Forward Kinematics Modeling of 5DOF Stationary Articulated Robots

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ABSTRACT

This paper present a direct kinematics modeling of 5 DOF stationary articulated robot arm which is used for educational tasks, and presents an adopted modeling method to represent and simulate the simultaneous positional coordinates for each joint of the robot while it moving from one target to another, where a Lab Volt R5150 robot arm has been taken as a case study. the Denavite – Hartenberg (D-H) model of representation is used to model robot links and joints in this paper. It utilizes Matlab 2010a software as the tools for manipulation and testing. The adopted modeling solution was found to be identical with the physical behaviors.

Keywords*:* LabVolt R5150 robot arm, forward kinematics, D-H parameters, modeling and simulation

التمثيل الحركي المباشر لروبوت ممفصل ذا خمسة درجات حرية

الخلاصة

يقدم هذا البحث موديل رياضي للتحليل الحركي الامامي (المباشر) لذراع روبوت ممفصل ذا خمسة درجات حرية يستخدم للاغراض التعليمية ، ومن خلال التمثيل الحركي فان البحث الحالي يقدم موديل لمحاكاة وتمثيل المواقع الانية لمفاصل الروبوت اثناء الحركة من موقع الى اخر . تم استخدام الروبوت التعليمي **Lab-Volt R5150** لاختبار الموديلات المقترحة وباعتماد متغيرات دينيفت-هارتنبيرك **D-H** القياسية لحساب احداثيات الموقع النهائي للماسكة فيما اعتمد برنامج (**Matlab 2010a**) للمحاكات والتقييم. اثبتت النتائج ولعدة تجارب دقة الموديل المقترح للتمثيل والمحاكاة.

INTRODUCTION

Robot manipulator field is one of the interested fields in industrial, educational and medical applications. It works in unpredictable, hazard and inhospitable circumstances which human cannot reach. For example, working in chemical or nuclear reactors is very dangerous, while when a robot instead human it involves no risk to human life [1]. The robot has become important equipment in manufacturing of a country's

manufacture level and technological level. Robot kinematics analysis is the basis of robot motion control [٢], therefore, modeling and analysis of the robot manipulators are very important before using them in these circumstances to work with high accuracy. Robots are operated with there motors in the joint space, whereas tasks are defined and objects are manipulated in the Cartesian space .The transformation between joint space and the Cartesian space of the robot is very important.

There is a large amount of literature which discusses the kinematics of industrial robot. Yang et al [3], have described that the placement of an open loop robotic manipulator in a working environment is characterized by defining the position and orientation of the manipulator's base with respect to a fixed reference frame. Bi and Lang [4], have proposed a forward kinematics model for determining workspace by using joint motion. Forward and Inverse equation analysis, were generated and implemented using a simulation program [5] and [6]. Annand [7] derived the kinematics analysis of PUMA 560, and calculated the equation of motion of the robot by deriving the so-called Euler-Lagrange equations.

ROBOT DESCRIPTION

LabVolt Robot 5150 is a small table top robotic arm manufactured by LabVolt Inc Fig(1) . It is a five articulated coordinate robotic manipulator that uses stepper motors for joint actuators, and its motion are controlled by RoboCIM software.

Lab-Volt R5150 robot arm has base, shoulder, elbow, tool pitch and tool roll which are all consisting rotary joints and provide 5 directions of motion (DOF) plus a grip movement These joints provide shoulder rotation, shoulder motion, elbow rotation, wrist up and down motion, wrist rotation and gripper motion. Lab-Volt R5150 has five rotational joints, five axis (three major axes : base – shoulder-elbow to position the wrist, and two minor axes : pitch and roll to orient the gripper) and a moving grip.

Figure (1) LabVolt R5150 robot arm.

