12- Friction

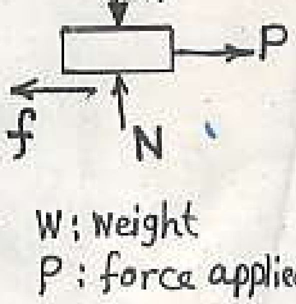
When a body slides or tends to slide on other body, the force tangent to the contact surface which resists the motion, or the tendency toward motion, of one body relative to the other is defined as friction.

It has advantages and disadvantages.

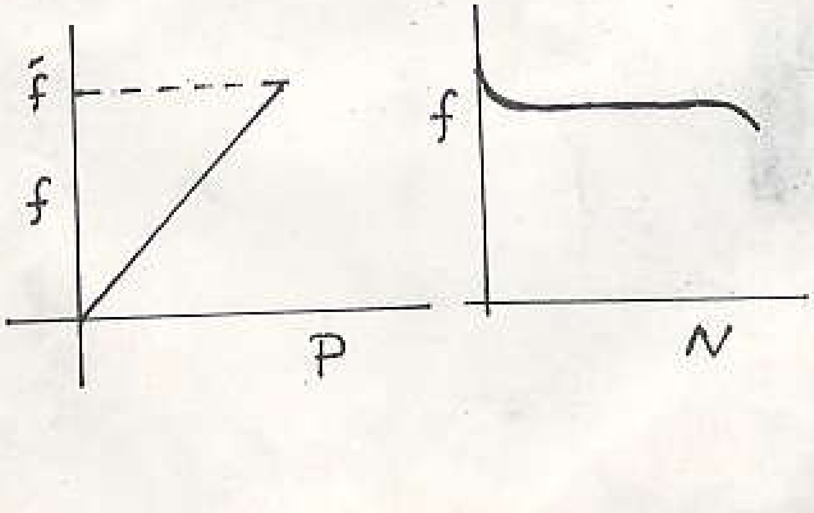
If the contact surface (area) between surfaces is assumed smooth then there is no friction. There are 2 types of friction.

1) static friction. When there is no relative motion between surfaces.
2) Kinetic friction. when there is relative motion between the bodies.

The static friction force is always the minimum force required to maintain equilibrium or prevent relative motion between the bodies, and the kinetic friction varies some what with velocity. see figure.



W: weight
P: force applied
N: Normal force
f: friction force



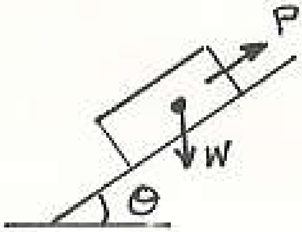
Example

What is the range of P to impend motion.

$W = 100N$

$\theta = 30^\circ$

$\mu = 0.2$

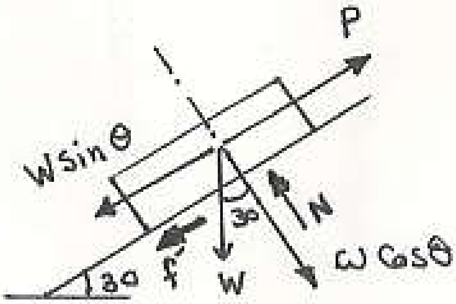


N.B

* the friction force is opposite to the direction of motion

Solution

to impends motion upward.



$P = W\sin\theta + f'$ —— ①

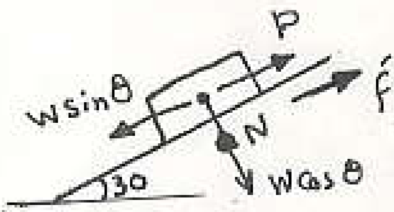
$N = W\cos\theta$ —— ②

$\mu = \frac{f'}{N} \Rightarrow f' = \mu N$

$\therefore P = 100 \sin 30 + 0.2\ (100) \cos 30$

$= 50 + 17.32 = 67.32\ N$

to impends motion downward



$P = W\sin\theta - f'$

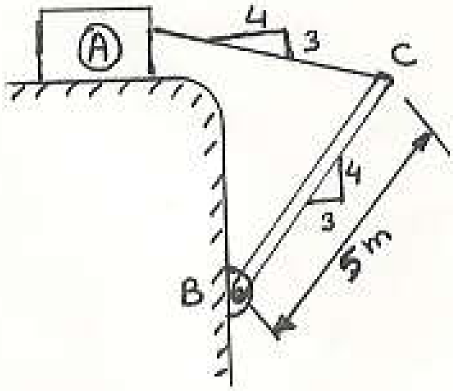
$= W\sin\theta - \mu N$

$= W\sin\theta - \mu W\cos\theta$

$= 50 - 0.2(100)\cos 30 = 32.68\ N$

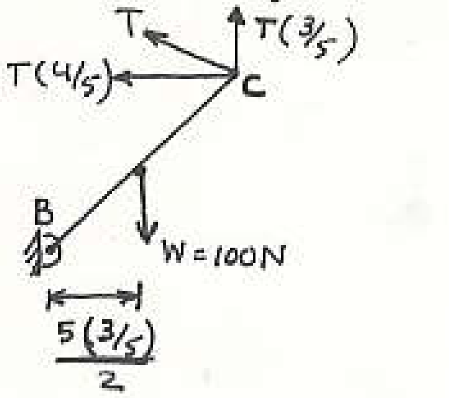
Example

The coefficient of friction μ between the 50 N body A and the plane is 0.5. The bar BC weight is 100 N. Determine the forces acting on body A.



Solution

First, draw the free body diagram (BC)



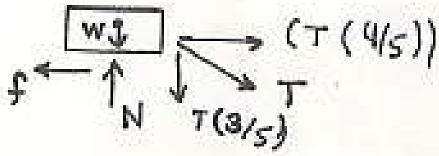
$\circlearrowleft^{+} \Sigma M_B = 0$

$$100(1.5) - T(4/5)(4) - T(3/5)(3) = 0$$

$$T = 30\,N.$$

F.B.D for (A)

\# (one case of motion is to the right) why?



