

المرونة و اللدونة

تعرف المرونة (elasticity) بانها دراسة العلاقات بين القوى الخارجية و نواتج هذه القوى من اجهادات (stresses) تودى الى تغير خطي تشوه (elastic -deformation) فى شكل الجسم الصلب. وفى حالة العلاقات الخطية بين الأجهاد و الأنفعال (strain) فى الأجسام الصلبة نستخدم قانون (Hook) فالأجهاد هو حاصل قسمة القوة (F) على المساحة (A) اما الأنفعال (strain) فهو حاصل قسمة التغير فى الطول (dL) على الطول الأصلي (L). نستخدم نظرية المرونة (elasticity) حالة التشوهات الصغيرة فى الأجسام الصلبة المتجانسة (Homogeneous) المتوحدة الخواص (isotropic) .

اما اللدونة (Plasticity) فهي نظرية تختص بدراسة العلاقات بين القوى الخارجية و نواتج هذه القوى من اجهادات تودى الى تشوهات غير خطية فى شكل الأجسام الصلبة تشوهات دائمية -لدنة . لماذا نحتاج الى نظرية اللدونة (theory of Plasticity) ؟

ان العلاقة الخطية بين الأجهاد والأنفعال تصبح علاقة غير خطية بعد مستوى معين من الجهد و بعد هذا المستوى يصبح من المتعذر ارجاع المادة لحالتها الاصلية بشكل كامل حتى لو ازيل الجهد الواقع على المادة. معظم الآلات تصمم على ان تعمل فى المدى المرن ولكن الاختبارات مطلوبة فى المدى اللدن ومن الأمثلة على ذلك .

- اماكن تركيز الأجهاد فى الآلات مثل الثقوب , الحزوز -notches- والحوامل .

- عمليات التصنيع مثل تشكيل المعادن هي عبارة عن تشوهات لدنة للمعادن .

لذلك يحتاج مهندسو الانتاج والمعادن الى تطبيق قوانين المرونة واللدونة فى عمليات الانتاج ومهندسو الميكانيك والانشاء فى عمليات التصميم والتحليل