Kinematics

Robot arm kinematics explain the analytical description of the motion geometry of the manipulator with the reference to a robot coordinate system fixed to a frame without the consideration of the forces or the moments causing the movements.[8]

For the direct kinematics the inputs are the joint angle vectors and the link length parameters, while the output of the problem is the orientation and the position of the tool or gripper. The block diagram representation of the direct kinematics shown by Fig.2

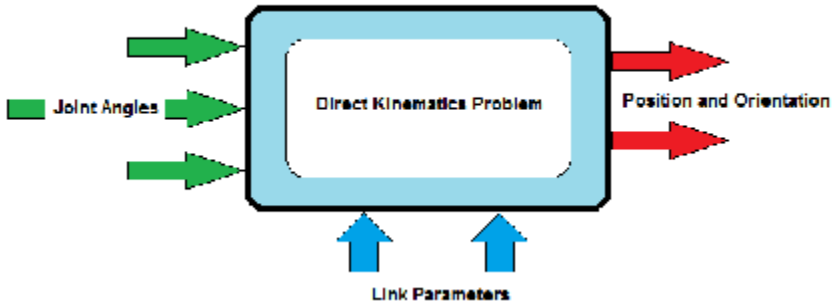


Figure (2). Direct Kinematic Block Diagram.

Many methods can be used in the direct kinematics calculation. The Denavit-Hartenberg (D-H) analyses is one of the most used, in this method the direct kinematics is determinate from some parameters that have to be defined, depending on each mechanism. However, it was chosen to use the homogeneous transformation matrix [9]. This transformation specifies the location (position and orientation) of the hand in space with respect to the base of the robot, but it does not tell us which configuration of the arm is required to achieve this location .

Applying the D-H algorithm, the link coordinate diagram can be represented as shown in fig, (3) , the dotted diagonal line between the origin of link3 and the origin of link4 indicates that the origin of these two coordinate frames coincide

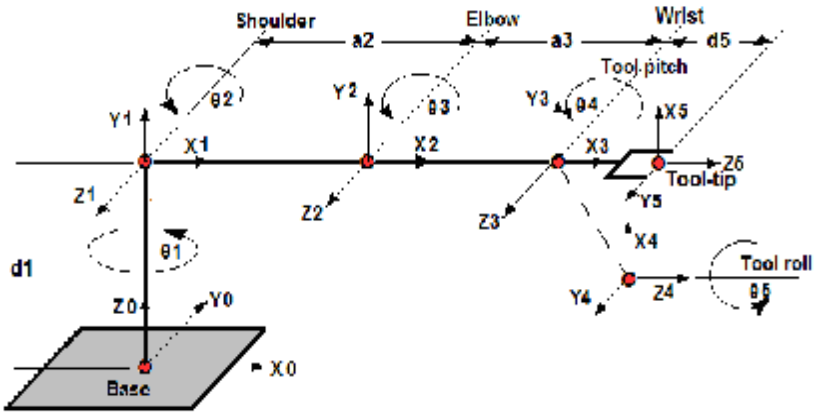


Figure (3) Link coordinate diagram of LabVolt 5150 robot arm.

Since LabVolt R5150 robot is a five-axis articulated coordinate robot as shown in figures (1) and (3) then:

The vector of joint variables is given by: $q = [q_1 \quad q_2 \quad q_3 \quad q_4 \quad q_5]^T$

The vector of joint distances is given by: $d = [d_1 \quad d_2 \quad d_3 \quad d_4 \quad d_5]^T$

The vector of link lengths is given by: $a = [a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5]^T$

The vector of the link twist angle is given by: $a = [a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5]^T$

These four parameters are called Denavit-Hartenberg (D-H) parameters. The D-H representation provides a systematic method for describing the relationship between adjacent links and gives a mathematical description for all serial manipulators depending on the robot geometry [5],[8],[9]. It defines the position and orientation of the current link with respect to previous one. Using fig. (3), the set of kinematics D-H parameters for LabVolt R5150 robot are shown in Table (1).

Table (1) Link parameters for LabVolt R5150 robot arm.

Axis	q	d	a	a
1	$q1$	255.5mm	0	90
2	$q2$	0	190mm	0
3	$q3$	0	190mm	0
4	$q4$	0	0	90
5	$q5$	115mm	0	0

As shown in fig. (3) the origins of the coordinate frames associated with the tool orientation coincide at the wrist because the robot has a spherical wrist with no tool yaw motion

THE ARM EQUATION

In physical applications, it is important to describe the position of the end effector of the robot manipulator in one global coordinates. In transforming, the coordinates of the end effector from the local position to the global position, the robot movements are represented by a series of movements of rigid links. Each link defines a proper transformation matrix relating the position of the current link to the previous one [10]

Once a set of link coordinates is assigned using D-H algorithm, then the transform from coordinate frame i to coordinate frame i-1 can be achieved using homogeneous coordinate transformation matrix. To determine the transformation matrix rotation and translation of the frame i-1 have to be done successively to render it coincident with coordinate frame i, this involves four fundamental operations as illustrated in Table (2) [11].

