

8. Material Properties

The prime objective of this chapter is to explore the electrical properties of materials, that is, their responses to an applied electric field. We begin with the phenomenon of electrical conduction: the parameters by which it is expressed, the mechanism of conduction by electrons, and how the electron energy band structure of a material influences its ability to conduct. These principles are extended to metals, semiconductors, and insulators. Particular attention is given to the characteristics of semiconductors and then to semiconducting devices. Also treated are the dielectric characteristics of insulating materials. The final sections are devoted to the peculiar phenomena of ferroelectricity and piezoelectricity.

8.1 Electrical Properties

8.1.1 Electrical Resistivity of Metals

As mentioned previously, most metals are extremely good conductors of electricity; room-temperature conductivities for several of the more common metals are contained in Table 8.1.

Table 8.1 Room-Temperature Electrical Conductivities for Nine Common Metals and Alloys.

<i>Metal</i>	<i>Electrical Conductivity</i> [$(\Omega\text{-m})^{-1}$]
Silver	6.8×10^7
Copper	6.0×10^7
Gold	4.3×10^7
Aluminum	3.8×10^7
Brass (70Cu-30Zn)	1.6×10^7
Iron	1.0×10^7
Platinum	0.94×10^7
Plain carbon steel	0.6×10^7
Stainless steel	0.2×10^7

Again, metals have high conductivities because of the large numbers of free electrons that have been excited into empty states above the Fermi energy. Thus n has a large value in the conductivity expression, Equation 8.1.

$$\sigma = n|e|\mu_e \quad (8.1)$$

At this point it is convenient to discuss conduction in metals in terms of the resistivity, the reciprocal of conductivity; the reason for this switch in topic should become apparent in the ensuing discussion.

Since crystalline defects serve as scattering centers for conduction electrons in metals, increasing their number raises the resistivity (or lowers the conductivity). The concentration of these imperfections depends on temperature, composition, and the degree of cold work of a metal specimen. In fact, it has been observed experimentally that the total resistivity of a metal is the sum of the contributions from thermal vibrations, impurities, and plastic deformation; that is, the scattering

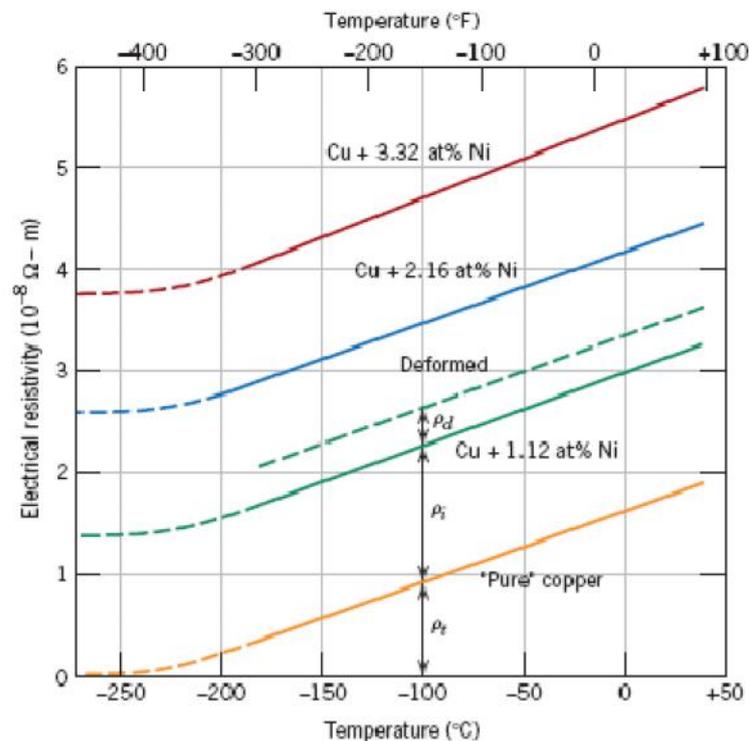


Figure 8.1 The electrical resistivity versus temperature for copper and three copper–nickel alloys, one of which has been deformed. Thermal, impurity, and deformation contributions to the resistivity are indicated at 100 °C.

mechanisms act independently of one another. This may be represented in mathematical form as follows:

$$\rho_{\text{total}} = \rho_t + \rho_i + \rho_d \quad (8.2)$$

in which ρ_t , ρ_i and ρ_d represent the individual thermal, impurity, and deformation resistivity contributions, respectively. Equation 8.2 is sometimes known as Matthiessen's rule. The ρ influence of each variable on the total resistivity is demonstrated in Figure 8.1, a plot of resistivity versus temperature for copper and several copper–nickel alloys in annealed and deformed states. The additive nature of the individual resistivity contributions is demonstrated at $-100\text{ }^\circ\text{C}$

8.1.1.1. Influence of Temperature

For the pure metal and all the copper–nickel alloys shown in Figure 8.1, the resistivity rises linearly with temperature above about $-200\text{ }^\circ\text{C}$. Thus,

$$\rho_t = \rho_0 + aT \quad (8.3)$$

Where ρ_0 and a are constants for each particular metal. This dependence of the thermal resistivity component on temperature is due to the increase with temperature in thermal vibrations and other lattice irregularities (e.g., vacancies), which serve as electron-scattering centers.

8.1.1.2. Influence of Impurities

For additions of a single impurity that forms a solid solution, the impurity ρ_i resistivity is related to the impurity concentration C_i in terms of the atom fraction (at%100) as follows:

$$\rho_i = Ac_i(1 - c_i) \quad (8.4)$$

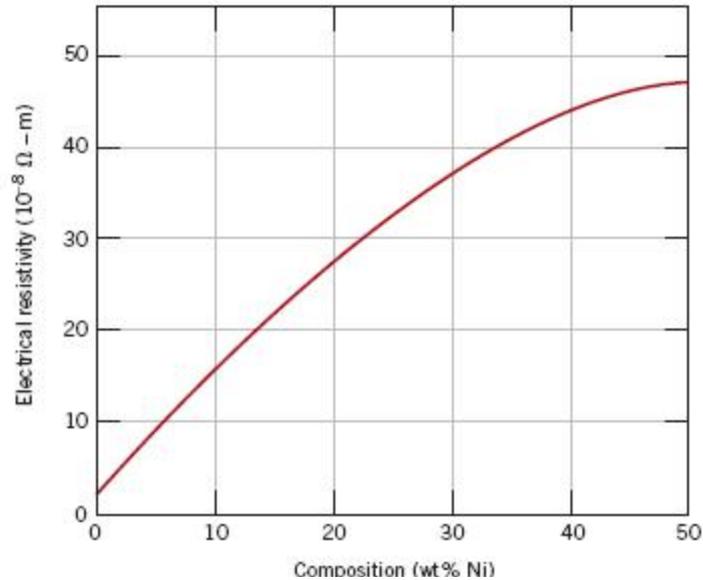


Figure 8.2 Room temperature electrical resistivity versus composition for copper–nickel alloys.

where A is a composition-independent constant that is a function of both the impurity and host metals. The influence of nickel impurity additions on the room temperature resistivity of copper is demonstrated in Figure 8.2, up to 50 wt% Ni; over this composition range nickel is completely soluble in copper (Figure 8.3).

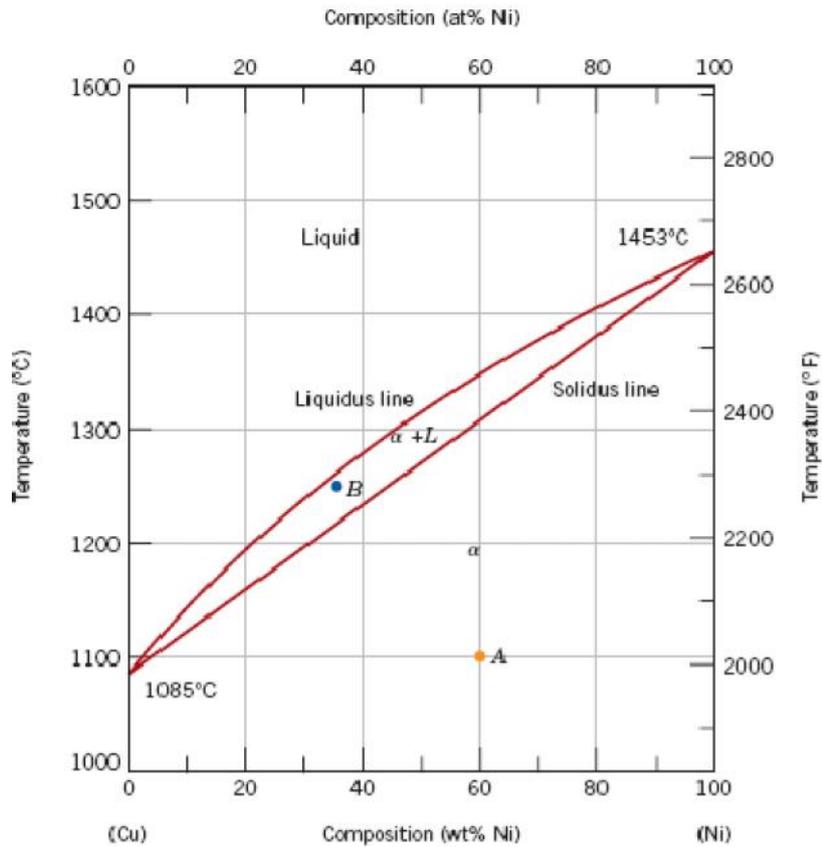


Figure 8.3 The copper–nickel phase diagram

Again, nickel atoms in copper act as scattering centers, and increasing the concentration of nickel in copper results in an enhancement of resistivity. For a two-phase alloy consisting of α and β phases, a rule-of-mixtures expression may be utilized to approximate the resistivity as follows:

$$\rho_i = \rho_\alpha V_\alpha + \rho_\beta V_\beta \quad (8.5)$$

where the V 's and ρ 's represent volume fractions and individual resistivities for the respective phases.

8.1.1.3. Influence of Plastic Deformation

Plastic deformation also raises the electrical resistivity as a result of increased numbers of electron-scattering dislocations. The effect of deformation on resistivity is also represented in Figure 8.1. Furthermore, its influence is much weaker than that of increasing temperature or the presence of impurities.

8.2 Mechanical and Thermal properties

8.2.1. Mechanical properties

Many materials, when in service, are subjected to forces or loads; examples include the aluminum alloy from which an airplane wing is constructed and the steel in an automobile axle. In such situations it is necessary to know the characteristics of the material and to design the member from which it is made such that any resulting deformation will not be excessive and fracture will not occur. The mechanical behavior of a material reflects the relationship between its response or deformation to an applied load or force. Important mechanical properties are strength, hardness, ductility, and stiffness.

The mechanical properties of materials are ascertained by performing carefully designed laboratory experiments that replicate as nearly as possible the service conditions. Factors to be considered include the nature of the applied load and its duration, as well as the environmental conditions. It is possible for the load to be tensile, compressive, or shear, and its magnitude may be constant with time, or it may fluctuate continuously. Application time may be only a fraction of a second, or it may extend over a period of many years. Service temperature may be an important factor.

Mechanical properties are of concern to a variety of parties (e.g., producers and consumers of materials, research organizations, government agencies) that have differing interests. Consequently, it is imperative that there be some consistency in the manner in which tests are conducted, and in the interpretation of their results. This consistency is accomplished by using standardized testing techniques. Establishment and publication of these standards are often coordinated by professional societies. In the United States the most active organization is the American Society for Testing and Materials (ASTM). Its Annual Book of ASTM Standards comprises numerous volumes, which are issued and updated yearly; a large number of these standards relate to mechanical testing techniques. Several of these are referenced by footnote in this and subsequent chapters. The role of structural engineers is to determine stresses and stress distributions within members that are subjected to well-defined loads. This may be accomplished by experimental testing techniques and/or by theoretical and mathematical stress analyses. These topics are treated in traditional stress analysis and strength of materials texts.

Materials and metallurgical engineers, on the other hand, are concerned with producing and fabricating materials to meet service requirements as predicted by these stress analyses. This necessarily involves an

understanding of the relationships between the microstructure (i.e., internal features) of materials and their mechanical properties.

Materials are frequently chosen for structural applications because they have desirable combinations of mechanical characteristics. The present discussion is confined primarily to the mechanical behavior of metals; polymers and ceramics are treated separately because they are, to a large degree, mechanically dissimilar to metals. This chapter discusses the stress–strain behavior of metals and the related mechanical properties, and also examines other important mechanical characteristics. Discussions of the microscopic aspects of deformation mechanisms and methods to strengthen and regulate the mechanical behavior of metals are deferred to later chapters.

8.2.2. Concept of Stress and Strain.

If a load is static or changes relatively slowly with time and is applied uniformly over a cross section or surface of a member, the mechanical behavior may be ascertained by a simple stress–strain test; these are most commonly conducted for metals at room temperature. There are three principal ways in which a load may be applied: namely, tension, compression, and shear (Figures 8.4a, b, c). In engineering practice many loads are torsional rather than pure shear; this type of loading is illustrated in Figure 8.4d.

8.2.2.a. Tension Test

One of the most common mechanical stress–strain tests is performed in tension. As will be seen, the tension test can be used to ascertain several mechanical properties of materials that are important in design. A specimen is deformed, usually to fracture, with a gradually increasing tensile load that is applied uniaxially along the long axis of a specimen. A standard tensile specimen is shown in Figure 8.5. Normally, the cross section is circular, but rectangular specimens are also used. This “dogbone” specimen configuration was chosen so that, during testing, deformation is confined to the narrow center region (which has a uniform cross section along its length), and, also, to reduce the likelihood of fracture at the ends of the specimen.

The standard diameter is approximately 12.8 mm (0.5 in.), whereas the reduced section length should be at least four times this diameter; 60 mm(2 ¼ in) is

common. Gauge length is used in ductility computations; the standard value is 50 mm (2.0 in.). The specimen is mounted by its ends into the holding grips of the testing apparatus (Figure 8.6). The tensile testing machine is designed to elongate the specimen at a constant rate, and to continuously and simultaneously measure the instantaneous applied load (with a load cell) and the resulting elongations (using an extensometer). A stress–strain test typically takes several minutes to perform and is destructive; that is, the test specimen is permanently deformed and usually fractured.

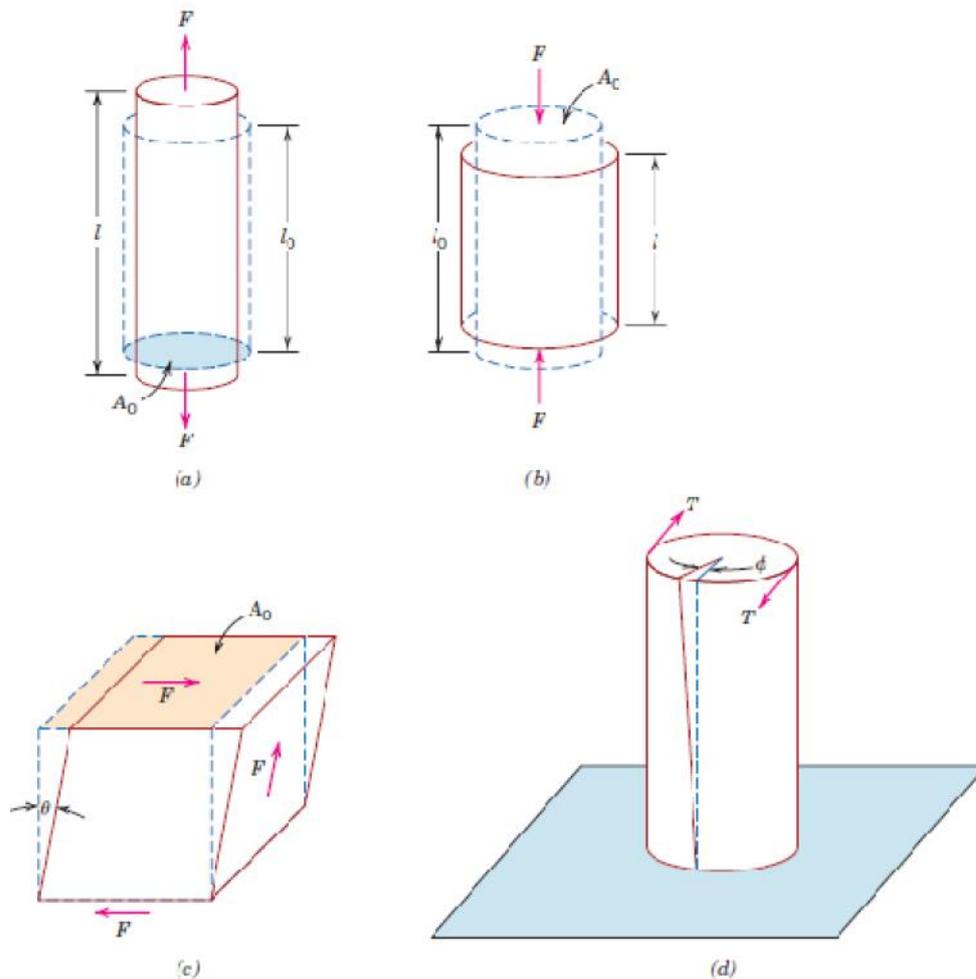


Figure 8.4

- (a) Schematic illustration of how a tensile load produces an elongation and positive linear strain. Dashed lines represent the shape before deformation; solid lines, after deformation.
- (b) Schematic illustration of how a compressive load produces contraction and a negative linear strain.
- (c) Schematic representation of shear strain γ where $\gamma = \tan \theta$.