$\Sigma F_y = 0$

$$W + 30(3/5) = N$$

$$N = 50 + 18 = 68\,N$$

$$f = T(4/5) = 24\,N$$

and for test if body impend motion.

$$f = \mu N = 0.5(50+18) = 34\,N$$

This is the value of friction force to impend motion

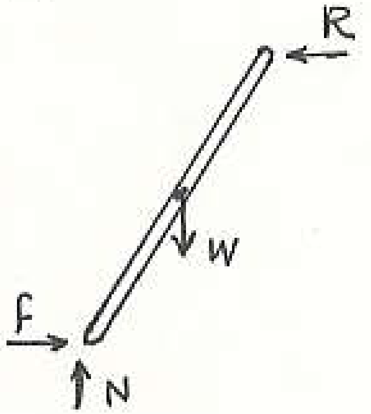
Then $T(4/5) = 34\,N \Rightarrow T = 42.5\,N$ (required T to impend motion).

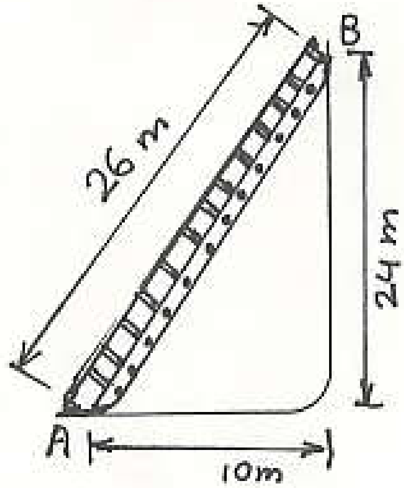
HW find the mass of BC to impend motion

(ans: 14.45 kg)

Example

A 26m ladder weighs 50 N is placed against a smooth vertical wall, its lower end is at 10 m from the wall. (μ) Coefficient of friction between the ladder and the floor is 0.3. Determine the frictional force at A acting on the ladder.





$\Sigma F_y = 0$

$N - W = 0 \Rightarrow N - 50 = 0 \Rightarrow N = 50 N \uparrow$

$\Sigma M_B = 0$

$f\,(24) + 50\,(5) - N\,(10) = 0$

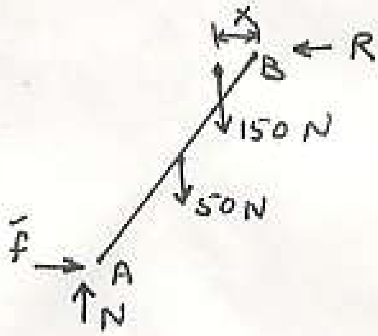
$f = 10.42 \text{ N} \longrightarrow$

to impend motion

$f \not\equiv \bar{f} = \mu N = 0.3\,(50) = 15 \text{ N}$

then the ladder will not slip.

If in the above example there is a boy of weight 150 N is Climb the ladder. Determine the distance from the boy to the wall when the ladder start to slip.



$\Sigma F_y = 0 \Rightarrow N - 50 - 150 = 0$

$N = 200 N \uparrow$

$\bar{F} = \mu N = 0.3\,(200) = 60 \text{ N}$

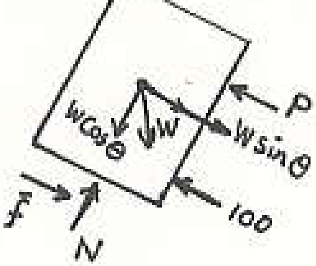
$\Sigma M_B = 0$

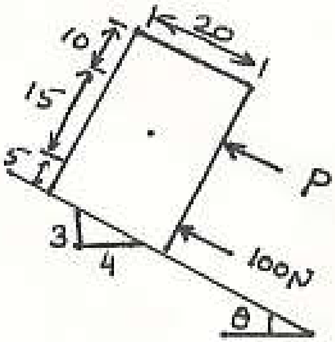
$150\,(x) + 50\,(5) - 10\,(200) + 24\,(60) = 0$

$x = 2.02 \text{ m}$

Example

The homogeneous block weighs 2500 N. The Coefficient of friction between the plane and the block is 0.3. Determine the range of values of P for which the block is in equilibrium.





All Dimensions in «Cm»

$\cos\theta = 4/5$

$\sin\theta = 3/5$

to Impend motion upward.

$\Sigma F_y = 0$

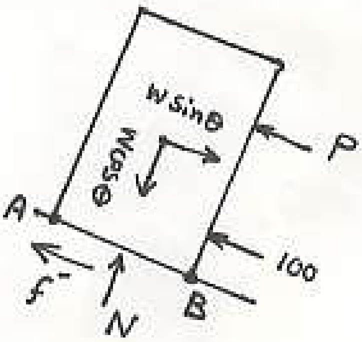
$N = W\cos\theta = 2500(4/5) = 2000\ N$

$f = \mu N = 0.3(2000) = 600\ N$

$\Sigma F_x = 0$

$P = 600 + 2500(3/5) - 100 = 2000\ N.$

Now, If P is small so that the body will impends motion downward then:



$\Sigma F_x = 0$

$f + P + 100 = W\sin\theta$

$P = 2500(3/5) - 100 - \mu N$

$= 1500 - 100 - 0.3(2500)(4/5)$

$= 1500 - 100 - 600 = 800\ N$

$\circlearrowleft \Sigma M_A = 0$ this is to test if the block will turn over.

$$-P(0.2) - 100(0.05) + 2500(4/5)(0.1) + 2500(3/5)(0.15) = 0$$

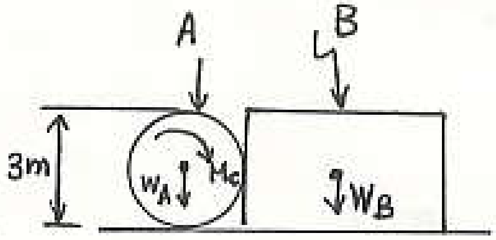
$$P = \frac{-5 + 200 + 225}{0.2} = 2100\ N.$$

$\because 2100 > 2000$ the the body will move upward before it turns over.

Example

Body A is a homogeneous cylinder weighing 500 N, and body B weighs 900 N. The coefficient of friction for all contact surfaces of body A is 0.4, and between body B and the plane it is 0.1

Determine couple M_C that will cause body B to have impending motion.