Table (2) Transferring from frame i to frame i-1

Operation	Description
1	**Rotation** $about\ \ z_{i-1}\ by\ \ q_i$
2	**Translate** $along\ \ z_{i-1}\ by\ a\ disance\ d_i$
3	**Translate** $along\ \ x_{i-1}\ through\ \ a\ length\ ai$
4	**Rotation** $about\ \ x_{i-1}\ by\ \ twist\ a_i$

This can be expressed as a product of four homogeneous transformation relating coordinate frame of i-1 link to that of link i this relation is known as Arm Matrix

$$A_{i-1}^{i} = R_{(Z,q)}.T_{(0,0,d)}.T_{(a,0,0)}.R_{(x,a)}$$
$$A_{i-1}^{i} = R_{(Z,q)}.T_{(a,0,d)}.R_{(x,a)} \qquad \text{...1}$$

Hence

$$A_{i-1}^{i} = \begin{bmatrix} cosq & -sinq & 0 & 0 \\ sinq & cosq & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & cos\,a & -sin\,a & 0 \\ 0 & sin\,a & cos\,a & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_{i-1}^{i} = \begin{bmatrix} cosq & -sinq.cos\,a & sinq.sin\,a & a.cosq \\ sinq & cosq.cos\,a & -cosq.sin\,a & a.sinq \\ 0 & sin\,a & cos\,a & d \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \text{....2}$$

After obtaining the table of DH convention, a series of homogeneous matrices can be derived depending on the number of the DOF. The transformation matrix for each joint from joint 1 to the joint *i* can be calculated as:

$$A_i = R_{(Z,qi)}.T_{(0,0,di)}.T_{(ai,0,0)}.R_{(x,ai)} \qquad \text{....3}$$

In order to compute the arm matrix, it is often helpful to partition the problem at the wrist, this effectively decomposes the problem into two smaller sub problems, one sub problem associated with the major axes used to position the tool and the other sub problem associated with the minor axes used to orient the tool that can be formulated as follows:

$$A_0^5 = A_0^1.A_1^2.A_2^3.A_3^4 A_4^5$$
$$A_{Base}^{Tool} = A_{Base}^{Wrist}.A_{Wrist}^{Tool}$$
$$A_{Base}^{Wrist} = A_0^1 A_1^2 A_2^3 \qquad \text{...4}$$
$$A_{Wrist}^{Tool} = A_3^4.A_5^4$$

$$A_{Base}^{Wrist} = \begin{bmatrix} C_1 & 0 & S_1 & 0 \\ S_1 & 0 & -C_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_2 & -S_2 & 0 & a_2C_2 \\ S_2 & C_2 & 0 & a_2S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_3 & -S_3 & 0 & a_3C_3 \\ S_3 & C_3 & 0 & a_3S_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_{Base}^{Wrist} = \begin{bmatrix} C_1C_{23} & -C_1S_{23} & S_1 & C_1(a_2C_2 + a_3C_{23}) \\ S_1C_{23} & -S_1S_{23} & -C_1 & S_1(a_2C_2 + a_3C_{23}) \\ S_{23} & C_{23} & 0 & d_1 + a_2S_2 + a_3S_{23} \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{...5}$$

Where

$$C_1 = Cosq_1 \ , \ S_1 = Sinq_1 \ , \ C_{23} = Cos(q_2 + q_3) \ and \ S_{23} = Sin(q_2 + q_3)$$

The final expression for A_{Base}^{Wrist} depends only on the first three joint variables
Computing the tool coordinates relative to the wrist:

$$A_{Wrist}^{Tool} = \begin{bmatrix} C_4 & 0 & S_4 & 0 \\ S_4 & 0 & -C_4 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_5 & -S_5 & 0 & 0 \\ S_5 & C_5 & 0 & 0 \\ 0 & 0 & 1 & d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{.....6}$$

$$A_{Wrist}^{Tool} = \begin{bmatrix} C_4C_5 & -C_4S_5 & S_4 & S_4d_5 \\ S_4C_5 & -S_4S_5 & -C_4 & -C_4d_5 \\ S_5 & C_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The final expression for A_{Wrist}^{Tool} depends only on the last two joint variables.