Tension test

Subjects of interest

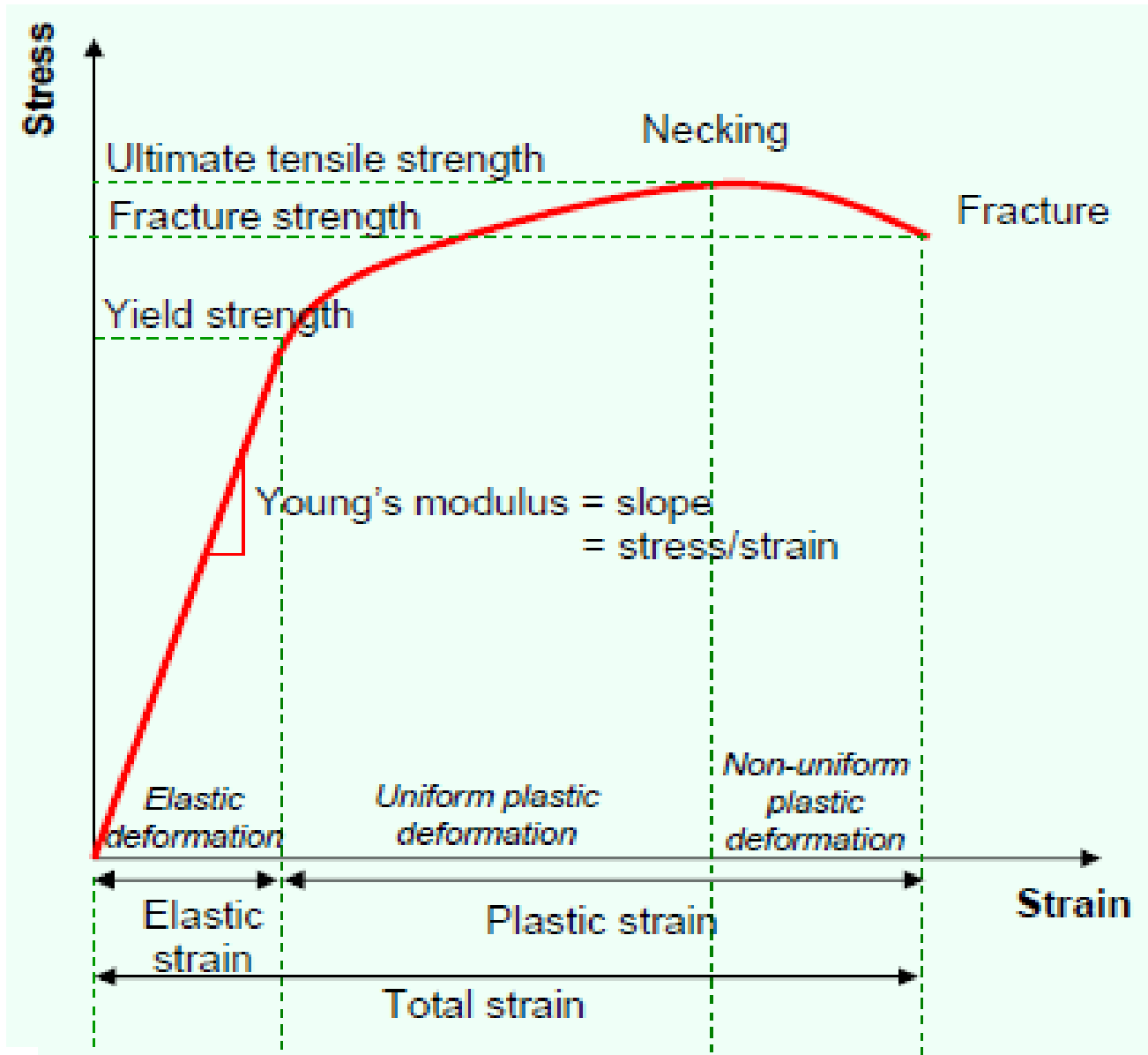
- Engineering stress-strain curve
- True stress-true strain curve
- Ductility measurement in tension tests
- Instability in tension
- Stress distribution at the neck
- Effect of strain rate on flow properties
- Effect of temperature on flow properties
- An Isotropy

Objectives

- This lecture provides fundamental backgrounds of tension tests where appropriate material parameters can be used for material selection.
- Differences between engineering stress-strain curve and true stress – true strain curve will be clearly understood.
- Effects of strain rate, test temperature and anisotropy on tensile properties

الهدف هوة تعرف الطلبة على مفهوم الإجهاد و الإنفعال و العلاقات الرياضية التي تربط بينهما و المفاهيم ذات العلاقة كالخواص الميكانيكية للمواد وأنواع الأحمال المسلطة بالاضافة الى اهمية اختبار الشد. و تكمن أهمية التعرف على الإجهاد و الإنفعال هو أن العلاقة الرياضية بينهما إذا رسمت على مخطط بياني سينتج منحنى الإجهاد و الإنفعال و هو منحنى خاص بالمادة الهندسية بذاتها يمكن أن يكون هذا المنحنى مفتاح التعامل مع المادة الهندسية.

Engineering stress-strain curve



Understanding the following terms will help us to understand the tensile test

Tensile test - Measures the response of a material to a slowly applied uniaxial force. The yield strength, tensile strength, modulus of elasticity, and ductility are obtained.

Elastic deformation - Deformation of the material that is recovered when the applied load is removed. Or Is the ability of the material to return to its original dimensions when the external applied load is removed.

Proportional limit - maximum stress in linear region.

elastic limit - based on microstrain measurement at strains on order of 2×10^{-6} . Very low value and is related to the motion of a few hundred dislocations.

And it is the greatest stress the material can withstand without any measurable permanent strain after unloading. Elastic limit > proportional limit.

Modulus of elasticity - Young's modulus, or the slope of the stress-strain curve in the elastic region.

