

## Gain Characterization of Praseodymium Doped Single Mode Optical Fiber Amplifier

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### ABSTRACT

A theoretical study for three level laser systems, describes the interaction process between optical fields and optical fiber material of amplifier. This study gives a simple approach expression for gain coefficient which depends on some parameters such as input power pump rate, number of doped atoms per unit volume, fiber length and core radius. It has been found that In order to keep the fiber amplifiers doped with praseodymium as short as possible with good efficiency, it is best to increasing the doped concentration. For 3m length and  $5 \times 10^{24}$  ion/m<sup>3</sup> doping concentration, the gain is 8dB, while for same length and pump power the gain is >10dB if doping Concentration  $8 \times 10^{24}$  ion/m<sup>3</sup>.

### صفات الربح لمضخم ليف بصري احادي النمط مطعم بالبراسوديوم

#### الخلاصة

دراسة نظرية لنظام ليزري ثلاثي المستوى ، تصف عملية التفاعل بين المجالات البصرية ومادة الليف البصري للمضخم. الدراسة تعطي تمثيل تقريبي مبسط لمعامل الربح المعتمد على بعض المعاملات مثل القدرة الداخلة ، عدد ذرات التطعيم لوحدة الحجم ، طول الليف ، نصف قطر اللب ومعدل الضخ. وجد انه لكي نبقي مضخم الليف المطعم بالبراسوديوم قصير قدر الامكان بكفائه عالية يستحسن زيادة تركيز التطعيم. للليف طوله 3m وتركيز التطعيم  $5 \times 10^{24}$  ion/m<sup>3</sup> الربح 8Db بينما لنفس الطول ونفس قدرة الضخ الربح اكبر من 10dB لتطعيم تركيزه  $8 \times 10^{24}$  ion/m<sup>3</sup>.

### INTRODUCTION

Optical fiber amplifiers are promising devices to be used in realizing the direct in-line amplification of optical signals in optical fiber transmission systems. In 2009 Chun Jiang and Li Jin derived a theoretical model which gives the dependence of the gain on fiber length [1]. Artro Chavez in 2010 focused on application of the highly doped phosphate fiber, this type of active fiber can lead to unique and superior performance [2]. Arlee and Jesse used a beam propagation model of CW fiber amplifier they showed that gain saturation by strong fundamental mode 2011 [3]. Peiyong Chen and etal in 2012 gives a numerical solution of Eulers method

to deal with initial value problems for fiber amplifier rate equations and their gain dependence [4].

**THEORY**

Theoretical study for continuous wave laser system for Praseodymium 1.3 μm doped fiber amplifier where the pump power 1030nm reached to higher energy level directly was achieved [5]. We will examine numerically the operating of Praseodymium doped fiber amplifier (PDFA), with transverse ( $^1G_4 \rightarrow ^3H_5$ ). Since population inversion in the system is only related to the population in the higher and lower energy levels, it is reasonable to simplify the four energy levels to three level as in  $Pr^{3+}$  without any intermediate energy levels [6,7]. The additional susceptibility induced by transitions of atoms in the fiber medium due to the pump action will cause a perturbation of the propagation constants on the transmission mode in the optical fiber. Maxwell's equations and scalar wave approximations, we have [8].

$$\nabla^2 E + (K_1^2 - K_z^2)E = 0 \quad r \leq a \quad \dots (1)$$

$$\nabla^2 E + (K_2^2 - K_z^2)E = 0 \quad r \geq a \quad \dots (2)$$

Where,

$$K_1^2 = \omega_{ab}^2 \mu_0 (\epsilon_1 + \epsilon_0 x') \quad \dots (3)$$

$$K_2^2 = \omega_{ab}^2 \mu_0 \epsilon_2, \quad K_z^2 = k_z^2 + i\Delta k_z, \quad \Delta K_z^2 = \omega_{ab}^2 \mu_0 \epsilon_0 x'' \quad \dots (4)$$

$K_z$ , is the propagation constant,  $K_1 > K_z > K_2$ ,  $\epsilon_1$  and  $\epsilon_2$  is the permittivity of the fiber core and cladding respectively,  $\omega_{ab}$  atomic transition frequency. Since  $\Delta K_z^2 \leq K_z^2$  is usually satisfied and using the wave-guide approximation, one can obtain the field amplitude equations in the fiber.

$$E = J_m\left(\frac{u}{a}r\right) \exp\left(-\frac{\Delta K_z^2}{2K_z}z\right) \exp[-i(\omega_{ab}t - K_z z + m\theta)] \quad r \leq a \quad \dots (5)$$

Where,  $\frac{u^2}{a^2} = K_1^2 - K_z^2$ ,  $J_m$ , is the Bessel function. Equation (5) shows that if