The matrix of the Lab Volt R5150 robot arm can be achieved by multiplying the two partitioned factors together:

$$A_{Base}^{Tool} = \begin{bmatrix} C_1C_{234}C_5 + S_1S_5 & -C_1C_{234}S_5 + S_1C_5 & C_1S_{234} & C_1(a_2C_2 + a_3C_{23} + d_5S_{234}) \\ S_1S_{234}C_5 - C_1S_5 & -S_1C_{234}S_5 - C_1C_5 & S_1S_{234} & S_1(a_2C_2 + a_3C_{23} + d_5S_{234}) \\ S_{234}C_5 & -S_{234}S_5 & -C_{234} & d_1 + a_2S_2 + a_3S_{234} - d_5C_{234} \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{...7}$$

Hence the coordinate of the tool-tip position can be found according to the following expression:

$$\begin{aligned} P_X &= C_1(a_2C_2 + a_3C_{23} + d_5S_{234}) \\ P_Y &= S_1(a_2C_2 + a_3C_{23} + d_5S_{234}) \\ P_Z &= d_1 + a_2S_2 + a_3S_{23} - d_5C_{234} \end{aligned} \qquad \text{...8}$$

Where $S_{234} = sin\ (q_1 + q_2 + q_3)$, $C_{234} = cos\ (q_1 + q_2 + q_3)$

The previous forward kinematics solution can be used for modeling the position of each joint of the manipulator as it moves.

$Base = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ **, as a reference point (origin) :** $P_x = 0,\ P_y = 0,\ P_z = 0$

$Shoulder = \begin{bmatrix} 0 \\ 0 \\ d_1 \\ 1 \end{bmatrix}$ **, as shown in Figure.(3) there is no change in axes direction (no rotation) at the shoulder joint but just translation along Z-axis so :** $P_x = 0,\ P_y = 0,\ P_z = d_1$

$Elbow = \begin{bmatrix} a_2C_1C_2 \\ a_2C_2S_1 \\ d_1 + a_2S_2 \\ 1 \end{bmatrix}$ **, the position of elbow joint can be deduced by multiplying the two matrices (fourth column):**

$$A_{elbow} = A_1^0.A_2^1 = \begin{bmatrix} C_1 & 0 & S_1 & 0 \\ S_1 & 0 & -C_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_2 & -S_2 & 0 & a_2C_2 \\ S_2 & C_2 & 0 & a_2S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} C_1C_2 & -C_1S_2 & S_1 & a_2C_1C_2 \\ S_1C_2 & -S_1S_2 & -C_1 & a_2S_1C_2 \\ S_2 & C_2 & 0 & d_1 + a_2S_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Wrist = \begin{bmatrix} a_2C_1C_2 + a_3C_1C_{23} \\ a_2S_1C_2 + a_3S_1C_{23} \\ d_1 + a_2S_2 + a_3S_{23} \\ 1 \end{bmatrix}$$

, the position of wrist joint can be deduced by multiplying the three matrices (fourth column represent the joint position):

$$A_1^0.A_2^1.A_3^2 = A_{Wrist} = \begin{bmatrix} C_1 & 0 & S_1 & 0 \\ S_1 & 0 & -C_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_2 & -S_2 & 0 & a_2C_2 \\ S_2 & C_2 & 0 & a_2S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_3 & -S_3 & 0 & a_3C_3 \\ S_3 & C_3 & 0 & a_3S_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_{Wrist} = \begin{bmatrix} C_1C_{23} & -C_1S_{23} & S_1 & C_1(a_2C_2 + a_3C_{23}) \\ S_1C_{23} & -S_1S_{23} & -C_1 & S_1(a_2C_2 + a_3C_{23}) \\ S_{23} & C_{23} & 0 & d_1 + a_2S_2 + a_3S_{23} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

While tool-tip position has been deduced by multiplying the five matrices $A_1^0.A_2^1.A_3^2.A_4^3.A_5^4$ as illustrated in equation (7) (fourth column in the final matrix):

$$Tool-tip = \begin{bmatrix} C_1(a_2C_2 + a_3C_{23} + d_5S_{234}) \\ S_1(a_2C_2 + a_3C_{23} + d_5S_{234}) \\ d_1 + a_2S_2 + a_3S_{23} - d_5C_{234} \\ 1 \end{bmatrix} \qquad \text{....9}$$

The above models have been invested to simulate the positional coordinates of each joint of the robot arm while it moves to a desired target.

