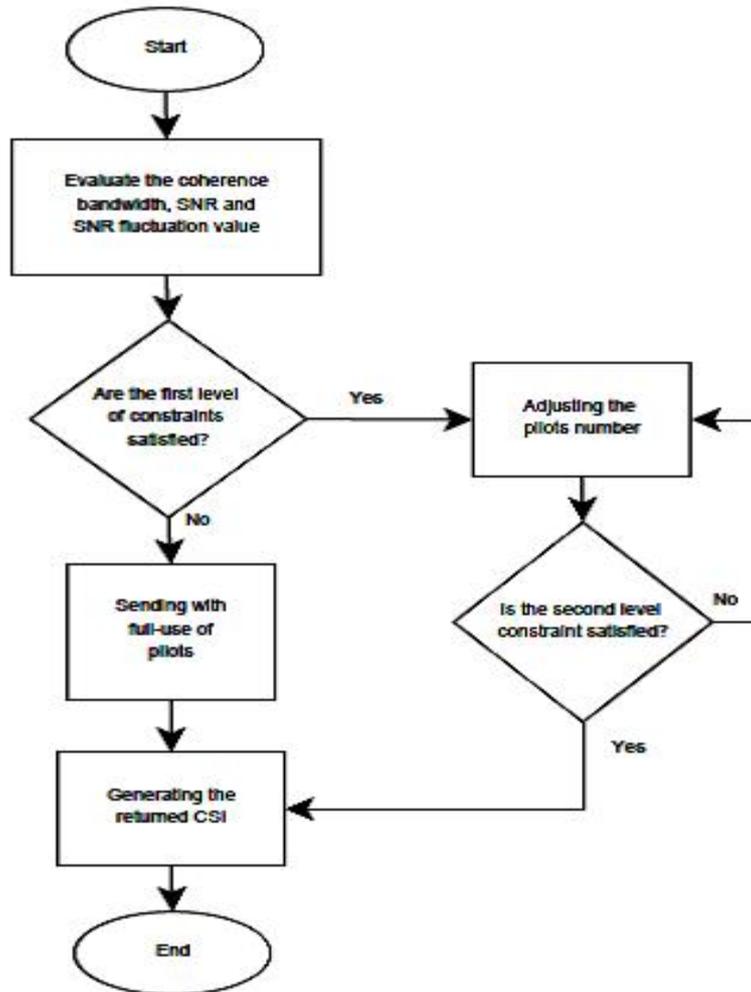


Figure (2) Proposed optimal pilot allocation algorithm.

The proposed system throughput can be evaluated semi-analytically by using (4). The computed SNR values from the simulation are considered for each user instead of the theoretical equivalents due to the diversity of the user conditions, which lead to select distinct MCS within the same OFDMA frame. The optimal number of pilots $\xi_{k,p}$ can be obtained using (9), while $\rho_{k,av}$ is based on the selected MCS, which is a function of SNR. P_{ek} is approximated by using the general OFDM BER formula [13]:

$$P_{e_k} = c_{1k} \exp[-c_{2k}\gamma_{min}], \dots (14)$$

Where c_{1k} and c_{2k} are constants used to fit the semi-analytical BER curve over the corresponding simulated. If $c_{1k} = 0.5$ is assumed, then c_{2k} can be empirically calculated for PSK based on the method in [11] as given below:

$$c_{1k} = 1.275 \frac{1}{(2^{w_k-1})R_{ck}v^{R_{ck}}}, \dots (15)$$

And for QAM:

$$c_{1k} = 1.16 \frac{1}{(2^{w_k-1})R_{ck}v^{R_{ck}}}, \dots (16)$$

Where $v = \{1, 2, 4\}$ for $R_{ck} = \{1/2, 2/3, 3/4\}$ represents the selected coding rate order, and $w_k = \log_2[Mk]$ denotes the number of bits per modulated symbol.

SIMULATION AND SEMI-ANALYTICAL RESULTS

The performance of the proposed algorithm based system is investigated in mobile vehicular ITU-A channels with mobility speed $v=60$ kmph, by generating distinct time varying coefficients for different users. The simulation model is adopted from the Mobile WiMAX IEEE 802.16e standard with $N_{FFT} = 2048$, which are divided into data $N_d = 1440$, pilots $N_p = 240$, and guard $N_g = 368$. Moreover, the system channel bandwidth of $BW_{sys} = 20$ MHz, and carrier frequency of $F_c = 2.3$ GHz are considered. Additionally, the simulation results are obtained for 30 users i.e. $K = 30$. Three schemes are considered for comparison: 1) conventional AMC, which adopts the same MCS for the whole OFDMA frame [1], 2) fixed subchannel, which represents the full use of the pilots [2], 3) the proposed pilot allocation algorithm. The semi-analytical throughput of the investigated schemes is also presented.