(d)Schematic representation of torsional deformation (i.e., angle of twist Φ) produced by an applied torque T.

The output of such a tensile test is recorded (usually on a computer) as load or force versus elongation. These load–deformation characteristics are dependent on the specimen size. For example, it will require twice the load to produce the same elongation if the cross-sectional area of the specimen is doubled. To minimize these geometrical factors, load and elongation are normalized to the respective parameters of engineering stress and engineering strain. Engineering stress σ is defined by the Relationship

$$\sigma = \frac{F}{A_0} \quad (8.6)$$

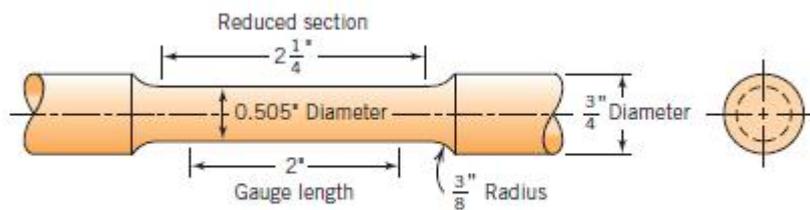


Figure (8.5) A standard tensile specimen with circular cross section.

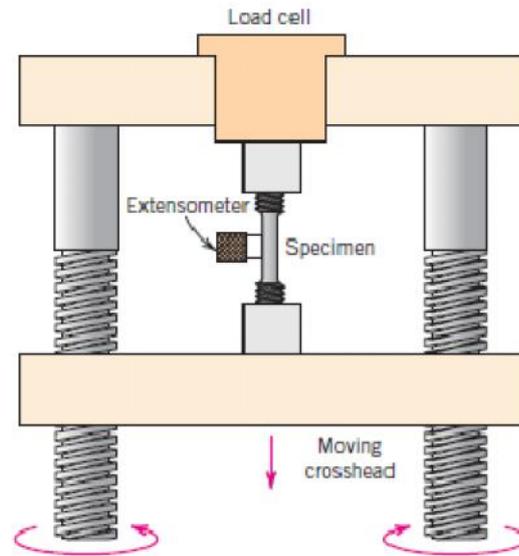


Figure (8.6) Schematic representation of the apparatus used to conduct tensile stress–strain tests. The specimen is elongated by the moving crosshead; load cell and extensometer measure, respectively, the magnitude of the applied load and the elongation.

in which F is the instantaneous load applied perpendicular to the specimen cross section, in units of newtons (N) or pounds force ($1b_f$), and A_0 is the original cross sectional area before any load is applied (m^2 or in^2). The units of engineering stress (referred to subsequently as just stress) are megapascals, MPa (SI) (where $1 \text{ MPa} = 10^6 \text{ N/m}^2$) and pounds force per square inch, psi (Customary U.S.)².

Engineering strain is defined according to

$$\epsilon = \frac{l_i - l_0}{l_0} = \frac{\Delta l}{l_0} \quad (8.7)$$

in which l_0 is the original length before any load is applied, and is the instantaneous length. Sometimes the quantity $l_i - l_0$ is denoted as ΔL and is the deformation elongation or change in length at some instant, as referenced to the original

length. Engineering strain (subsequently called just strain) is unitless, but meters per meter or inches per inch are often used; the value of strain is obviously independent of the unit system. Sometimes strain is also expressed as a percentage, in which the strain value is multiplied by 100.

8.2.2.b Compression Test.

Compression stress–strain tests may be conducted if in-service forces are of this type. A compression test is conducted in a manner similar to the tensile test, except that the force is compressive and the specimen contracts along the direction of the stress. Equations 8.6 and 8.7 are utilized to compute compressive stress and strain, respectively. By convention, a compressive force is taken to be negative, which yields a negative stress. Furthermore, since l_0 is greater than l_i compressive strains computed from Equation 8.7 are necessarily also negative. Tensile tests are more common because they are easier to perform; also, for most materials used in structural applications, very little additional information is obtained from compressive tests.

Compressive tests are used when a material's behavior under large and permanent (i.e., plastic) strains is desired, as in manufacturing applications, or when the material is brittle in tension.

8.2.2.c Shear and Tensional Test.

For tests performed using a pure shear force as shown in Figure 8.4c, the shear stress τ is computed according to

$$\tau = \frac{F}{A_0} \quad (8.8)$$

Where F is the load or force imposed parallel to the upper and lower faces, each of which has an area of A_0 . The shear strain γ is defined as the tangent of the strain angle, as indicated in the figure. The units for shear stress and strain are the same as for their tensile counterparts.

Torsion is a variation of pure shear, wherein a structural member is twisted in the manner of Figure 8.4d; torsional forces produce a rotational motion about the longitudinal axis of one end of the member relative to the other end. Examples of torsion are found for machine axles and drive shafts, and also for twist drills. Torsional tests are normally performed on cylindrical solid shafts or tubes. A shear stress τ is a function of the applied torque T , whereas shear strain is related to

the angle of twist , Φ in Figure 8.4d.

8.2.3 Geometric Considerations of the Stress State

Stresses that are computed from the tensile, compressive, shear, and torsional force states represented in Figure 8.4 act either parallel or perpendicular to planar faces of the bodies represented in these illustrations. Note that the stress state is a function of the orientations of the planes upon which the stresses are taken to act. For example, consider the cylindrical tensile specimen of Figure 8.7 that is subjected to a tensile stress σ applied parallel to its axis. Furthermore, consider also the plane $p-p'$ that is oriented at some arbitrary angle relative to the plane of the specimen end-face. Upon this plane $p-p'$, the applied stress is no longer a pure tensile one. Rather, a more complex stress state is present that consists of a tensile (or normal) stress σ' that acts normal to the $p-p'$ Plane and, in addition ,a shear stress τ' that acts parallel to this plane; both of these stresses are represented in the figure. Using mechanics of materials principles, it is possible to develop equations for σ' and τ' in terms of σ and θ , as follows:

$$\sigma' = \sigma \cos^2 \theta = \sigma \left(\frac{1 + \cos 2\theta}{2} \right) \quad (8.9a)$$

$$\tau' = \sigma \sin \theta \cos \theta = \sigma \left(\frac{\sin 2\theta}{2} \right) \quad (8.9b)$$

These same mechanics principles allow the transformation of stress components from one coordinate system to another coordinate system that has a different orientation. Such treatments are beyond the scope of the present discussion

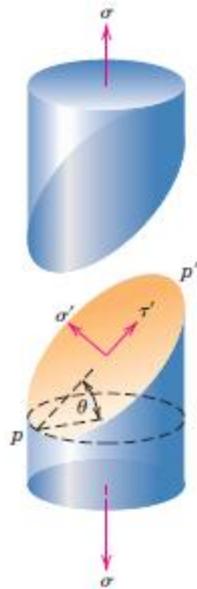


Figure 8.7 Schematic representation showing normal (σ') and shear (τ') stresses that act on a plane oriented at an angle θ relative to the plane taken perpendicular to the direction along which a pure tensile stress (σ) is applied.

8.2.4 Elastic Deformation

8.2.4.1 Stress Strain Behavior

The degree to which a structure deforms or strains depends on the magnitude of an imposed stress. For most metals that are stressed in tension and at relatively low levels, stress and strain are proportional to each other through the relationship

$$\sigma = E\epsilon \quad (8.10)$$

This is known as Hooke's law, and the constant of proportionality E (GPa or psi)⁶ is the modulus of elasticity, or *Young's modulus*. For most typical metals the magnitude of this modulus ranges between 45 GPa (6.5×10^6 psi), for magnesium, and 407 GPa (59×10^6 psi), for tungsten. Modulus of elasticity values for several metals at room temperature are presented in Table 6.1.

Table 8.2 Room-Temperature Elastic and Shear Moduli, and Poisson's Ratio for Various Metal Alloys

<i>Metal Alloy</i>	<i>Modulus of Elasticity</i>		<i>Shear Modulus</i>		<i>Poisson's Ratio</i>
	<i>GPa</i>	<i>10⁶ psi</i>	<i>GPa</i>	<i>10⁶ psi</i>	
Aluminum	69	10	25	3.6	0.33
Brass	97	14	37	5.4	0.34
Copper	110	16	46	6.7	0.34
Magnesium	45	6.5	17	2.5	0.29
Nickel	207	30	76	11.0	0.31
Steel	207	30	83	12.0	0.30
Titanium	107	15.5	45	6.5	0.34
Tungsten	407	59	160	23.2	0.28

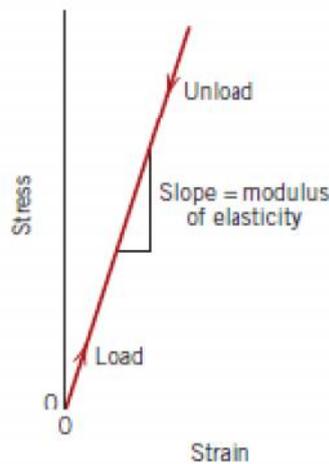


Figure 8.8 Schematic stress–strain diagram showing linear elastic deformation for loading and unloading cycles.

Deformation in which stress and strain are proportional is called elastic deformation; a plot of stress (ordinate) versus strain (abscissa) results in a linear relationship, as shown in Figure 8.8. The slope of this linear segment corresponds to the modulus of elasticity E . This modulus may be thought of as stiffness, or a material's resistance to elastic deformation. The greater the modulus, the stiffer the material, or the smaller the elastic strain that results from the application of a given stress. The modulus is an important design parameter used for computing elastic deflections. Elastic deformation is nonpermanent, which means that when the applied load is released, the piece returns to its original shape. As shown in the stress–strain plot (Figure 8.8), application of the load corresponds to

moving from the origin up and along the straight line. Upon release of the load, the line is traversed in the opposite direction, back to the origin.

There are some materials (e.g., gray cast iron, concrete, and many polymers) for which this elastic portion of the stress–strain curve is not linear (Figure 8.9); hence, it is not possible to determine a modulus of elasticity as described above. For this nonlinear behavior, either tangent or secant modulus is normally used. Tangent modulus is taken as the slope of the stress–strain curve at some specified level of stress, while secant modulus represents the slope of a secant drawn from the origin to some given point of the σ - ϵ curve. The determination of these moduli is illustrated in Figure 8.9.

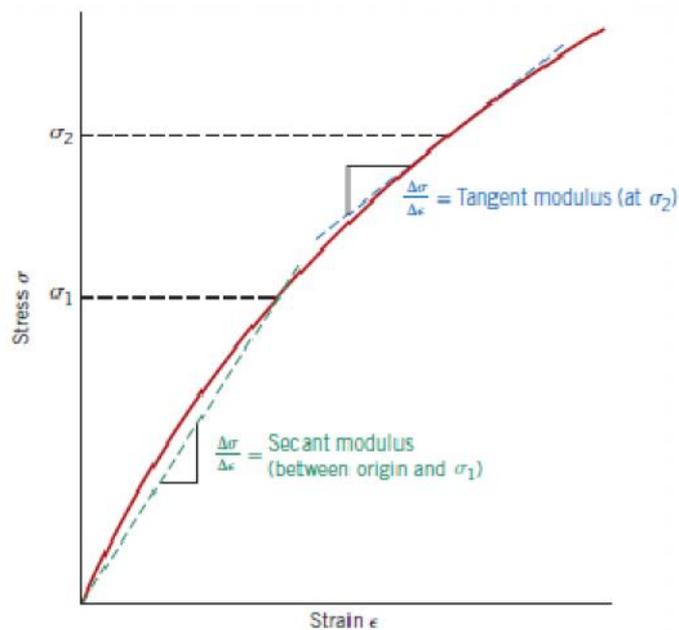


Figure 8.9 Schematic stress–strain diagram showing non-linear elastic behavior, and how secant and tangent moduli are determined.

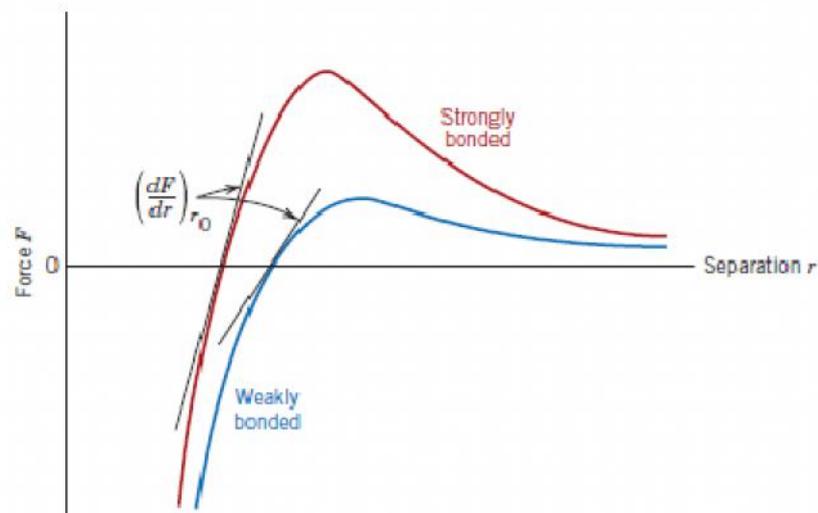


Figure 8.10 Force versus interatomic separation for weakly and strongly bonded atoms. The magnitude of the modulus of elasticity is proportional to the slope of each curve at the equilibrium interatomic separation r_0 .

On an atomic scale, macroscopic elastic strain is manifested as small changes in the interatomic spacing and the stretching of interatomic bonds. As a consequence, the magnitude of the modulus of elasticity is a measure of the resistance to separation of adjacent atoms, that is, the interatomic bonding forces. Furthermore, this modulus is proportional to the slope of the interatomic force–separation curve (Figure 8.11) at the equilibrium spacing:

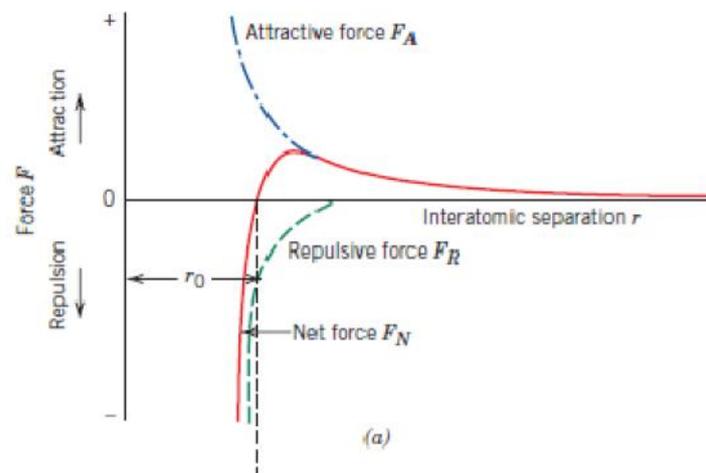


Figure 8.11

$$E \propto \left(\frac{dF}{dr} \right)_{r_0} \quad (8.11)$$

Figure 8.10 shows the force–separation curves for materials having both strong and weak interatomic bonds; the slope at r_0 is indicated for each. Values of the modulus of elasticity for ceramic materials are about the same as for metals; for polymers they are lower (Figure 8.12). These differences are a direct consequence of the different types of atomic bonding in the three materials types.

Furthermore, with increasing temperature, the modulus of elasticity diminishes, as is shown for several metals in Figure 8.13.