RESULTS AND DISCUSSION

The adopted model has been tested in real environment at university of technology /Dep. of production engineering and metallurgy for five different cases using a Lab-Volt R5150 robot arm, also the method has been tested by linking the output of the mathematical formulation with Matlab Software for simulation using robot toolbox.

The simulation algorithm includes the following three steps:

<u>Input:</u>

1- identify each link parameters (D-H) of the robot as in Table (1)

2- identify the value of each angle (q_1, q_2, q_3, q_4, q_5)

Processing:

mathematical processing according to the equation (7)

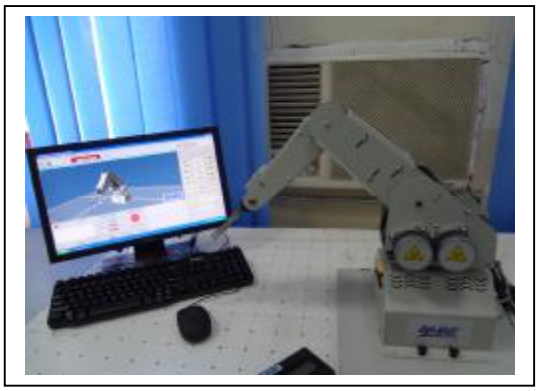
Output :

1- graphical representation of the robot as in Figure.(4) (b, d, f, h and j)

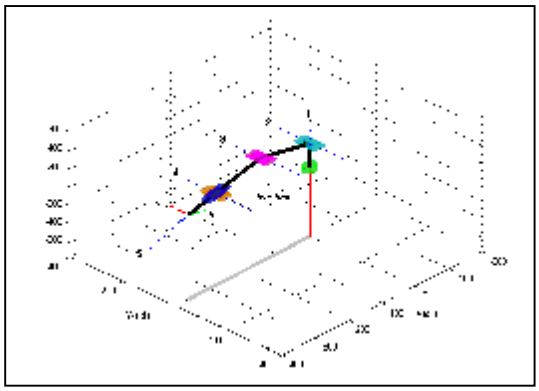
2- tool-tip position as in Table (3)

In both tests the adopted model gives good results for all the tested cases as shown in the Figure (4) and Table (3).

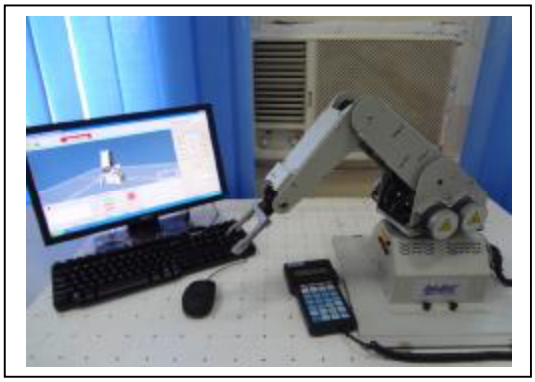
The results of comparison in five different cases as illustrated in table (3) shows that the derived solution is very accurate, it was found that the maximum error in the tool pose position do not exceed (0.001) mm. The derived analytical forward kinematics solution provides the correct joint position of the end-effector to any given reachable position.