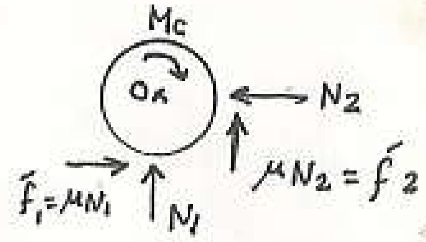


Solution

⊛ If the cylinder will sliding before it moves rightward

$\circlearrowleft + \Sigma M_O = 0$

$$-\mu N_1 (1.5) - \mu N_2 (1.5) + M_C = 0 \quad \text{—} ①$$



$\Sigma F_y = 0$

$$-W_A + N_1 + \mu N_2 = 0 \quad \text{—} ②$$

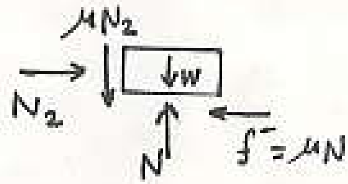
$\Sigma F_x = 0$

$$\mu N_1 = N_2 \quad \text{—} ③$$

$\therefore N_1 = 431.03 N \; ; \; N_2 = 172.413 N \; ; \; M_C = 362 \; N.m$.

⊛ If Block B move first.

$$N_2 = \bar{f} = \mu N \quad \text{—} ④$$



$$\mu N_2 + W - N = 0 \quad \text{—} ⑤$$

$N_2 = 187.5 N \; ; \; N = 937.5 N$

Back to cylinder (eqn. ①)

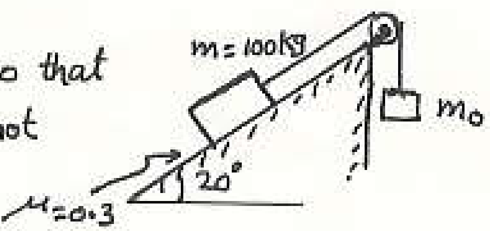
$$M_C - N_2 (1.5) - 0.6 (N_2) = 0$$

$$M_C = 2.1 N_2 = 393.75 \; N.m$$

$\because M_{C\,slip} < M_{C\,move} \Rightarrow$ the body will slip before impends motion of block B.

Problems

1. What is the range of mass m_o so that the 100 kg mass shown in figure cannot move upward or downward.



Ans: (m_o 6kg → 62.4kg).

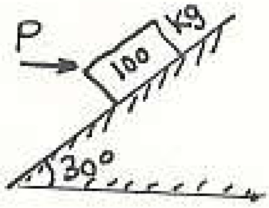
2- Find the friction on the body if

a) P = 500 N

b) P = 100 N

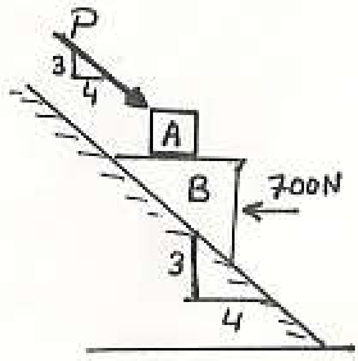
take $\mu = 0.17$

ans: ⓐ 134N ↙ ⓑ 163N ↗

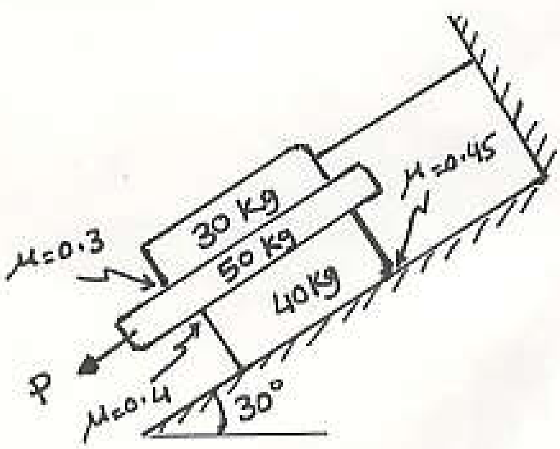


3. Body A weighs 700N, Body B weighs 900N. The coefficient of friction for all surfaces of contact is 0.3. Determine the force P that will cause motion of A to impend.

Ans: P = 110N ↘

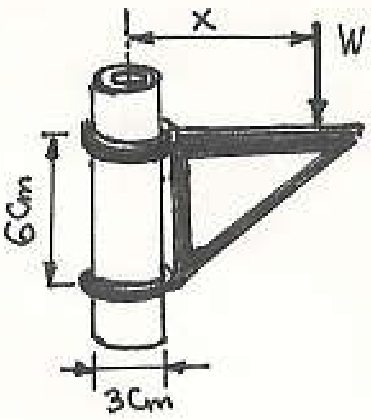


4. Three blocks are arranged as shown in figure below. The upper one is forbeden from movement by a rope. Determine the maximum value of P to impend motion. (Ans: P = 93.8N)



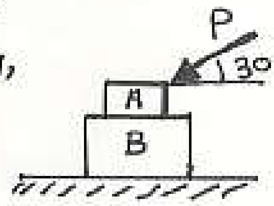
5. The movable bracket shown may be placed at any height on the 3 Cm diamater pipe. If the Coefficient of static friction between the pipe and bracket is 0.25, determine the minimum distance X at which the load W can be supported. Neglect the weight of bracket.

((Ans: 12 Cm))



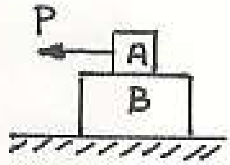
6. Find P so that block B will move first assume all surface friction $\mu = 0.3$, $W_A = 200$ N, $W_B = 100$ N.

(Ans: P = 83.8N)



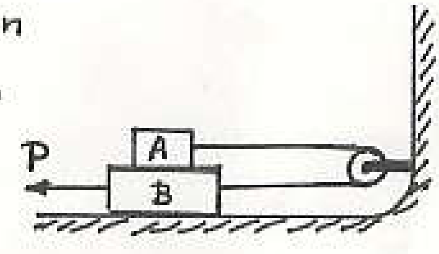
7. Find P so that block A will move first, $\mu_s = 0.3$, $W_A = 200$ N & $W_B = 100$ N. What is the magnitude of P required to move block B firstly?

Ans: (P = 60N, 90N).



