A New Fast Adjusted Step Size Affine Projection Algorithm Designed for Adaptive Filtering System

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Abstract: *This paper presents an adjusted step size method for affine projection algorithm (APA). This proposed algorithm named as (ASSAPA). The main goal of this algorithm is performance enhancement of adaptive colored filtering system in terms of fast convergence and low level of misadjustment. The adjusted step size of the algorithm is adjusted recursively from the maximum step size to the minimum value based on rough estimation of the performance surface gradient square. Furthermore, an appropriate time varying value of the maximum step size is chosen to be inversely proportional to the instantaneous energy of the input signal vector. Then this time varying upper bound value of the step size is used to guarantee the stability of the algorithm. The proposed algorithm is tested with colored input signal. The result shows that the proposed algorithm is better than the traditional APA and another variable step size LMS (VSSLMS) algorithm in terms of convergence rate and misadjustment.*

Keywords: *adaptive filter, adjusted step size, affine projection algorithm*

1. Introduction

The most common adaptive algorithms are Least Mean Square (LMS) but this algorithm suffers from slow convergence speed if it is driven by colored input signals as is with speech. One method presented to overcome this problem is the Ozeki/ Umeda Affine Projection Algorithm (APA) ([4], [10], and [5]). APA is a useful family of adaptive filters whose main purpose is to accelerate the convergence of LMS-type filters, especially for correlated data at a computational cost that is comparable to that of LMS. While LMS updates the weights based on the current input vector, APA updates the weights based on previous input vectors [4].

An APA with a constant step-size parameter has to compromise between the performance's criteria of fast convergence rate, and low misadjustment. Therefore, a variable step-size APA represents a more reliable solution [9]. Therefore, several proposed variable step-size APAs algorithms were developed ([1],[4],[6] ,[8] ,[9], [11], and[12]).

In this paper, time varying step size method is chosen due to its powerful effect on the performance of the system. In addition, the structure of the adaptive filter will not be changed, and this technique requires fewer overheads in computations, which are an important factor for hardware implementation. The proposed algorithm in this paper is called Adjusted Step Size Affine Projection Algorithm (ASSAPA) . This algorithm is regarded as modified version of Variable Step Size LMS (VSSLMS) algorithm, which was developed by R.H.Kang and E.W.Johnstone [6]. The new proposed algorithm shows good performance and also gets rid of the main drawback of the VSSLMS algorithm which is the trial and error in the selection of the maximum value of the step size (μ_{MAX}). The value of the maximum of the step size in this paper is adjusted according to the input power of the signal. This step size is proportional to the inverse of the total expected energy of the instantaneous values of the coefficients of the input vector.

The main goal of this new proposed algorithm is achieving fast convergence time and low level of misadjustment when colored input signal was used in the adaptive filtering system.

This paper is organized as follows: section 2 presents the basic concept of the APA, while section 3 gives the proposed algorithm. Section 4 demonstrates the performance of the proposed algorithm by carrying out experiments in system identification and finally section 5 gives important conclusions from this research.

2. Basic Concept of Affine Projection Algorithm (APA)

In this section, the basic concept of the APA will be introduced for the adaptive filter system shown in Figure 1. The input vector $\boldsymbol{x}(k)$ can be defined as:

$$\boldsymbol{x}(k) = [x(k), x(k-1), \dots\dots\dots x(k-L+1)]^T \qquad (1)$$

Where k is the discrete time index, ^{T} denotes to transpose. The adaptive Finite Impulse Response (FIR) filter coefficient vector is defined by [7]:

$$\boldsymbol{w}(k) = [w_1(k), w_2(k), \dots\dots\dots\dots w_L(k)]^T \qquad (2)$$

