Absolute Maximum Bending Moment in Short Simply Supported Beams under Moving Point Loads

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ABSTRACT

This paper represents a theoretical investigation of the location of the absolute maximum bending moment in short simply supported beams under the influence of several moving point loads. However, for decades it was considered that the absolute maximum bending moment is determined by positioning the beam center-line mid-way between the resultant of the loads and the nearer heavy load. Obviously, other point loads may also be required to be mirror imaged with the resultant to ensure that the obtained maximum bending moment is an absolute maximum. This method was assumed to be working for all spans of simply supported beams without any limitations.

In this paper it is shown that this traditional method is not always valid. Examples of short span beams subjected to moving point loads having distance between two loads exceeding half the length of the beam indicate absolute maximum bending moments greater than those obtained by the traditional method.

Keywords: Absolute maximum bending moment; Short simply supported beams; Moving point loads; location of maximum moment.

عزم الإنحناءالأقصى المطلق في العتبات الخرسانية ذات الإسناد البسيط تحت تأثير احمال مركزة متحركة

الخلاصة

هذا البحث يقدم تحليلا نظريا لموقع عزم الإنحناء الأقصى المطلق في العتبات الخرسانية القصيرة ذات الإسناد البسيط تحت تأثير مجموعة من الأحمال المركزة المتحركة. منذ عقود كان الإعتقاد السائد ان عزم الانحناء الأقصى المطلق يتم احتسابه من خلال جعل الخط الوسطي للعتبة في وسط المسافة بين محصلة القوى للاحمال واقرب حمل ثقيل لها. وقد يتطلب الأمر تكرار مثل هذه المناظرة لمحصلة الأحمال مع احمال اخرى لضمان الحصول على العزم الأقصى المطلق.

هذه الطريقة كان من المفترض ان تعمل لكل العتبات ذات الإسناد البسيط وبمختلف الأطوال وبدون محددات.

في هذا البحث تم توضيح ان الطريقة التقليدية قد لا تصح دائما. امثلة من عتبات ذات اطوال قصيرة معرضة لاحمال متحركة فيها المسافة بين حملين منها يزيد على نصف طول العتبة تعطي عزم انحناء اقصى مطلق اعلى من العزم المحتسب بالطريقة التقليدية.

NOTATIONS

M_{abs} : Absolute Maximum Bending Moment

M_{CL} : Moment at Mid-span of the Beam

R : P_1/P_2, (r>1)

P_1 : Heaviest Moving Point Load

P_2 : Load adjacent to Heaviest Moving Point Load

P_{l1}, P_{l2}, P_{li} : Other Moving Point Loads

x : Distance between Heaviest Load and Resultant

R : Resultant of moving point loads

L : Length of Beam

d, d_{l1}, d_{l2}, d_{li} : Distances between Point Loads and Heaviest Load

d_{cr} : Critical Distance between moving point loads

d_{out} : Minimum Distance that keeps P_2 within the vicinity of the beam

w : d/L

w_{cr} : d_{cr}/L

w_{out} : d_{out}/L

S : R/P_2

S_1 : $\sum P_{li}d_{li}/P_2L$

S_2 : $\sum P_{li}/P_2$

INTRODUCTION

For simply supported beams the critical positions of the loading and the associated absolute maximum bending moment cannot, in general, be determined by inspection.

Russell C. Hibbeler [1] concluded that the critical position of the absolute maximum moment in a simply supported beam occurs under one of the concentrated forces such that this force is positioned on the beam so that it and the resultant force of the system are equidistant from the beam's center line. As a general rule, the absolute maximum moment often occurs under the highest force lying nearest to the resultant force of the system.

W. R. Spillers [2] developed a simple result which can be useful in determining how loads should be positioned in order to produce maximum bending moment. He selected one load of a group of moving loads and found the location of the load group so that the moment under that load is considered to be the maximum. For maximum effect, the load group should be positioned so that the distance from the centroid of the load group to the right support equals the distance from the left support to the load under consideration.

R. L. Jindal [3] considered that the moment at the center is generally not the absolute maximum bending moment and in short beams the difference is appreciable. The maximum moment under any load will occur when that load and the center of gravity of all the loads are equidistant from the center of the beam.

Research Significance

The present theoretical investigation examines some limitations in the calculation and location of the absolute maximum bending moment in short simply supported beams; since most of the moving loads in Iraq are due to vehicles moving on roads, and the main vehicle characteristics are limited by the state or highway authorities to suit both transportation services and highway network conditions. It states that the maximum distance between wheels in a semitrailer combination is limited to (7.93 m)[4], therefore, the term short beam in this research will refer to beams with spans less than (15 m).

THEORETICAL INVESTIGATION

Two types of loadings are considered in this research, two point loads and more than two point loads.

Simply Supported Beams Under The Influence of Two Moving Point Loads

A simply supported beam of length (L) loaded with two moving point loads (P_1) and (P_2) where (P_1) is the heaviest load; by applying the traditional absolute maximum bending moment procedure, the resultant force would be ($\boldsymbol{R} = \boldsymbol{P_1} + \boldsymbol{P_2}$) and the location of the absolute maximum moment would be under one of the point loads such that this force is positioned on the beam so that it and the resultant force of the system are equidistant from the beam's mid-span, as shown in Figure (1).

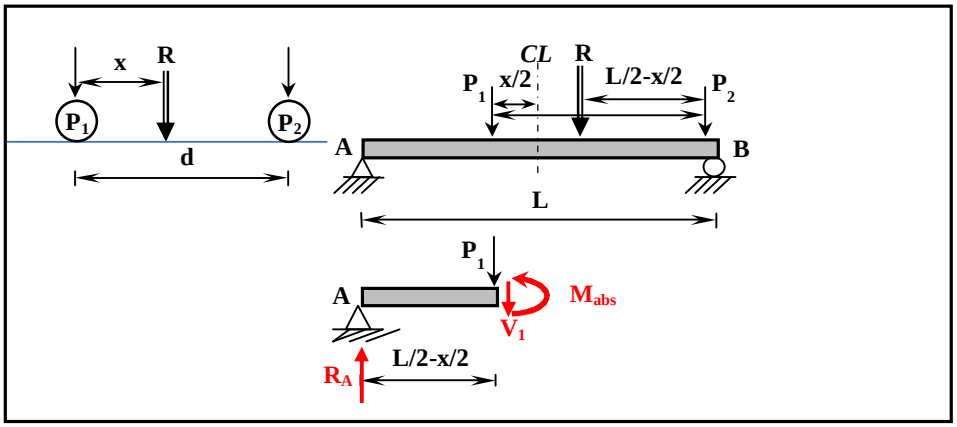


Figure (1) – A Simply Supported Beam Loaded with Two Point Loads with (d>L/2).