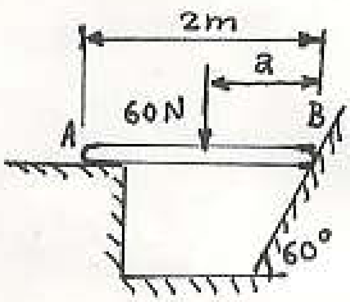
8. Find force P required to impend motion of block B. $W_A = 200$N, $W_B = 100$N, $\mu = 0.2$, neglect friction in pully.

(Ans: 140N)



9. The rod AB rests on a horizontal surface at A and against slipping surface at B if $\mu_s = 0.25$ at A & B. Find the minimum distance a for equilibrium, neglect the weight of the rod.

(Ans a = 1.61 m)



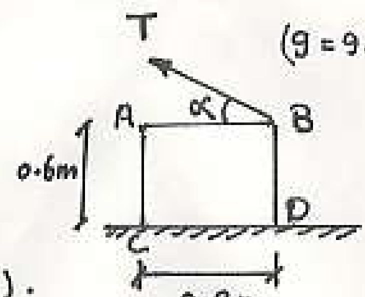
10. A packing Crate of mass 30 kg. The Coefficient of friction between the crate and the floor is 0.35 if $\alpha = 30°$ determine.

a) The tension T required to move the crate.

b) Whether the crate will slide or tip

ans: (98.9 N, Slides since T = 140N to tip).

($g = 9.8$ m/sec)



2. Dynamics

It is a part of mechanics dealing with the analysis of bodies in motion.

It is divided into two parts.

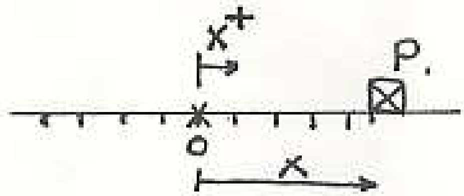
1. Kinematics, which is the study of motion without reference to the cause of the motion.
2. Kinetics, which is the study of the relation existing between the force acting on a body and the motion of the body.

Kinematics

2.1 Rectilinear motion of particles:

If a particle moving a long a straight line is said to be in rectilinear motion.

then,



$$\text{Average velocity} = \bar{v} = \frac{\Delta x}{\Delta t} \quad \left(\frac{m}{sec}\right)$$

$$\text{Instanteneous velocity} = v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Also,

$$\text{Average acceleration} = \bar{a} = \frac{\Delta v}{\Delta t} \quad (m/sec^2)$$

$$\text{Instanteneous acceleration} = a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

Example

Consider a particle moving in a straight line and assume that its position is defined by:

$$x = 6t^2 - t^3$$

where, t in sec, x in meters.

Determine ① the time for max. velocity.

② max. velocity.

③ displacement at max. velocity.

④ Distance traveled by the particle from $t = 2$ sec to $t = 6$ sec.

Solution

① $x = 6t^2 - t^3$ ——— ①

$$v = \frac{dx}{dt} = 12t - 3t^2 \quad \text{---} \ ②$$

If $v = f(t)$ and we went to find time for max. velocity then

$$\frac{dv}{dt} = 0 = 12 - 6t \Rightarrow t = \frac{12}{6} = \underline{\underline{2 \text{ sec}}}$$ (this is the time for max velocity).

② put $t = 2$ sec in eqn ② above to get max. velocity.

$$v_{max} = 12(2) - 3(2^2) = 24 - 12 = \underline{\underline{12 \text{ m/sec}}}$$

③ $x = 6(2^2) - (2^3) = 6 \times 4 - 8 = 24 - 8 = 16$ m.

④ test motion (ذهاب وعودة)

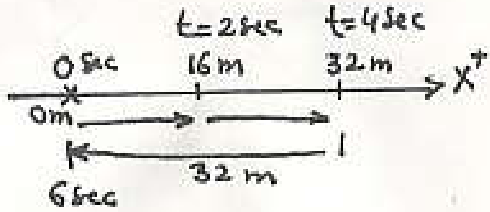
$$\frac{dx}{dt} = 0 = 12t - 3t^2 \Rightarrow t = 4 \text{ sec for max displacement.}$$

$$X_{\substack{at \\ 2sec}} = 6(2^2) - (2^3) = 24 - 8 = 16 \text{ m.}$$

$$X_{\substack{at \\ 4sec}} = 6(4^2) - (4^3) = 96 - 64 = 32 \text{ m.}$$

$$X_{\substack{at \\ 6sec}} = 6(6^2) - (6^3) = 0 \text{ m.}$$

Distance $= |16| + |32| = \underline{\underline{48 \text{ m}}}$



Example

The position of a particle which moves along a straight line is defined by the relation

$$x = t^3 - 6t^2 - 15t + 40$$

where ; t in sec, x in meters.

Determine ① The time at which the velocity will be zero.

② the position and distance traveled by the particle at that time.

③ The acceleration at that time.

④ distance traveled by the particle from $t = 4$ sec to $t = 6$ sec.

<u>Solution</u>

① $\frac{dx}{dt} = 0 = 3t^2 - 12t - 15 \Rightarrow t^2 - 4t - 5 = 0$

$(t-5)(t+1) = 0$

$\underline{\underline{t = 5 \text{ sec}}}$ $\quad t = -1$ sec (ignore).

② $x = t^3 - 6t^2 - 15t + 40 = (5)^3 - 6(5)^2 - 15(5) + 40$

$= 125 - 150 - 75 + 40$

$= -60$ m.

motion test. $\frac{dx}{dt} = 0$ & from ① $t = 5$ sec, so

distance traveled $= X_{t=5} - X_{t=0} = -60 - 40 = -100$ m.

where, $X_{t=5} = -60$ m & $X_{t=0} = 40$ m.

③ $a = \frac{dv}{dt} = \frac{d^2x}{dt^2} \Rightarrow \frac{dx}{dt} = 3t^2 - 12t - 15$

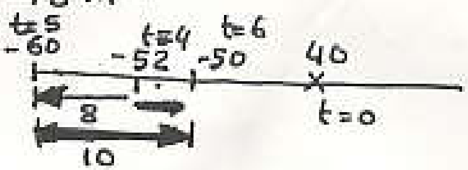
$a = \frac{d^2x}{dt^2} = 6t - 12$ then $a_{t=5\,sec} = 6(5) - 12 = 18 \text{ m/sec}^2$

④ Distance traveled $= (X_{t=5} - X_{t=4}) + (X_{t=6} - X_{t=5})$

$X_{t=4} = -52$ m, $X_{t=5} = -60$ m, $X_{t=6} = -50$ m.