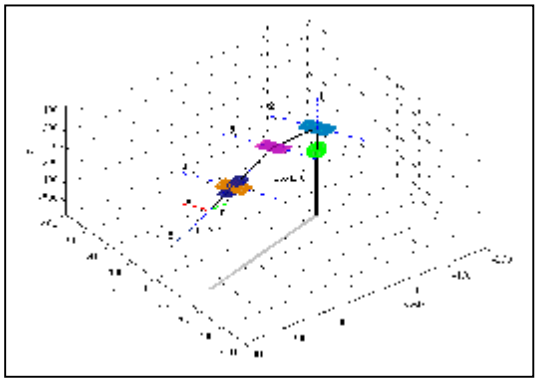
a-Real - test No.1-



b- simulated - test No.1-

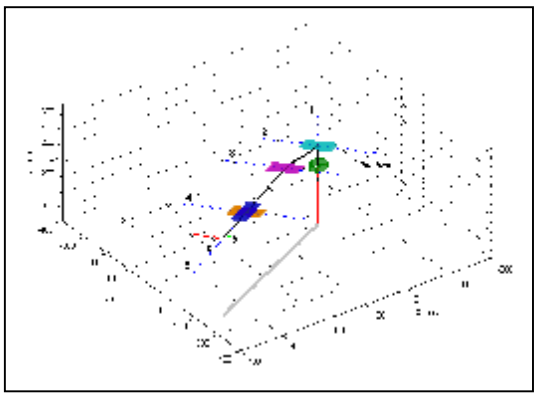


c- real - test No.2 -



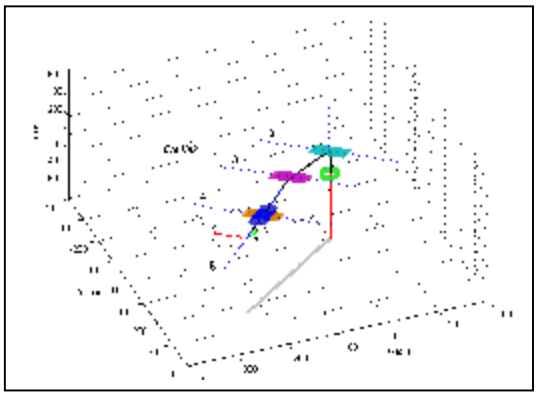
d- simulated - test No.2-

e- real - test No.3 -

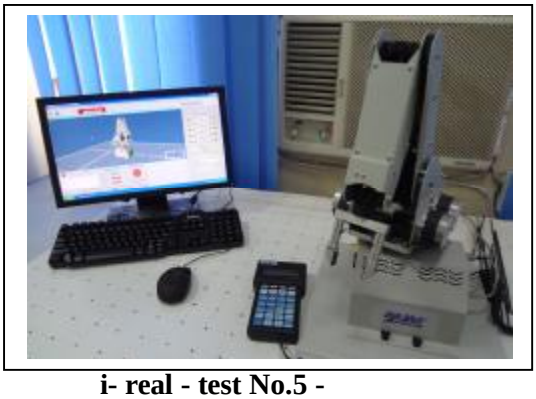


f- simulated - test No.3 –

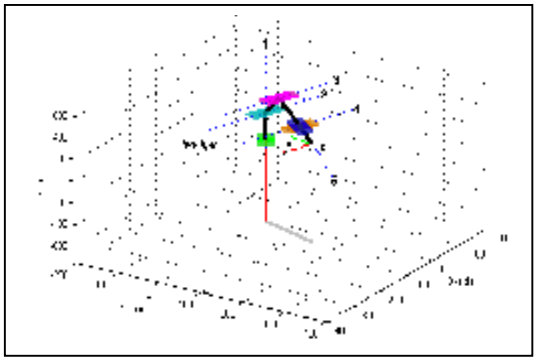
g- real - test No.4 -



h- simulated - test No.4



i- real - test No.5 -



j- simulated - test No.5 –

Figure (4) The real and the simulation of the five selected tests.

Table (3) the real and simulated positional coordinates of the robot joints.