Hooke's law - the linear relationship between stress and strain in the elastic portion of the stress-strain curve.

Yield strength - The stress applied to a material that just causes permanent plastic deformation.

Offset yield strength - The offset yield strength can be determined by the stress corresponding to the intersection of the stress-strain curve and a line parallel to the elastic line offset by a strain of 0.2 or 0.1%. ($\epsilon = 0.002$ or 0.001).

Plastic deformation - Is the property which permits materials to undergo permanent change in shape without fracture, i.e. the material does not return to its original dimensions

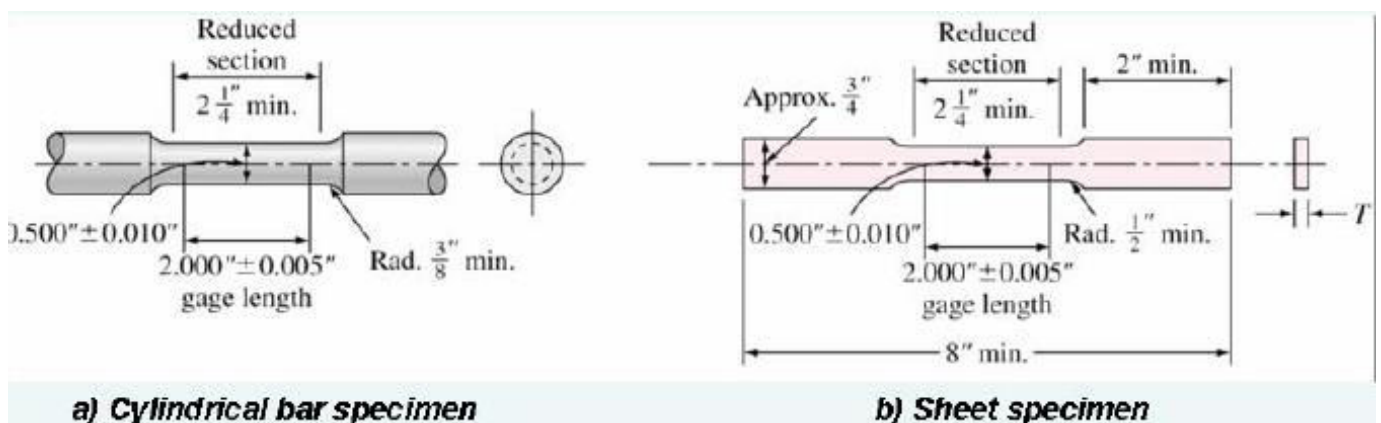
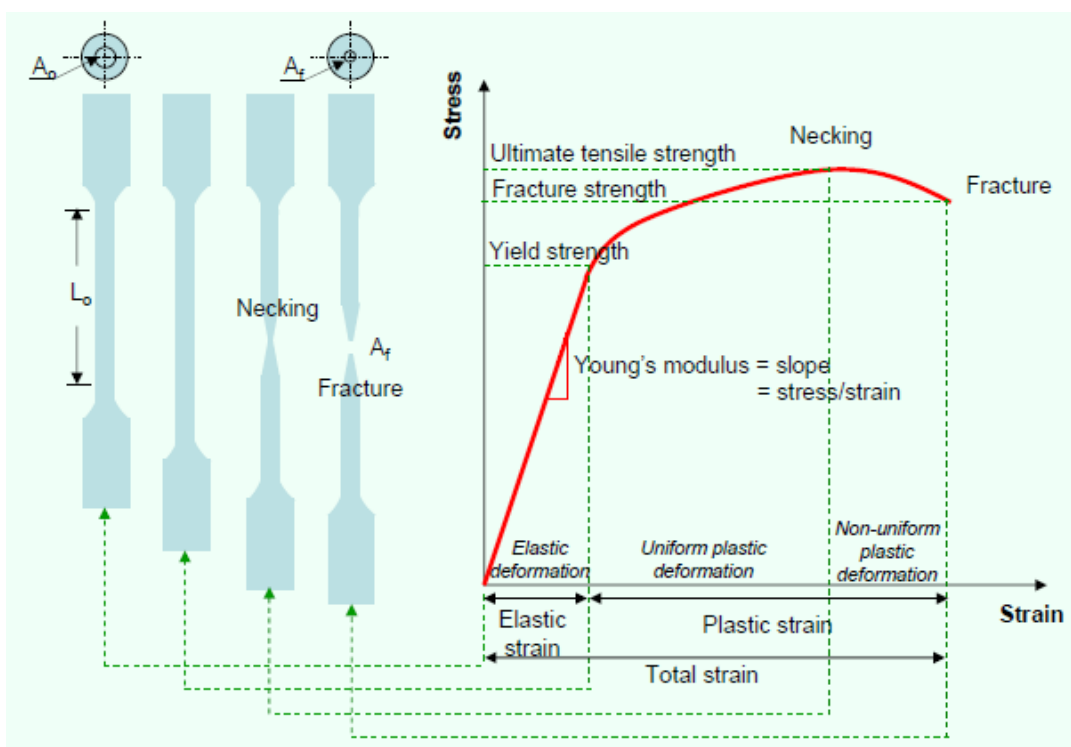
Tensile strength - The maximum engineering stress experienced by a material during a tensile test (ultimate tensile strength).

