Bezier curve:-

- This Method was developed by P. Bezier of French Renault Company. It has become the most popular curve design Method used in graphic packages and CAD system.

- To derive the Matrix of Bezier curve we will take a cubic curve defined by four control points as follows:-

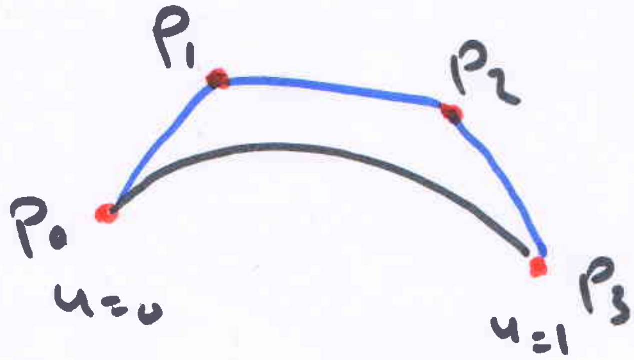
Bezier

$P(0) = P_0$

$P(1) = P_3$

$\bar{P}(0) = 3(P_1 - P_0)$

$\bar{P}(1) = 3(P_3 - P_2)$



where P_0, P_1, P_2 & P_3 are four user-defined Control Points.

The general Parametric eqn. is:-

$$P(u) = a_0 + a_1 u + a_2 u^2 + a_3 u^3$$

for $P(0) = a_0$ ——— (1)

$$P(1) = a_0 + a_1 + a_2 + a_3 \quad \text{--- (2)}$$

The tangent vector of Parametric curve is :- (2)

$$\acute{P}(u) = a_1 + 2a_2u + 3a_3u^2$$

$\therefore$ $\acute{P}(0) = a_1$ ——— (3)

$\acute{P}(1) = a_1 + 2a_2 + 3a_3$ — (4)

$\therefore$ $P_0 = P(0) = a_0$ ——— (5)

$P_3 = P(1) = a_0 + a_1 + a_2 + a_3$ — (6)

$3(P_1 - P_0) = \acute{P}(0) = a_1$ ——— (7)

$3(P_3 - P_2) = \acute{P}(1) = a_1 + 2a_2 + 3a_3$ — (8)

Sub. (5) into (6)

$P_3 = P_0 + \underline{a_1} + a_2 + a_3$ ——— (9)

Sub. (7) into (9)

$P_3 = P_0 + 3P_1 - 3P_0 + a_2 + a_3$

$\Rightarrow a_2 = P_3 - 3P_1 + 2P_0 - a_3$ —— (10)

Sub. (10) into (8)

$3P_3 - 3P_2 = 3P_1 - 3P_0 + 2P_3 - 6P_1 + 4P_0$
$-2a_3 + 3a_3$

$\therefore$ $3P_3 - 2P_3 - 3P_2 - 3P_1 + 6P_1 + 3P_0 - 4P_0 = a_3$

$\therefore$ $P_3 - 3P_2 + 3P_1 - P_0 = a_3$ —— (11)

sub. (11) into (10)

$$a_2 = P_3 - 3P_1 + 2P_0 - P_3 + 3P_2 - 3P_1 + P_0$$

$$a_2 = 3P_2 - 6P_1 + 3P_0 \text{ --- } (12)$$

$$a_1 = 3P_1 - 3P_0 \text{ --- } (13)$$

$$a_0 = P_0 \text{ --- } (14)$$

sub eq. (11) $\longrightarrow$ (14) into general parametric eq.

$$\therefore P(u) = P_0 + (3P_1 - 3P_0)u + (3P_2 - 6P_1 + 3P_0)u^2 + (P_3 - 3P_2 + 3P_1 - P_0)u^3$$

$$= P_0 + 3P_1u - 3P_0u + 3P_2u^2 - 6P_1u^2 + 3P_0u^2 + P_3u^3 - 3P_2u^3 + 3P_1u^3 - P_0u^3$$

$$= (1 - 3u + 3u^2 - u^3)P_0 + (3u - 6u^2 + 3u^3)P_1 + (3u^2 - 3u^3)P_2 + (u^3)P_3$$

$$= (-u^3 + 3u^2 - 3u + 1)P_0 + (3u^3 - 6u^2 + 3u)P_1 + (-3u^3 + 3u^2)P_2 + u^3P_3$$

$$= [u^3\ u^2\ u\ 1] \underbrace{\begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}}_{\text{Bezier Matrix}} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