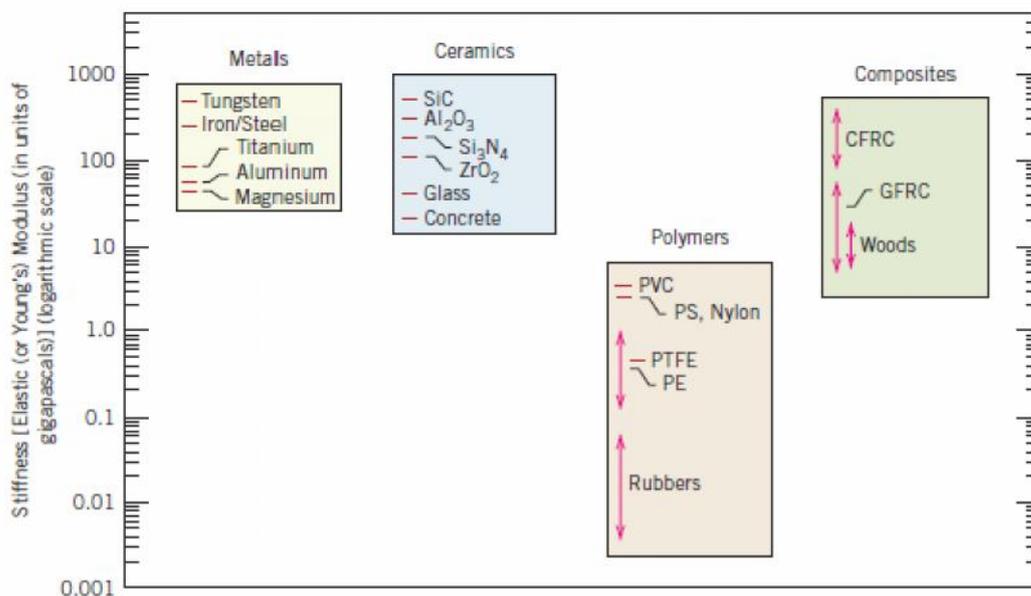


Figure 8.12

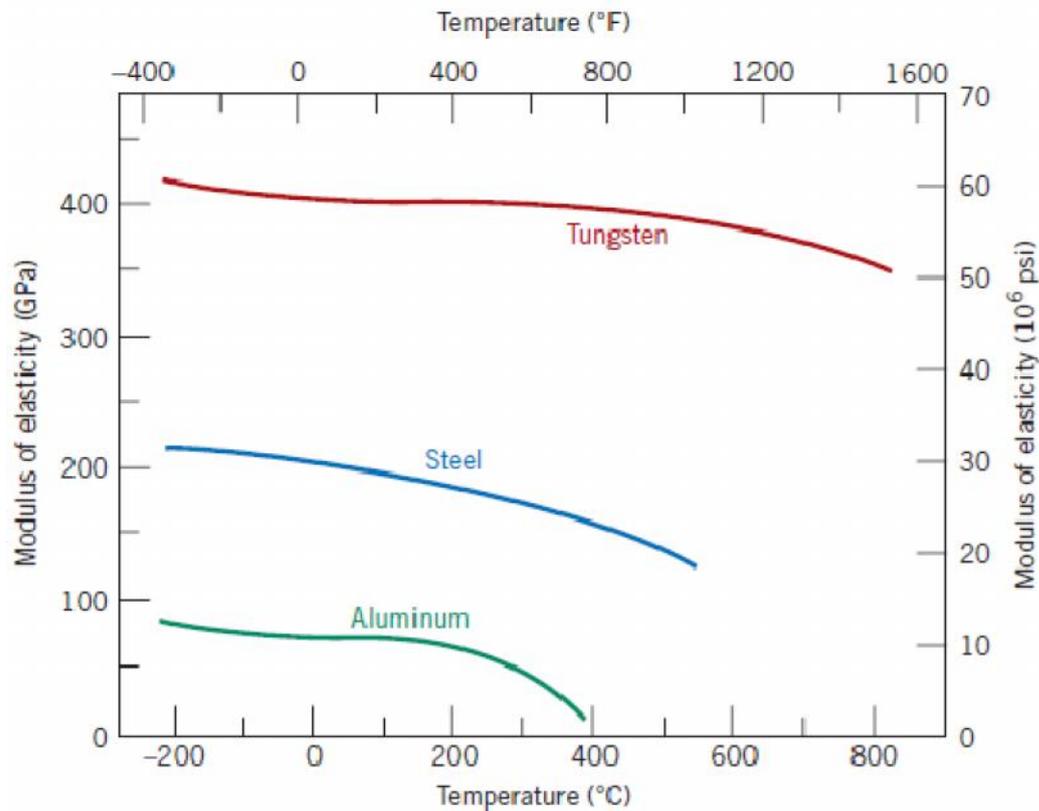


Figure 8.13 Plot of modulus of elasticity versus temperature for tungsten, steel, and aluminum.

As would be expected, the imposition of compressive, shear, or torsional stresses also evokes elastic behavior. The stress–strain characteristics at low stress levels are virtually the same for both tensile and compressive situations, to include the magnitude of the modulus of elasticity. Shear stress and strain are proportional to each other through the expression.

$$\tau = G\gamma \quad (8.12)$$

where G is the shear modulus, the slope of the linear elastic region of the shear stress–strain curve. Table 8.2 also gives the shear moduli for a number of the common metals.

8.2.5 Anelasticity.

Up to this point, it has been assumed that elastic deformation is time independent—that is, that an applied stress produces an instantaneous elastic strain that remains constant over the period of time the stress is maintained. It has also been assumed that upon release of the load the strain is totally recovered—that is, that the strain immediately returns to zero. In most engineering materials, however, there will also exist a time-dependent elastic strain component. That is, elastic deformation will continue after the stress application, and upon load release some finite time is required for complete recovery. This time-dependent elastic behavior is known as anelasticity, and it is due to time-dependent microscopic and atomistic processes that are attendant to the deformation. For metals the anelastic component is normally small and is often neglected. However, for some polymeric materials its magnitude is significant; in this case it is termed viscoelastic behavior.

***Example Problem (6.a)**

Elongation (Elastic) Computation

A piece of copper originally 305 mm (12 in.) long is pulled in tension with a stress of 276 MPa (40,000 psi). If the deformation is entirely elastic, what will be the resultant elongation?

Solution

Since the deformation is elastic, strain is dependent on stress according to Equation 8.10. Furthermore, the elongation Δl is related to the original length l_0 through Equation 8.7. Combining these two expressions and solving for Δl yields

$$\sigma = \epsilon E = \left(\frac{\Delta l}{l_0}\right)E$$
$$\Delta l = \frac{\sigma l_0}{E}$$

The values of σ and l_0 are given as 276 MPa and 305 mm, respectively, and the magnitude of E for copper from Table 8.2 is 110 GPa (16×10^6 psi). Elongation is obtained by substitution into the expression above as

$$\Delta l = \frac{(276 \text{ MPa})(305 \text{ mm})}{110 \times 10^3 \text{ MPa}} = 0.77 \text{ mm (0.03 in.)}$$

8.2.6 Elastic Properties of Materials.

When a tensile stress is imposed on a metal specimen, an elastic elongation and accompanying strain ϵ_z result in the direction of the applied stress (arbitrarily taken to be the z direction), as indicated in Figure 8.14. As a result of this elongation, there will be constrictions in the lateral (x and y) directions perpendicular to the applied stress; from these contractions, the compressive strains ϵ_x and ϵ_y may be determined. If the applied stress is uniaxial (only in the z direction), and the material is isotropic, then $\epsilon_x = \epsilon_y$. A parameter termed Poisson's ratio ν is defined as the ratio of the lateral and axial strains, or

$$\nu = -\frac{\epsilon_x}{\epsilon_z} = -\frac{\epsilon_y}{\epsilon_z} \quad (8.13)$$

The negative sign is included in the expression so that ν will always be positive, since ϵ_x and ϵ_z will always be of opposite sign. Theoretically, Poisson's ratio for isotropic materials should be $\frac{1}{2}$; furthermore, the maximum value for ν (or that value for which there is no net volume change) is 0.50. For many metals and other alloys, values of Poisson's ratio range between 0.25 and 0.35. Table 8.2 shows values for several common metallic materials.

For isotropic materials, shear and elastic moduli are related to each other and to Poisson's ratio according to

$$E = 2G(1 + \nu) \quad (8.14)$$

In most metals G is about $0.4E$; thus, if the value of one modulus is known, the other may be approximated. Many materials are elastically anisotropic; that is, the elastic behavior (e.g., the magnitude of E) varies with crystallographic direction (see Table 8.3). For these materials the elastic properties are completely characterized only by the specification.

Table 8.3 Modulus of Elasticity Values for Several Metals at Various Crystallographic Orientations.

<i>Metal</i>	<i>Modulus of Elasticity (GPa)</i>		
	<i>[100]</i>	<i>[110]</i>	<i>[111]</i>
Aluminum	63.7	72.6	76.1
Copper	66.7	130.3	191.1
Iron	125.0	210.5	272.7
Tungsten	384.6	384.6	384.6

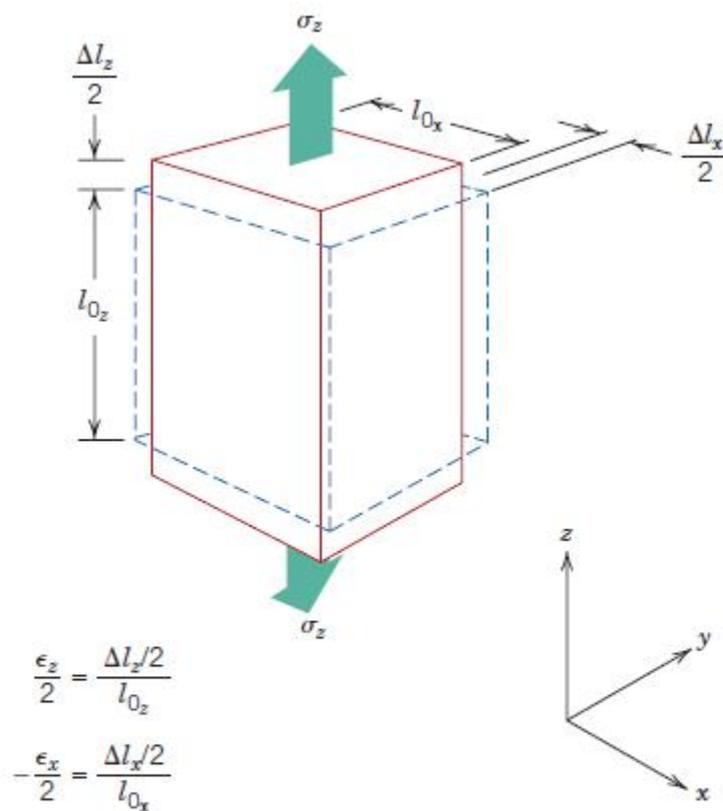


Figure 8.14 Axial (z) elongation (positive strain) and lateral (x and y) contractions (negative strains) in response to an imposed tensile stress. Solid lines represent dimensions after stress application; dashed lines, before.

of several elastic constants, their number depending on characteristics of the crystal structure. Even for isotropic materials, for complete characterization of the elastic properties, at least two constants must be given. Since the grain orientation is random in most polycrystalline materials, these may be considered to be isotropic; inorganic ceramic glasses are also isotropic. The remaining discussion of mechanical behavior assumes isotropy and polycrystallinity because such is the character of most engineering materials.

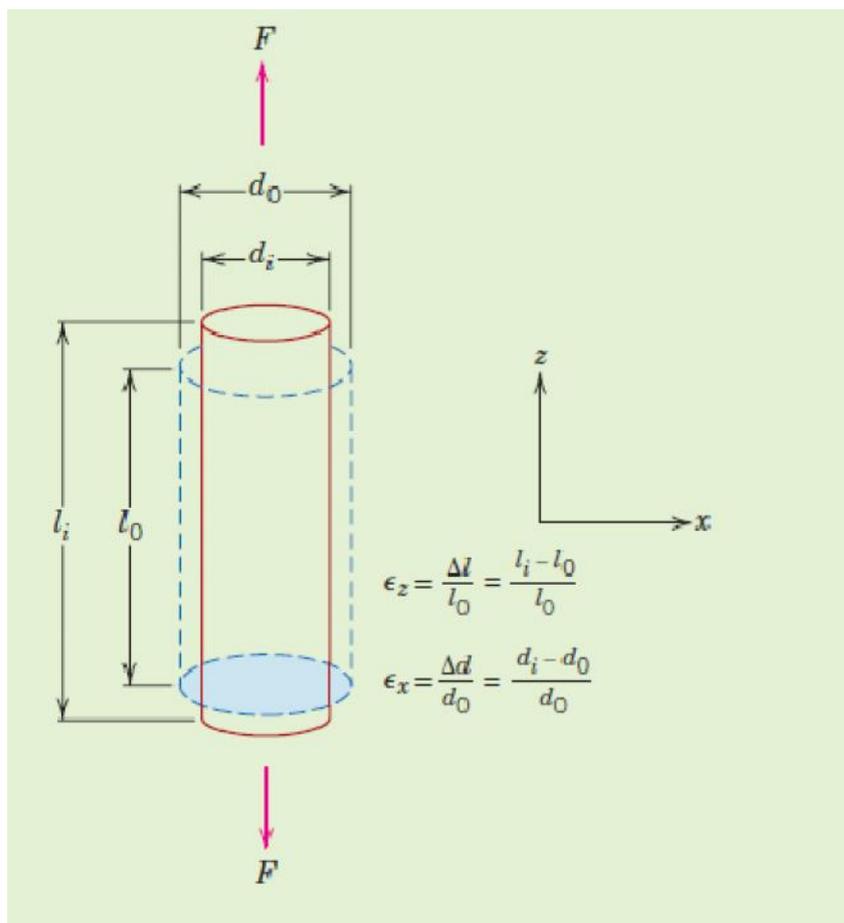
****Example Problem (6.b)***

Computation of Load to Produce Specified Diameter Change

A tensile stress is to be applied along the long axis of a cylindrical brass rod that has a diameter of 10 mm (0.4 in.). Determine the magnitude of the load required to produce a 2.5×10^{-3} mm (10^{-4} in) change in diameter if the deformation is entirely elastic.

Solution

This deformation situation is represented in the accompanying drawing



When the force F is applied, the specimen will elongate in the z direction and at the same time experience a reduction in diameter, $\Delta d, 2.5 \times 10^{-3}$ mm in the x direction. For the strain in the x direction,

$$\epsilon_x = \frac{\Delta d}{d_0} = \frac{-2.5 \times 10^{-3} \text{ mm}}{10 \text{ mm}} = -2.5 \times 10^{-4}$$

Which is negative, since the diameter is reduce.

It next necessary to calculate the strain in the z direction using equation 8.13 .The value for Poisson's ratio for brass is 0.34 (Table 8.2),and thus

$$\epsilon_z = -\frac{\epsilon_x}{\nu} = -\frac{(-2.5 \times 10^{-4})}{0.34} = 7.35 \times 10^{-4}$$

The applied stress may now be computed using equation 8.10 and the modulus of elasticity is given by table 8.2 as 97 GPa (14×10^6 psi), as

$$\sigma = \epsilon_z E = (7.35 \times 10^{-4})(97 \times 10^3 \text{ MPa}) = 71.3 \text{ MPa}$$

Finally, from equation 8.8 the applied force may be determined as

$$F = \sigma A_0 = \sigma \left(\frac{d_0}{2} \right)^2 \pi$$

$$= (71.3 \times 10^6 \text{ N/m}^2) \left(\frac{10 \times 10^{-3} \text{ m}}{2} \right)^2 \pi = 5600 \text{ N} (1293 \text{ lb}_f)$$

8.2.7 Plastic Deformation

For most metallic materials, elastic deformation persists only to strains of about 0.005. As the material is deformed beyond this point, the stress is no longer proportional to strain (Hooke's law, Equation 8.10, ceases to be valid), and permanent, nonrecoverable, or plastic deformation occurs. Figure 8.15a plots schematically the tensile stress–strain behavior into the plastic region for a typical metal. The transition from elastic to plastic is a gradual one for most metals; some curvature results at the onset of plastic deformation, which increases more rapidly with rising stress.