Necking - Local deformation of a tensile specimen. Necking begins at the tensile point.

Ductility - The ability of a material to be permanently deformed without breaking when a force is applied.

% Elongation - The total percent increase in the length of a specimen during a tensile test.

% Reduction in area - The total percent decrease in the cross-sectional area of a specimen during the tensile test.



Standard tensile specimens

Type specimen	United State (ASTM)	Great Britain	Germany
Sheet ($L_o / \sqrt{A_o}$)	4.5	5.65	11.3
Rod ($L_o / \sqrt{D_o}$)	4.0	5.0	10.0

Dimensional relationships of tensile specimens used in different countries

When a specimen is subjected to an external tensile loading, the metal will undergo elastic and plastic deformation. Initially, the metal will elastically deform giving a linear relationship of load and extension. These two parameters are then used for the calculation of the engineering stress and engineering strain to give a relationship as illustrated below.

Strain

Is a dimensionless value, it is the ratio between the change of length (L) to the original length (L_0):

$$e = \frac{\delta}{L_o} = \frac{\Delta L}{L_o} = \frac{L - L_o}{L_o}$$

Stress

is the load (P) divided by the original cross-sectional area(A_0)

$$\sigma = \frac{P}{A_o}$$

True-stress and true-strain

- True stress-strain curve gives a true indication of deformation characteristics because it is based on the instantaneous dimension of the specimen.
- The true stress-strain curve is also known as the flow curve.

Engineering stress - The applied load, or force, divided by the original cross-sectional area of the material.

$$\frac{\text{الحمل المؤثر عند أي لحظة}}{\text{مساحة المقطع عند لحظة تأثير الحمل}} = \text{الإجهاد الحقيقي}$$

$$\sigma = \frac{P}{A_0} \quad \sigma = \text{Engineering stress (nominal stress)} \quad \text{الاجهاد الهندسي}$$

Engineering strain: - Increase in sample length at a given load divided by the original length. Or

$$e_0 = \frac{L - L_0}{L_0} \quad \mathbf{e = extension of gauge length / original gauge length}$$

e = engineering strain (nominal strain) الانفعال الهندسي

Instability in tension

Necking or localized deformation begins at maximum load, where the increase in stress due to decrease in the cross-sectional area of the specimen becomes greater than the increase in the load-carrying ability of the metal due to strain hardening. This conditions of instability leading to localized deformation is defined by the condition $dP=0$.