$\Delta K_z^2 < 0$  the field amplitudes in the core and cladding will increase exponentially with Z. As we know that different propagation in the fiber have different field distributions, then they also have different power distributions in the core and cladding. Since the fiber core is effective amplification area in the doped fiber, the power distribution in core section will strongly affect the gain and other properties of amplifier. The waveguide efficiency is used to describe this effect and is defined as the ratio of power in core to the total power for the  $ij$ th mode that is [8]

$$\eta_{ij} = 1 - \left( \frac{u^2}{u^2 + w^2} \right) \left( 1 - \frac{K_m^2(w)}{K_{m-1}(w)K_{m+1}(w)} \right) \quad \dots (6)$$

Where,  $\eta_{mn}$  is the waveguide efficiency of the signal and  $\eta_{kt}$  is the waveguide efficiency for pump light. The average waveguide efficiency,

$$\eta = \frac{\sum_{ij} \eta_{ij}}{\sum_{ij} 1} \quad \dots (7)$$

The average waveguide efficiency as the function of the normalized frequency  $V$  under the condition of uniform illumination, and the normalized frequency  $V$  is related to pump wavelength and radius of core  $a$  by equation [9]

$$V = K \cdot a \cdot NA \quad \dots (8)$$

Where,  $K$  is the wave number of light and  $N_A$  Numerical aperture. High waveguide efficiencies would be obtained with ( $V=2.4$ ) for single mode operation, with ( $V > 2.4$ ) for multimode operation. The population inversion in the fiber is also in proportion to the waveguide efficiency of the pump light  $\mu_{kt}$ . Hence, the pump rate can be written as [7].

$$w_p = \frac{\sigma_a \eta_q \eta_{kt}}{h\omega_p N_0} P_p \quad \dots (9)$$

Where  $\sigma_a$ , is the cross-section for absorption,  $\omega_p$  is the angular frequency of pump power,  $P_p$  is the pump power and  $\eta_q$  is quantum efficiency. The absorbed pump power ( $P_{abs}$ ) of the doped per unit length can be expressed as [7].

$$P_{abs} = \sigma_a \eta_q \eta_{kt} P_p M \quad \dots (10)$$

Where  $M$  is the volume of the interaction region of the fiber core, which can be expressed as:

$$M = A_{eff} \cdot L \quad \dots (11)$$

Where  $L$  is the length of optical fiber,  $A_{eff}$  is the active area of the fiber core which can be written as.

$$A_{eff} = \pi a^2 \quad \dots (12)$$

Substituting equation (10) into equation (9) yields.

$$w_p = \frac{P_{abs}}{h\omega_p N_0 M} \quad \dots (13)$$

From equation (5) the gain coefficient given by [10] .

$$g = -\frac{\Delta K_z^2}{K_z} \quad \dots (14)$$

From the approximation of equation (3) we have.

$$K_1^2 = K_z^2 = \omega_{ab}^2 \mu_0 \epsilon_1 \quad \dots (15)$$

Substituting equation (4) and equation (15) into equation (14).

$$g = -\frac{\omega_{ab}^2 \mu_0 \epsilon_0}{\omega_{ab} (\mu_0 \epsilon_1)^{1/2}} x'' \quad \dots (16)$$

$$g = -\frac{n_1 \omega_{ab}}{c} x'' \quad \dots (17)$$

Where:  $(c = \frac{1}{(\mu_0 \epsilon_0)^{1/2}})$  and  $(k = \frac{\epsilon_1}{\epsilon_0})$  ,  $(n_1 = \sqrt{k})$  . The parameter k is the

dielectric constant and c is the speed of light in vacuum. From induced susceptibility definitions, we have [7]

$$x'' = \frac{p^2}{\epsilon_0 \hbar} \left[ \frac{\gamma}{(\omega_{ab} - \omega)^2 + \gamma^2} f(s) \frac{1}{\gamma_a + n_q \omega_p} n_q \omega_p N_0 \right] \quad \dots (18)$$

Where,  $(\omega_{ab})$  is the circular frequency of input field,  $N_0$  is the number of doped atoms in unit volume; p is the electric dipole which can be calculated from the output power and the threshold power of laser action. Substituting equation (18) into equation (17) yield [11]

$$g = \frac{n_1 \omega_{ab} p^2}{\epsilon_0 \hbar} \frac{\gamma}{(\omega_{ab} - \omega)^2 + \gamma^2} f(s) \frac{1}{\gamma_a + n_q \omega_p} n_q \omega_p N_0 \quad \dots (19)$$

We did not pay much attention to the function f(s) yet in the expression for gain coefficient. Now expanding f(s) under the condition of small signal amplification  $(R < R_s)$ .