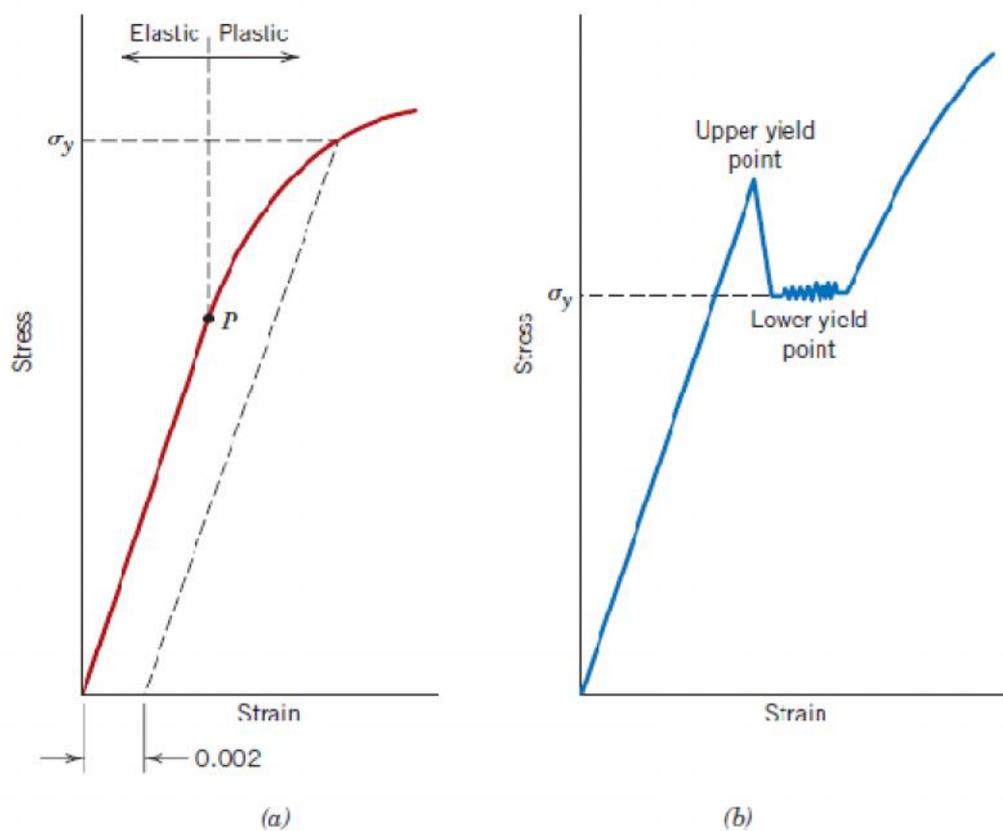


Figure 8.15

(a) Typical stress–strain behavior for a metal showing elastic and plastic deformations, the proportional limit P , and the yield strength σ_y as determined using the 0.002 strain offset method.

(b) Representative stress–strain behavior found for some steels demonstrating the yield point phenomenon.

From an atomic perspective, plastic deformation corresponds to the breaking of bonds with original atom neighbors and then reforming bonds with new neighbors as large numbers of atoms or molecules move relative to one another; upon removal of the stress they do not return to their original positions. The mechanism of this deformation is different for crystalline and amorphous materials. For crystalline solids, deformation is accomplished by means of a process called slip, which involves the motion of dislocations. Plastic deformation in noncrystalline solids (as well as liquids) occurs by a viscous flow mechanism.

8.2.8 Tensile Properties

8.2.8.1 Yielding and Yield Strength

Most structures are designed to ensure that only elastic deformation will result when a stress is applied. A structure or component that has plastically deformed, or experienced a permanent change in shape, may not be capable of functioning as intended.

It is therefore desirable to know the stress level at which plastic deformation begins, or where the phenomenon of yielding occurs. For metals that experience this gradual elastic–plastic transition, the point of yielding may be determined as the initial departure from linearity of the stress–strain curve; this is sometimes called the *proportional limit*, as indicated by point *P* in Figure 8.15a. In such cases the position of this point may not be determined precisely. As a consequence, a convention has been established wherein a straight line is constructed parallel to the elastic portion of the stress–strain curve at some specified strain offset, usually 0.002.

The stress corresponding to the intersection of this line and the stress–strain curve as it bends over in the plastic region is defined as the *yield strength* σ_y . This is demonstrated in Figure 8.15a. Of course, the units of yield strength are MPa or psi.

For those materials having a nonlinear elastic region (Figure 8.9), use of the strain offset method is not possible, and the usual practice is to define the yield strength as the stress required to produce some amount of strain (e.g., $\epsilon=0.005$).

Some steels and other materials exhibit the tensile stress–strain behavior as shown in Figure 8.15b. The elastic–plastic transition is very well defined and occurs abruptly in what is termed a yield point phenomenon. At the upper yield point, plastic deformation is initiated with an actual decrease in stress. Continued deformation fluctuates slightly about some constant stress value, termed the lower yield point;

stress subsequently rises with increasing strain. For metals that display this effect, the yield strength is taken as the average stress that is associated with the lower yield point, since it is well defined and relatively insensitive to the testing procedure.

Thus, it is not necessary to employ the strain offset method for these materials. The magnitude of the yield strength for a metal is a measure of its resistance to plastic deformation. Yield strengths may range from 35 MPa (5000 psi) for a low strength aluminum to over 1400 MPa (200,000 psi) for high-strength steels.

8.2.8.2 Tensile Strength

After yielding, the stress necessary to continue plastic deformation in metals increases to a maximum, point M in Figure 8.16, and then decreases to the eventual fracture, point F. The *tensile strength* TS (MPa or psi) is the stress at the maximum on the engineering stress–strain curve (Figure 8.16). This corresponds to the maximum stress that can be sustained by a structure in tension; if this stress is applied and maintained, fracture will result. All deformation up to this point is uniform throughout the narrow region of the tensile specimen. However, at this maximum stress, a small constriction or neck begins to form at some point, and all subsequent deformation is confined at this neck, as indicated by the schematic specimen insets in Figure 8.16. This phenomenon is termed “necking,” and fracture ultimately occurs at the neck. The fracture strength corresponds to the stress at fracture.

Tensile strengths may vary anywhere from 50 MPa (7000 psi) for an aluminum to as high as 3000 MPa (450,000 psi) for the high-strength steels. Ordinarily, when the strength of a metal is cited for design purposes, the yield strength is used. This is because by the time a stress corresponding to the tensile strength has been applied, often a structure has experienced so much plastic deformation that it is useless. Furthermore, fracture strengths are not normally specified for engineering design purposes.

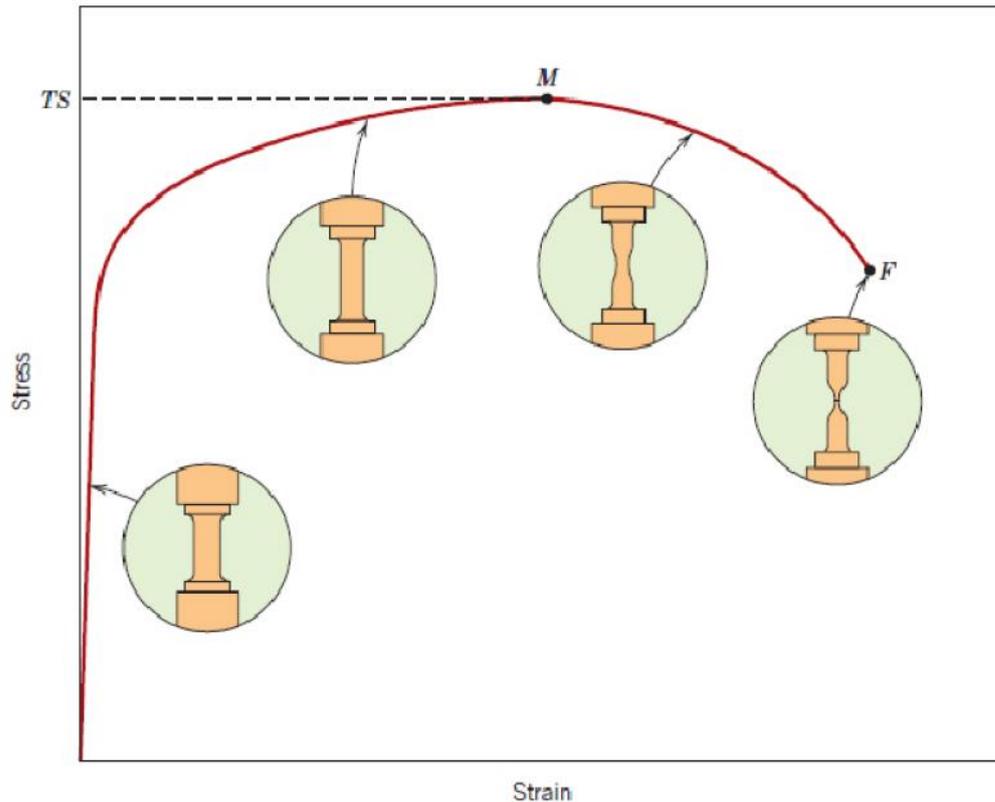


Figure 8.16 Typical engineering stress– strain behavior to fracture, point F . The tensile strength TS is indicated at point M . The circular insets represent the geometry of the deformed specimen at various points along the curve.

****Example Problem (6.c)***

Mechanical property for determinations from stress-strain plot.

From the tensile stress-strain behavior for the brass specimen shown in figure 8.a ,determine the following:

- The modulus of elasticity.
- The yield strength at a strain offset of 0.002.
- The maximum load that can sustained by a cylinder specimen having an original diameter of 12.8 mm (0.505 in).
- The change in the length of specimen originally 250mm (10 in)long that is subject to a tensile stress of 345 MPa(50.000 psi).

Solution

a)The modulus of elasticity is the slope of the elastic or initial linear properties of the stress-strain curve. The strain axis has been expanded in the inset, Figure 8.b ,to facilitate this computation. The slope of this linear region is the rise over the run, or the change in stress divided by the corresponding change in strain ;in mathematical terms,

$$E = \text{slope} = \frac{\Delta\sigma}{\Delta\epsilon} = \frac{\sigma_2 - \sigma_1}{\epsilon_2 - \epsilon_1} \quad (6.a)$$

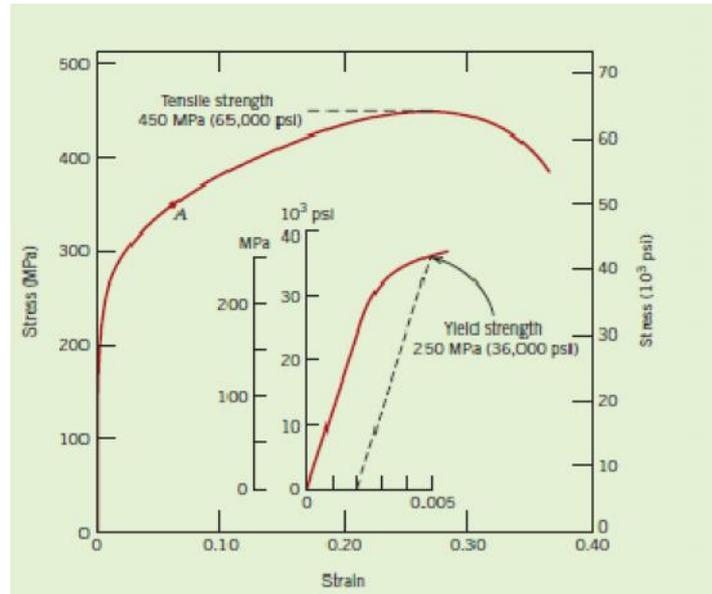


Figure 8.a The stress-strain behavior for the brass specimen discussed in example problem 6.c.

Inasmuch as the line segment passes through the origin, it is convenient to take both σ_1 and ϵ_1 as zero. If σ_2 is arbitrarily taken as 150 MPa, then ϵ_2 will have a value of 0.0016. Therefore,

$$E = \frac{(150 - 0) \text{ MPa}}{0.0016 - 0} = 93.8 \text{ GPa} (13.6 \times 10^6 \text{ psi})$$

Which is very close to the value of 97 GPa (14×10^6 psi) given for brass in Table 8.2.

b) The 0.002 strain offset line is constructed as shown in the inset, its intersection with the stress-strain curve is at approximately 250 MPa (36,000 psi), which is the yield strength of the brass.

c) The maximum load that can be sustained by the specimen is calculated by using equation 8.6, in which σ is taken to the

tensile strength ,from figure 8.a ,450MPa(65.000 psi).Solving for F,the maximum load, yields

$$F = \sigma A_0 = \sigma \left(\frac{d_0}{2} \right)^2 \pi$$

$$= (450 \times 10^6 \text{ N/m}^2) \left(\frac{12.8 \times 10^{-3} \text{ m}}{2} \right)^2 \pi = 57,900 \text{ N (13,000 lbf)}$$

d)To compute the change in length , Δl ,in equation 8.7,it is first necessary to determine the strain that is produced by a stress of 345 MPa.This is accomplished by locating the stress point on the stress-strain curve, point A and reading the corresponding strain from the strain axis, which is approximately 0.06.Inasmuch as $l_0 = 250 \text{ mm}$,we have

$$\Delta l = \epsilon l_0 = (0.06)(250 \text{ mm}) = 15 \text{ mm (0.6 in.)}$$

8.2.8.3 Ductility

Ductility is another important mechanical property. It is a measure of the degree of plastic deformation that has been sustained at fracture. A material that experiences very little or no plastic deformation upon fracture is termed brittle. The tensile stress–strain behaviors for both ductile and brittle materials are schematically illustrated in Figure 8.17.

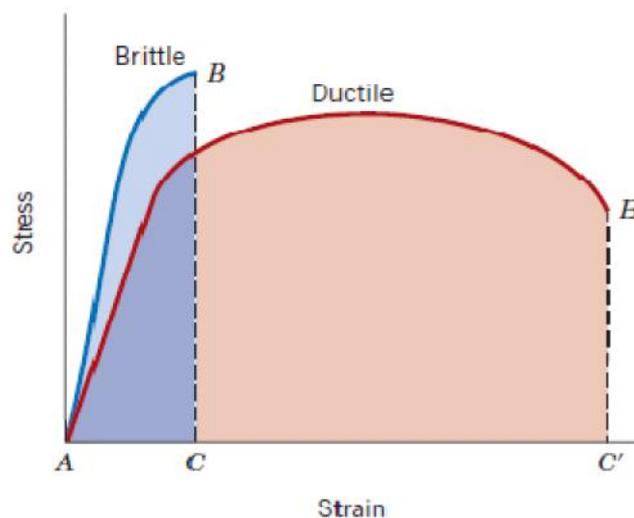


Figure 8.17 Schematic representations of tensile stress–strain behavior for brittle and ductile materials loaded to fracture.

Ductility may be expressed quantitatively as either *percent elongation* or *percent reduction in area*. The percent elongation %EL is the percentage of plastic strain at fracture, or

$$\% \text{EL} = \left(\frac{l_f - l_0}{l_0} \right) \times 100 \quad (8.15)$$

Where l_f is the fracture length and l_0 is the original gauge length as above. Inasmuch as a significant proportion of the plastic deformation at fracture is confined to the neck region, the magnitude of %EL will depend on specimen gauge length. The shorter l_0 the greater is the fraction of total elongation from the neck and, consequently, the higher the value of %EL. Therefore, l_0 should be specified when percent elongation values are cited; it is commonly 50 mm (2 in.). Percent reduction in area %RA is defined as

$$\% \text{RA} = \left(\frac{A_0 - A_f}{A_0} \right) \times 100 \quad (8.16)$$

Where A_0 is the original cross-sectional area and A_f is the cross-sectional area at the point of fracture. Percent reduction in area values are independent of both l_0 and A_0 .

Furthermore, for a given material the magnitudes of %EL and %RA will, in general, be different. Most metals possess at least a moderate degree of ductility at room temperature; however, some become brittle as the temperature is lowered.

A knowledge of the ductility of materials is important for at least two reasons. First, it indicates to a designer the degree to which a structure will deform plastically before fracture. Second, it specifies the degree of allowable deformation during fabrication operations. We sometimes refer to relatively ductile materials as being “forgiving,” in the sense that they may experience local deformation without fracture should there be an error in the magnitude of the design stress calculation. Brittle materials are *approximately* considered to be those having a fracture strain of less

than about 5%. Thus, several important mechanical properties of metals may be determined from tensile stress–strain tests. Table 8.4 presents some typical room-temperature values of yield strength, tensile strength, and ductility for several of the common metals. These properties are sensitive to any prior deformation, the presence of impurities, and/or any heat treatment to which the metal has been subjected. The modulus of elasticity is one mechanical parameter that is insensitive to these treatments. As with modulus of elasticity, the magnitudes of both yield and tensile strengths decline with increasing temperature; just the reverse holds for ductility—it usually increases with temperature. Figure 8.18 shows how the stress–strain behavior of iron varies with temperature.