$$R = P_1 + P_2 \;\; and \;\; M_1 = P_2 d$$

$$x = \frac{M_1}{R} = \frac{P_2 d}{R}$$

$$R_A = \frac{R}{L}\left(\frac{L}{2} - \frac{x}{2}\right) = \frac{R}{L}\left(\frac{L-x}{2}\right)$$

$$M_{abs} = \frac{R}{4L}(L-x)^2$$

If ($d>L/2$) then the moment at the center of the beam due to (P_1) would be: $M_{CL}=P_1L/4$.

The relationship between M_{abs} and M_{CL} would be:

$$\frac{M_{abs}}{M_{CL}} = \frac{\frac{R}{4L}(L-x)^2}{\frac{P_1 L}{4}} \Rightarrow \frac{M_{abs}}{M_{CL}} = \frac{r+1}{r}\left(1 - \frac{w}{r+1}\right)^2$$

Where: $r = \frac{P_1}{P_2}$, $w = \frac{d}{L}$ and $r \geq 1$,

The relationship between (M_{abs}/M_{CL}) and (w) is shown in (Fig. 2); the error area in the traditional absolute maximum bending moment procedure lies under the line

$(M_{abs}/M_{CL} = 1)$.

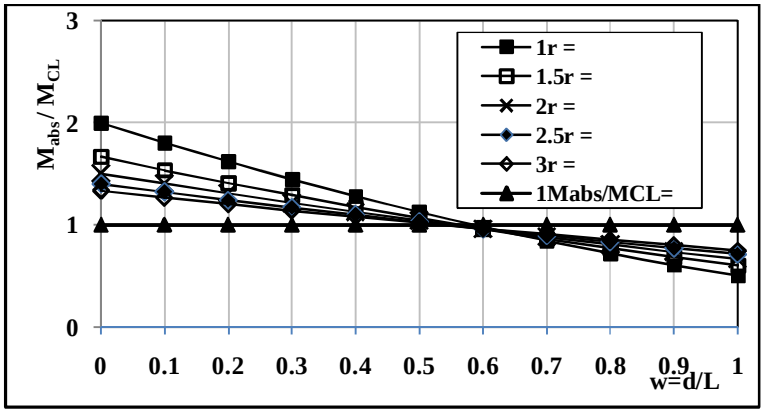


Figure (2) – The Relationship between (M_{abs}/M_{CL}) and (w).

The critical distance (d_{cr}) between the two moving point loads can be obtained by substituting $M_{abs} = M_{CL}$

$$\frac{R}{4L}(L - x)^2 = \frac{P_1 L}{4}$$

Then by substituting, ($R = P_1 + P_2$) and ($x = \frac{P_2 d}{R}$) the following is obtained:

$$\frac{L(P_1 + P_2)}{4} - \frac{P_2 \times d}{2} + \frac{P_2^2 \times d^2}{4L(P_1 + P_2)} = \frac{P_1 \times L}{4}$$

$$d = \left(\frac{(P_1 + P_2) \pm \sqrt{(P_1 + P_2) \times P_1}}{P_2} \right) \times L$$

Since the minimum d is required, and R is much greater than P_2, then the positive sign will be ignored and d_{cr} would be:

$$d_{cr} = \left(\frac{(P_1 + P_2) - \sqrt{(P_1 + P_2) \times P_1}}{P_2} \right) \times L$$

$$d_{cr} = \sqrt{r+1}\left(\sqrt{r+1} - \sqrt{r}\right) \qquad \text{..... (1)}$$

$$w_{cr} = \frac{d_{cr}}{L} = \sqrt{r+1} \times \left(\sqrt{r+1} - \sqrt{r}\right) \qquad \text{..... (2)}$$

Any distance between the two point loads larger than the amount calculated by Eq. (1), (d_{cr}), leads to a failure in the traditional absolute maximum moment procedure and the maximum moment location would be at the middle of the beam under the influence of the heaviest point loads (P_1).

If the distance ($d > \frac{L}{2} + \frac{x}{2}$), then P_2 will be positioned outside the beam, therefore the minimum distance (d_{out}) that keeps (P_2) within the vicinity of the beam would be:

$$d_{out} = \frac{r+1}{2r+1} \times L$$

$$w_{out} = \frac{d_{out}}{L} = \frac{r+1}{2r+1} \qquad \text{..... (3)}$$

$$w_{cr} < \frac{d}{L} < w_{out}$$

$$\sqrt{r+1} \times \left(\sqrt{r+1} - \sqrt{r}\right) < \frac{d}{L} < \frac{r+1}{2r+1}$$

The distance between the moving point loads with relative to the length of the beam depends on the value of (r) which is always greater than or equal to (1).

$$w_c = f(r) = \sqrt{r+1} \times \left(\sqrt{r+1} - \sqrt{r}\right), \; r \geq 1$$

If ($r = 1$) then:

$$f(r) = 0.5858$$

If ($r \rightarrow \infty$) then:

$$Lim_{(r \rightarrow \infty)} \; f(r) = 0.5$$

This relationship is shown in (Figure 3).