$\therefore$ Distance traveled $= |-60 + 52| + |-50 + 60|$

$= 8 + 10 = 18$ m.



H.W

① The motion of a particle is defined by the relation.

$$x = \frac{1}{3}t^3 - 3t^2 + 8t + 2$$

where, x in meters, t in sec.

Determine

① when the velocity is zero. (2 sec, 4 sec)

② the position and the total distance traveled when the acceleration is zero (8m, 7.33m)

② The motion of a particle is defined by the relation.

$$x = t^2 - 10t + 30$$

where, x in meters, t in sec.

Determine

① when the velocity is zero. (5 sec)

② the position and total distance traveled when $t = 8$ sec.

($x_{t=8} = 14$ m, Distance = 39m).

2.2 Determination of the motion of a particle

From (1) we see that the motion as a function between position and time. In practice It is defined as a relation between acceleration and time, position or velocity. These relations are usually obtained from experiment.

$\boxed{A}$ if $a = f(t)$

$$a = \frac{dv}{dt} \Rightarrow dv = a\,dt = f(t)\,dt \quad \text{and by} \int$$

$$\int_{v_0}^{v} dv = \int_0^t f(t)\,dt \quad \text{———} \boxed{A} \quad \text{see } Ex_A.$$

$$\therefore v - v_0 = \int_0^t f(t)\,dt$$

note that this relation applied for $a = 0$, $a = $ Constant.

$\boxed{B}$ $a = f(x)$

$$a = \frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt} = v\frac{dv}{dx}$$

$$v\,dv = a\,dx = f(x)\,dx$$

$$\int_{v_0}^{v} v\,dv = \int_{x_0}^{x} f(x)\,dx \quad \text{———} \boxed{B} \quad \text{see } Ex_B.$$

$$\frac{1}{2}(v^2 - v_0^2) = \int_{x_0}^{x} f(x)\,dx$$

$\boxed{C}$ $a = f(v)$

$$a = f(v) = \frac{dv}{dt} \Rightarrow dt = \frac{dv}{f(v)}$$

$$\int_0^t dt = \int_{v_0}^{v} \frac{dv}{f(v)} \quad \text{———} \boxed{C}$$

or $f(v) = v\frac{dv}{dx} \Rightarrow dx = \frac{v\,dv}{f(v)}$

$$\int_{x_0}^{x} dx = \int_{v_0}^{v} \frac{v\,dv}{f(v)} \quad \text{———} \boxed{C} \quad \text{see } Ex_C.$$

Ex A The acceleration of a particle is defined by the relation

$$a = 18 - 6t^2$$

where, a - acceleration in m/s^2, t - time in sec.

assume at $t=0$, $v=0$, $X = 100$ m Determine

(a) the time when the velocity is again zero.

(b) the position and velocity when $t = 4$ sec.

(c) total distance traveled by particle from $t=0$ to $t=4$ sec.

Solution

$$a = \frac{dv}{dt} = 18 - 6t^2 \Rightarrow dv = 18dt - 6t^2dt$$

$$\int_{v_0}^{v} dv = \int_0^t 18\,dt - \int_0^t 6t^2dt \Rightarrow v - v_0 = 18t - 2t^3$$

but at $t=0$, $v=0 \Rightarrow v_0 = 0$

$$\therefore v = 18t - 2t^3 \quad \text{—— (1)}$$

(a) for $v=0$ sub in (1)

$$0 = 18t - 2t^3 \Rightarrow t^2 - 9 = 0 \Rightarrow \underline{\underline{t = 3\,\text{sec}}}$$ (Ignore -ve value).

(b) $v = 18t - 2t^3$ to find v at $t = 4$ sec.

$$v = 18(4) - 2(4)^3 = \underline{\underline{-56\ \text{m/sec}}}$$

$$v = \frac{dx}{dt} = 18t - 2t^3 \Rightarrow dx = 18t\,dt - 2t^3dt$$

$$\int_{x_0}^{x} dx = \int_0^t 18t\,dt - \int_0^t 2t^3dt$$

$$X - X_0 = 9t^2 - \frac{t^4}{2} \Rightarrow X - 100 = 9t^2 - \frac{t^4}{2} \text{ —}$$

$$X = 100 + 9t^2 - \frac{t^4}{2} \quad \text{—— (2)}$$

$$X_{4sec} = 100 + 9(4)^2 - \frac{(4)^4}{2} = 100 + 144 - 128 = 116 \text{ m}$$

(c) Since $\frac{dx}{dt} = 0$ in (2) lead to $t = 3$ sec see (a)

then total distance traveled =

$$|X_{t=4} - X_{t=3}| + |X_{t=3} - X_{t=0}| = |116 - 140.5| + |140.5 - 100|$$
$$= 65 \text{ m}.$$

Ex B The acceleration of a particle is defined by the relation

$$a = 21 - 12X^2$$

where a - acceleration m/s^2, X - distance in meters.

The particle starts with no initial velocity at $X=0$

Determine (a) the velocity when $X = 1.5$ m.

(b) the position where the velocity is again zero.

(c) the position where the velocity is maximum.

Solution

$$a = \frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt} = v\frac{dv}{dx}$$

$$\int_{v_0}^{v} v\,dv = \int_{x_0}^{x} f(x)\,dx = \int_{x_0}^{x} (21-12x^2)\,dx$$

$$\frac{1}{2}(v^2 - v_0^2) = \int_{x_0}^{x} (21-12x^2)\,dx$$

but $v_0 = 0$ at $x=0$ then.

$$v^2 = 42x - 8x^3 \quad \text{——} (1)$$

(a) $v^2 = 42(1.5) - 8(1.5)^3 = 63 - 27 = 36$

$\therefore v = \pm 6$ m/sec.

(b) $v = 0$ sub in (1)

$0 = 42x - 8x^3 \Rightarrow 21 - 4x^2 = 0$, $x^2 = \frac{21}{4}$, $x = \pm\sqrt{\frac{21}{4}}$

$\therefore X = \pm 1.73$ m, the value is $X = 1.73$ m since -ve value is impossible. (see eqn (1)) (!)