Table 8.4 Typical Mechanical Properties of Several Metals and Alloys in an Annealed State.

<i>Metal Alloy</i>	<i>Yield Strength MPa (ksi)</i>	<i>Tensile Strength MPa (ksi)</i>	<i>Ductility, %EL [in 50 mm (2 in.)]</i>
Aluminum	35 (5)	90 (13)	40
Copper	69 (10)	200 (29)	45
Brass (70Cu–30Zn)	75 (11)	300 (44)	68
Iron	130 (19)	262 (38)	45
Nickel	138 (20)	480 (70)	40
Steel (1020)	180 (26)	380 (55)	25
Titanium	450 (65)	520 (75)	25
Molybdenum	565 (82)	655 (95)	35

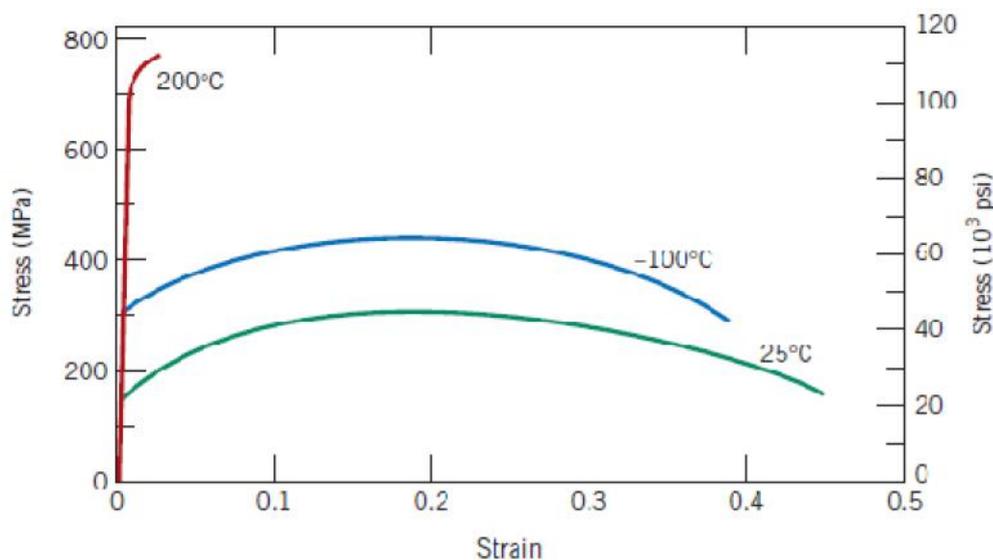


Figure 8.18 Engineering stress– strain behavior for iron at three Temperatures.

8.2.8.4 Resilience

Resilience is the capacity of a material to absorb energy when it is deformed elastically and then, upon unloading, to have this energy recovered. The associated property is the *modulus of resilience*, U_r , which is the strain energy per unit volume required to stress a material from an unloaded state up to the point of yielding. Computationally, the modulus of resilience for a specimen subjected to a uniaxial tension test is just the area under the engineering stress–strain curve taken to yielding figure 8.19 or

$$U_r = \int_0^{\epsilon_y} \sigma d\epsilon \quad (8.17 a)$$

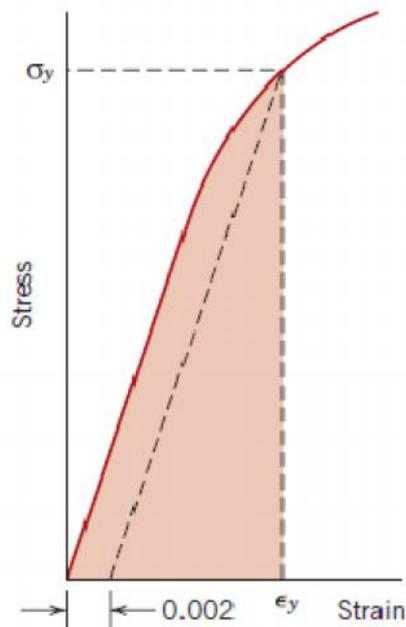


Figure 8.19 Schematic representation showing how modulus of resilience (corresponding to the shaded area) is determined from the tensile stress–strain behavior of a material.

Assuming a linear elastic region,

$$U_r = \frac{1}{2}\sigma_y\epsilon_y \quad (8.17 \text{ b})$$

in which ϵ_y is the strain at yielding.

The units of resilience are the product of the units from each of the two axes of the stress–strain plot. For SI units, this is joules per cubic meter (J/m^3 , equivalent to Pa), whereas with Customary U.S. units it is inch-pounds force per cubic inch ($\text{in.-lb}_f/\text{in.}^3$, equivalent to psi). Both joules and inch-pounds force are units of energy, and thus this area under the stress–strain curve represents energy absorption per unit volume (in cubic meters or cubic inches) of material. Incorporation of Equation 8.10 into Equation 8.17b yields

$$U_r = \frac{1}{2}\sigma_y\epsilon_y = \frac{1}{2}\sigma_y\left(\frac{\sigma_y}{E}\right) = \frac{\sigma_y^2}{2E} \quad (8.18)$$

Thus, resilient materials are those having high yield strengths and low moduli of elasticity; such alloys would be used in spring applications.

8.2.8.5 Toughness

Toughness is a mechanical term that is used in several contexts; loosely speaking, it is a measure of the ability of a material to absorb energy up to fracture. Specimen geometry as well as the manner of load application are important in toughness determinations. For dynamic (high strain rate) loading conditions and when a notch (or point of stress concentration) is present, notch toughness is assessed by using an impact test. Furthermore, fracture toughness is a property indicative of a material's resistance to fracture when a crack is present.

For the static (low strain rate) situation, toughness may be ascertained from the results of a tensile stress–strain test. It is the area under the $\sigma - \epsilon$ curve up to the point of fracture. The units for toughness are the same as for resilience (i.e., energy per unit volume of material). For a material to be tough, it must display both strength and ductility; often, ductile materials are tougher than brittle ones.

This is demonstrated in Figure 8.17, in which the stress–strain curves are plotted for both material types. Hence, even though the brittle material has higher yield and tensile strengths, it has a lower toughness than the ductile one, by virtue of lack of ductility; this is deduced by comparing the areas ABC and AB'C' in figure 8.17.

Table 8.5 Tensile Stress–Strain Data for Several Hypothetical Metals to be Used with Concept Checks 8.4 and 8.6

<i>Material</i>	<i>Yield Strength (MPa)</i>	<i>Tensile Strength (MPa)</i>	<i>Strain at Fracture</i>	<i>Fracture Strength (MPa)</i>	<i>Elastic Modulus (GPa)</i>
A	310	340	0.23	265	210
B	100	120	0.40	105	150
C	415	550	0.15	500	310
D	700	850	0.14	720	210
E	Fractures before yielding			650	350

8.2.9 True Stress and Strain

From Figure 8.16, the decline in the stress necessary to continue deformation past the maximum, point M, seems to indicate that the metal is becoming weaker. This is not at all the case; as a matter of fact, it is increasing in strength. However, the cross-sectional area is decreasing rapidly within the neck region, where deformation is occurring. This results in a reduction in the load-bearing capacity of the specimen.

The stress, as computed from Equation 8.6, is on the basis of the original cross-sectional area before any deformation, and does not take into account this reduction in area at the neck.

Sometimes it is more meaningful to use a true stress–true strain scheme. True stress σ_T is defined as the load F divided by the instantaneous cross-sectional area A_i over which deformation is occurring (i.e., the neck, past the tensile point), or

$$\sigma_T = \frac{F}{A_i} \quad (8.19)$$

Furthermore, it is occasionally more convenient to represent strain as true strain defined by ϵ_T

$$\epsilon_T = \ln \frac{l_i}{l_0} \quad (8.20)$$

If no volume change occurs during deformation—that is, if

$$A_i l_i = A_0 l_0 \quad (8.21)$$

true and engineering stress and strain are related according to

$$\sigma_T = \sigma(1 + \epsilon) \quad (8.22 \text{ a})$$

$$\epsilon_T = \ln(1 + \epsilon) \quad (8.22 \text{ b})$$

Equations 8.22a and 8.22b are valid only to the onset of necking; beyond this point true stress and strain should be computed from actual load, cross-sectional area, and gauge length measurements.

A schematic comparison of engineering and true stress–strain behaviors is made in Figure 8.20. It is worth noting that the true stress necessary to sustain increasing strain continues to rise past the tensile point M' .

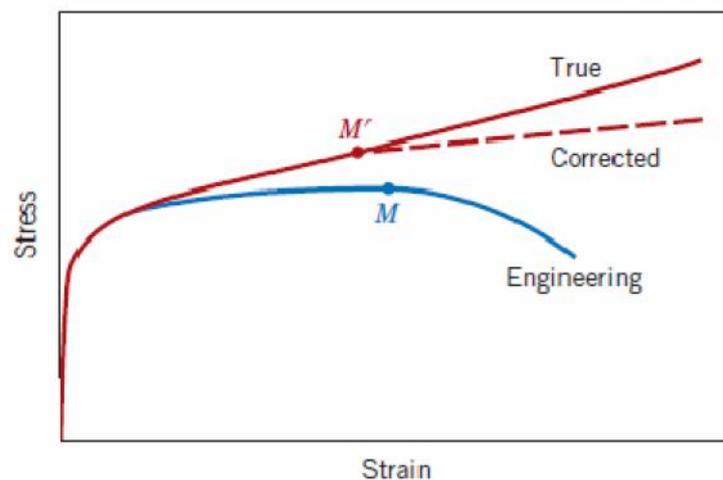


Figure 8.20 A comparison of typical tensile engineering stress–strain and true stress–strain behaviors. Necking begins at point M on the engineering curve, which corresponds to M' on the true curve. The

“corrected” true stress– strain curve takes into account the complex stress state within the neck region.

Coincident with the formation of a neck is the introduction of a complex stress state within the neck region (i.e., the existence of other stress components in addition to the axial stress). As a consequence, the correct stress (*axial*) within the neck is slightly lower than the stress computed from the applied load and neck crosssectional area. This leads to the “corrected” curve in Figure 8.20.

For some metals and alloys the region of the true stress–strain curve from the onset of plastic deformation to the point at which necking begins may be approximated by

$$\sigma_T = K\epsilon_T^n \quad (8.23)$$

In this expression, K and n are constants; these values will vary from alloy to alloy, and will also depend on the condition of the material (i.e., whether it has been plastically deformed, heat treated, etc.). The parameter n is often termed the *strainhardening exponent* and has a value less than unity. Values of n and K for several alloys are contained in Table 8.6.

Table 8.6 Tabulation of n and K Values (Equation 8.23) for Several Alloys.

<i>Material</i>	<i>n</i>	<i>K</i>	
		<i>MPa</i>	<i>psi</i>
Low-carbon steel (annealed)	0.21	600	87,000
4340 steel alloy (tempered @ 315°C)	0.12	2650	385,000
304 stainless steel (annealed)	0.44	1400	205,000
Copper (annealed)	0.44	530	76,500
Naval brass (annealed)	0.21	585	85,000
2024 aluminum alloy (heat treated—T3)	0.17	780	113,000
AZ-31B magnesium alloy (annealed)	0.16	450	66,000

Example problem (6.d)**Ductility and True-Stress-At-Fracture Computations***

A cylindrical specimen of steel having an original diameter of 12.8 mm (0.505 in).is tensile tested to fracture and found to have an engineering fracture strength σ_f of 460 MPa(67.000psi).If its cross-sectional diameter at fracture is 10.7 mm (0.422 in),determine:

- The ductility in terms of percent reduction in area.
- The true stress at fracture

Solution

- Ductility is computed using equation 8.16

$$\begin{aligned} \%RA &= \frac{\left(\frac{12.8 \text{ mm}}{2}\right)^2 \pi - \left(\frac{10.7 \text{ mm}}{2}\right)^2 \pi}{\left(\frac{12.8 \text{ mm}}{2}\right)^2 \pi} \times 100 \\ &= \frac{128.7 \text{ mm}^2 - 89.9 \text{ mm}^2}{128.7 \text{ mm}^2} \times 100 = 30\% \end{aligned}$$

b)True stress is defined by equation 8.19,where in this case the area is taken as the fracture area A_f However, the load at fracture must first be computed from the fracture strength as

$$F = \sigma_f A_0 = (460 \times 10^6 \text{ N/m}^2)(128.7 \text{ mm}^2) \left(\frac{1 \text{ m}^2}{10^6 \text{ mm}^2}\right) = 59,200 \text{ N}$$

Thus ,the true stress is calculated as

$$\begin{aligned} \sigma_T &= \frac{F}{A_f} = \frac{59,200 \text{ N}}{(89.9 \text{ mm}^2) \left(\frac{1 \text{ m}^2}{10^6 \text{ mm}^2}\right)} \\ &= 6.6 \times 10^8 \text{ N/m}^2 = 660 \text{ MPa (95,700 psi)} \end{aligned}$$

Example problem(6.e)**Calculation of Strain-Hardening Exponent***

Compute the strain-hardening exponent n in equation 8.23 for an alloy in which a true stress of 415 MPa (60.000 psi) produces a true strain of 0.10; assume a value of 1035 MPa(150.000psi) for K .

Solution

The requires some algebraic manipulation of equation 8.23 so that n becomes the dependent parameter. This is accomplished by taking logarithms and rearranging .Solving for n yields

$$n = \frac{\log \sigma_T - \log K}{\log \epsilon_T}$$

$$= \frac{\log(415 \text{ MPa}) - \log(1035 \text{ MPa})}{\log(0.1)} = 0.40$$

8.2.10 Elastic Recovery after Plastic deformation

Upon release of the load during the course of a stress–strain test, some fraction of the total deformation is recovered as elastic strain. This behavior is demonstrated in Figure 8. 21, a schematic engineering stress–strain plot. During the unloading cycle, the curve traces a near straight-line path from the point of unloading (point D), and its slope is virtually identical to the modulus of elasticity, or parallel to the initial elastic portion of the curve. The magnitude of this elastic strain, which is regained during unloading, corresponds to the strain recovery, as shown in Figure 8.21. If the load is reapplied, the curve will traverse essentially the same linear portion in the direction opposite to unloading; yielding will again occur at the unloading stress level where the unloading began. There will also be an elastic strain recovery associated with fracture.

8.2.11 Compressive ,Shear, and Torsional Deformation

Of course, metals may experience plastic deformation under the influence of applied compressive, shear, and torsional loads. The resulting stress–strain behavior into the plastic region will be similar to the tensile counterpart (Figure 8.15a: yielding and the associated curvature). However, for compression, there will be no maximum, since necking does not occur; furthermore, the mode of fracture will be different from that for tension.

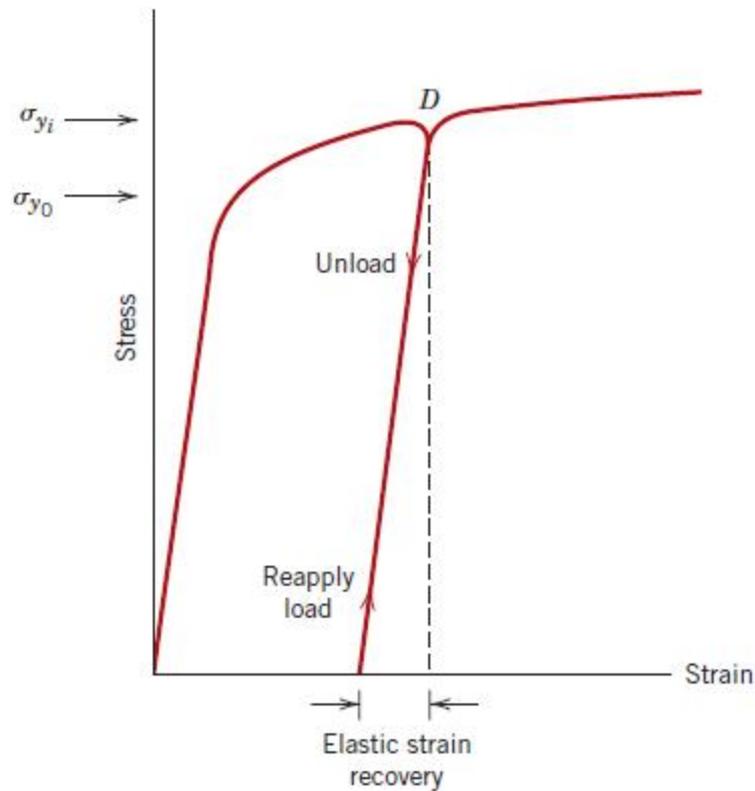


Figure 8.21 Schematic tensile stress–strain diagram showing the phenomena of elastic strain recovery and strain hardening. The initial yield strength is designated as σ_{y0} , σ_{yi} is the yield strength after releasing the load at point D, and then upon reloading.