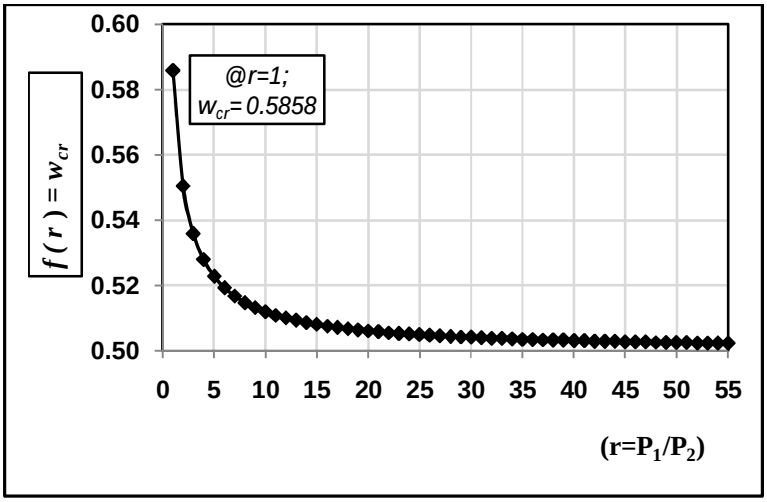


Figure (3) – The Relationship between ($r=P_1/P_2$) and ($f(r) = w_{cr}$)

According to the above function, in the case of two moving point loads if the distance between the two point loads (*d*) accedes *0.5L*, no matter what the values of (P_1) and (P_2) are; if ($w_{cr}<w< w_{out}$), then the traditional absolute maximum moment procedure is invalid and the maximum moment occurs when the heaviest point load is positioned at mid-span.

Figure (4) shows the validity of the traditional absolute maximum moment procedure for a simply supported beam with regard to w, w_{cr} and w_{out}.

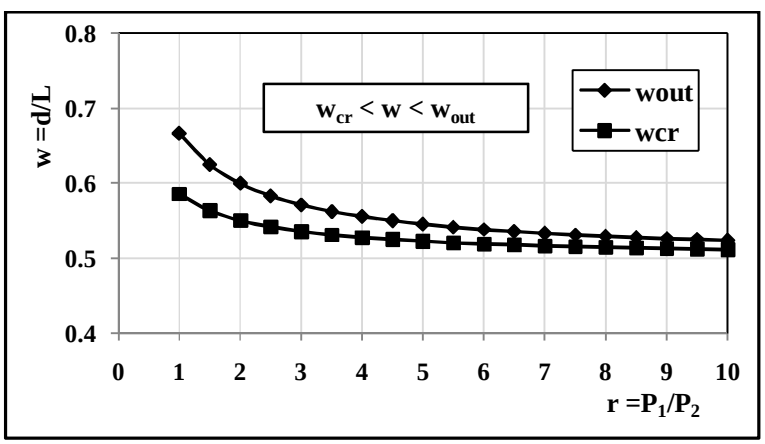


Figure (4) – The Validity of the Traditional Absolute Maximum Moment Procedure for a Simply Supported Beam.

SIMPLY SUPPORTED BEAMS UNDER THE INFLUENCE OF MORE THAN TWO MOVING POINT LOADS

For a simply supported beam of length (L) under the influence of more than two moving point loads (P_1, P_2, P_{l1}, P_{l2},….,P_{li}), three cases must be considered regarding the location of the heaviest moving point load, these cases are:

a) If the heaviest load is in the middle, then the traditional absolute maximum moment procedure may be applied to calculate the maximum bending moment.

b) If the heaviest load is the second load, then the resultant would be nearer the heaviest load; by applying the traditional absolute maximum moment procedure, the resultant force would be ($\boldsymbol{R} = \boldsymbol{P_1} + \boldsymbol{P_2} + \sum \boldsymbol{P_{li}}$) and the location of the absolute maximum moment would be under the heaviest closer point load ($\boldsymbol{P_1}$) such that this force is positioned on the beam so that it and the resultant force of the system are equidistant from the beam's mid-span, as shown in (Figure 5).

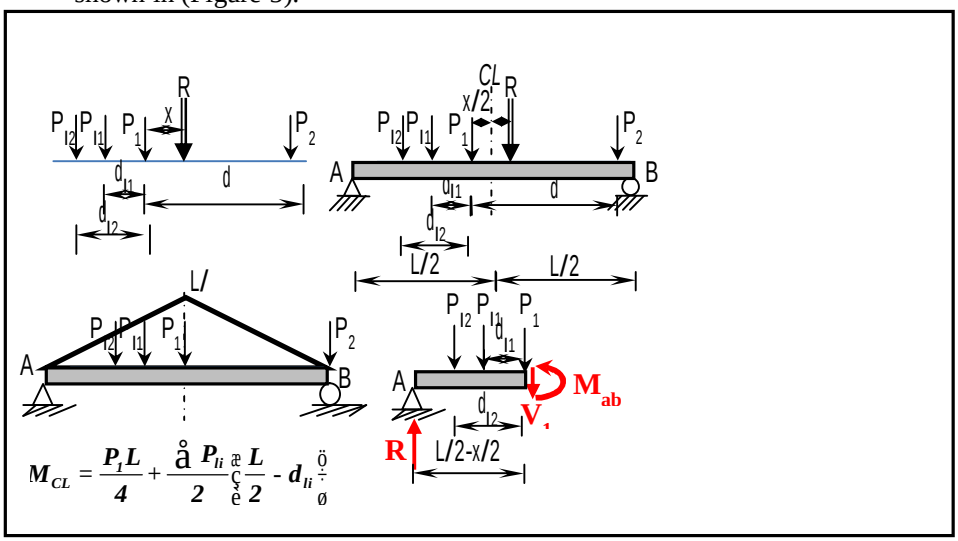


Figure (5) - A Simply Supported Beam Loaded with more than Two Point Loads, the 2nd Load is the Heaviest.

$$R = P_1 + P_2 + \sum P_{li}$$

$$M = P_2 d - \sum P_{li} \boldsymbol{d_{li}}$$

$$x = \frac{M}{R} = \frac{P_2 d - \sum P_{li} \boldsymbol{d_{li}}}{\boldsymbol{R}}$$

$$M_{abs} = \frac{R}{4L}(L - x)^2 - \sum P_{li} d_{li}$$

But:

$$M_{CL} = \frac{P_1 L}{4} + \frac{\sum P_{li}}{2}\left(\frac{L}{2} - d_{li}\right)$$

Then by substituting $M_{abs}=M_{CL}$

$$\left(\frac{d}{L}\right)^2 - 2(S + S_1)\left(\frac{d}{L}\right) + \left(S^2 + S_1^{\,2} - Sr - SS_2\right) = 0$$

$$\left(\frac{d}{L}\right) = w_{cr} = (S + S_1) - \sqrt{S(2S_1 + S_2 + r)} \qquad \text{.....(4)}$$