- Let us start by considering the amount of force (dF) that is required to deform a specimen by $d\varepsilon$.

$$F = \sigma A$$

The slope of the stress strain curve is:

$$\frac{dF}{d\varepsilon} = \left[\sigma \left(\frac{dA}{d\varepsilon} \right) \right] + \left[A \left(\frac{d\sigma}{d\varepsilon} \right) \right]$$

NOTE: We are using true stress and strain (i.e., σ, ε) here rather than engineering stress and strain (s, e)

$(d\sigma/d\varepsilon)$ is the Work Hardening Rate. It is the slope of the stress-strain curve. It is always positive.

$(dA/d\varepsilon)$ is the Rate of Geometrical Softening. It is the rate at which the cross-sectional area of the specimen decreases with increasing strain due to constancy of volume. It is always negative.

- Local ↓ in A (i.e., deformation) causes that region to strain harden locally (relative to the rest of the cross section). The remainder of the cross section then deforms until a uniform cross-section is re-established.
- The **rates balance at the UTS** [$(dA/d\varepsilon) = (d\sigma/d\varepsilon)$].
- When $(dA/d\varepsilon) > (d\sigma/d\varepsilon)$, deformation becomes unstable. The material cannot strain harden fast enough to inhibit necking.
- The criteria for **instability** is defined by the condition where the slope of the force distance curve equals zero ($dF = 0$):

$$F = \sigma A$$

where

$$F = \text{load,}$$

$$\sigma = \text{true stress,}$$

$$A = \text{area at max load}$$

NOTE: We are using true stress and strain here rather than engineering

$$dF = \sigma dA + A d\sigma = 0 \dots\dots\dots (*)$$

- Recall that deformation is a constant volume process.
Thus:

$$L_0 A_0 = LA = \text{constant}$$

$$\frac{dL}{L} = -\frac{dA}{A} = d\varepsilon$$

- If we invoke the instability criteria from above (*) then we get:

$$-\frac{dA}{A} = \frac{d\sigma}{\sigma} = d\varepsilon$$

- Thus, at the point of tensile instability,

$$\frac{d\sigma}{d\varepsilon} = \sigma \quad \text{When "necking" occurs.}$$

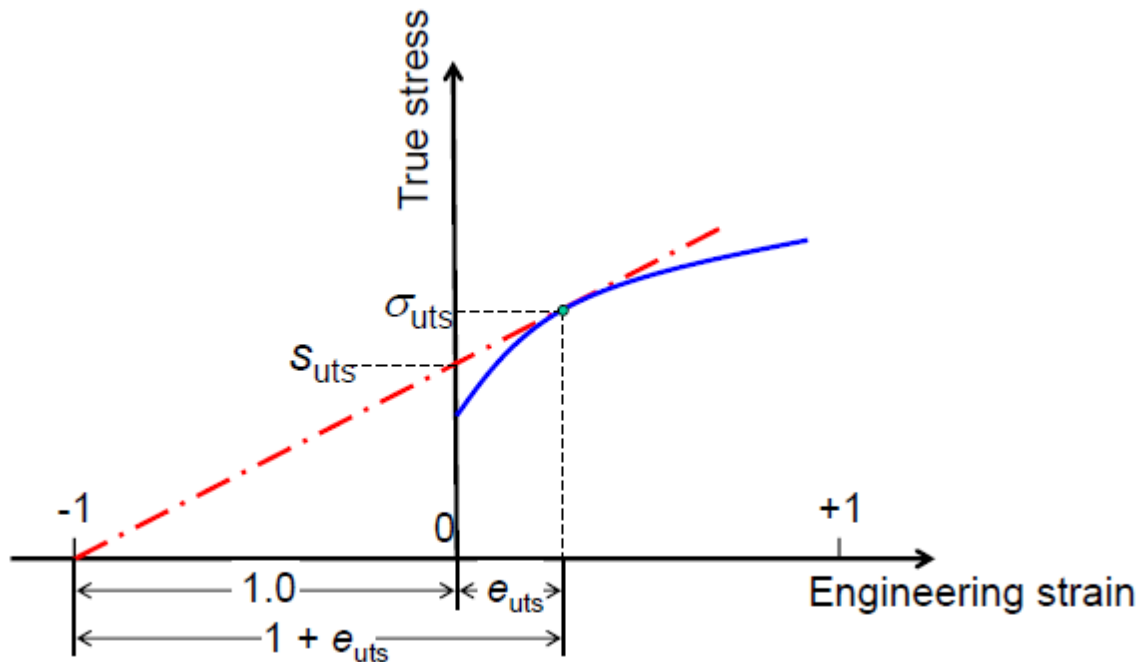
- If we incorporate engineering strain e , into the equation presented above, we can develop a more explicit expression:

$$\frac{d\sigma}{d\varepsilon} = \frac{d\sigma}{de} \frac{de}{d\varepsilon} = \frac{d\sigma}{de} \frac{dL/L_0}{dL/L} = \frac{d\sigma}{de} \frac{L}{L_0} = \frac{d\sigma}{de} (1+e) = \sigma$$

or

$$\boxed{\frac{d\sigma}{de} = \frac{\sigma}{(1+e)}}$$

- This is known as Considère's construction.



Strain-hardening exponent

$$n = \frac{d(\log \sigma)}{d(\log \varepsilon)} = \frac{d(\ln \sigma)}{d(\ln \varepsilon)} = \frac{\varepsilon}{\sigma} \frac{d\sigma}{d\varepsilon}$$

$n = 0$ for perfectly plastic solids

$n = 1$ for perfectly elastic solids

$n = 0.1 - 0.5$ for most metals

Strain-hardening rate

$$\frac{d\sigma}{d\varepsilon} = n \frac{\sigma}{\varepsilon}$$

Factors affecting on stress-strain curve

Metallurgical factors

1. Composition
2. Heat treatment
3. History of plastic deformation

Test condition

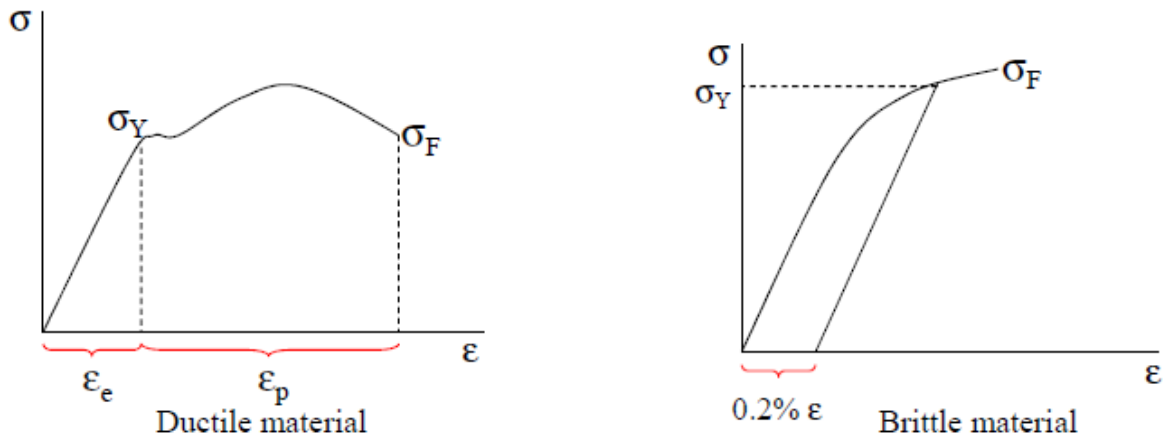
1. Strain rate
2. Temperature
3. State of stress

The unit $\dot{\varepsilon} = \frac{d\varepsilon}{dt}$ is per second, s^{-1}

Theories of Failure

- Failure occurs when material starts exhibiting inelastic behavior
- Brittle and ductile materials – different modes of failures – mode of failure – depends on loading
- Ductile materials – exhibit yielding – plastic deformation before failure
- Yield stress – material property
- Brittle materials – no yielding – sudden failure
- Factor of safety (FS)