Case	Target coord. Actual	Base	Shoulder Position	Elbow Position	Wrist Position	Tool-tip coord. Simulated	Error %
1 $q_1 = 0$ $q_2 = 40$ $q_3 = -85$ $q_4 = 90$ $q_5 = 90$	$P_x : 361.45$ $P_y : 0$ $P_z : 161.98$	0 0 0	0 0 255.5	415.548 0 377.12	279.898 0 242.779	$P_x : 361.216$ $P_y : 0$ $P_z : 162.012$	0.06% - 0,02%
2 $q_1 = 21$ $q_2 = 40$ $q_3 = -85$ $q_4 = 90$ $q_5 = 90$	$P_x : 337.45$ $P_y : 129.53$ $P_z : 161.98$	0 0 0	0 0 255.5	114.017 52.16 377.12	261.308 100.307 242.779	$P_x : 337.224$ $P_y : 129.448$ $P_z : 162.012$	0.062% 0.063% 0.018%
3 $q_1 = 33$ $q_2 = 40$ $q_3 = -67$ $q_4 = 90$ $q_5 = 90$	$P_x : 350.26$ $P_y : 227.46$ $P_z : 239.31$	0 0 0	0 0 255.5	122.067 27.27 377.12	364.046 171.474 272.223	$P_x : 350.98$ $P_y : 227.281$ $P_z : 239.212$	0.08% 0.08% 0.04%
4 $q_1 = 45$ $q_2 = 35$ $q_3 = -90$ $q_4 = 90$ $q_5 = 90$	$P_x : 233.89$ $P_y : 233.89$ $P_z : 114.57$	0 0 0	0 0 255.5	110.05 110.05 389.35	187.113 187.113 233.711	$P_x : 233.755$ $P_y : 233.755$ $P_z : 114.688$	0.06% 0.06% 0.1%
5 $q_1 = 72$ $q_2 = 77$ $q_3 = -147$ $q_4 = 90$ $q_5 = 90$	$P_x : 45,38$ $P_y : 139.65$ $P_z : 154.05$	0 0 0	0 0 255.5	13.207 40.648 440.13	32,289 107.452 157.159	$P_x : 45,445$ $P_y : 139.859$ $P_z : 154.074$	0.13% 0.14% 0.01%

CONCLUSIONS

A complete analytical solution to the forward kinematics of the 5 DOF LabVolt robots is derived in this paper by partitioning the kinematics problem into two sub-problems at the wrist joint which greatly simplified the solution of the robot arm. An analytical formulation for determining the position of each joint of a 5 DOF articulated robot was presented, the results have been linked with the Matlab 2010a software to simulate the robot joint coordinates as the robot moves from one position to another, as it can be clearly seen from the formulation and the results that the roll angle (q_5) has no effect on the final position of the tool pose, but it effects the orientation of the end effector.

The results of comparison in five different cases as illustrated in table (3) shows that the adopted model is very effective, efficient and accurate, it was found that the maximum error in the tool pose position do not exceed (0.001) mm. We believe that the solution developed in this paper will make the 5 DOF LabVolt robot more useful in applications .

REFERENCES

[1].Pasan Kulvanit," Implementation of resolved motion rate controller with 5-axis robot manipulator", International Conference of Mechanical Engineering, 2010.

[2].Shan Junfeng, Zhang Shangxing and Zhu Xuejun," Analysis of Motion Control with Trajectory Planning for HP20 Manipulator", Third International Conference on Measuring Technology and Mechatronics Automation, 2010.

[3].Yang, J, Yu, W., Kim ,J and Malek ,K.A.," On placement of open loop robotic manipulator for reachability ", Journal of Mechanism and Machine Theory, Vol.44, pp671 - 684 , 2009.

[4]. Lang, Bi, Z.M., S.Y.T.,"Joint workspace of parallel kinematic machines ", Journal of Robotics and Computer Integrated Manufacturing, Vol.25, pp 57-63, 2009.

[5].Baki koyunen and Mehmet Guzel, " Software development for the kinematic analysis of a Lynx 6 Robot Arm ", World Academy of Science , Engineering and Technology. Vol.30, pp 252-257, 2007.

[6].Gürdal, M. Albayrak and T. Aydogan, "Computer Aided Control and Simulation of Robot Arm Moving in Three Dimension", Suleiman Demirel University.

[7]. Annand, S. "Software for Control and Dynamic Simulation of Unimate Puma 560 Robot",MS Thesis, Ohio University, 1993.

[8]. Abu Qassem, M. I. Abuhadrous, and H. Elaydi, "Modeling and Simulation of 5 DOF Educational Robot Arm". IEEE, 2010.

[9]. Manjunath, T.C. "Kinematic modeling and maneuvering of 5-axis articulated robot arm", World Academy of Science, Engineering and Technology. Vol.28, pp 364-370, 2007.

[10].Brandi House, Jonathan Malkin and Jeff Bilmes." The voice bot: A Voice Controlled Robot Arm ", ACME, 2009.

[11].Yujie Cui, Pu Shi, and Jianning Rua. "Kinematics Analysis and Simulation of a 6-DOF Humanoid Robot Manipulator", 2nd International Asia Conference on Informatics in Control, Automation and Robotics, 2010.