Where:

$$S = \frac{R}{P_2},\ S_1 = \frac{\sum P_{li} d_{li}}{P_2 L},\ S_2 = \frac{\sum P_{li}}{P_2} \ \ and\ r = \frac{P_1}{P_2}$$

To calculate $w_{out}=d_{out}/L$, substitute $d=(L/2+x/2)$

$$d_{out} = \frac{L}{2} + \frac{x}{2}$$

$$\therefore \frac{d_{out}}{L} = w_{out} = \frac{S - S_1}{2S - 1} \qquad \text{..... (5)}$$

So if ($w_{cr} < w = d/L < w_{out}$) , then the traditional absolute maximum moment procedure does not work and the maximum moment occurs when the heaviest load is positioned at the center of the beam.

c) If the heaviest load is the first load, then two cases must be tested:

– *When x<d/2:*
In this case the resultant of the system would be closer to the heaviest load. By applying the traditional absolute maximum moment procedure, the resultant force would be:

($R = P_1 + P_2 + \sum P_{li}$) and the location of the absolute maximum moment would be under the heaviest point load (P_1) such that this force is positioned on the beam so

that it and the resultant force of the system are equidistant from the beam's mid-span, as shown in Figure (6).

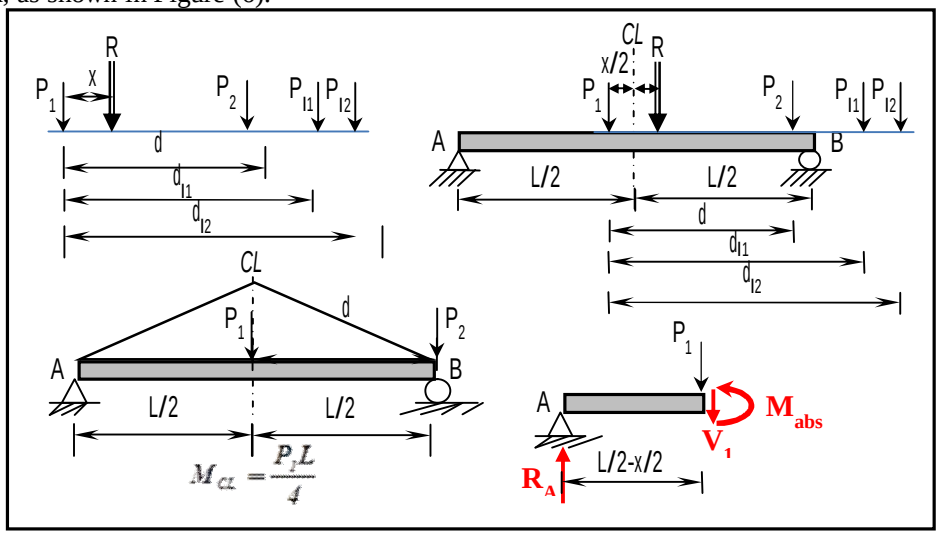


Figure (6) - A Simply Supported Beam Loaded with more than Two Point Loads the 1st Load is Heaviest - Case1 (x<d/2).

$$R = P_1 + P_2 + \sum P_{li}$$

$$M = P_2 d + \sum P_{li} d_{li}$$

$$x = \frac{M}{R} = \frac{P_2 d + \sum P_{li} d_{li}}{R}$$

$$M_{abs} = \frac{R}{4L}(L - x)^2, \quad M_{CL} = \frac{P_1 L}{4}$$

Then by substituting $M_{abs}=M_{CL}$

$$\frac{R}{4L}(L - x)^2 = \frac{P_1 L}{4}$$

$$\begin{cases} \left(\frac{d}{L}\right)^2 - 2(S - S_1)\left(\frac{d}{L}\right) + \\ (S - S_1)^2 - Sr = 0 \end{cases}$$

$$\frac{d}{L} = w_{cr} = (S - S_1) - \sqrt{Sr} \qquad \text{.....(6)}$$

Where:

$$S = \frac{R}{P_2},\ S_1 = \frac{\sum P_{li} d_{li}}{P_2 L} \text{ and } r = \frac{P_1}{P_2}$$

To calculate $w_{out}=d_{out}/L$, substitute $d=(L/2+x/2)$

$$d_{out} = \frac{L}{2} + \frac{x}{2}$$

$$\therefore \frac{d_{out}}{L} = w_{out} = \frac{S + S_1}{2S - 1} \qquad \text{..... (7)}$$

So if ($w_{cr} < d/L < w_{out}$) , then the traditional absolute maximum moment procedure does not work and the maximum moment occurs when the heaviest load is positioned at the center of the beam.

– *When x>d/2:*

In this case the resultant of the system would be closer to the second load (not the heaviest load), and that happen when the value of the second load is approaching the value of the heaviest load; by applying the traditional absolute maximum moment procedure, the resultant force would be: ($R = P_1 + P_2 + \sum P_{li}$) and the location of the absolute maximum moment would be either under the closer point load (P_2) or under the farther point load (P_1) whichever the heaviest as shown below:

a) The closer point load (P_2) is positioned on the beam such that this force and the resultant force of the system are equidistant from the beam's mid-span, as shown in (Fig.7).