Where $w_1(k)$, $w_2(k)$, …… $w_L(k)$ are filter coefficients at time k, L is the number of taps (or coefficients). The output of the adaptive filter $y(k)$ is calculated by

$$y(k) = \boldsymbol{x}^T(k)\,\boldsymbol{w}(k) \qquad (3)$$

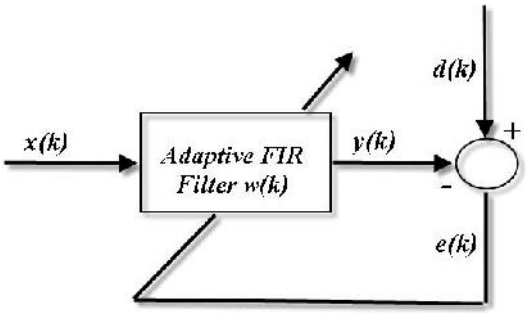

Figure 1. Block diagram of adaptive filtering system

The objective of an adaptive filter is to generate the output signal $y(k)$ as close as possible to the desired signal $d(k)$. To do this, the adaptive filter adjusts its coefficient $\mathbf{w}(k)$ at every sampling time k by [7]

$$\boldsymbol{w}(k+1) = \boldsymbol{w}(k) + \mu\, \boldsymbol{\Delta w}(k) \tag{4}$$

Where $\boldsymbol{\Delta w}(k)$ is an adjustment vector and μ is called the step size, which controls the amount of adjustments. The adjustment vector $\boldsymbol{\Delta w}(k)$ is determined in the projection algorithm to satisfy the following equations [14]:

$$d(k) = \boldsymbol{x}^T(k)(w(k) + \Delta\, \boldsymbol{w}(k))$$

$$d(k-1) = \boldsymbol{x}^T(k-1)(w(k) + \Delta\, \boldsymbol{w}(k))$$

$$\cdots\cdots$$

$$d(k-p+1) = \boldsymbol{x}^T(k-p+1)(w(k) + \Delta\, \boldsymbol{w}(k)) \tag{5}$$

Where p $(p < L)$ is called the projection order. These equations means that the adjusted vector $\mathbf{w}(k+1)$ with $\mu = 1$ (that is $\mathbf{w}(k) + \Delta\, \boldsymbol{w}(k)$) generates $d(k), \ldots d(k-p+1)$ for the past p input vectors $\boldsymbol{x}(k), \ldots \boldsymbol{x}(k-p+1)$. Equation (5) is rewritten in matrix form as [14]:

$$\boldsymbol{d}_p(k) = \boldsymbol{X}_p^T(k)\left(\boldsymbol{w}(k) + \Delta\, \boldsymbol{w}(k)\right) \tag{6}$$

Or equivalently, as

$$\boldsymbol{X}_p^T(k)\Delta\, \boldsymbol{w}(k) = \boldsymbol{e}_p(k) \tag{7}$$

Where $\boldsymbol{d}_p(k)$ and $\boldsymbol{e}_p(k)$ are vectors with p elements, and $\boldsymbol{X}_p(k)$ is an $L \times p$ matrix defined by [14]:

$$\boldsymbol{d}_p(k) = [d(k), d(k-1), \ldots\ldots, d(k-p+1)]^T \tag{8}$$

$$\boldsymbol{X}_p(k) = [\boldsymbol{x}(k), \boldsymbol{x}(k-1), \ldots\ldots \boldsymbol{x}(k-p+1)] \tag{9}$$

$$\boldsymbol{e}_p(k) = \boldsymbol{d}_p(k) - \boldsymbol{X}_p^T(k)\boldsymbol{w}(k) \tag{10}$$

We can write the prior error equation vector as:-

$$\boldsymbol{e}_p(k) = [e(k), e(k-1), \ldots\ldots e(k-p+1)]^T \tag{11}$$

Since the number of simultaneous equations p in (5) is smaller than the number of unknowns L, these equations are indeterminate. Among the many solutions, the minimum norm solution is derived from (7) by [14]:

$$\Delta\boldsymbol{w}(k) = \boldsymbol{X}_p(k)(\boldsymbol{X}_p^T(k)\boldsymbol{X}_p(k))^{-1}\boldsymbol{e}_p(k) \tag{12}$$

Substituting (12) into (4) gives the adjustment formula for the original projection algorithm that first time proposed by Ozeki/ Umeda [14]:

$$\boldsymbol{w}(k+1) = \boldsymbol{w}(k) + \mu\, \boldsymbol{X}_p(k)(\boldsymbol{X}_p^T(k)\boldsymbol{X}_p(k))^{-1}\boldsymbol{e}_p(k) \tag{13}$$

Table 1 shows the step's sequence of APA algorithm.