$$F = 1 - \frac{3}{4}S + \frac{5}{8}S^2 - \frac{35}{64}S^3 + \dots \quad \dots (20)$$

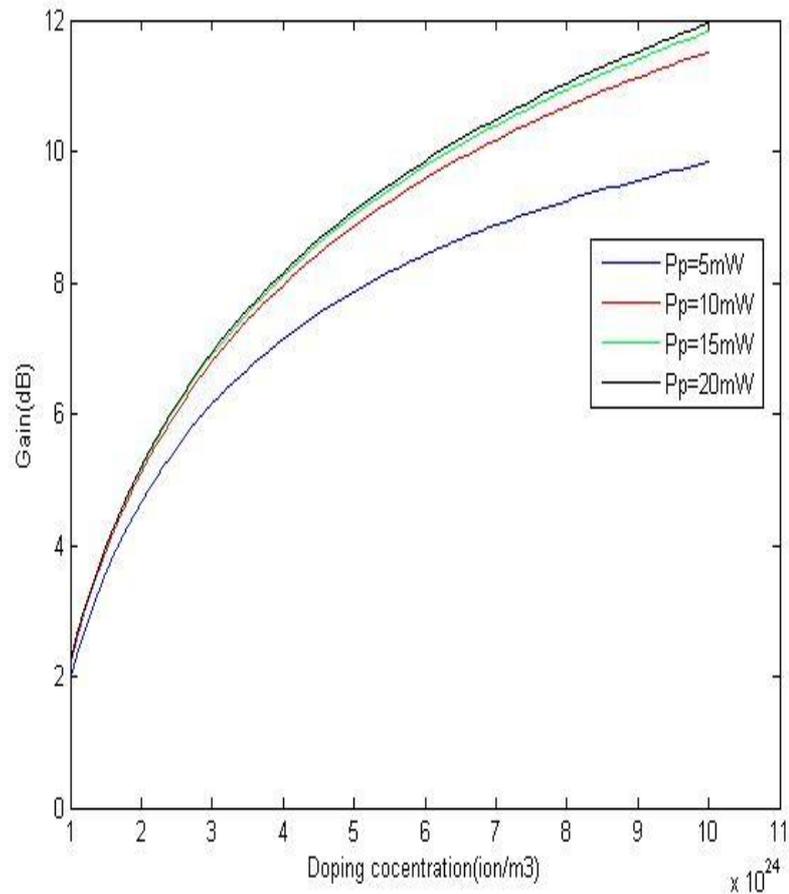
The gain coefficient for a small signal can be written as [7].

$$G = \frac{n_1 \omega_{ab} p^2}{c \epsilon_0 \hbar} \frac{\gamma}{(\omega_{ab} - \omega)^2 + \gamma^2} \frac{1}{\gamma_a + \eta_q w_p} \eta_q w_p N_0 \quad \dots (21)$$