8.2.12 Hardness

Another mechanical property that may be important to consider is hardness, which is a measure of a material's resistance to localized plastic deformation (e.g., a small dent or a scratch). Early hardness tests were based on natural minerals with a scale constructed solely on the ability of one material to scratch another that was softer.

A qualitative and somewhat arbitrary hardness indexing scheme was devised, termed the Mohs scale, which ranged from 1 on the soft end for talc to 10 for diamond. Quantitative hardness techniques have been developed over the years in which a small indenter is forced into the surface of a material to be tested, under controlled conditions of load and rate of application. The depth or size of the resulting indentation is measured, which in turn is related to a hardness number; the softer the material, the larger and deeper is the indentation, and the lower the hardness index number. Measured hardnesses are only relative (rather

than absolute), and care should be exercised when comparing values determined by different techniques.

Hardness tests are performed more frequently than any other mechanical test for several reasons:

1. They are simple and inexpensive—ordinarily no special specimen need be prepared, and the testing apparatus is relatively inexpensive.
2. The test is nondestructive—the specimen is neither fractured nor excessively deformed; a small indentation is the only deformation.
3. Other mechanical properties often may be estimated from hardness data, such as tensile strength (see Figure 8.23).

8.2.12.(A) Rockwell Hardness Tests:

The Rockwell tests constitute the most common method used to measure hardness because they are so simple to perform and require no special skills. Several different scales may be utilized from possible combinations of various indenters and different loads, which permit the testing of virtually all metal alloys (as well as some polymers).

Indenters include spherical and hardened steel balls having diameters of $\frac{1}{16}$, $\frac{1}{8}$, $\frac{1}{4}$ and $\frac{1}{2}$ in. (1.588, 3.175, 6.350, and 12.70 mm), and a conical diamond (Brale) indenter, which is used for the hardest materials.

With this system, a hardness number is determined by the difference in depth of penetration resulting from the application of an initial minor load followed by a larger major load; utilization of a minor load enhances test accuracy. On the basis of the magnitude of both major and minor loads, there are two types of tests: Rockwell and superficial Rockwell. For Rockwell, the minor load is 10 kg, whereas major loads are 60, 100, and 150 kg. Each scale is represented by a letter of the alphabet; several are listed with the corresponding indenter and load in Tables 8.6 and 8.7a.

For superficial tests, 3 kg is the minor load; 15, 30, and 45 kg are the possible major load values. These scales are identified by a 15, 30, or 45 (according to load), followed by N, T, W, X, or Y, depending on indenter. Superficial tests are frequently performed on thin specimens. Table 6.6b presents several superficial scales.

When specifying Rockwell and superficial hardnesses, both hardness number and scale symbol must be indicated. The scale is designated by the symbol HR

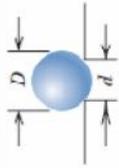
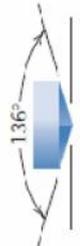
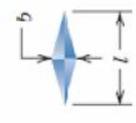
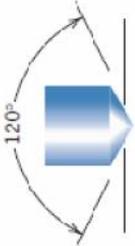
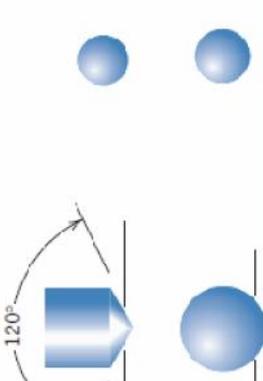
Test	Shape of Indentation			Formula for Hardness Number ^a
	Indenter	Side View	Top View	
Brinell	10-mm sphere of steel or tungsten carbide			$HB = \frac{2P}{\pi D [D - \sqrt{D^2 - d^2}]}$
Vickers microhardness	Diamond pyramid			$HV = 1.854P/d_1^2$
Knoop microhardness	Diamond pyramid			$HK = 14.2P/l^2$
Rockwell and Superficial Rockwell	Diamond cone; $\frac{1}{16}, \frac{1}{8}, \frac{1}{4}, \frac{1}{2}$ in. diameter steel spheres			$\left. \begin{matrix} 60 \text{ kg} \\ 100 \text{ kg} \\ 150 \text{ kg} \end{matrix} \right\} \text{Rockwell}$ $\left. \begin{matrix} 15 \text{ kg} \\ 30 \text{ kg} \\ 45 \text{ kg} \end{matrix} \right\} \text{Superficial Rockwell}$

Table 8.6

Hardness – Testing Techniques

Table 8.7a Rockwell Hardness Scales

<i>Scale Symbol</i>	<i>Indenter</i>	<i>Major Load (kg)</i>
A	Diamond	60
B	$\frac{1}{16}$ -in. ball	100
C	Diamond	150
D	Diamond	100
E	$\frac{1}{8}$ -in. ball	100
F	$\frac{1}{16}$ -in. ball	60
G	$\frac{1}{16}$ -in. ball	150
H	$\frac{1}{8}$ -in. ball	60
K	$\frac{1}{8}$ -in. ball	150

Table 8.7b Superficial Rockwell Hardness Scales

<i>Scale Symbol</i>	<i>Indenter</i>	<i>Major Load (kg)</i>
15N	Diamond	15
30N	Diamond	30
45N	Diamond	45
15T	$\frac{1}{16}$ -in. ball	15
30T	$\frac{1}{16}$ -in. ball	30
45T	$\frac{1}{16}$ -in. ball	45
15W	$\frac{1}{8}$ -in. ball	15
30W	$\frac{1}{8}$ -in. ball	30
45W	$\frac{1}{8}$ -in. ball	45

followed by the appropriate scale identification. For example, 80 HRB represents a Rockwell hardness of 80 on the B scale, and 60 HR30W indicates a superficial hardness of 60 on the 30W scale.

For each scale, hardnesses may range up to 130; however, as hardness values rise above 100 or drop below 20 on any scale, they become inaccurate; and because the scales have some overlap, in such a situation it is best to utilize the next harder or softer scale.

Inaccuracies also result if the test specimen is too thin, if an indentation is made too near a specimen edge, or if two indentations are made too close to one another. Specimen thickness should be at least ten times the indentation depth, whereas allowance should be made for at least three indentation diameters between the center of one indentation and the specimen edge, or to the center of a second indentation. Furthermore,

testing of specimens stacked one on top of another is not recommended. Also, accuracy is dependent on the indentation being made into a smooth flat surface. The modern apparatus for making Rockwell hardness measurements (see the chapter-opening photograph for this chapter) is automated and very simple to use; hardness is read directly, and each measurement requires only a few seconds. The modern testing apparatus also permits a variation in the time of load application. This variable must also be considered in interpreting hardness data.

8.2.12.(B) Brinell Hardness Tests:

In Brinell tests, as in Rockwell measurements, a hard, spherical indenter is forced into the surface of the metal to be tested. The diameter of the hardened steel (or tungsten carbide) indenter is 10.00 mm (0.394 in.). Standard loads range between 500 and 3000 kg in 500-kg increments; during a test, the load is maintained constant for a specified time (between 10 and 30 s). Harder materials require greater applied loads. The Brinell hardness number, HB, is a function of both the magnitude of the load and the diameter of the resulting indentation (see Table 8.6).

This diameter is measured with a special low-power microscope, utilizing a scale that is etched on the eyepiece. The measured diameter is then converted to the appropriate HB number using a chart; only one scale is employed with this technique. Semiautomatic techniques for measuring Brinell hardness are available. These employ optical scanning systems consisting of a digital camera mounted on a flexible probe, which allows positioning of the camera over the indentation. Data from the camera are transferred to a computer that analyzes the indentation, determines its size, and then calculates the Brinell hardness number. For this technique, surface finish requirements are normally more stringent than for manual measurements.

Maximum specimen thickness as well as indentation position (relative to specimen edges) and minimum indentation spacing requirements are the same as for Rockwell tests. In addition, a well-defined indentation is required; this necessitates a smooth flat surface in which the indentation is made.

8.2.12. (C) Knoop and Vickers Microindentation Hardness Tests:

Two other hardness-testing techniques are Knoop (pronounced *nūp*) and Vickers (sometimes also called diamond pyramid). For each test a very small diamond indenter having pyramidal geometry is forced into the

surface of the specimen. Applied loads are much smaller than for Rockwell and Brinell, ranging between 1 and 1000 g. The resulting impression is observed under a microscope and measured; this measurement is then converted into a hardness number (Table 8.6). Careful specimen surface preparation (grinding and polishing) may be necessary to ensure a well-defined indentation that may be accurately measured. The Knoop and Vickers hardness numbers are designated by HK and HV, respectively,

and hardness scales for both techniques are approximately equivalent. Knoop and Vickers are referred to as microindentation-testing methods on the basis of indenter size. Both are well suited for measuring the hardness of small, selected specimen regions; furthermore, Knoop is used for testing brittle materials such as ceramics. The modern microindentation hardness-testing equipment has been automated by coupling the indenter apparatus to an image analyzer that incorporates a computer and software package. The software controls important system functions to include indent location, indent spacing, computation of hardness values, and plotting of data.

Other hardness-testing techniques are frequently employed but will not be discussed here; these include ultrasonic microhardness, dynamic (Scleroscope), durometer (for plastic and elastomeric materials), and scratch hardness tests.

8.2.13. Hardness Conversion

The facility to convert the hardness measured on one scale to that of another is most desirable. However, since hardness is not a well-defined material property, and because of the experimental dissimilarities among the various techniques, a comprehensive conversion scheme has not been devised. Hardness conversion data have been determined experimentally and found to be dependent on material type and characteristics. The most reliable conversion data exist for steels, some of which are presented in Figure 8.22 for Knoop, Brinell, and two Rockwell scales; the Mohs scale is also included. Detailed conversion tables for various other metals and

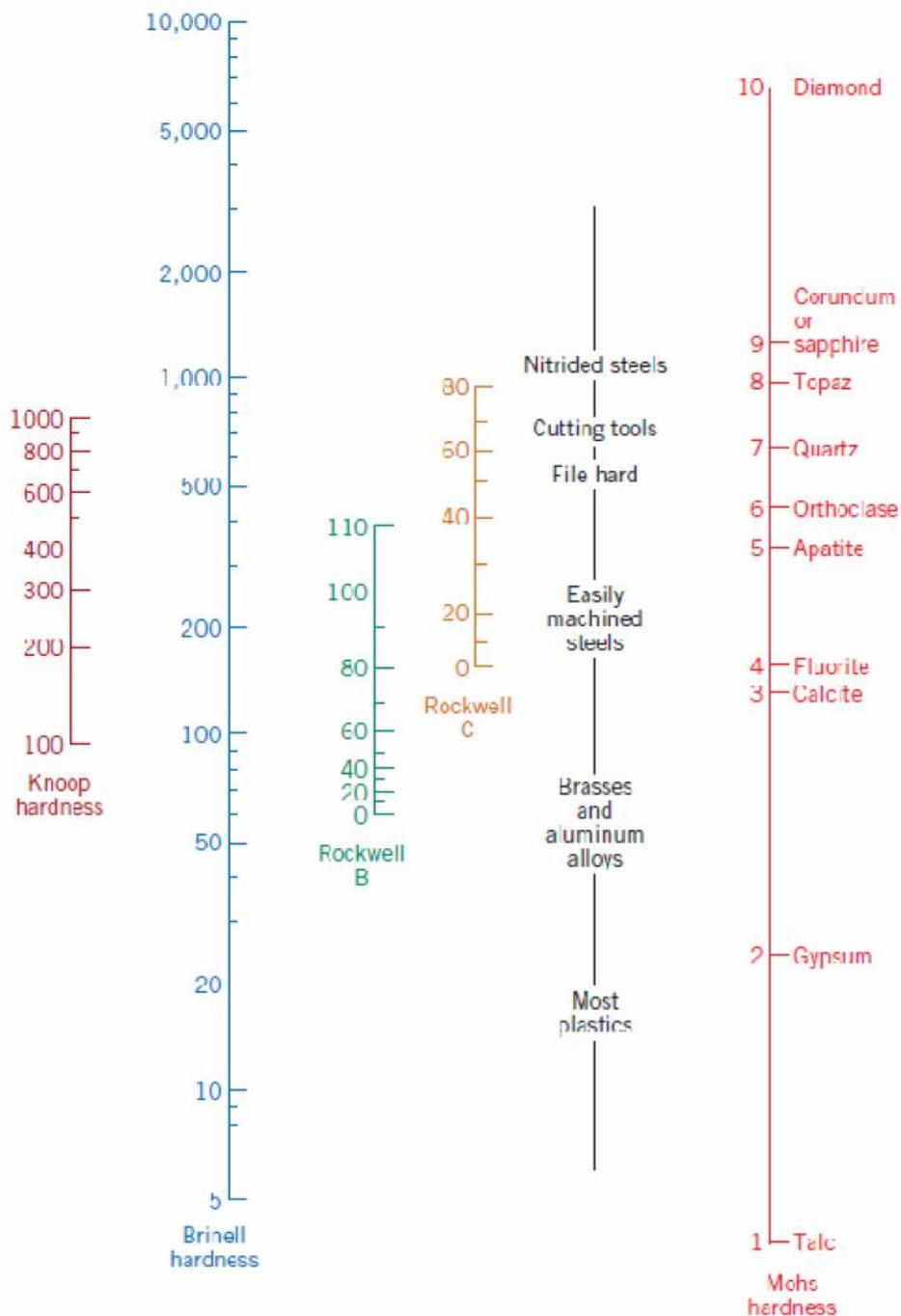


Figure 8.22 Comparison of several hardness scales.

alloys are contained in ASTM Standard E 140, “Standard Hardness Conversion Tables for Metals.” In light of the preceding discussion, care should be exercised in extrapolation of conversion data from one alloy system to another.

8.2.14 Correlation Between Hardness and Tensile Strength

Both tensile strength and hardness are indicators of a metal's resistance to plastic deformation. Consequently, they are roughly proportional, as shown in Figure 8.23, for tensile strength as a function of the HB for cast iron, steel, and brass. The same proportionality relationship does not hold for all metals, as Figure 8.23 indicates. As a rule of thumb for most steels, the HB and the tensile strength are related according to

$$TS(\text{MPa}) = 3.45 \times \text{HB} \quad (8.24)$$

$$TS(\text{psi}) = 500 \times \text{HB} \quad (8.24)$$

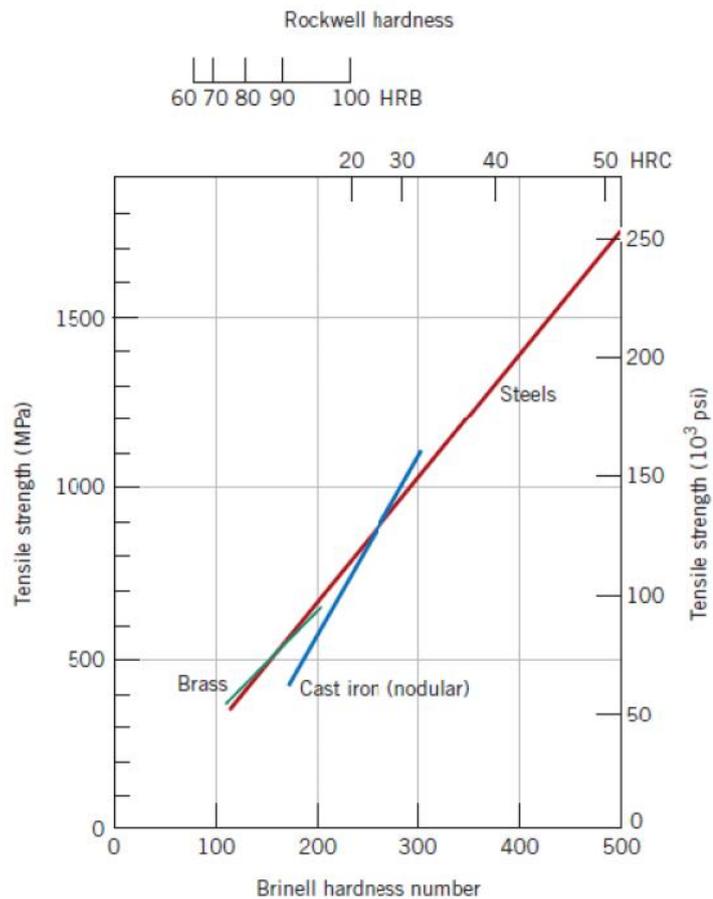


Figure 8.23 Relationships between hardness and tensile strength for steel, brass, and cast iron.