Table 1.APA algorithm

Initialization:- Set $\boldsymbol{w}(0) = [0, 0, \ldots\ldots 0]^T$, and set the parameters L is the order of taps FIR filter, p is the projection order, and μ is the step size.
Given the input vector:-
$\boldsymbol{x}(k) = [x(k), x(k-1), \ldots\ldots\ldots x(k-L+1)]^T$, with dimension vector LXp, and d(k) is the desired response at time k .

$\boldsymbol{X}_p(k) = [\boldsymbol{x}(k), \boldsymbol{x}(k-1), \ldots\ldots \boldsymbol{x}(k-p+1)]$

$\boldsymbol{e}_p(k) = [e(k), e(k-1), \ldots\ldots e(k-p+1)]^T$, is the prior error equation vector with dimension px1.

$\boldsymbol{d}_p(k) = [d(k), d(k-1), \ldots\ldots, d(k-p+1)]^T$, is the desired response vector with dimension px1.

For k= 0,1,2,..... Iteration
Compute the following:-
$\boldsymbol{e}_p(k) = \boldsymbol{d}_p(k) - \boldsymbol{X}_p^T(k)\boldsymbol{w}(k)$
$\boldsymbol{w}(k+1) = \boldsymbol{w}(k) + \mu\, \boldsymbol{X}_p(k)(\boldsymbol{X}_p^T(k)\boldsymbol{X}_p(k))^{-1}\boldsymbol{e}_p(k)$

3. Proposed Adjusted Step Size APA Algorithm (ASSAPA)

The step size of the proposed algorithm was adjusted as follows:-

$$\mu(k+1) = \alpha\mu(k) + \delta\,(\boldsymbol{e}_p(k)\,\boldsymbol{x}(k))^2 \tag{14}$$

Where $0 < \alpha < 1$ and $\delta > 0$, then:-

$$\left.\begin{array}{l} \mu(k+1) = \mu_{MAX} \quad if \quad \mu(k+1) > \mu_{MAX} \\ \text{or} \\ \mu(k+1) = \mu_{min} \quad if \quad \mu(k+1) < \mu_{min} \\ \text{Otherwise} \\ \mu(k+1) = \mu(k+1) \end{array}\right\} \tag{15}$$

Equation (14) is a formula to adjust the step size which is modified from the original equation which proposed by R.H. Kwong and E.W. Johnston in 1992 [6]. The method in [6] provides an important theoretical support of all error based variable step size LMS methods. The step size is adjusted according to the square of the gradient of the performance surface (i.e. $(\boldsymbol{e}_p(k)\,\boldsymbol{x}(k))^2$. The terms $\boldsymbol{x}(k)$ and $\boldsymbol{e}_p(k)$ are calculated from (1) and (11) respectively.

To ensure stability, the variable step size $\mu(k+1)$ is constrained to the pre-determined maximum and minimum step size values .While α and δ are parameters that are controlling the recursion. The parameter δ controls the convergence time as well as the level of misadjustment of the algorithm at the steady state. However, there is no

formula or equation to calculate α and δ in all papers, including the original paper [9] but usually they assigned high value for α which is very close to 1 (i.e. 0.97-to-0.99) and very small value for δ [3].

Moreover, $\mu(k+1)$ is set to μ_{min} or μ_{MAX} when it falls below or above these lower and upper bounds, respectively. The constant μ_{MAX} is normally selected near the point of instability of the conventional LMS to provide the maximum possible convergence speed. While the value of μ_{min} is chosen as a compromise between the desired level of steady state misadjustment and the required tracking capabilities of the algorithm.