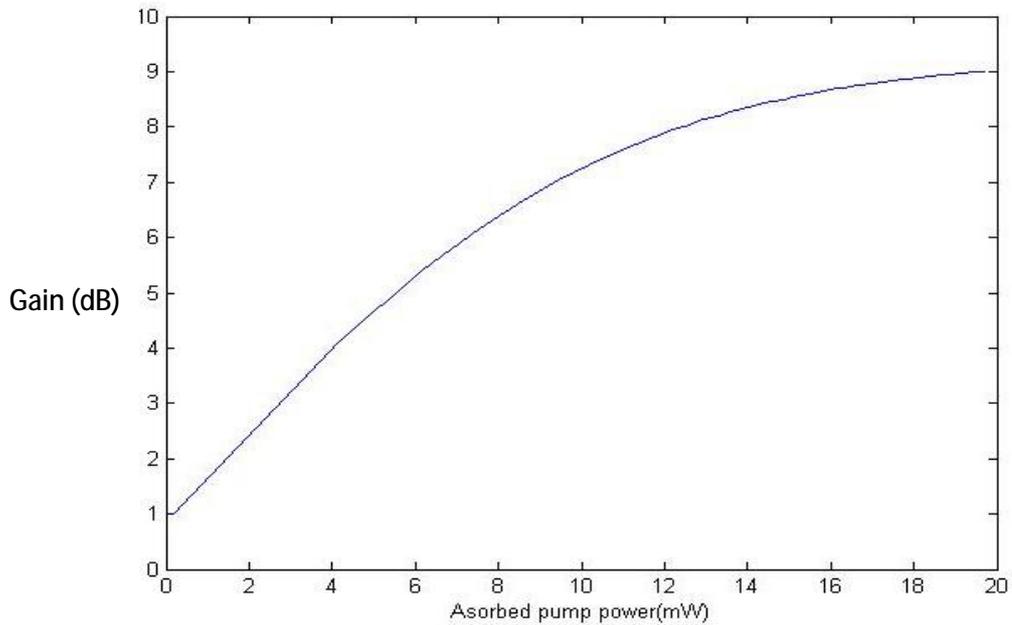
### RESULTS AND DISCUSSION

Analyzing the characteristics of the (Pr<sup>3+</sup>) CO-doped telluride amplifiers co-pumping lasers, by employing an amplifier which is based on propagation and population of theoretical model. Any increasing in do pant concentration means increasing in a transition cross section which causes high gain. The following data are used in simulation program[9] , where ,N<sub>0</sub> is 4\*10<sup>24</sup> - 4\* 10<sup>25</sup> ion/m<sup>3</sup> ,NA is 0.13, n<sub>i</sub> is 1.446,L is ( 3-10)m, a is (2-9)m,μ<sub>q</sub> is 0.8 , σ is 1.5\* 10<sup>-21</sup> m<sup>2</sup> ,μ<sub>kt</sub> is 0.78 ,p is 2.78 \*10<sup>-29</sup> - 2.78\*10<sup>32</sup> mC , λ<sub>p</sub> is 1030nm, λ<sub>s</sub> is 1300nm ,λ<sub>c</sub> is 1280nm ,τ<sub>a</sub> is 10ms , and Y is 1.4\*10<sup>12</sup>S<sup>-1</sup> .Figure (1) shows the effect of the variation of the Pr<sup>3+</sup> ions concentration on the gain coefficient. Hence employing amplifiers with Pr<sup>3+</sup> ions of 1×10<sup>25</sup> ions/m<sup>3</sup> and active fiber length of 5m, achieved high gain about 12dB.Figure (2) shows the gain coefficient as a function of absorbed pump power. For a given amplifier pump power, the gain increases exponentially with the absorbed pump power and reached saturation value 9dB.This result agrees with the results of [7]. The effect of variation of praseodymium doped fiber length on gain coefficient is shown in Figure (3). As the fiber length increase for low pumping powers, the gain starts to decrease after a certain length about 5m because the pump does not enough to create a complete population inversion in downward have portion of the amplifier. In this case, the un pumped region of the fiber absorbs the signal thus, resulting loss in signal rather than gain in that section. Figure (4) shows the gain coefficient as a function of the input power for different values of fiber length. This figure shows that for fiber length L=10m the gain reaches a saturation level. For the available input power of 20mW, maximum gain of 7.5 dB is reached, this result is in agreement with [12, 7]. Figure (5) shows the gain coefficient as a function of effective core radius for different values of pumping power. Any increase in pump power caused an increase in the gain of stimulated emission, but any increase in the effective core radius to large value will reducing in gain coefficient, and large gain can be achieved about 20dB for small core radius(2-5) μm in an certain pump power. This result is in agreement with the result of [12] Figure (6) shows the gain coefficient as function of the input power for different value of core radius the calculation includes the relation between the gain coefficient and input power for deferent value of core radius. The results of this calculation are capable to get gain up to 12dB or more for small core radius 5μm. This result is in agreement with published work [13, 14]. Figure (7) shows gain coefficient as function of pump power by using some typical parameter values.

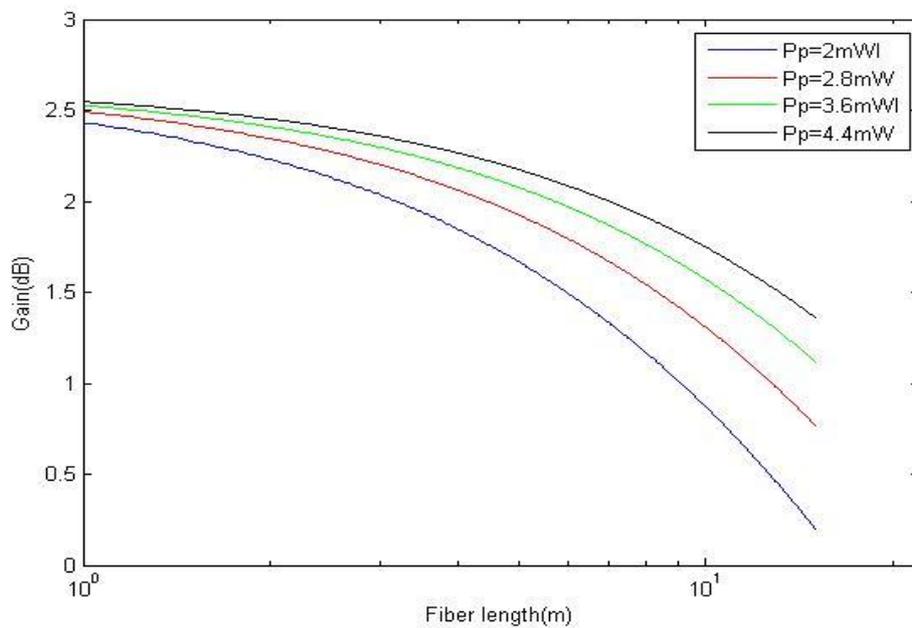
At a certain value of core radius and fiber length the gain coefficient will increase exponentially with pumping rate as shown in Figures (8,9,10 and 11).