$$R = P_1 + P_2 + \sum P_{li}$$

$$M_{(P_1)} = P_2 d + \sum P_{li} d_{li}$$

$$x = \frac{M}{R} = \frac{P_2 d + \sum P_{li} d_{li}}{R}$$

$$R_B = \frac{R}{L}\left(\frac{L}{2} - \frac{d - x}{2}\right)$$

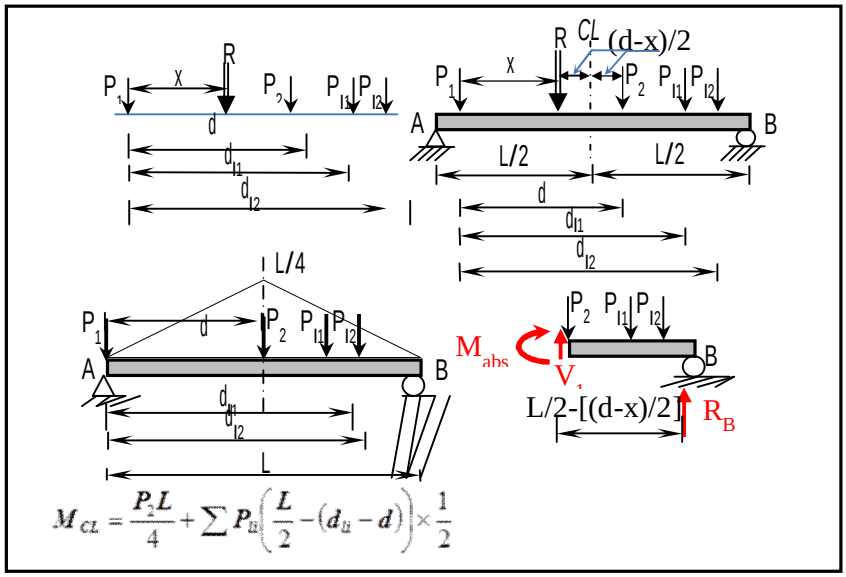


Figure (7) - A Simply Supported Beam Loaded with more than Two Point Loads the 1st Load is Heaviest -Case2-a (x>d/2).

$$M_{abs} = \frac{R}{4L}(L - d + x)^2 - \sum P_{Ii}(d_{Ii} - d) \qquad \text{.....(8)}$$

$$M_{CL} = \frac{P_2 L}{4} + \frac{\sum P_{Ii}}{2}\left(\frac{L}{2} - (d_{Ii} - d)\right)$$

Then by substituting $M_{abs}=M_{CL}$

$$\frac{R}{4L}(L - d + x)^2 - \sum P_{Ii}(d_{Ii} - d) = \frac{P_2 L}{4} + \sum P_{Ii}\left(\frac{L}{2} - (d_{Ii} - d)\right) \times \frac{1}{2}$$

$$(S-1)^2\left(\frac{d}{L}\right)^2 - 2\begin{bmatrix}(S-1)(S+S_1)\\ -SS_2\end{bmatrix}\left(\frac{d}{L}\right) + \left(S^2 + S_1^{\,2} - S(1+S_2)\right) = 0$$

$$\frac{d}{L} = w_{cr} = \frac{(S+S_1)}{(S-1)} - \frac{SS_2}{(S-1)^2} - \sqrt{\frac{S(2S_1+S_2+1)}{(S-1)^2} - \frac{2SS_2(S+S_1)}{(S-1)^3} + \frac{S^2 S_2^{\,2}}{(S-1)^4}} \qquad \text{... (9)}$$

Where:

$$S=\frac{R}{P_2},\ S_1=\frac{\sum P_{li}d_{li}}{P_2L},\ S_2=\frac{\sum P_{li}}{P_2}\ \ and\ r=\frac{P_1}{P_2}$$

To calculate $w_{out}=d_{out}/L$, substitute d=[L/2+(d-x)/2]

$$d_{out}=\frac{L+d-x}{2}$$

$$\frac{d_{out}}{L}=w_{out}=\frac{(S-S_1)}{(S+1)} \qquad \text{.....(10)}$$

So if ($w_{cr} < d/L < w_{out}$) , then the traditional absolute maximum moment procedure does not work and the maximum moment occurs when (P_2) is positioned at the center of the beam.

b) The farther point load (P_1) is positioned on the beam such that this force and the resultant force of the system are equidistant from the beam's mid-span, as shown in (Figure 8).

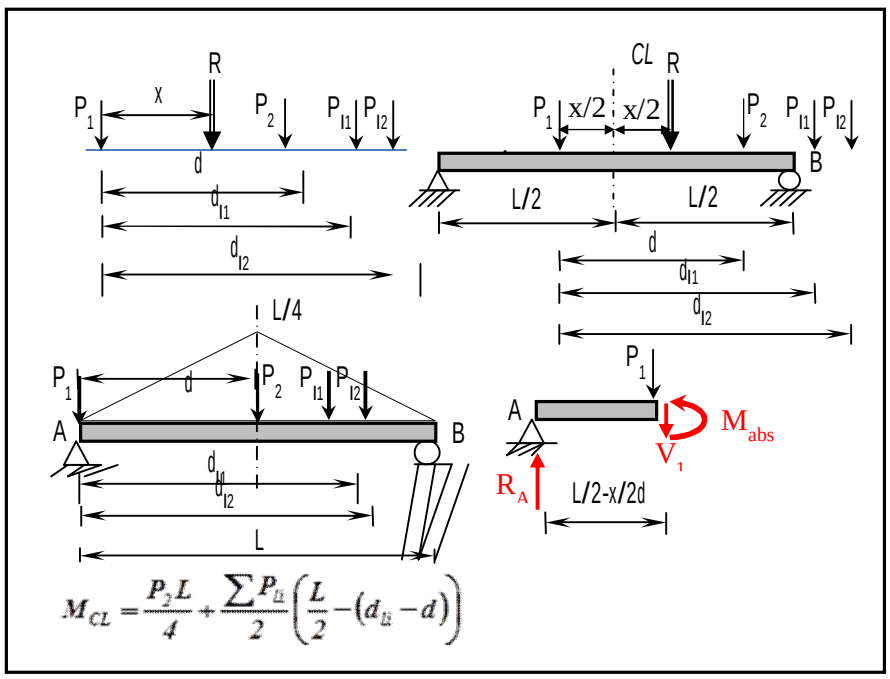


Figure (8) - A Simply Supported Beam Loaded with more than Two Point Loads the 1st Load is Heaviest-Case2-b (x>d/2).