Equation (13) of APA algorithm can be rewritten or modified according to ASSAPA as:

$$\boldsymbol{w}(k+1) = \boldsymbol{w}(k) + \mu(k)\,\boldsymbol{X}_p(k)(\boldsymbol{X}_p^T(k)\boldsymbol{X}_p(k))^{-1}\boldsymbol{e}_p(k) \tag{16}$$

The main drawback of the (VSSLMS) algorithm is how to select the value of the upper bound of step size , i.e. μ_{MAX} . In other words, this drawback is the requirement of a statistical knowledge of the input signal prior to the starting learning of the algorithm which is necessary to determine the fixed value of the maximum step size. μ_{MAX} (i.e. the upper bound value) in the initialization stage of the algorithm. Therefore, an appropriate time varying value of the maximum step size is calculated based upon inversely proportional to the instantaneous energy of the input signal vector.

$$\mu_{MAX} = \frac{1}{2\boldsymbol{x}^T(k)\boldsymbol{x}(k)} \tag{17}$$

Equation (17) is implemented as follows [14]:-

$$\mu_{MAX} = \frac{\beta}{2\boldsymbol{x}^T(k)\boldsymbol{x}(k)+\varphi} \tag{18}$$

Where the value of φ is a small positive constant in order to avoid division by zero when the values of the input vector are zero and β is within the range of 0<β>2, usually it is equal to 1. Table 2 shows the step's sequence of new proposed ASSAPA algorithm.

In this paper in addition to APA, a comparison between the performance of the ASSAPA and another Variable Step Size (VSSLMS) [6] algorithm is introduced. The steps required by the VSSLMS algorithm are as follows:

$$\mu(k+1) = \alpha\mu(k) + \delta\,(\boldsymbol{e}_p(k))^2 \tag{19}$$

Where $0 < \alpha < 1$ and $\delta > 0$, then:-

$$\left.\begin{array}{l} \mu(k+1) = \mu_{MAX} \quad if \quad \mu(k+1) > \mu_{MAX} \\ \text{or} \\ \mu(k+1) = \mu_{min} \quad if \quad \mu(k+1) < \mu_{min} \\ \text{Otherwise} \\ \mu(k+1) = \mu(k+1) \end{array}\right\} \tag{20}$$

The term $\boldsymbol{e}_p(k)$ in (19) is calculated according to the (11) which means that the VSSLMS algorithm is modified in this paper according to the affine projection method.

Table 2. New Proposed ASSAPA Algorithm

tialization:- Set $w(0) = [0, 0, \ldots\ldots 0]$, and set the parameters L is the order of taps FIR filter, p is the projection order , α, δ, μ_{min} and φ for adjusting step size.

Given the input vector:-

$\boldsymbol{x}(k) = [x(k), x(k-1), \ldots\ldots\ldots x(k-L+1)]^T$, with dimension vector(LXp) , and d(k) is the desired response at time k .

$\boldsymbol{X}_p(k) = [\boldsymbol{x}(k), \boldsymbol{x}(k-1), \ldots\ldots \boldsymbol{x}(k-p+1)]$

$\boldsymbol{e}_p(k) = [e(k), e(k-1), \ldots\ldots e(k-p+1)]^T$, is the prior error equation vector with dimension px1.

$\boldsymbol{d}_p(k) = [d(k), d(k-1), \ldots\ldots, d(k-p+1)]^T$, is the desired response vector with dimension px1.