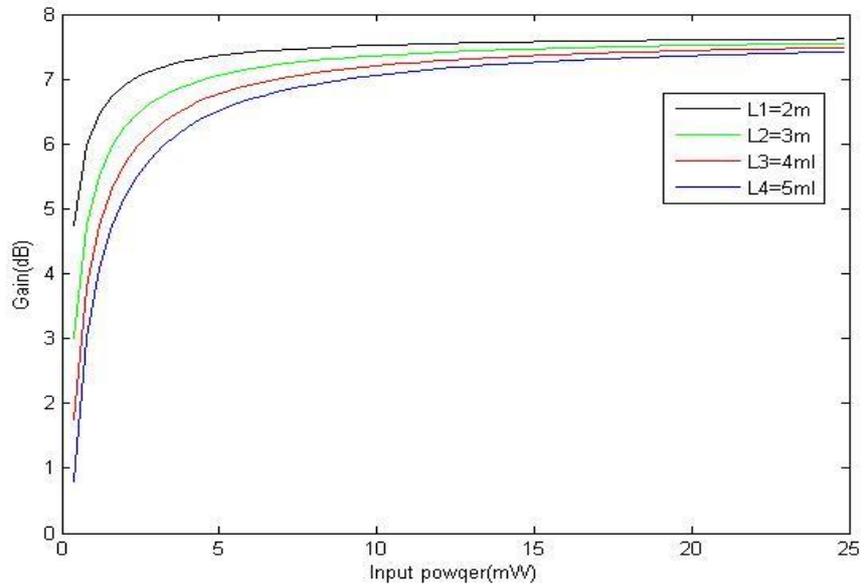
**Figure (1) Gain vs. doped concentration for different pump power with L=3m, a=3.75μm.**



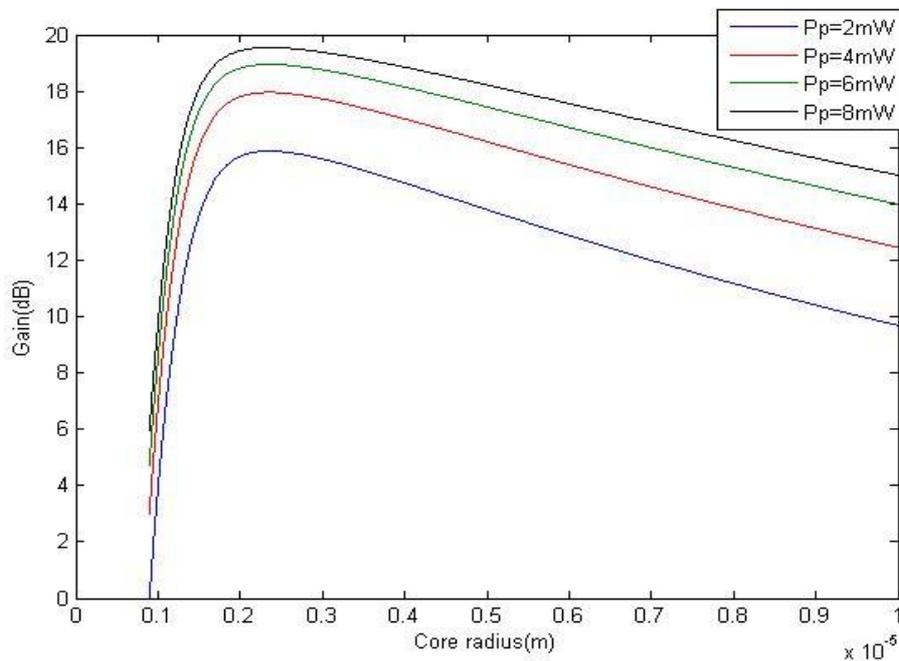
**Figure (2) Gain vs. absorbed pump power with  $L=3\text{m}$ ,  $a=3.75\mu\text{m}$ , ( $p = 2.78 \times 10^{-31} \text{ mC}$ ) and input power =  $50\text{mW}$ .**