■ Ductile and brittle materials

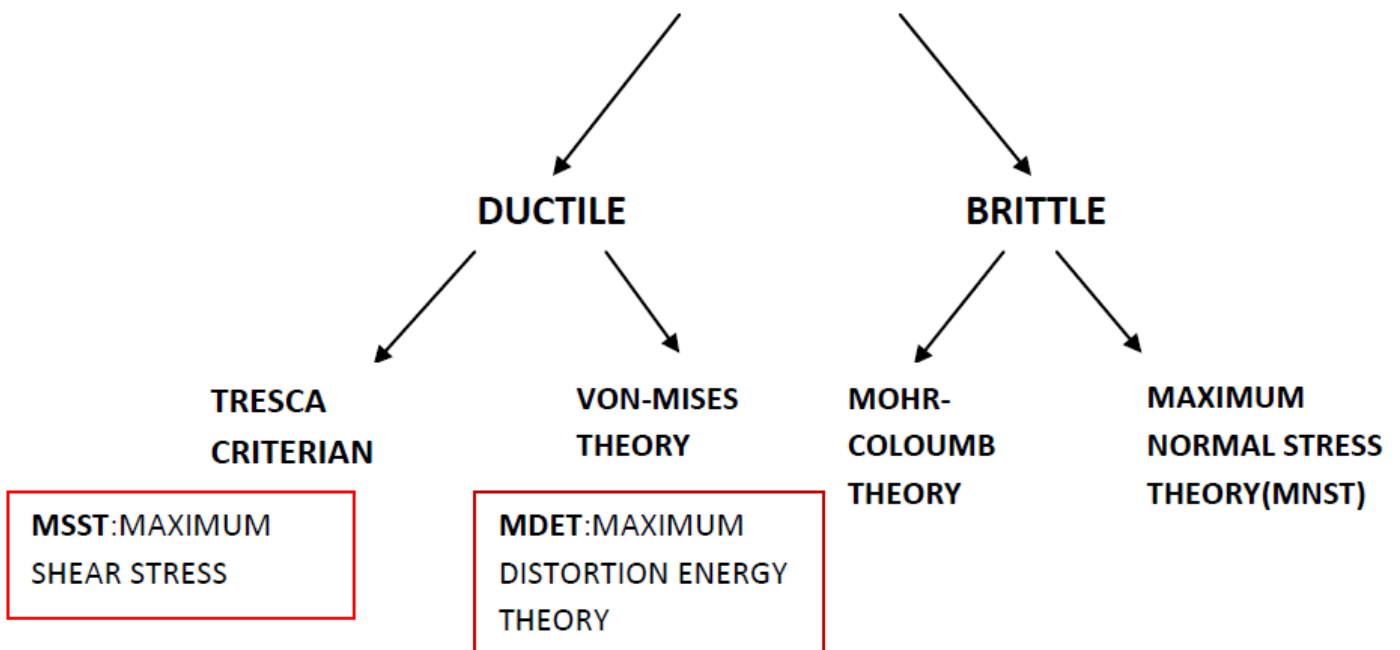


Well – defined yield point in ductile materials – FS on yielding

No yield point in brittle materials sudden failure – FS on failure load

The “theory” behind the various failure theories is that *whatever is responsible for failure in the standard tensile test will also be responsible for failure under all other conditions of static loading.*

STATIC FAILURE THEORIES



- Theories of failure
 - Max. principal stress theory – Rankine
 - Max. principal strain theory – St. Venants
 - Max. strain energy – Beltrami
 - Distortional energy – von Mises
 - Max. shear stress theory – Tresca
 - Octahedral shear stress theory

MAXIMUM NORMAL STRESS THEORY

For maximum normal stress theory, the failure occurs when one of the principal stresses (σ_1, σ_2 and σ_3) equals to the yield strength.

$$\sigma_1 > \sigma_2 > \sigma_3$$

Failure occurs when either $\sigma_1 = S_t$ or $\sigma_3 = -S_c$, where S_t is strength in tension and S_c is strength in compression.

MAXIMUM DISTORTION ENERGY THEORY (VON-MISES THEORY)

The maximum distortion energy theory, also known as the Von Mises theory,