2.For k= 0,1,2,….. Iterations

Compute the following:-

$\boldsymbol{e}_p(k) = \boldsymbol{d}_p(k) - \boldsymbol{X}_p^T(k)\boldsymbol{w}(k)$

$\mu_{MAX} = \frac{\beta}{2\boldsymbol{x}^T(k)\boldsymbol{x}(k)+\varphi}$,

$\mu(0) = \mu_{MAX}$,

$\boldsymbol{w}(k+1) = \boldsymbol{w}(k) + \mu(k)\,\boldsymbol{X}_p(k)(\boldsymbol{X}_p^T(k)\boldsymbol{X}_p(k))^{-1}\boldsymbol{e}_p(k)$

$\mu(k+1) = \alpha\mu(k) + \delta\,(\boldsymbol{e}_p(k)\boldsymbol{X}_p(k))^2$

$\mu(k+1) = \mu_{MAX}$ if $\mu(k+1) > \mu_{MAX}$

Or

$\mu(k+1) = \mu_{min}$ if $\mu(k+1) < \mu_{min}$

Otherwise $\mu(k+1) = \mu(k+1)$

4. Simulation Results

The performance of proposed, APA, and VSSLMS algorithms are validated by simulation of a system identification model as shown in Figure 2 which shown an adaptive filter used in the system identification model. The VSSLMS algorithm is modified according to the Affine Projection method. The adaptive filter and unknown system are FIR, and both have the same number of taps (L= 36). The unknown system has 36 randomly selected taps. The goal of the adaptive processing is to estimate unknown system parameters (weight coefficients W) by optimizing the adaptive filter parameters (weight coefficients W^) iteratively using APA, new proposed ASSAPA, and VSSLMS algorithms.

The input signal x(k) (colored signal) is obtained by filtering a white Gaussian random noise x(k) (zero mean, unit variance) through a 1*st* order filter (first-order autoregressive system) , $A(z) = 1/(1 - 0.9z^{-1})$, where z^{-1} is the sample delay operator, the value of $0.9\,z^{-1}$ is chosen to generate a highly colored Gaussian signal.

The step size μ used for APA is equal to 0.005 and the parameters of VSSLMS algorithm are as follows: α = 0.97, δ= 1e-3, μ_{MAX} = 0.05, μ_{min} = 1e-5 . While the parameters of ASSAPA algorithm are as follows: α = 0.97, δ= 1e-3, μ_{min} = 1e-5 , φ=0.001 and β = 0.5 . All the values of these parameters were chosen to achieve better performance in terms of fast convergence time and low level misadjustment for all algorithms in order to make fairly comparison between these algorithms.

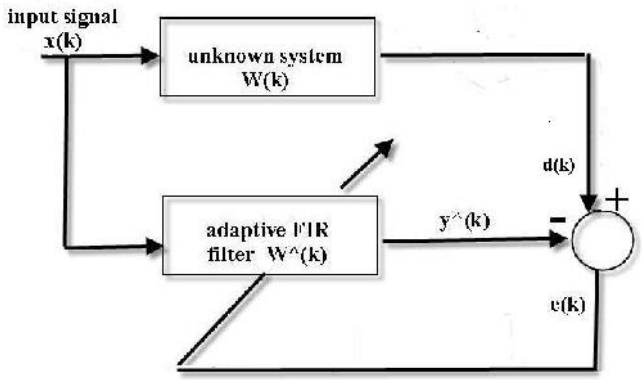

Figure 2. Adaptive System Identification Structure

Figure 3 shows the squared estimation error between unknown system weight coefficients (W) and the corresponding FIR weight coefficients (W^) i.e. (W-W^) for all algorithms when the projection order (p) is equal to 2.

It is clear from Figure 3 ; all algorithms have the same convergence time (200 iterations) , but ASSAPA algorithm shows a better low misadjustment in a steady state compared with the other algorithms.

Figure 4 shows the performance of all algorithms when the projection order was 4. In this case, the proposed algorithm ASSAPA has fast convergence time (600 iterations) compared with APA (1200 iterations) and VSSLMS (800 iterations).