It can also expressed in Polynomial form :-

$$P(u) = \sum_{i=0}^{n} B_i^n(u)\, P_i \qquad 0 \leq u \leq 1$$

where

$$B_i^n(u) = \frac{n!}{i!\,(n-i)!}\, u^i\, (1-u)^{n-i}$$

for example :

P(u) =

; $n = 3$

$CP = n+1 = 4$

$$\therefore B_0^3(u) = \frac{3!}{0!\,(3-0)!}\, u^0\, (1-u)^{3-0}$$

$$= \frac{\cancel{3\times2\times1}}{1\times\cancel{3\times2\times1}} * 1 * (1-u)^3 = \boxed{(1-u)^3}$$

$$B_1^3(u) = \frac{3!}{1!\,(3-1)!}\, u^1\, (1-u)^{3-1}$$

$$= \frac{3\times\cancel{2\times1}}{1\times\cancel{2\times1}}\, u\, (1-u^2) = \boxed{3u(1-u)^2}$$

$$B_2^3 = \frac{3!}{2!\,(3-2)!}\, u^2\, (1-u)^{3-2}$$

$$= \frac{3\times\cancel{2\times1}}{\cancel{2\times1}\times1}\, u^2\, (1-u) = \boxed{3u^2(1-u)}$$

$$B_3^3 = \frac{\cancel{3!}}{\cancel{3!}\,(3-3)!}\, u^3 \,\cancel{(1-u)^{3-3}} = \boxed{u^3}$$

$$\therefore P(u) = (1-u)^3 P_0 + 3u(1-u)^2 P_1 + 3u^2(1-u)P_2 + u^3 P_3$$

$$= \left[(1-u)(1-2u+u^2)\right] P_0 + \left[3u(1-2u+u^2)\right] P_1 + (3u^2-3u^3) P_2 + u^3 P_3$$

$$= (1-2u+u^2-u+2u^2-u^3) P_0 + (3u-6u^2+3u^3) P_1 + (3u^2-3u^3) P_2 + u^3 P_3$$

$$= (-u^3+3u^2-3u+1) P_0 + (3u^3-6u^2+3u) P_1 + (-3u^3+3u^2) P_2 + u^3 P_3$$

$$= \left[u^3\ u^2\ u\ 1\right] \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

Bezier Matrix

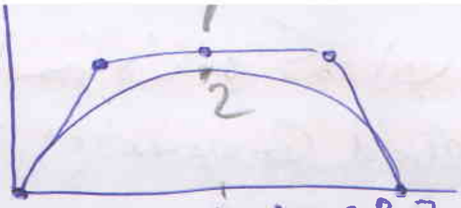
Same

①

$$\begin{bmatrix}0\\1\\2\\3\\4\end{bmatrix} \quad \begin{bmatrix}0\\3\\3\\3\\0\end{bmatrix} = 2\ \text{باسلب}$$

$\boxed{n=4}$

$\boxed{CP=5}$



$u=0.5$

$y=[0\ 0\ 0\ 0;\ 1\ 1\ 1\ 2\ 2 \ldots 2\ 2\ 3;\ 4\ 4]$

$$P(u)=[u^4\ u^3\ u^2\ u\ 1]\begin{bmatrix}1&-4&6&-4&1\\-4&12&-12&4&0\\6&-12&6&0&0\\-4&4&0&0&0\\1&0&0&0&0\end{bmatrix}\begin{bmatrix}P_0\\P_1\\P_2\\P_3\\P_4\end{bmatrix}$$