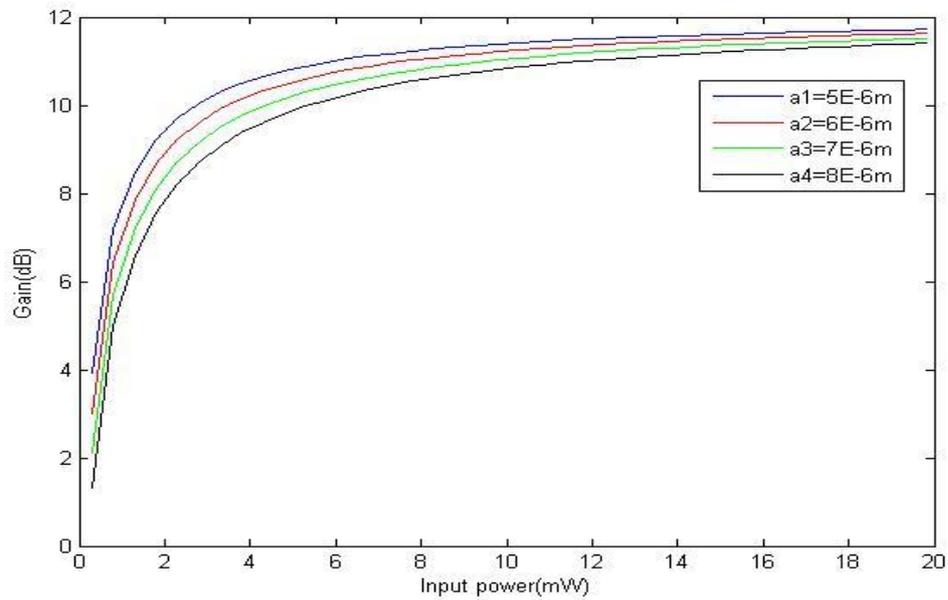
**Figure (3) Gain vs. fiber length for different value of pumping power with  $a=5\mu\text{m}$ ,  $p = 2.78 \times 10^{-31} \text{ mC}$  and  $N = 4 \times 10^{25} \text{ ion/m}^3$ .**



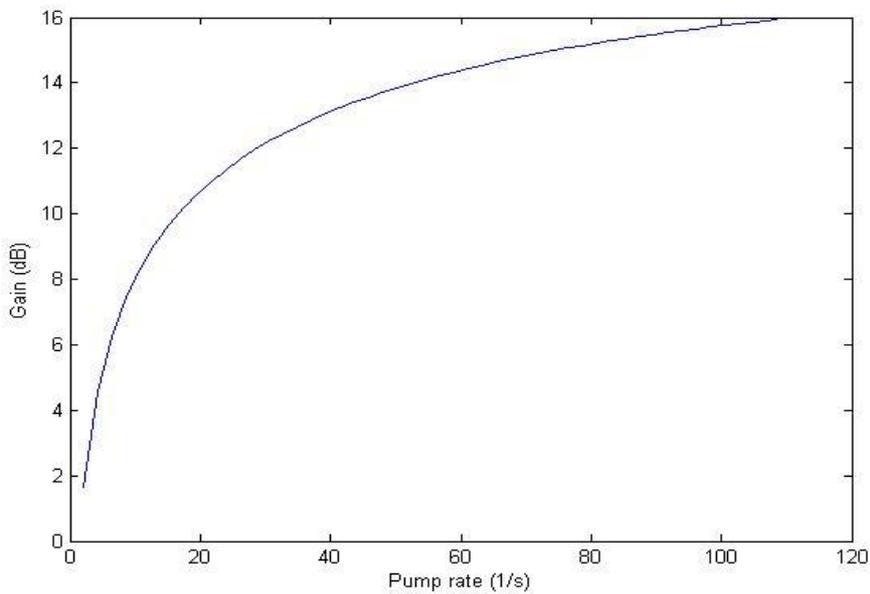
**Figure (4) Gain vs. pump power for different value of fiber length, with  $a=3.75\mu\text{m}$ ,  $p = 2.78 \times 10^{-31} \text{ mC}$  and  $N = 4 \times 10^{24} \text{ ion/m}^3$ .**



**Figure (5) gain vs. core radius for different value of pumping power with  $L=5\text{m}$ ,  $p = 2.78 \times 10^{-30} \text{ mC}$  and  $N = 4 \times 10^{24} \text{ ion/m}^3$ .**



**Figure (6) Gain vs. pump power for different value of core radius with  $a=3.75\mu\text{m}$ ,  $p = 2.78 \times 10^{-31} \text{ mC}$  and  $N = 4 \times 10^{24} \text{ ion/m}^3$ .**



**Figure (7) Gain vs. Pump rate for Praseodymium doped fiber amplifier With  $L=5\text{m}$ ,  $a= 3.75\mu\text{m}$ ,  $p = 2.78 \times 10^{-30} \text{ mC}$ ,  $N = 4 \times 10^{24} \text{ ion/m}^3$**