8.3 Thermal properties

8.3.1 Heat Capacity

A solid material, when heated, experiences an increase in temperature signifying that some energy has been absorbed. Heat capacity is a property that is indicative of a material's ability to absorb heat from the external surroundings; it represents the amount of energy required to produce a unit temperature rise. In mathematical terms, the heat capacity C is expressed as follows:

$$C = \frac{dQ}{dT} \quad (8.25)$$

where dQ is the energy required to produce a dT temperature change. Ordinarily, heat capacity is specified per mole of material (e.g., J/mol-K, or cal/mol-K). Specific heat (often denoted by a lowercase c) is sometimes used; this represents the heat capacity per unit mass and has various units (J/kg-K, cal/g-K, Btu/lb_m °F).

There are really two ways in which this property may be measured, according to the environmental conditions accompanying the transfer of heat. One is the heat capacity while maintaining the specimen volume constant, C_v ; the other is for constant external pressure, which is denoted C_p . The magnitude of C_p is almost always greater than C_v ; however, this difference is very slight for most solid materials at room temperature and below.

8.3.2 Vibrational Heat Capacity

In most solids the principal mode of thermal energy assimilation is by the increase in vibrational energy of the atoms. Again, atoms in solid materials are constantly vibrating at very high frequencies and with relatively small amplitudes. Rather than being independent of one another, the vibrations of adjacent atoms are coupled by virtue of the atomic bonding. These vibrations are coordinated in such a way that traveling lattice waves are produced, a phenomenon represented in Figure 24. These may be thought of as elastic waves or simply sound waves, having short

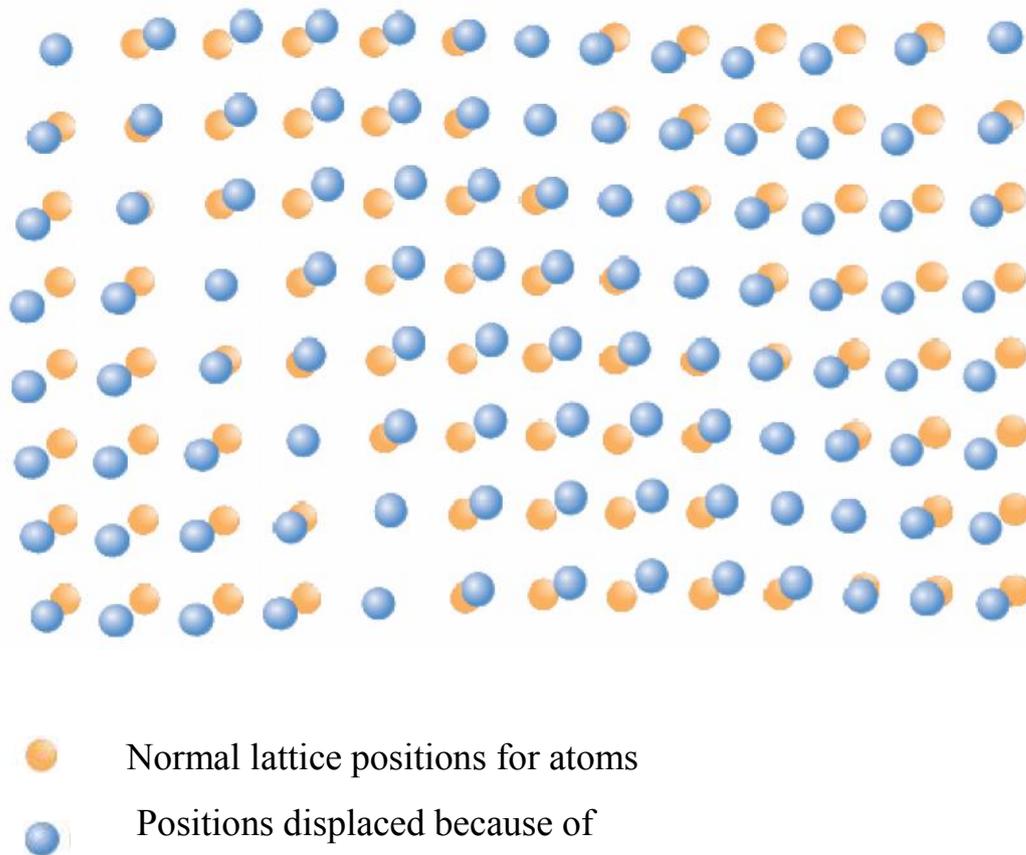


Figure 8.24 Schematic representation of the generation of lattice waves in a crystal by means of atomic vibrations.

wavelengths and very high frequencies, which propagate through the crystal at the velocity of sound. The vibrational thermal energy for a material consists of a series of these elastic waves, which have a range of distributions and frequencies. Only certain energy values are allowed (the energy is said to be quantized), and a single quantum of vibrational energy is called a phonon. (A phonon is analogous to the quantum of electromagnetic radiation, the photon.) On occasion, the vibrational waves themselves are termed phonons.

The thermal scattering of free electrons during electronic conduction is by these vibrational waves, and these elastic waves also participate in the transport of energy during thermal conduction .

8.3.3 Temperature Dependence of the Heat Capacity

The variation with temperature of the vibrational contribution to the heat capacity at constant volume for many relatively simple crystalline solids is shown in Figure 25. The C_v is zero at 0 K, but it rises rapidly with temperature; this corresponds to an increased ability of the lattice waves to enhance their average energy with ascending temperature. At low temperatures the relationship between C_v and the absolute temperature T is

$$C_v = AT^3 \quad (8.26)$$

where A is a temperature-independent constant. Above what is called the *Debye temperature* θ_D , C_v levels off and becomes essentially independent of temperature at a value of approximately $3R$, R being the gas constant. Thus even though the total energy of the material is increasing with temperature, the quantity of energy required to produce a one-degree temperature change is constant. The value of θ_D is

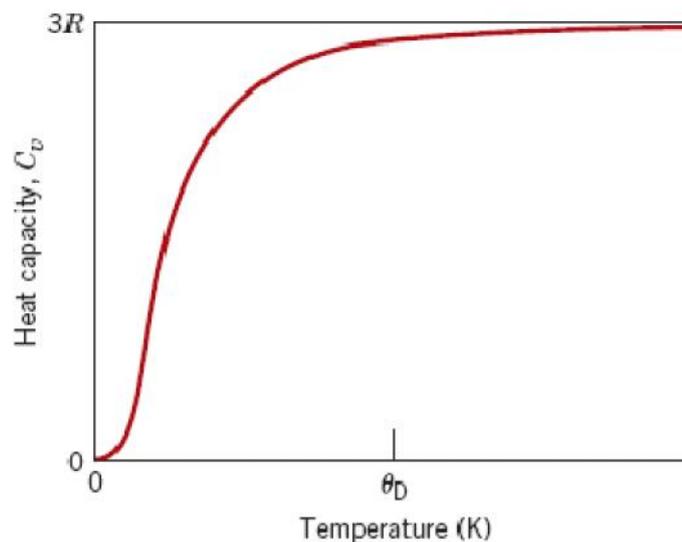


Figure 8.25 The temperature dependence of the heat capacity at constant volume; θ_D is the *Debye temperature*.

below room temperature for many solid materials, and 25 J/mol-K is a reasonable room-temperature approximation for C_v . Table 8.8 presents experimental specific heats for a number of materials.

Table 8.8 Tabulation of the Thermal Properties for a Variety of Materials

<i>Material</i>	c_p (J/kg-K) ^a	α_l [(°C) ⁻¹ × 10 ⁻⁶] ^b	k (W/m-K) ^c	L [Ω-W/(K) ² × 10 ⁻⁸]
<i>Metals</i>				
Aluminum	900	23.6	247	2.20
Copper	386	17.0	398	2.25
Gold	128	14.2	315	2.50
Iron	448	11.8	80	2.71
Nickel	443	13.3	90	2.08
Silver	235	19.7	428	2.13
Tungsten	138	4.5	178	3.20
1025 Steel	486	12.0	51.9	—
316 Stainless steel	502	16.0	15.9	—
Brass (70Cu–30Zn)	375	20.0	120	—
Kovar (54Fe–29Ni–17Co)	460	5.1	17	2.80
Invar (64Fe–36Ni)	500	1.6	10	2.75
Super Invar (63Fe 32Ni 5Co)	500	0.72	10	2.68
<i>Ceramics</i>				
Alumina (Al ₂ O ₃)	775	7.6	39	—
Magnesia (MgO)	940	13.5 ^d	37.7	—
Spinel (MgAl ₂ O ₄)	790	7.6 ^d	15.0 ^e	—
Fused silica (SiO ₂)	740	0.4	1.4	—
Soda-lime glass	840	9.0	1.7	—
Borosilicate (Pyrex TM) glass	850	3.3	1.4	—
<i>Polymers</i>				
Polyethylene (high density)	1850	106–198	0.46–0.50	—
Polypropylene	1925	145–180	0.12	—
Polystyrene	1170	90–150	0.13	—
Polytetrafluoroethylene (Teflon TM)	1050	126–216	0.25	—
Phenol-formaldehyde, phenolic	1590–1760	122	0.15	—
Nylon 6,6	1670	144	0.24	—
Polyisoprene	—	220	0.14	—

8.3.4 Thermal Expansion

Most solid materials expand upon heating and contract when cooled. The change in length with temperature for a solid material may be expressed as follows:

$$\frac{l_f - l_0}{l_0} = \alpha_l(T_f - T_0) \quad (8.27)$$

Or

$$\frac{\Delta l}{l_0} = \alpha_l \Delta T \quad (8.28)$$

where l_0 and l_f represent, respectively, initial and final lengths with the temperature change from T_0 , T_f . The parameter α_l is called the linear coefficient of thermal expansion; it is a material property that is indicative of the extent to which a material expands upon heating, and has units of reciprocal temperature $[(^{\circ}\text{C})^{-1}$ or $(^{\circ}\text{F})^{-1}]$. Of course, heating or cooling affects all the dimensions of a body, with a resultant change in volume. Volume changes with temperature may be computed from

$$\frac{\Delta V}{V_0} = \alpha_v \Delta T \quad (8.29)$$

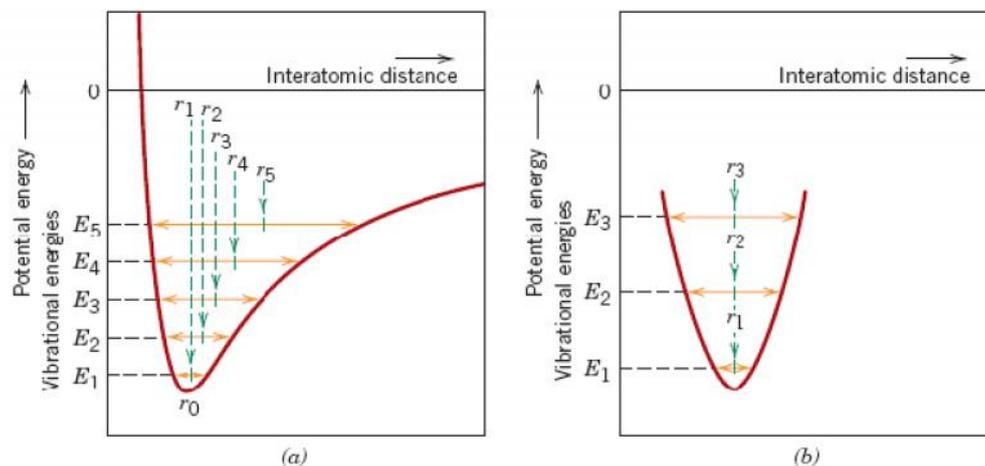


Figure 8.26 (a) Plot of potential energy versus interatomic distance, demonstrating the increase in interatomic separation with rising temperature. With heating, the interatomic separation increases from r_0 to r_1 to r_2 and so on.

(b) For a symmetric potential energy versus-interatomic distance curve, there is no increase in interatomic separation with rising temperature (i.e., $r_1 = r_2 = r_3$).

Where ΔV and V_0 are the volume change and the original volume, respectively, and symbolizes the volume coefficient of thermal expansion. In many materials, the value of α_v is anisotropic; that is, it depends on the crystallographic direction along which it is measured. For materials in which the thermal expansion is isotropic, α_v is approximately $3\alpha_l$.

From an atomic perspective, thermal expansion is reflected by an increase in the average distance between the atoms. This phenomenon can best be understood by consultation of the potential energy-versus-interatomic spacing curve for a solid material introduced previously (Figure 8.27), and reproduced in Figure 8.26(a). The curve is in the form of a potential energy trough, and the equilibrium interatomic spacing at 0 K, r_0 corresponds to the trough minimum. Heating to successively higher temperatures (T_1, T_2, T_3 , etc.) raises the vibrational energy from E_1 to E_2 to E_3 , and so on. The average vibrational amplitude of an atom corresponds to the trough width at each temperature, and the average interatomic distance is represented by the mean position, which increases with temperature from r_0 to r_1 to r_2 and so on.

Thermal expansion is really due to the asymmetric curvature of this potential energy trough, rather than the increased atomic vibrational amplitudes with rising temperature. If the potential energy curve were symmetric (Figure 8.26(b)), there would be no net change in interatomic separation and, consequently, no thermal expansion.

For each class of materials (metals, ceramics, and polymers), the greater the atomic bonding energy, the deeper and more narrow this potential energy trough. As a result, the increase in interatomic separation with a given rise in temperature will be lower, yielding a smaller value of α_l .

Table 8.8 lists the linear coefficients of thermal expansion for several materials. With regard to temperature dependence, the magnitude of the coefficient of expansion increases with rising temperature. The values in Table 8.8 are taken at room temperature unless indicated otherwise.

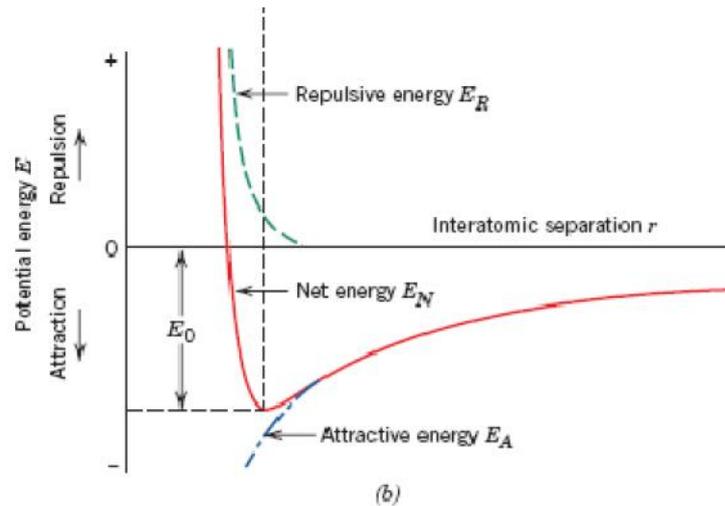


Figure 8.27 The dependence of repulsive, attractive, and net potential energies on interatomic separation for two isolated atoms.

8.4 Optical properties

By virtue of their electron energy band structures, nonmetallic materials may be transparent to visible light. Therefore, in addition to reflection and absorption, refraction and transmission phenomena also need to be considered.