Figure (3) shows the throughput comparison between the investigated approaches. In this plot, the average system throughput for N_{frame} OFDMA frames is based on equation (4) with respect to the effective bandwidth. It is observed that the throughput related performance of the proposed scheme exhibits more gain over the fixed and conventional approaches for SNR range between 5 and 20 dB. In addition, the fixed scheme outperforms the conventional approach over the same SNR range. The enhancement in throughput performance, with increasing SNR, for the proposed method over conventional AMC is due to the use of the proposed algorithm that exploits the diversity of the user's channel conditions to construct the transmitted OFDMA frame, which contains different MCS levels for all users. Furthermore, the replacement of redundant pilots across the OFDMA frame produces a significant improvement in the system throughput. From the plot, it is also evident that the semi-analytical approach provides a satisfactory approximation to the results obtained via simulation.

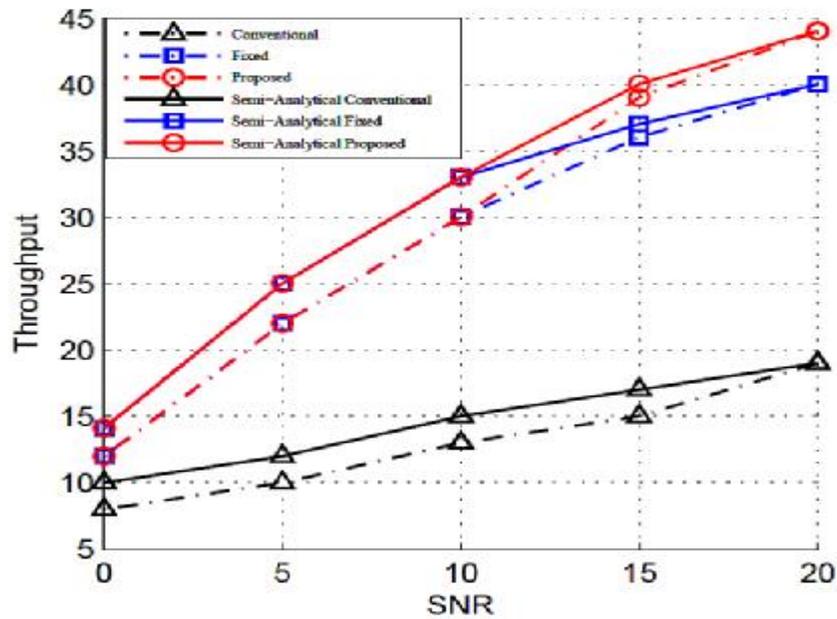


Figure (3) Average system throughput of fixed, proposed, and conventional approaches.

Figure (4) demonstrates the BER for the investigated approaches. The figure shows that all approaches achieve the same performance in terms of BER. This can support that the proposed algorithm enhances the throughput of the system, while keeping the BER same.

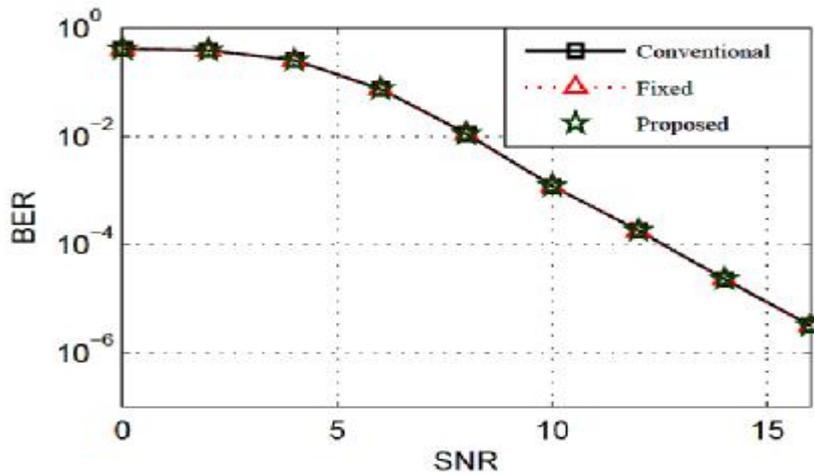


Figure (4) BER for fixed, proposed, and conventional approaches.

CONCLUSIONS

An efficient pilot allocation algorithm has been proposed and investigated in this paper. In order to improve the throughput performance, the detected number of users has been utilized as a metric to divide the OFDMA frame into distinct subchannels and subsequently assign individual modulation schemes and coding rates suited to the corresponding channel conditions. Additionally, the proposed pilot adjusting algorithm replaces redundant pilots with additional data subcarriers based on the received SNR levels, the measured coherence bandwidth and the SNR fluctuation values for each user, in which the channel estimation error is within an acceptable range. Simulation and semi-analytical results have demonstrated that the proposed system throughput is improved in comparison with conventional AMC schemes.

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