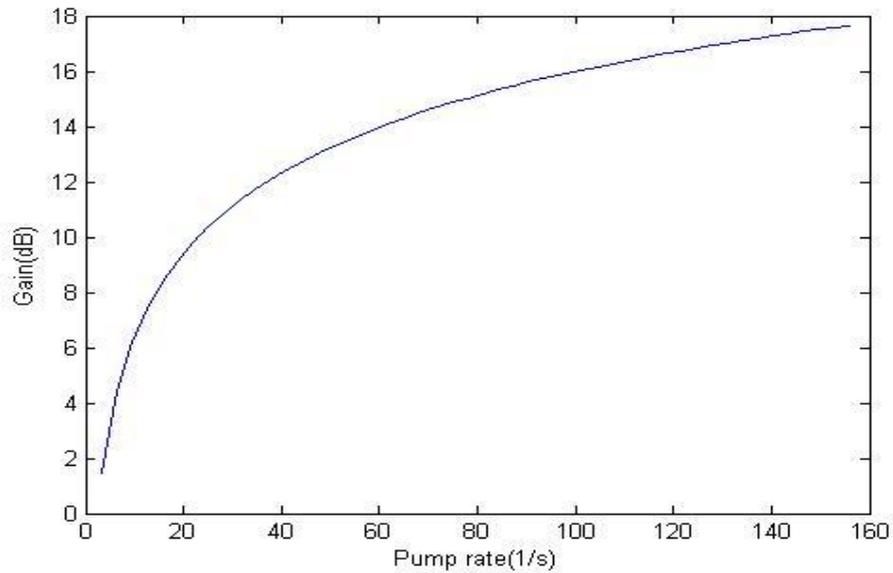


Figure (8) Gain vs. pump rate for (PDFA) with  $L=5\text{m}$ ,  $a=3.75\mu\text{m}$ ,  $p=2.78 \times 10^{-30} \text{ mC}$ ,  $N = 2 \times 10^{24} \text{ ion/m}^3$

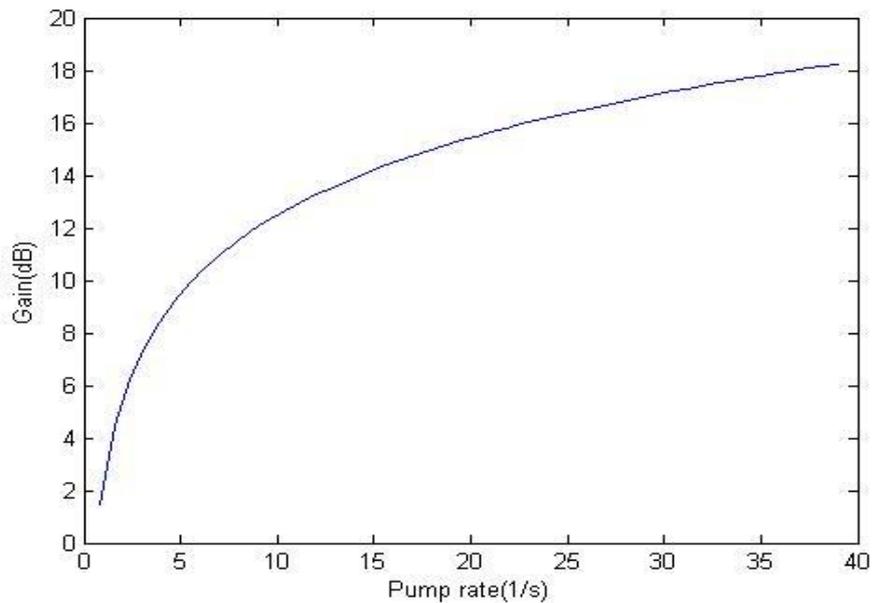


Figure (9) Gain vs. pump rate for (PDFA) with  $L=5\text{m}$ ,  $a=3.75\mu\text{m}$ ,  $p=2.78 \times 10^{-30} \text{ mC}$ ,  $N = 8 \times 10^{24} \text{ ion/m}^3$ .