$[0.0625\ 0.125\ 0.25\ 0.5\ 1]$ $\{M\}$ $\{P\}$

$x=2$

$y=2.625$

① $(0,0),(2,2.625),(4,0)$

$P'(u)=[4u^3\ 3u^2\ 2u\ 1\ 0]\ [M]\ \{P\}$

$P'(0)=[0\ 0\ 0\ 1\ 0] * [M]\ \{P\}$

$0.0625-0.5+1.5-2+1$

$P''(u)=[12u^2\ 6u\ 2\ 0\ 0]\ \{M\}\ \{P\}$

$0.0625\ P_0$

$P''(0)=[0\ 0\ 2\ 0\ 0] * [M]\ \{P\}$

$-0.25+1.5-3+2$

حاصل ضرب $[M]*$

$$P'_x(u)=[-4\ 4\ 0\ 0\ 0]\begin{bmatrix}0\\1\\2\\3\\4\end{bmatrix}=4i$$

$0.25P_1$

$$P'_y(u)=[-4\ 4\ 0\ 0\ 0]\begin{bmatrix}0\\3\\3\\3\\0\end{bmatrix}=12j$$

$0.375\ \cancel{-1.5}\ \cancel{+1.5}$

$$\underline{T}=P'_x(u)+P'_y(u)=4i+12j$$

$$P''_x(u)=[0\ 0\ 2\ 0\ 0]\begin{bmatrix}0\\1\\2\\3\\4\end{bmatrix}=4i$$

$0.375P_2$

$$P''_y(u)=[0\ 0\ 2\ 0\ 0]\begin{bmatrix}0\\3\\3\\3\\0\end{bmatrix}=6j$$

$-0.25+0.5\ \ 2\ 0.25P_3$

$$\underline{N}=P''_x(u)+P''_y(u)=4i+6j$$

$0.0625P_4$

$$T\times N=[4i+12j][4i+6j]$$
$$=24K-48K=\boxed{-24K}$$

$0.0625P_0+10.75P_1+0.375P_2+0.25P_3$

كفية

$$K=\frac{|T\times N|}{|T|^3}=\frac{\sqrt{(-24)^2}}{\left(\sqrt{(4)^2+(12)^2}\right)^3}=\frac{24}{2023.8}=0.0118$$

$+0.0625P_4$

$$Ra=\frac{1}{K}=84.325$$

هذا الحل الاول

طريقة تكنا لورل لا يطابق اطالب

قوانين اخرى في كتاب

[Applied Geometry for CAD]

$$k = \frac{|\dot{c} \times \ddot{c}|}{|\dot{c}|^3}$$

$$t = \frac{\dot{c}}{|\dot{c}|} \quad , \quad b = \frac{\dot{c} \times \ddot{c}}{|\dot{c} \times \ddot{c}|}$$

$$n = b \times t$$

Sub. in the previ~~ose~~ Solution ⇒

$$t = \frac{4i + 12j}{\sqrt{(4)^2 + (12)^2}} = \frac{4i + 12j}{12.6}$$

(2)

$$b = \frac{(4i + 12j) \times (4i + 6j)}{\sqrt{(-24)^2}} = \frac{-\cancel{24}K}{\cancel{24}} = \boxed{-K}$$

$$n = b \times t$$

Normal $[-K] \times \left[\frac{4i + 12j}{12.6}\right]$

$$\boxed{n} = \frac{-4j + 12i}{12.6} = -\underline{0.31j} + \underline{\underline{0.95i}}$$

in matlab the result for $\underline{\underline{u = 0}}$

$N_x = 0$ بسبب ان الكتاب اعطى [illegible]

$$\boxed{\begin{aligned} N_y &= -0.9470 \\ N_z &= 0.3211 \end{aligned}}$$

⇒ the same ~~value~~ value but diff in direction ???

(2-303) رأي اللجنة العلمية في القسم أو مايناظرها ومدى تأييدها سفر الطالب من عدمه

(3-303) رأي قسم العلاقات الثقافية في الجامعة أو مايناظرها والاجراءات التي قامت بها باتجاه سفر الطالب :

1.
2.
3.
4.
5.
6.
7.

3

$$K = \frac{|\dot{c} \times \ddot{c}|}{|\dot{c}|^3}$$

$$= \frac{\sqrt{(-24)^2}}{\left[\sqrt{(4)^2+(12)^2}\right]^3} = \frac{24}{2023.8} = 0.0118$$

$$Ra = \frac{1}{0.0118} = 84.325$$