(c) derive eq (1) with respect to x

$$2v\,dv = 42\,dx - 24x^2\,dx$$

$$2v\frac{dv}{dx} = 42 - 24x^2 = 0$$

$\therefore 42 - 24x^2 = 0 \Rightarrow x^2 = \frac{42}{24} \Rightarrow x = \pm\sqrt{\frac{42}{24}}$

$X = \pm 1.205$ m But the -ve value is impossible see eq(1). then

$X = 1.205$ m

2-3 Uniform Rectilinear motion

It means that acceleration is Zero (i.e $a=0$) which means $v=$ Constant.

$$v = \frac{dx}{dt} = \text{const.}$$

$$dx = v\,dt \Rightarrow \int_{x_0}^{x} dx = \int_0^t v\,dt$$

$$x - x_0 = vt \Rightarrow x = x_0 + vt \quad \text{———} \; ①$$

2-4 Uniform accelerated rectilinear motion

$$a = \text{Constant} = \frac{dv}{dt} \Rightarrow \int_{v_0}^{v} dv = \int_0^t a\,dt$$

$$v - v_0 = at$$

$$v = v_0 + at \quad \text{———} \; ②$$

But $v = \frac{dx}{dt}$

$$\frac{dx}{dt} = v_0 + at \Rightarrow \int_{x_0}^{x} dx = \int_0^t v_0\,dt + \int_0^t at\,dt$$

$$\therefore x - x_0 = v_0 t + \frac{at^2}{2}$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2 \quad \text{———} \; ③$$

Also $a = v\frac{dv}{dx}$

$$v\,dv = a\,dx$$

$$\int_{v_0}^{v} v\,dv = a\int_{x_0}^{x} dx$$

$$\frac{1}{2}(v^2 - v_0^2) = a(x - x_0)$$

$$v^2 = v_0^2 + 2a(x - x_0) \quad \text{———} \; ④$$

It is to be note that eqs 2,3 & 4 are used when the acceleration of a particle is known to be Constant and when the velocity is Constant we use eq ①

Ex:C

The acceleration of a particle falling through atmosphere is defined by the relation $a = g(1 - K^2 V^2)$.

Knowing that the particle starts at $t = 0$ & $X = 0$ with no initial velocity.

ⓐ write an equation for the velocity $\rightarrow f(t)$

ⓑ write an equation for the velocity $\rightarrow f(X)$

Solution

ⓐ $a = f(v)$

$$f(v) = \frac{dv}{dt} \Rightarrow dt = \frac{dv}{f(v)}$$

$$\int_0^t dt = \int_{v_0}^{v} \frac{dv}{f(v)} = \int_0^V \frac{dv}{g(1-K^2V^2)},$$

$$\therefore gt = \int_0^V \frac{dv}{1-K^2V^2}$$

(HW: Complete the integration)
Hint: Let $K^2V^2 = \cos^2\theta$ or $\sin^2\theta$)

ⓑ $a = f(v) = v\frac{dv}{dx}$

$$dx = \frac{v\,dv}{f(v)} = \frac{v\,dv}{g(1-K^2V^2)}$$

$$\int_{x_0}^{X} g\,dx = \int_{v_0}^{v} \frac{v\,dv}{1-K^2V^2}$$

$$gX = \int_0^v \frac{v\,dv}{1-K^2V^2}$$

(HW: Complete the integration)

HW① The acceleration of a particle is defined by the relation

$a = -KX^{-2}$ where a in m/s^2, X in meters.

the particle starts with no initial velocity at $X = 12$ m, and its velocity is 8 m/sec when $X = 6$ m.

Determine ⓐ the value of K. [384 m^3/s^2]

ⓑ the velocity of the particle when $X = 3$ m [13.86 m]

Example

The acceleration of a particle is defined by the relation:

$a = -0.0125\, v^2$

where a - acceleration m/s^2, v - velocity m/sec.

If the particle is given an initial velocity v_0 at $x=0$ find the distance traveled

a) before the velocity drops to half the initial one.
b) before it comes to rest (i.e $v_f = 0$)

Solution

$$a = f(v) = v\frac{dv}{dx}$$

$$\int_{x_0}^{x} dx = \int_{v_0}^{v} \frac{v\,dv}{f(v)} \Rightarrow x - x_0 = -\int_{v_0}^{v} \frac{v\,dv}{0.0125\,v^2}$$

$$\therefore x = -\frac{1}{0.0125}\left[\ln v - \ln v_0\right]$$

(a)

If $v = \frac{1}{2}v_0$ then

$$x = -\frac{1}{0.0125}\left[\ln\left(\frac{\frac{1}{2}v_0}{v_0}\right)\right] = 55.45\, m$$

(b)

If $v=0$ then $\ln 0 = \infty$

$$x = +\infty$$

HW(2): The acceleration of a particle is defined by the relation.

$$a = -10v$$

Knowing that at $t=0$ the velocity is 30 m/sec find

a) the distance that the particle will travel before coming to rest.
b) time required for the particle to come to rest.
c) the time required for the velocity to be reduced 1% of its initial value.

These two situation (3, 4) is define the movement of a projectile, it may be an electron or a solid rockets. Two component of the velocity one with x-axis where $a_x = 0$ (eqn 1) and with respect to y-axis where a_y = Const = $-g$ (eq 2, 3 & 4).

Projectile

In case of the motion of a projectile it may be shown.

$a_x = 0$, $a_y = -g$ where $g = 9.81 \simeq 10\ m/sec^2$

neglect the resistance of the air to the body of projectile then in x- direction

$$x = x_o + v_{ox} t \quad — (5)$$ i.e constant acceleration

in y - direction.

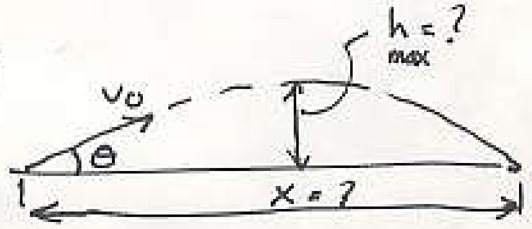
$$v = v_{oy} - gt \quad — (6)$$ since $a_y = -g$

$$y = y_o + v_{oy} t - \frac{1}{2} g t^2 \quad — (7)$$

$$v^2 = v_{oy}^2 - 2g(y - y_o) \quad — (8)$$

HW A projectile is fired with an initial velocity of v_o (m/sec) upward at an angle (θ) with the horizontal. Find the horizontal distance covered before the projectile returns to its original Level. Also determine the maximum height attained by the projectile.