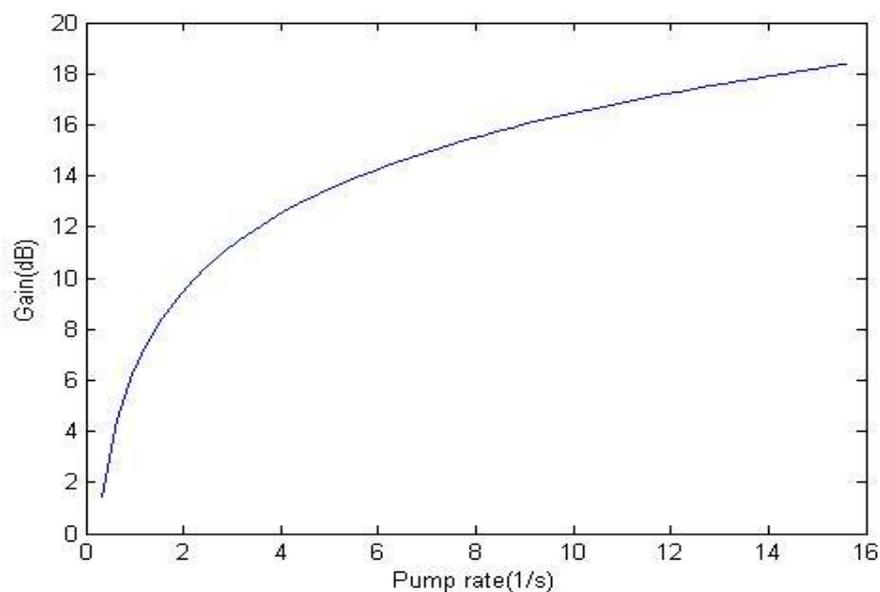


Figure (10) Gain vs. pump rate for (PDFA) with  $L=5\text{m}$ ,  $a=3.75\mu\text{m}$ ,  $p=2.78 \times 10^{-30} \text{ mC}$ ,  $N = 20 \times 10^{24} \text{ ion/m}^3$ .

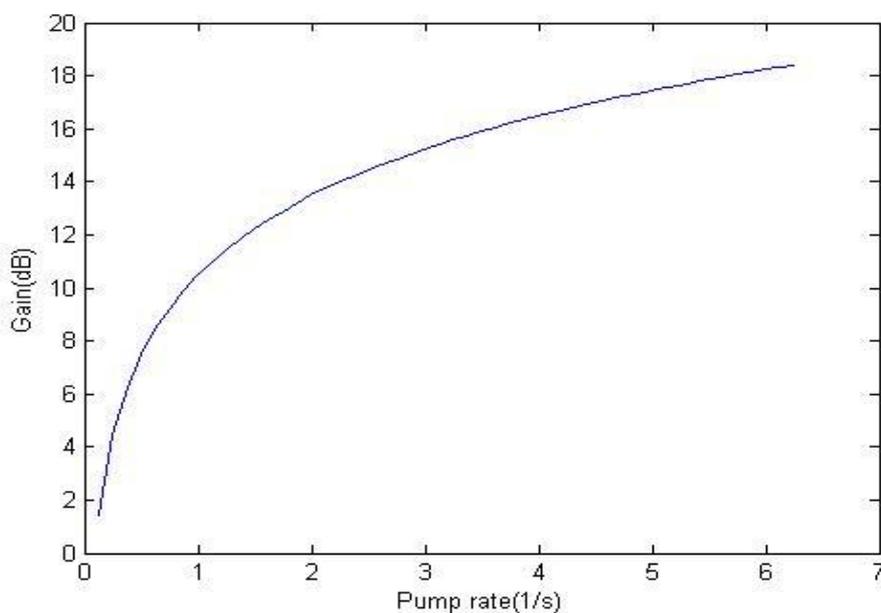


Figure (11) Gain vs. pump rate for (PDFA) with  $L=5\text{m}$ ,  $a=3.75\mu\text{m}$ ,  $p=2.78 \times 10^{-30} \text{ mC}$ ,  $N = 50 \times 10^{24} \text{ ion/m}^3$ .

## CONCLUSIONS

The interaction between the input light and atoms causes a change of susceptibility of the fiber material and that the value change depends on the characteristics of the fiber material ( $p$ ,  $\gamma$ ,  $\omega$ ,  $N_0$ ), frequency of input light ( $\omega_{ab}$ ) and pump rate ( $wp$ ). The certain fiber length can produce a higher gain because it can absorb a great of pumping power but the maximum gain does not scale linearly with the fiber length because of fiber attenuation losses. In order to keep the fiber amplifiers as short as possible with good efficiency, it is best to increasing the doped concentration. For 3m length and ( $5 \times 10^{24}$  ion/m<sup>3</sup>) doping concentration, the gain is 8dB, while for same length and pump power the gain is >10dB if doping concentration  $8 \times 10^{24}$  ion/m<sup>3</sup>. High gain for small core radius result in active fiber device which can combine the excellent properties of standard laser material . For 5m length and (2-5)  $\mu\text{m}$  core radius, the gain is 12dB and reach saturation value and any increase in core radius causes reduce in gain or output power. Lastly we can achieve high gain by increasing pumping rate. For  $L=5\text{m}$ ,  $a=5\mu\text{m}$ ,  $N = 1 \times 10^{25}$  ion / m<sup>3</sup> the gain is 20dB at  $100\text{s}^{-1}$ .

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