8.4.1. Refraction

Light that is transmitted into the interior of transparent materials experiences a decrease in velocity, and, as a result, is bent at the interface; this phenomenon is termed refraction. The index of refraction n of a material is defined as the ratio of the velocity in a vacuum c to the velocity in the medium v or

$$n = \frac{c}{v} \quad (8.30)$$

The magnitude of n (or the degree of bending) will depend on the wavelength of the light. This effect is graphically demonstrated by the familiar dispersion or separation of a beam of white light into its component colors by a glass prism. Each color is deflected by a different

amount as it passes into and out of the glass, which results in the separation of the colors. Not only does the index of refraction affect the optical path of light, but also, as explained below, it influences the fraction of incident light that is reflected at the surface. Just as Equation 8.32 defines the magnitude of c , an equivalent expression gives the velocity of light in a medium as

$$v = \frac{1}{\sqrt{\epsilon\mu}} \quad (8.31)$$

$$c = \frac{1}{\sqrt{\epsilon_0\mu_0}} \quad (8.32)$$

Where ϵ and μ are, respectively, the permittivity and permeability of the particular substance. From Equation 8.30, we have

$$n = \frac{c}{v} = \frac{\sqrt{\epsilon\mu}}{\sqrt{\epsilon_0\mu_0}} = \sqrt{\epsilon_r\mu_r} \quad (8.33)$$

where ϵ_r and μ_r are the dielectric constant and the relative magnetic permeability, respectively. Since most substances are only slightly magnetic; $\mu_r \approx 1$, and

$$n \cong \sqrt{\epsilon_r} \quad (8.34)$$

Thus, for transparent materials, there is a relation between the index of refraction and the dielectric constant. As already mentioned, the phenomenon of refraction is related to electronic polarization at the relatively high frequencies for visible light; thus, the electronic component of the dielectric constant may be determined from index of refraction measurements using Equation 8.34.

Since the retardation of electromagnetic radiation in a medium results from electronic polarization, the size of the constituent atoms or ions has

a considerable influence on the magnitude of this effect—generally, the larger an atom or ion, the greater will be the electronic polarization, the slower the velocity, and the greater the index of refraction. The index of refraction for a typical soda–lime glass is approximately 1.5. Additions of large barium and lead ions (as BaO and PbO) to a glass will increase n significantly. For example, highly leaded glasses containing 90 wt% PbO have an index of refraction of approximately 2.1.

For crystalline ceramics that have cubic crystal structures, and for glasses, the index of refraction is independent of crystallographic direction (i.e., it is isotropic). Noncubic crystals, on the other hand, have an anisotropic n ; that is, the index is greatest along the directions that have the highest density of ions.

Table 8.9 gives refractive indices for several glasses, transparent ceramics, and polymers. Average values are provided for the crystalline ceramics in which n is anisotropic.

Table 8.9 Refractive Indices for Some Transparent Materials

<i>Material</i>	<i>Average Index of Refraction</i>
Ceramics	
Silica glass	1.458
Borosilicate (Pyrex) glass	1.47
Soda–lime glass	1.51
Quartz (SiO ₂)	1.55
Dense optical flint glass	1.65
Spinel (MgAl ₂ O ₄)	1.72
Periclase (MgO)	1.74
Corundum (Al ₂ O ₃)	1.76
Polymers	
Polytetrafluoroethylene	1.35
Polymethyl methacrylate	1.49
Polypropylene	1.49
Polyethylene	1.51
Polystyrene	1.60

8.4.2 Reflection

When light radiation passes from one medium into another having a different index of refraction, some of the light is scattered at the interface between the two media even if both are transparent. The reflectivity R represents the fraction of the incident light that is reflected at the interface, or

$$R = \frac{I_R}{I_0} \quad (8.35)$$

where I_0 and I_R are the intensities of the incident and reflected beams, respectively. If the light is normal (or perpendicular) to the interface, then

$$R = \left(\frac{n_2 - n_1}{n_2 + n_1} \right)^2 \quad (8.36)$$

where n_1 and n_2 are the indices of refraction of the two media. If the incident light is not normal to the interface, R will depend on the angle of incidence. When light is transmitted from a vacuum or air into a solid s , then

$$R = \left(\frac{n_s - 1}{n_s + 1} \right)^2 \quad (8.37)$$

since the index of refraction of air is very nearly unity. Thus, the higher the index of refraction of the solid, the greater is the reflectivity. For typical silicate glasses, the reflectivity is approximately 0.05. Just as the index of refraction of a solid depends on the wavelength of the incident light, so also does the reflectivity vary with wavelength. Reflection losses for lenses and other optical instruments are minimized significantly by coating the reflecting surface with very thin layers of dielectric materials such as magnesium fluoride (MgF₂).

8.4.3 Absorption

Nonmetallic materials may be opaque or transparent to visible light; and, if transparent, they often appear colored. In principle, light radiation is absorbed in this group of materials by two basic mechanisms, which also influence the transmission characteristics of these nonmetals. One of these is electronic polarization. Absorption by electronic polarization is important only at light frequencies in the vicinity of the relaxation frequency of the constituent atoms. The other mechanism

involves valence band-conduction band electron transitions, which depend on the electron energy band structure of the material;

Absorption of a photon of light may occur by the promotion or excitation of an electron from the nearly filled valence band, across the band gap, and into an empty state within the conduction band, as demonstrated in Figure 8.27a; a free

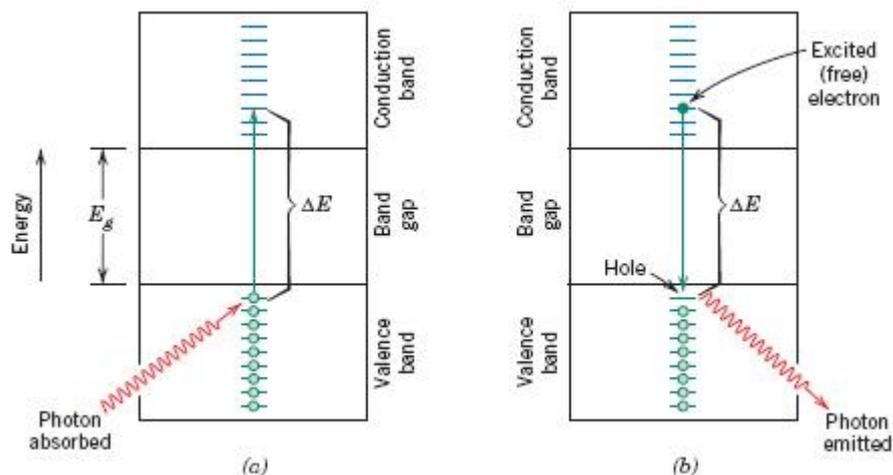


Figure 8.27

(a) Mechanism of photon absorption for nonmetallic materials in which an electron is excited across the band gap, leaving behind a hole in the valence band. The energy of the photon absorbed is ΔE , which is necessarily greater than the band gap energy E_g .

(b) Emission of a photon of light by a direct electron transition across the band gap

electron in the conduction band and a hole in the valence band are created. Again, the energy of excitation ΔE is related to the absorbed photon frequency through Equation 8.38.

$$\Delta E = h\nu \quad (8.38)$$

These excitations with the accompanying absorption can take place only if the photon energy is greater than that of the band gap E_g that is, if

$$h\nu > E_g \quad (8.39)$$

or, in terms of wavelength,

$$\frac{hc}{\lambda} > E_g \quad (8.40)$$

The minimum wavelength for visible light, (min), is about 0.4 m, and since $c=3 \times 10^8$ m/s and $h=4.13 \times 10^{-15}$ eV-s the maximum band gap energy E_g (max) for which absorption of visible light is possible is just

$$E_g(\text{max}) = \frac{hc}{\lambda(\text{min})} \quad (8.41a)$$

$$= \frac{(4.13 \times 10^{-15} \text{ eV-s})(3 \times 10^8 \text{ m/s})}{4 \times 10^{-7} \text{ m}}$$

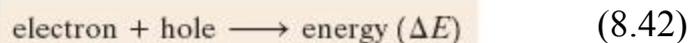
$$= 3.1 \text{ eV}$$

Or, no visible light is absorbed by nonmetallic materials having band gap energies greater than about 3.1 eV; these materials, if of high purity, will appear transparent and colorless. On the other hand, the maximum wavelength for visible light, (max), is about 0.7 m; computation of the minimum band gap energy E_g (min) for which there is absorption of visible light is according to

$$E_g(\text{min}) = \frac{hc}{\lambda(\text{max})} \quad (8.41 b)$$

$$= \frac{(4.13 \times 10^{-15} \text{ eV}\cdot\text{s})(3 \times 10^8 \text{ m/s})}{7 \times 10^{-7} \text{ m}} = 1.8 \text{ eV}$$

This result means that all visible light is absorbed by valence band to conduction band electron transitions for those semiconducting materials that have band gap energies less than about 1.8 eV; thus, these materials are opaque. Only a portion of the visible spectrum is absorbed by materials having band gap energies between 1.8 and 3.1 eV; consequently, these materials appear colored. Every nonmetallic material becomes opaque at some wavelength, which depends on the magnitude of its E_g . For example, diamond, having a band gap of 5.6 eV, is opaque to radiation having wavelengths less than about 0.22 μm . Interactions with light radiation can also occur in dielectric solids having wide band gaps, involving other than valence band-conduction band electron transitions. If impurities or other electrically active defects are present, electron levels within the band gap may be introduced, such as the donor and acceptor levels, except that they lie closer to the center of the band gap. Light radiation of specific wavelengths may be emitted as a result of electron transitions involving these levels within the band gap. For example, consider Figure 8.28a, which shows the valence band conduction band electron excitation for a material that has one such impurity level. Again, the electromagnetic energy that was absorbed by this electron excitation must be dissipated in some manner; several mechanisms are possible. For one, this dissipation may occur via direct electron and hole recombination according to the reaction



which is represented schematically in Figure 8.27b. In addition, multiple-step electron transitions may occur, which involve impurity levels lying within the band gap. One possibility, as indicated in Figure 8.28b, is the emission of two photons; one is emitted as the electron drops from a state in the conduction band to the impurity level, the other as it decays back into the valence band. Alternatively, one of the transitions may involve the generation of a phonon (Figure 8.28c), wherein the associated energy is dissipated in the form of heat.

The intensity of the net absorbed radiation is dependent on the character of the medium as well as the path length within. The intensity of transmitted or

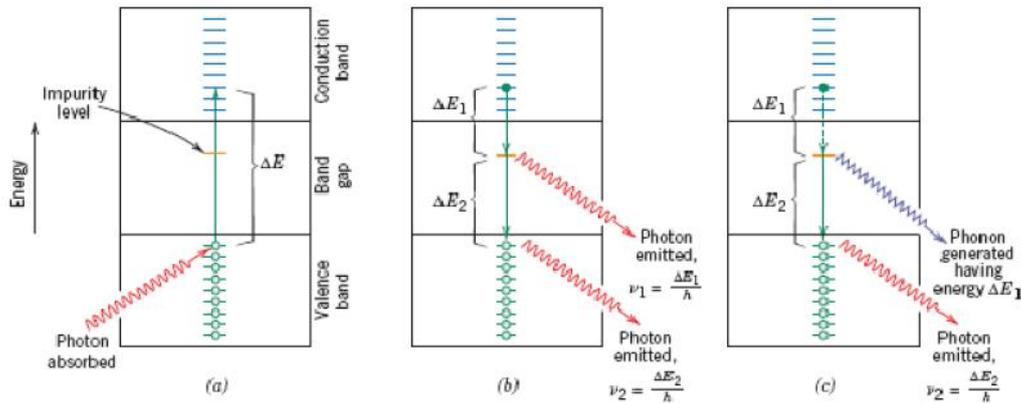


Figure 8.28

(a) Photon absorption via a valence band-conduction band electron excitation for a material that has an impurity level that lies within the band gap.

(b) Emission of two photons involving electron decay first into an impurity state, and finally to the ground state.

(c) Generation of both a phonon and a photon as an excited electron falls first into an impurity level and finally back to its ground state.

nonabsorbed radiation I_T continuously decreases with distance x that the light traverses:

$$I_T' = I_0' e^{-\beta x} \quad (8.43)$$

where I_0' is the intensity of the non reflected incident radiation and β the *absorption coefficient* (in mm^{-1}) is characteristic of the particular material; furthermore, β varies with wavelength of the incident radiation. The distance parameter x is measured from the incident surface into the material. Materials that have large β values are considered to be highly absorptive.

Example Problem (6.f)**Computation of the Absorption Coefficient for Glass**

The fraction of non reflected light that is transmitted through a 200 mm thickness of glass is 0.98. Calculate the absorption coefficient of this material.

Solution

This problem calls for us to solve for β in Equation 8.43 We first of all rearrange this expression as

$$\frac{I'_T}{I'_0} = e^{-\beta x}$$

Now taking logarithms of both sides of the above equation leads to

$$\ln\left(\frac{I'_T}{I'_0}\right) = -\beta x$$

And, finally ,solving for β realizing that $I'_T/I'_0 = 0.98$ and $x=200$ mm yields,

$$\begin{aligned} \beta &= -\frac{1}{x} \ln\left(\frac{I'_T}{I'_0}\right) \\ &= -\frac{1}{200 \text{ mm}} \ln(0.98) = 1.01 \times 10^{-4} \text{ mm}^{-1} \end{aligned}$$

8.4.5 Transmission

The phenomena of absorption, reflection, and transmission may be applied to the passage of light through a transparent solid, as shown in Figure 8.29. For an incident beam of intensity I_0 that impinges on the front surface of a specimen of thickness l and absorption coefficient β the transmitted intensity at the back face I_T is

$$I_T = I_0(1 - R)^2 e^{-\beta l} \quad (8.44)$$

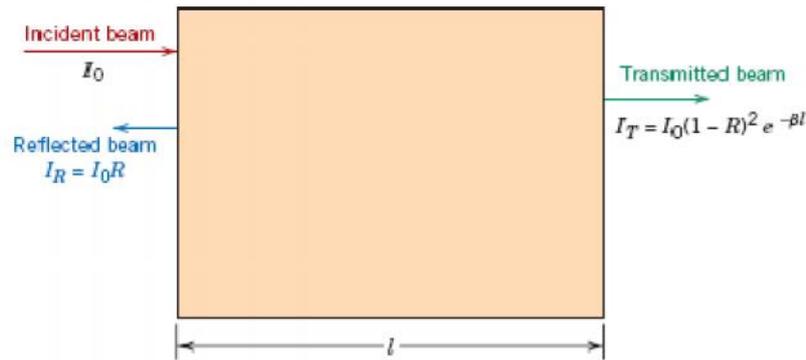


Figure 8.29 The transmission of light through a transparent medium for which there is reflection at front and back faces, as well as absorption within the medium

where R is the reflectance; for this expression, it is assumed that the same medium exists outside both front and back faces. The derivation of Equation 8.44 is left as a homework problem. Thus, the fraction of incident light that is transmitted through a transparent material depends on the losses that are incurred by absorption and reflection. Again, the sum of the reflectivity R , absorptivity A , and transmissivity T , is unity according to Equation 8.45.

$$T + A + R = 1 \quad (8.45)$$

Also, each of the variables R , A , and T depends on light wavelength. This is demonstrated over the visible region of the spectrum for a green glass in Figure 8.30. For example, for light having a wavelength of $0.4\mu\text{m}$, the fractions transmitted, absorbed, and reflected are approximately 0.90, 0.05, and 0.05, respectively. However, at $0.55\mu\text{m}$, the respective fractions have shifted to about 0.50, 0.48, and 0.02.

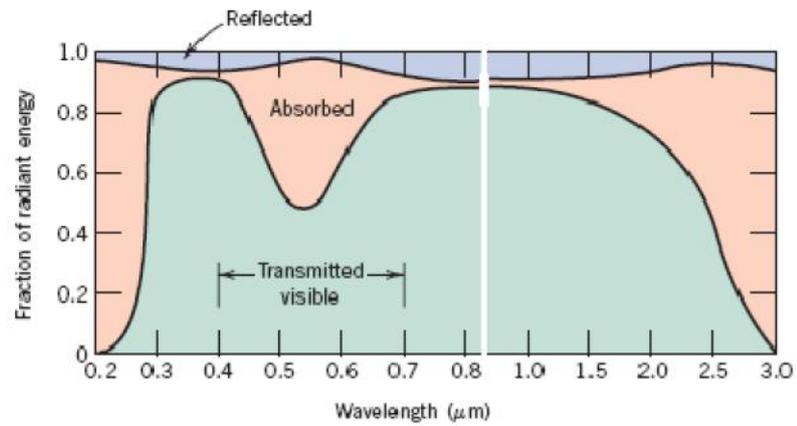


Figure 8.30 The variation with wavelength of the fractions of incident light transmitted, absorbed, and reflected through a green glass.