[Hint]



Ans: $\left[h = \frac{v_o^2 \sin^2\theta}{2g} \ ; \ x = \frac{v_o^2 \sin 2\theta}{g} \right]$

Example

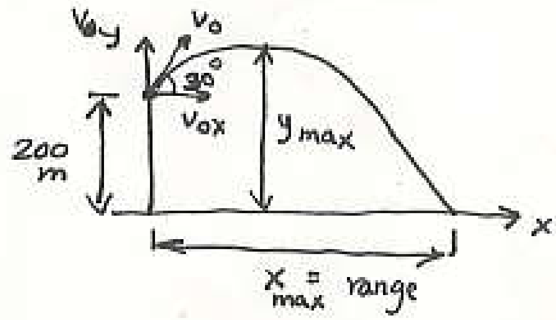
A projectile is fired from the edge of 200 m Cliff with an initial velocity of 180 m/sec at an angle of 30° with horizontal neglect air resistance find.

ⓐ the horizontal distance from the gun to the point where the projectile hits the ground.

ⓑ the greatest elevation above the ground reached by the projectile.

ⓒ velocity at which it hits the ground (take $g = 10\ m/sec^2$)

Solution



$V_{ox} = V_0 \cos 30 = 155.88\ m/s$.

$V_{oy} = V_0 \sin 30 = 90\ m/s$.

$a_x = 0, \quad a_y = -10\ m/s^2$

$y = y_0 + v_{oy} t - \frac{1}{2} g t^2$

to hit the ground

$y = 0 = 200 + 90t - 5t^2$ —①

$t^2 - 18t - 40 = 0$

$(t - 20)(t + 2) = 0$

$\therefore t = 20\ sec$ (time to hit the ground)

Range = X_{max}

$X_{max} = \cancel{X_0}^{0} + v_{ox} t = 155.8(20) = 3117.6\ m$.

max elevation at $\frac{dy}{dt} = 0$ (from eq ①)

$\frac{dy}{dt} = 0 = 90 - 10t \Rightarrow t = \frac{90}{10} = 9\ sec$.

$y_{max} = 200 + 90(9) - 5(9)^2 = 605\ m$.

$V_y = V_{oy} - gt = 90 - 10(20) = -110\ m/s$

$V_x = 155.88\ m/s$.

$V = \sqrt{V_x^2 + V_y^2} = 190\ m/sec$

$\theta = \tan^{-1} \frac{V_y}{V_x} = 35°$.

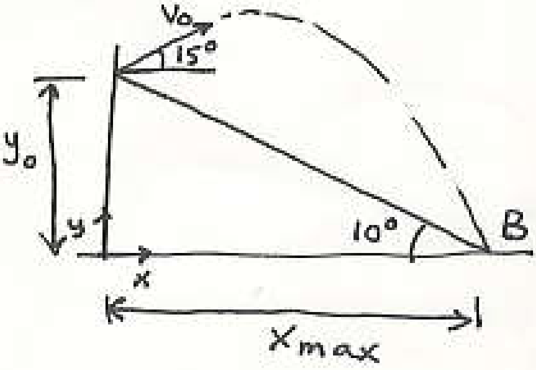
Example

Standing on the side of the hill, a person shoots an arrow with an initial velocity of 76 m/sec at an angle of 15° with the horizontal. Find.

ⓐ The horizontal distance x_{max} traveled by the arrow before it strikes the ground at B.

ⓑ max elevation with respect to B reached by the arrow.

ⓒ velocity at which the arrow hits the ground.



Solution

$$\tan 10 = \frac{y_o}{X_{max}}$$

$$y_o = X_{max} \tan 10$$

$$X_{max} = v_{ox} t + \cancel{X_o}^{0} = v_{ox} t$$

$$y = y_o + v_{oy} t - \tfrac{1}{2} g t^2 \quad \text{——} ①$$

$$v_{ox} = 76 \cos 15 = 73.41 \text{ m/s}$$

$$v_{oy} = 76 \sin 15 = 19.67 \text{ m/s}$$

when the arrow hits ground. $(y = 0)$

$$\therefore 0 = y_o + v_{oy} t - 5t^2$$

$$x_{max} \tan 10 + V_{oy} t - 5t^2 = 0$$

$$v_{ox} t \tan 10 + v_{oy} t - 5t^2 = 0$$

$$73.41\, t \tan 10 + 19.67 t - 5t^2 = 0$$

$$32.614\, t - 5t^2 = 0$$

$$t = 6.52 \text{ sec.}$$

$$\therefore X_{max} = 73.41\,(6.5) = 478.8 \text{ m.}$$

Differentiate eq 1 with t & ≡ to zero.

$$\frac{dy}{dt} = 0 = 0 + v_{oy} - gt$$

$$t = \frac{v_{oy}}{g} = \frac{19.67}{10} = 1.967 \text{ sec.}$$

$$y_{max} = (X_{max} \tan 10) + v_{oy} t - 5t^2$$

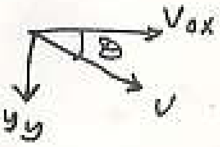
$$= 84.4326 + 19.67 * 1.967 - 5\,(1.967)^2 = 103.77 \text{ m.}$$