$$R = P_1 + P_2 + \sum P_{li}$$

$$M = P_2 d + \sum P_{li} \boldsymbol{d_{li}}$$

$$x = \frac{M}{R} = \frac{P_2 d + \sum P_{li} \boldsymbol{d_{li}}}{\boldsymbol{R}}$$

$$\boldsymbol{R_A} = \frac{\boldsymbol{R}}{\boldsymbol{L}}\left(\frac{\boldsymbol{L}}{2} - \frac{\boldsymbol{x}}{2}\right)$$

$$\boldsymbol{M_{abs}} = \frac{\boldsymbol{R}}{\boldsymbol{4L}}(\boldsymbol{L}-\boldsymbol{x})^2 \qquad \textbf{.....(11)}$$

$$\boldsymbol{M_{CL}} = \frac{\boldsymbol{P_2 L}}{\boldsymbol{4}} + \sum \boldsymbol{P_{li}}\left(\frac{\boldsymbol{L}}{\boldsymbol{2}} - (\boldsymbol{d_{li}} - \boldsymbol{d})\right) \times \frac{\boldsymbol{1}}{\boldsymbol{2}}$$

Then by substituting $M_{abs}=M_{CL}$

$$\frac{\boldsymbol{R}}{\boldsymbol{4L}}(\boldsymbol{L}-\boldsymbol{x})^2 = \frac{\boldsymbol{P_2 L}}{\boldsymbol{4}} + \frac{\sum \boldsymbol{P_{li}}}{\boldsymbol{2}}\left(\frac{\boldsymbol{L}}{\boldsymbol{2}} - (\boldsymbol{d_{li}} - \boldsymbol{d})\right)$$

$$\left(\frac{\boldsymbol{d}}{\boldsymbol{L}}\right)^2 - \boldsymbol{2}\left[\boldsymbol{S}(\boldsymbol{S_2}+\boldsymbol{1}) - \boldsymbol{S_1}\right]\left(\frac{\boldsymbol{d}}{\boldsymbol{L}}\right) + \left[\left(\boldsymbol{S}^2 + \boldsymbol{S_1}^2\right) - \boldsymbol{S}(\boldsymbol{S_2}+\boldsymbol{1})\right] = \boldsymbol{0}$$

$$\frac{\boldsymbol{d}}{\boldsymbol{L}} = \boldsymbol{w_{cr}} = \boldsymbol{S}(\boldsymbol{S_2}+\boldsymbol{1}) - \boldsymbol{S_1} - \sqrt{\begin{matrix}\boldsymbol{S^2 S_2}(\boldsymbol{S_2}+\boldsymbol{2}) \\ -\boldsymbol{S}(\boldsymbol{S_2}+\boldsymbol{1})(\boldsymbol{2S_1}-\boldsymbol{1})\end{matrix}} \qquad \textbf{...(12)}$$

Where:

$$\boldsymbol{S} = \frac{\boldsymbol{R}}{\boldsymbol{P_2}},\ \boldsymbol{S_1} = \frac{\sum \boldsymbol{P_{li} d_{li}}}{\boldsymbol{P_2 L}}\ \textbf{and}\ \boldsymbol{S_2} = \frac{\sum \boldsymbol{P_{li}}}{\boldsymbol{P_2}}$$

To calculate $w_{out}=d_{out}/L$, substitute d=[L/2+x/2]

$$\frac{\boldsymbol{d_{out}}}{\boldsymbol{L}} = \frac{\boldsymbol{S}+\boldsymbol{S_1}}{\boldsymbol{2S}-\boldsymbol{1}} \qquad \textbf{... (13)}$$

So if ($w_{cr} < d/L < w_{out}$) , then the traditional absolute maximum moment procedure does not work and the maximum moment occurs when the second point load is positioned at the center of the beam.

CONCLUSIONS AND RECOMMENDATIONS

From the theoretical investigation done in this research the following can be concluded:

1) For short simply supported beams subjected to two point loads, if the distance between these two loads is more than half the length of the beam, then the maximum bending moment occurs when the heaviest load is positioned at mid-span.

2) For such beams (w_{cr}) and (w_{out}) must be calculated according to Eq. (2) and Eq. (3) respectively. If ($w_{cr}<w<w_{out}$), then the traditional absolute maximum moment procedure is invalid and the absolute maximum moment occurs when the heaviest load is positioned at mid-span.

3) For short simply supported beams subjected to more than two point loads, and the distance between the heaviest load and the load adjacent to is more than half the length of the beam, then three cases must be checked:

   a) If the heaviest load occurs in the middle of the moving loads, then the traditional absolute maximum bending moment procedure may be applied to calculate the amount and location of the absolute maximum bending moment.

   b) If the heaviest load is the second load from either side of the beam, then (w_{cr}) and (w_{out}) must be calculated according to Eq.(4) and Eq.(5) respectively, to check the validity of the traditional absolute maximum bending moment procedure.
   If ($w_{cr}<w<w_{out}$), then the traditional absolute maximum bending moment procedure would be invalid and the absolute maximum moment occurs when the heaviest load is positioned at mid-span.

   c) If the heaviest load is the first load from either side of the beam, then two cases must be checked:

   – If the distance between the first load and the resultant of system of moving loads (x) is less than half the distance between the first two loads ($x<d/2$),

then (w_{cr}) and (w_{out}) must be calculated according to Eq.(6) and Eq.(7) respectively, to check the validity of the traditional absolute maximum bending moment procedure. If ($w_{cr}<w<w_{out}$), then the absolute max moment theory would be invalid and the maximum moment occurs when the heaviest load is positioned at mid-span.

– If the distance between the first load and the resultant of moving loads (x) is more than half the distance between the first two loads (x>d/2), the resultant would be close to the second load (not the heaviest one), then two cases must be checked for the absolute maximum moment location:

* Case-1: When the resultant is positioned on the beam such that it and the closest load (not the heaviest one) are equidistant from the beam's mid-span, in this case M_{abs} must be calculated according to Eq. (8).

* Case-2: When the resultant is positioned on the beam such that it and the farther load (the heaviest one) are equidistant from the beam's mid-span, M_{abs} must be calculated according to Eq. (11).

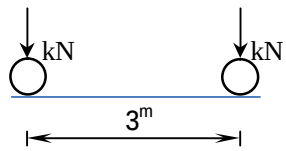
If the moment in case-1 is larger than the moment in case-2, then case-1 would be dominating and (w_{cr}) and (w_{out}) must be calculated according **to** Eq. (9) and Eq. (10) respectively, to check the validity of the traditional absolute maximum bending moment procedure. If ($w_{cr}<w<w_{out}$), then the traditional absolute maximum bending moment procedure would be invalid and the maximum moment occurs when the second point load (P_2) is positioned at mid-span. Otherwise case-2 would be the dominating case, then (w_{cr}) and (w_{out}) must be calculated according to Eq. (12) and Eq. (13) respectively, to check the validity of the traditional absolute maximum bending moment procedure. If ($w_{cr}<w<w_{out}$), then the traditional absolute maximum bending moment procedure would be invalid and the maximum moment occurs when the second point load (P_2) is positioned at mid-span.

