ex Bezier curve is defined by four control points (3 0 1) (4 0 4) (8 0 4) & (10 0 1), find the equation of the curve using Matrix form.

Sol:-

equation of curve in Matrix form for four CP is:-

$$P(u) = [u^3 \quad u^2 \quad u \quad 1] \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

$$\therefore P(u) = \underset{1\times4}{[u^3 \; u^2 \; u \; 1]} \underset{4\times4}{\begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}} \underset{4\times3}{\begin{bmatrix} 3 & 0 & 1 \\ 4 & 0 & 4 \\ 8 & 0 & 4 \\ 10 & 0 & 1 \end{bmatrix}}$$

$$= [u^3 \; u^2 \; u \; 1] \begin{bmatrix} -5 & 0 & 0 \\ 9 & 0 & -9 \\ 3 & 0 & 9 \\ 3 & 0 & 1 \end{bmatrix}$$

$$\therefore P(u) = \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} = \begin{bmatrix} -5u^3 + 9u^2 + 3u + 3 \\ 0 \\ -9u^2 + 9u + 1 \end{bmatrix}$$

ex

Generate a Bezier curve using the following CP: (2,0), (4,3), (5,2), (4,-2), (5,-3) and (6,-2).

sol :-

There are 6 CP (n+1) → Hence n = 5

It can generate a Bezier curve using the polynomial form :-

$$P(u) = \sum_{i=0}^{n} B_i^n(u) P_i \qquad 0 \leq u \leq 1$$

where $B_i^n(u) = \dfrac{n!}{i!\,(n-i)!} u^i (1-u)^{n-i}$

$n = 5$

$$B_0^5(u) = \frac{5!}{0!\,(5-0)!} u^0 (1-u)^{5-0}$$

$$= (1-u)^5$$

$$B_1^5(u) = \frac{5!}{1!\,(5-1)!} u^1 (1-u)^{5-1} = \frac{5\times4\times3\times2\times1}{1\times4\times3\times2\times1} \times u \times (1-u)^4$$

$$= 5u(1-u)^4$$

$$B_2^5(u) = \frac{5!}{2!\,(5-2)!} u^2 (1-u)^{5-2}$$

$$= \frac{5\times4\times3\times2\times1}{2\times1\times3\times2\times1} u^2 (1-u)^3 = 10u^2(1-u)^3$$

$$B_3^5 = \frac{5!}{3!\,(5-3)!} u^3 (1-u)^{5-3}$$

$$= \frac{5\times4\times3\times2\times1}{3\times2\times1\times2\times1} u^3 (1-u)^2 = 10u^3(1-u)^2$$

$$B_4^5 = \frac{5!}{4!\,(5-4)!} u^4 (1-u)^{5-4} = \frac{5\times4\times3\times2\times1}{4\times3\times2\times1\times1} u^4 (1-u)^1$$

$$= 5u^4(1-u)$$

$$B_5^5 = \frac{\cancel{5!}}{\cancel{5!}\,(\cancel{5-5})!} u^5 \cancel{(1-u)^{5-5}} = u^5$$

$$\therefore P(u) = (1-u)^5 P_0 + 5u(1-u)^4 P_1 + 10u^2(1-u)^3 P_2$$
$$+ 10u^3(1-u)^2 P_3 + 5u^4(1-u) P_4 + u^5 P_5$$

$\therefore$ using the CP for x & y, so as to simplify the Parametric eq:-

$$P(u)_x = (1-u)^5 * 2 + 5u(1-u)^4 * 4 + 10u^2(1-u)^3 * 5$$
$$+ 10u^3(1-u)^2 * 4 + 5u^4(1-u) * 5 + u^5 * 6$$

