number system :-

A **number system** defines how a number can be represented using distinct symbols. A number can be represented differently in different systems.

Several number systems have been used in the past and can be categorized into two groups: **positional** and **non-positional** systems. Our main goal is to discuss the positional number systems, but we also give examples of non-positional systems.

In a **positional number system**, the position a symbol occupies in the number determines the value it represents. In this system, a number represented as:

$$\pm (S_{k-1} \ldots S_2\, S_1\, S_0 .\, S_{-1}\, S_{-2} \ldots S_{-l})_b$$

has the value of:

$$n = \pm \quad S_{k-1} \times b^{k-1} + \ldots + S_1 \times b^1 + S_0 \times b^0 \quad + \quad S_{-1} \times b^{-1} + S_{-2} \times b^{-2} + \ldots + S_{-l} \times b^{-l}$$

in which S is the set of symbols, b is the base (or radix).

The decimal system (base 10)

The word decimal is derived from the Latin root **decem** (ten). In this system the **base $b = 10$** and we use ten symbols.

$S = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

The symbols in this system are often referred to as **decimal digits** or just **digits**.

Integers

$$N = \pm \quad S_{k-1} \times 10^{k-1} + S_{k-2} \times 10^{k-2} + \ldots + S_2 \times 10^2 + S_1 \times 10^1 + S_0 \times 10^0$$

	10^{k-1}	10^{k-2}	• • •	10^2	10^1	10^0	Place values
$\pm$	S_{k-1}	S_{k-2}	• • •	S_2	S_1	S_0	Number
	↓	↓		↓	↓	↓	

$$N = \pm S_{k-1} \times 10^{k-1} + S_{k-2} \times 10^{k-2} + \bullet\bullet\bullet + S_2 \times 10^2 + S_1 \times 10^1 + S_0 \times 10^0 \quad \text{Values}$$

The binary system (base 2)

The word binary is derived from the Latin root **bini** (or two by two). In this system the **base b = 2** and we use only two symbols,

$$\mathbf{S = \{0, 1\}}$$

The symbols in this system are often referred to as **binary digits** or **bits** (binary digit).

Reals

$$R = \pm \underbrace{S_{k-1} \times 10^{k-1} + \ldots + S_1 \times 10^1 + S_0 \times 10^0}_{\text{Integral part}} + \underbrace{S_{-1} \times 10^{-1} + \ldots + S_{-l} \times 10^{-l}}_{\text{Fractional part}}$$

Example 3: The following shows the place values for the real number +24.13.

	10^1		10^0		10^{-1}		10^{-2}	Place values
	2		4	•	1		3	Number
$R = +$	2×10	+	4×1	+	1×0.1	+	3×0.01	Values

Example 4

The following shows that the number $(11001)_2$ in binary is the same as 25 in decimal. The subscript 2 shows that the base is 2.

		2^4		2^3		2^2		2^1		2^0	Place values
		1		1		0		0		1	Number
N	=	1×2^4	+	1×2^3	+	0×2^2	+	0×2^1	+	1×2^0	Decimal

The equivalent decimal number is $N = 16 + 8 + 0 + 0 + 1 = 25$.

Integers

$$N = \pm\ S_{k-1} \times 2^{k-1} + S_{k-2} \times 2^{k-2} + \dots + S_2 \times 2^2 + S_1 \times 2^1 + S_0 \times 2^0$$

	2^{k-1}	2^{k-2}	• • •	2^2	2^1	2^0	Place values
$\pm$	S_{k-1}	S_{k-2}	• • •	S_2	S_1	S_0	Number
	↓	↓		↓	↓	↓	

$$N = \pm\ S_{k-1} \times 2^{k-1} + S_{k-2} \times 2^{k-2} + \bullet\bullet\bullet + S_2 \times 2^2 + S_1 \times 2^1 + S_0 \times 2^0 \quad \text{Values}$$

The octal system (base 8)

The word octal is derived from the Latin root **octo** (eight). In this system the **base b = 8** and we use eight symbols to represent a number. The set of symbols is

$$S = \{0, 1, 2, 3, 4, 5, 6, 7\}$$

Integers

$$N = \pm S_{k-1} \times 8^{k-1} + S_{k-2} \times 8^{k-2} + \ldots + S_2 \times 8^2 + S_1 \times 8^1 + S_0 \times 8^0$$

	8^{k-1}	8^{k-2}	$\cdots$	8^2	8^1	8^0	Place values
$\pm$	S_{k-1}	S_{k-2}	$\cdots$	S_2	S_1	S_0	Number
	↓	↓		↓	↓	↓	

$$N = \pm S_{k-1} \times 8^{k-1} + S_{k-2} \times 8^{k-2} + \cdots + S_2 \times 8^2 + S_1 \times 8^1 + S_0 \times 8^0 \quad \text{Values}$$

Example 7

The following shows that the number $(1256)_8$ in octal is the same as 686 in decimal.

	8^3	8^2	8^1	8^0	Place values
	1	2	5	6	Number
N =	1×8^3	$+ \ 2 \times 8^2$	$+ \ 5 \times 8^1$	$+ \ 6 \times 8^0$	Values

Note that the decimal number is $N = 512 + 128 + 40 + 6 = 686$.

Conversion

We need to know how to convert a number in one system to the equivalent number in another system. Since the decimal system is more familiar than the other systems, we first show how to covert from any base to decimal. Then we show how to convert from decimal to any base. Finally, we show how we can easily convert from binary to hexadecimal or octal and vice versa.

The hexadecimal system (base 16)

The word **hexadecimal** is derived from the Greek root **hex** (six) and the Latin root **decem** (ten). In this system the **base** $\mathbf{b = 16}$ and we use sixteen symbols to represent a number. The set of symbols is

$$\mathbf{S = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F\}}$$

Note that the symbols A, B, C, D, E, F are equivalent to 10, 11, 12, 13, 14, and 15 respectively. The symbols in this system are often referred to as **hexadecimal digits**.

Reals

	Integral part	.	Fractional part
$R = \pm$	$S_{k-1} \times 2^{k-1} + \ldots + S_1 \times 2^1 + S_0 \times 2^0$	+	$S_{-1} \times 2^{-1} + \ldots + S_{-l} \times 2^{-l}$

Example 5

The following shows that the number $(101.11)_2$ in binary is equal to the number 5.75 in decimal.

		2^2		2^1		2^0		2^{-1}		2^{-2}		Place values
		1		0		1	•	1		1		Number
R	=	1×2^2	+	0×2^1	+	1×2^0	+	1×2^{-1}	+	1×2^{-2}		Values

Example 6

The following shows that the number (2AE)16 in hexadecimal is equivalent to 686 in decimal.

		16^2		16^1		16^0	Place values
		2		A		E	Number
N	=	2×16^2	+	10×16^1	+	14×16^0	Values

The equivalent decimal number is N = 512 + 160 + 14 = 686.

Integers

$$N = \pm S_{k-1} \times 16^{k-1} + S_{k-2} \times 16^{k-2} + \ldots + S_2 \times 16^2 + S_1 \times 16^1 + S_0 \times 16^0$$

	16^{k-1}	16^{k-2}	•••	16^2	16^1	16^0	Place values
$\pm$	S_{k-1}	S_{k-2}	•••	S_2	S_1	S_0	Number
	↓	↓		↓	↓	↓	

$$N = \pm S_{k-1} \times 16^{k-1} + S_{k-2} \times 16^{k-2} + \cdots + S_2 \times 16^2 + S_1 \times 16^1 + S_0 \times 16^0 \quad \text{Values}$$