4) In short simply supported beams, designers must check the correct location and amount for the absolute maximum bending moment, following the cases mentioned in this research; the highest moment must be taken into consideration for the design.

APPLICATIONS

1) Example on short simply supported beam subjected to two point loads:

– Calculate the location and amount of the absolute maximum bending moment in the simply supported beam of 5m length subjected to the moving loads shown in the figure,belov:



Upon applying the method proposed in this research according to Eq.(2) and Eq.(3), the following is found:

$$r = \frac{P_1}{P_2} = \frac{30}{20} = 1.5$$

$$w = \frac{d}{L} = \frac{3}{5} = 0.6$$

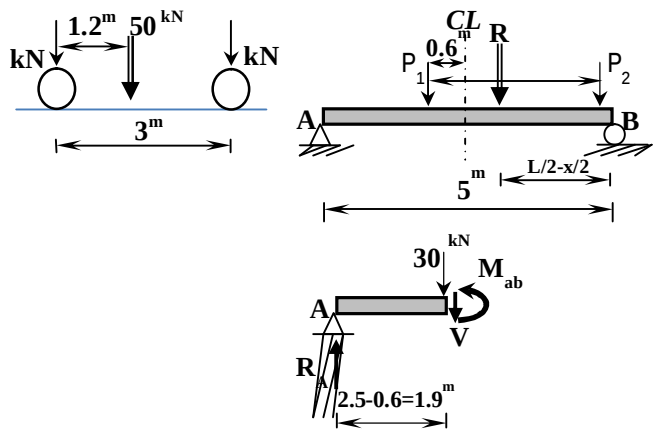
$$w_{cr} = \sqrt{r+1} \cdot \left(\sqrt{r+1} - \sqrt{r}\right) = 0.625$$

$$w_{out} = \frac{r+1}{2r+1} = 0.625$$

Since ($w_{cr} < w < w_{out}$), then the traditional absolute maximum moment procedure is invalid, therefore the maximum bending moment occurs when the heaviest load is positioned at mid-span:

$$M_{max} = \frac{PL}{4} = \frac{(30)(5)}{4} = 37.5^{kN.m}$$

To check that, below is the calculation of moment according to the traditional absolute maximum bending moment procedure:



$$R = 30 + 20 = 50\,kN$$

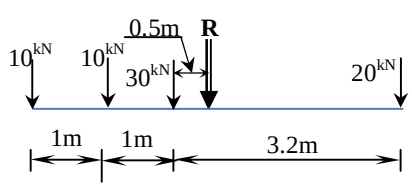
$$M = 20 \times 3 = 60\,kN.m$$

$$x = \frac{60}{50} = 1.2\,m\,,\quad \frac{x}{2} = 0.6\,m$$

$$M_{abs} = \frac{R}{L} \times \left(\frac{L}{2} - \frac{x}{2}\right)^2 = 36.1\ kN.m$$

2) Examples on short simply supported beam subjected to more than two moving point loads:

a) Calculate the location and amount of the maximum bending moment in the simply supported beam of 6m length subjected to the moving loads shown in figure.



The second load from the right of the moving loads given is the heaviest one; upon applying the method proposed in this research using Eq. (4) and Eq. (5) the following is found:

$$R = 10 + 10 + 30 + 20 = 70\,kN$$

$$M_{(P_1 = 30^{kN})} = 34\,kN.m$$

$$x = \frac{34}{70} = 0.5m, \quad \frac{x}{2} = 0.25m$$

$$w = \frac{d}{L} = \frac{3.2}{6} = 0.53,$$

$$S = \frac{70}{20} = 3.5, \quad r = \frac{30}{20} = 1.5$$

$$S_1 = \frac{(10)(2)+(10)(1)}{(20)(6)} = 0.25, \quad S_2 = \frac{(10+10)}{20} = 1$$

$$w_{cr} = (S + S_1) - \sqrt{S(2S_1 + S_2 + r)} = 0.51$$

$$w_{out} = \frac{S - S_1}{2S - 1} = 0.54$$

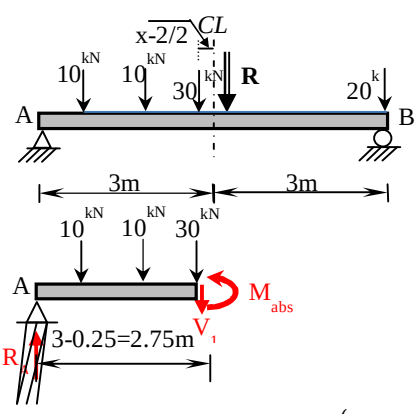
$$w_{cr} < w < w_{out}$$

Then the traditional absolute maximum moment procedure is invalid, and the max moment occurs when (P_1) is positioned at mid-span.

$$M_{max} = \frac{P_1 L}{4} + \frac{\sum P_{li}}{2}\left(\frac{L}{2} - d_{li}\right)$$

$$M_{max} = 45 + 15 + 5 = 65 \ kN.m$$

While the moment according to the traditional absolute maximum procedure will be:

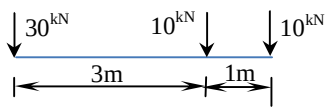


$$R_A = \frac{70(3 - 0.25)}{6} = 32.1 \ kN.m$$

$$M_{abs} = 58.275 \ kN.m$$

b) Calculate the location and amount of the maximum bending moment in the simply supported beam of 5m length subjected to the moving loads shown in figure.

c)