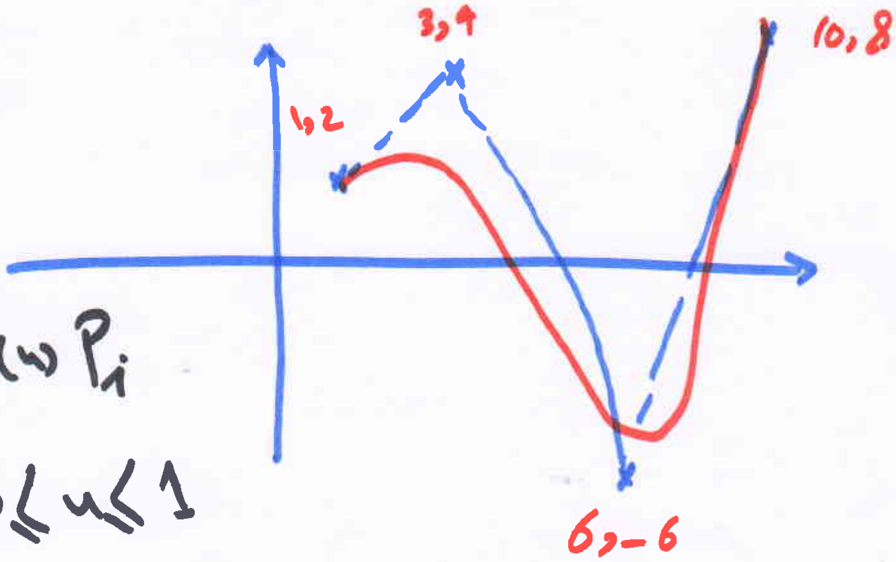
$$P(u)_y = (1-u)^5 * 0 + 5u(1-u)^4 * 3 + 10u^2(1-u)^3 * 2$$
$$+ 10u^3(1-u)^2 * -2 + 5u^4(1-u) * -3 + u^5 * -2$$

for $0 \leq u \leq 1$ it can find the Points of curve in x & y- axis.

Q4 Draw Bezier curve with following CP:- (1,2), (3,4), (6,−6) and (10,8) in polynomial form.

Sol:-

To draw Bezier curve, it must found x & y points, then it can draw it, as follows



There are 4 CP hence $n = 3$

$$P(u) = \sum_{i=0}^{n} B_i^n(u)\, P_i$$

$$0 \le u \le 1$$

$$B_i^n(u) = \frac{n!}{i!\,(n-i)!}\, u^i (1-u)^{n-i}$$

$n = 3$ $\therefore B_0^3 = \frac{3!}{0!\,(3-0)!}\, u^0 (1-u)^{3-0} = (1-u)^3$

$$B_1^3 = \frac{3!}{1!\,(3-1)!}\, u^1 (1-u)^{3-1} = 3u(1-u)^2$$

$$B_2^3 = \frac{3!}{2!\,(3-2)!}\, u^2 (1-u)^{3-2} = 3u^2(1-u)$$

$$B_3^3 = \frac{3!}{3!\,(3-3)!}\, u^3 (1-u)^{3-3} = u^3$$

$$P(u) = (1-u)^3 P_0 + 3u(1-u)^2 P_1 + 3u^2(1-u) P_2 + u^3 P_3$$

using the CP for x & y axis:-

$$P(u)_x = (1-u)^3 * 1 + 3u(1-u)^2 * 3 + 3u^2(1-u) * 6 + u^3 * 10$$

$$= (1-u)^3 + 9u(1-u)^2 + 18u^2(1-u) + 10u^3$$

$$P(u)_y = (1-u)^3 * 2 + 3u(1-u)^2 * 4 + 3u^2(1-u) * -6 + u^3 * 8$$

$$= 2(1-u)^3 + 12u(1-u)^2 - 18u^2(1-u) + 8u$$

∴ To draw Bezier curve; it must substitute The value of u from $0 \longrightarrow 1$ - as follows:-

u	x	y
0	1	2
0.2	2.32	2.048
0.4	3.88	0.944
0.6	5.68	0.416
0.8	7.72	2.192
1	10	8

ex

Generate a three dimensional Bezier curve using the following CP. $(5,4,2)$ $(6,2,3)$, $(5,-2,4)$ & $(6,-4,3)$.

Then draw this curve.