$$V = \sqrt{V_{ox}^2 + V_y^2}$$

$$V_y = V_{oy} - gt$$

$$= 19.67 - 10\,(6.523)$$

$$= -45.56 \text{ m/sec}$$



$$V = \sqrt{(45.56)^2 + (73.41)^2}$$

$$= 86.398 \text{ m/sec}$$

$$\theta = \tan^{-1} \frac{V_y}{V_x}$$

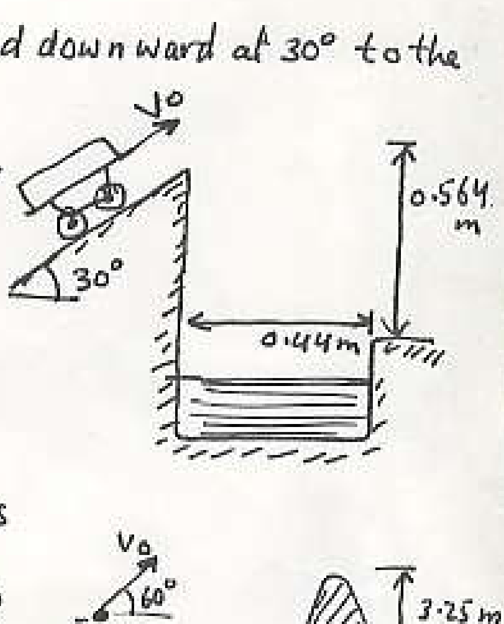
$$= 31.8^{0}$$

Problems

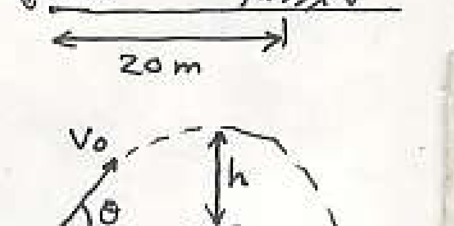
1. A stone is thrown from a hill at an angle of $60°$ to the horizontal with an initial velocity of 30 m/sec. After hitting level ground at the base of the hill the stone has covered a horizontal distance of 150 m. How high is the hill? (-230.69 m)

2. A shell leaves a mortar with a muzzle velocity of 150 m/s directed upward at $60°$ with the horizontal. Determine the position of the shell and its resultant velocity 20 sec after firing. How high will it rise? [1500 m, 100 m/s, 860 m].

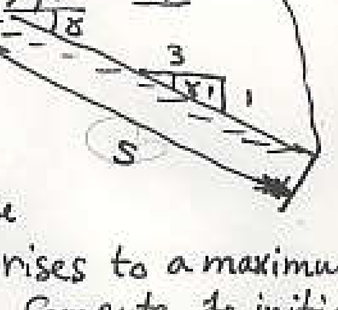
3. A projectile is fired with an initial velocity of 63 m/sec upward at an angle of $30°$ to the horizontal from a point 85 m above a level plain. What horizontal distance will it cover before it strikes the level plain? [462 m]
Repeat the problem if the projectile is fired downward at $30°$ to the horizontal level [111.65 m].



4. The car shown in fig is just to clear the wate -filled gap.
Find the take-off velocity v_0.
[1.244 m/sec].



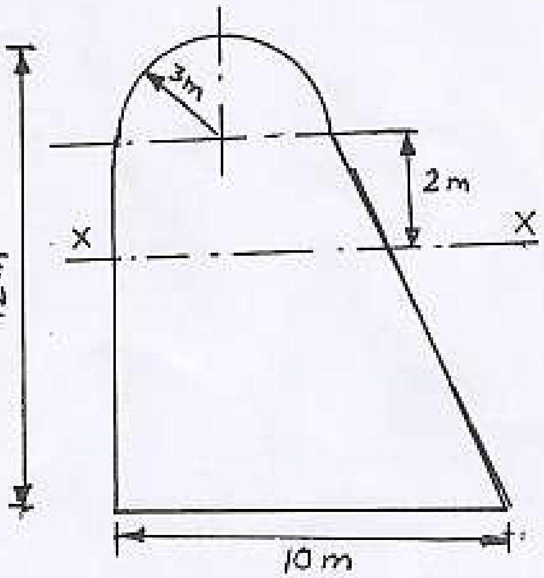
5. A ball is thrown so that it just clears a 3.25 m fence 20 m away. If it left the hand 1.65 m above the ground & at angle of $60°$ to the horizontal, what was the initial velocity of the ball? [15.41 m/sec]



6. Determine the distance S at which a ball thrown with a velocity v_0 of 32.8 m/sec. at an angle $\theta = \tan^{-1}\frac{3}{4}$ will strike the incline shown in figure. [160 m]
For the same figure, a ball thrown down the incline strikes it at a distance S = 83.5 m. If the ball rises to a maximum height h = 21.31 m above the point of release, Compute its initial velocity v_0 and inclination θ. [25.45 m/sec, 53.1°]

Example

Determine the moment of inertia for the Composite area shown in figure w.r.t the x-axis.



Solution

$I_{xx} = I_{\Delta} + I_{\square} + I_{\text{semicircle}}$

$I_{\Delta} = I'_{\Delta} + (Ah^2)_{\Delta}$

$I_{\square} = I'_{\square} + (Ah^2)_{\square}$

$I_{\text{semicircle}} = I'_{\text{semicircle}} + (Ah^2)_{\text{semicircle}}$

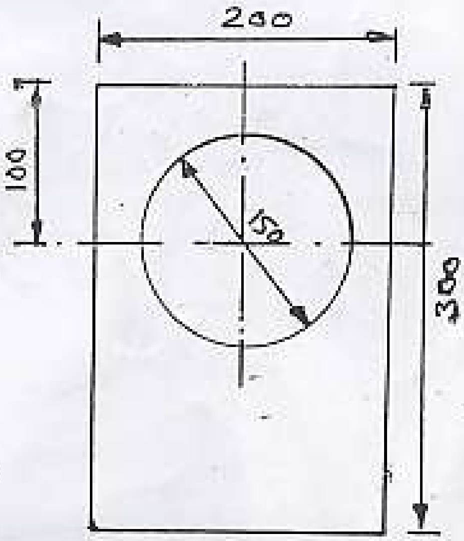
$I'_{\square} = \frac{bd^3}{12}$, $I'_{\Delta} = \frac{bh^3}{36}$, $I'_{\text{semicircle}} = 0.11 r^4$

Item	I'	Area	h	h^2	Ah^2
rectangle	364.5	54	-2.5	6.25	337.5
Semicircle	8.91	14.14	3.272	10.706	151.38
triangle	81	18	-4	16	288
SUM	454.41				776.88

$I_{xx} = 454.41 + 776.88 = 1231.29 \ m^4$

HW-1 Find the moment of inertia of a hollow section shown in figure about an axis passing through its Centroid and parallel to x-axis.

(all dimensions in mm).



HW-2 Find the Centroid for the figure in the above example w.r.t x-axis shown

[Ans: $\bar{x} = 0.9054$ m, $\bar{y} = -1.866$ m]