The first load of the moving loads given is the highest one; upon applying the method proposed in this research using Eq. (6) and Eq. (7) the following is found:

$$R = 30 + 10 + 10 = 50\,kN$$

$$M_{(P_1=30kN)} = 70\ kN.m$$

$$x = \frac{70}{50} = 1.4m,\ \frac{d}{2} = 1.5m,$$

$$\backslash\ x < \frac{d}{2}$$

$$w = \frac{d}{L} = \frac{3}{5} = 0.6$$

$$S = \frac{50}{10} = 5, \qquad S_1 = \frac{(10)\ (4)}{(10)\ (5)} = 0.8, \qquad r = \frac{30}{10} = 3$$

$$w_{cr} = (S - S_1) - \sqrt{Sr} = 0.33$$

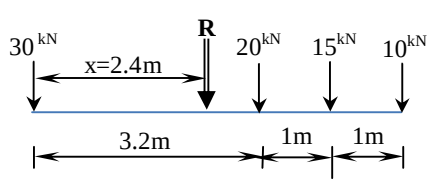
$$w_{out} = \frac{S + S_1}{2S - 1} = 0.64$$

$$w_{cr} < w < w_{out}$$

Then the traditional absolute maximum moment procedure is invalid, and the max moment occurs when P_1 is positioned at mid-span.

$$M_{max} = M_{CL} = \frac{P_1 L}{4} = 37.5\ kN.m$$

d) Calculate the location and amount of the maximum bending moment in the simply supported beam of 6m length subjected to the moving loads shown in figure.



The first load of the moving loads given is the highest one; first of all check x and d/2:

$$R = 30 + 20 + 15 + 10 = 75 \ kN$$

$$M_{(P_1=30kN)} = 179 \ kN.m$$

$$x = \frac{179}{75} = 2.4m, \ \frac{d}{2} = \frac{3.2}{2} = 1.6m,$$

$$\backslash \ x > \frac{d}{2}$$

Upon applying the method proposed in this research, two cases must be checked:

- Case-1: When the resultant is positioned on the beam such that it and the closest load (P_2) are equidistant from the beam's mid-span; calculate M_{abs} according to Eq. (8):

$$M_{abs} = \frac{R}{4L}(L - d + x)^2 - \sum P_{li}(d_{li} - d) = 49.5 \ kN.m$$

- Case-2: When the resultant is positioned on the beam such that it and the heaviest load (P_1) are equidistant from the beam's mid-span, calculate M_{abs} according to Eq. (11):

$$M_{abs} = \frac{R}{4L}(L - x)^2 = 40.5 \ kN.m$$

$$M_{abs\ (case-1)} \tilde{n} M_{abs\ (case-2)}$$

Then case-1 is dominating, therefore using Eq. (10) and Eq. (11) the following is found:

$$w = \frac{d}{L} = \frac{3.2}{6} = 0.53$$

$$S = \frac{75}{20} = 3.75, \ S_1 = \frac{(15)(4.2)+(10)(5.2)}{(20)(6)} = 0.96, \ S_2 = \frac{(15+10)}{20} = 1.25, \ r = \frac{30}{20} = 1.5$$

$$w_{cr} = \frac{(S+S_1)}{(S-1)} - \frac{SS_2}{(S-1)^2} - \sqrt{\frac{S(2S_1+S_2+1)}{(S-1)^2} - \frac{2SS_2(S+S_1)}{(S-1)^3} + \frac{S^2 S_2^{\ 2}}{(S-1)^4}} = 0.52, \qquad w_{out} = \frac{(S-S_1)}{(S+1)} = 0.59$$

$$w_{cr} < w < w_{out}$$

Then the traditional absolute maximum bending moment procedure is invalid, and the max moment occurs when (P_2) is at mid-span.

$$M_{max} = \frac{P_2 L}{4} + \sum \left(\frac{P_{li}}{2} \left(\frac{L}{2} - (d_{li} - d) \right) \right)$$

$$M_{max} = 50 \ kN.m$$

A Summary of the Applications' Results and the Percentage of Error between the Traditional Method and the Proposed Method is shown in Table-1, while a comparison between the traditional method and the proposed method is shown in Table-2.

Table (1) – Shows the Percentage of Error between the Traditional Method and the Proposed Method

Cases	M_T kN.m	M_P kN.m	% of Error
Simply Supported Beam (2-Point Loads)	36.1	37.5	3.7
Simply Supported Beam (3-Point Loads) 1st Load is the Heaviest (x<d/2)	26.8	37.5	28.5
Simply Supported Beam (4-Point Loads) 2st Load is the Heaviest	58.2	65	10.5
Simply Supported Beam (4-Point Loads) 1st Load is the Heaviest (x>d/2)	49.5	50	1

Table(2) – Shows a Comparison between the Traditional Method and the Proposed Method

Cases	Method Used	
Simply Supported Beam (Two Loads)	$d \le L/2$	$d > L/2$
	Traditional Method	Proposed Method: Calculate w_{cr} and w_{out} using Eqs. (2&3) $w_{cr} < w < w_{out}$ M_{abs} = When P_1 is located at Mid-Span
Simply Supported Beam (> Two Loads) Heaviest Load in the Middle	Traditional Method	
Simply Supported Beam (> Two Loads) 2nd Load is the Heaviest	$d \le L/2$	$d > L/2$
	Traditional Method	Proposed Method: Calculate w_{cr} and w_{out} using Eqs. (4&5) $w_{cr} < w < w_{out}$ M_{abs} = When P_1 is located at Mid-Span
Simply Supported Beam (> Two Loads) 1st Load is the Heaviest	$d \le L/2$	Traditional Method
	$d > \frac{L}{2}$ and $x < \frac{d}{2}$	Proposed Method: Calculate w_{cr} and w_{out} using Eqs. (6&7) $w_{cr} < w < w_{out}$ M_{abs} = When P_1 is located at Mid-Span
	$d > \frac{L}{2}$ and $x > \frac{d}{2}$ Calculate M_{abs} using Eqs. (8&11) The largest moment will dominate	If M_{abs} (Eq.8) is Dominating calculate w_{cr} and w_{out} using Eqs. (9&10) $w_{cr} < w < w_{out}$ $Mabs$ = When P_2 is located at Mid-Span
		If M_{abs} (Eq.11) is Dominating calculate w_{cr} and w_{out} using Eqs. (12&13) $w_{cr} < w < w_{out}$ $Mabs$ = When P_2 is located at Mid-Span

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