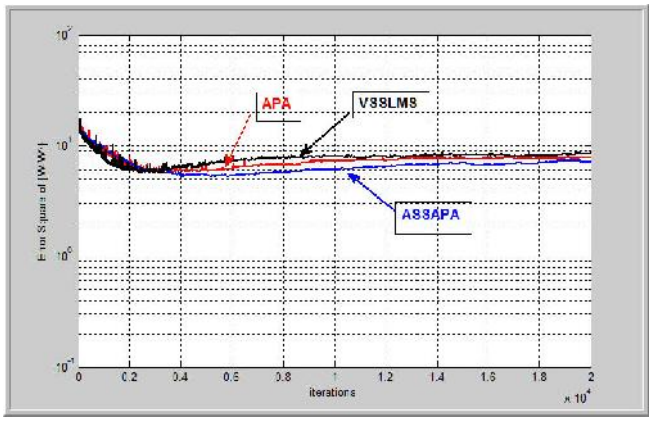

Figure 3. Squared estimation error (W-W^) for APA, VSSLMS, and ASSAPA algorithms when the projection order (P) is equal to 2

Figure 5 shows the performance of all algorithms when the projection order was increased to 8. In this case, the proposed algorithm ASSAPA also has fast convergence time (800 iterations) compared with the APA (1400 iterations), and VSSLMS (1000 iterations). Moreover, the proposed algorithm shows a low level of misadjustment compared with the other algorithms. Figure 6 shows the performance of all algorithms when the projection order was increased to 16. In this case, the proposed ASSAPA and VSSLMS algorithms has fast convergence time (1000 iterations) compared with APA which has convergence time equals to 1400 iterations. Moreover, the proposed algorithm shows a low level of misadjustment compared the other algorithms.

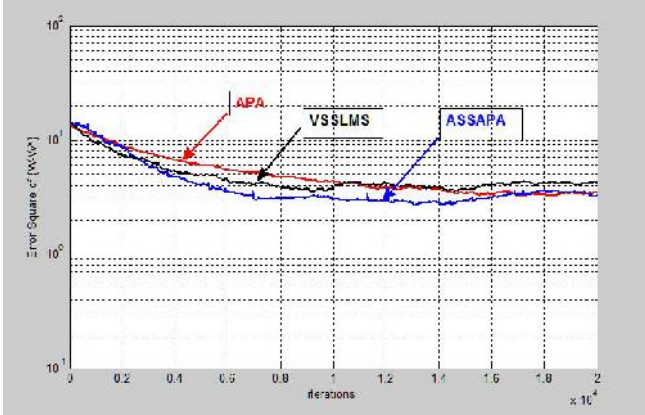

Figure 4. Squared estimation error (W-W^) for APA, VSSLMS, and ASSAPA algorithms when the projection order (P) is equal to 4.

As shown from Figures 5 and 6, that the performance of ASSAPA is improved very well compared with APA and VSSLMS algorithms when projection order's values are 8 and 16 respectively. The reason of that is the number of input signal vectors used for adaptation is increased, and the algorithm applies to update directions that are orthogonal to the last p input vectors and thus in turns allows decorrelation of an input process, speeding up the convergence [2].

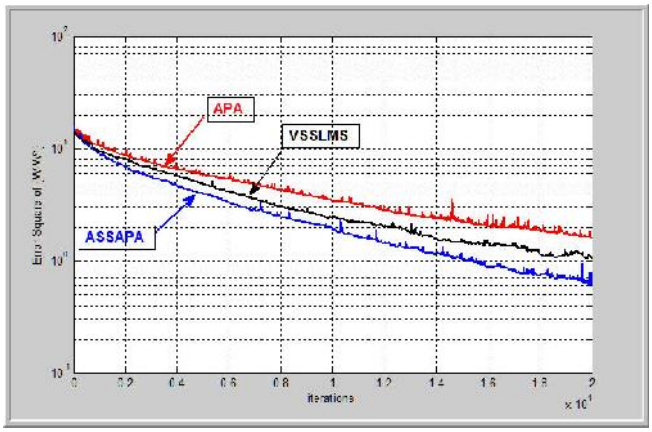

Figure 5. Squared estimation error (W-W^) for APA, VSSLMS, and ASSAPA algorithms when the projection order (p) is equal to 8

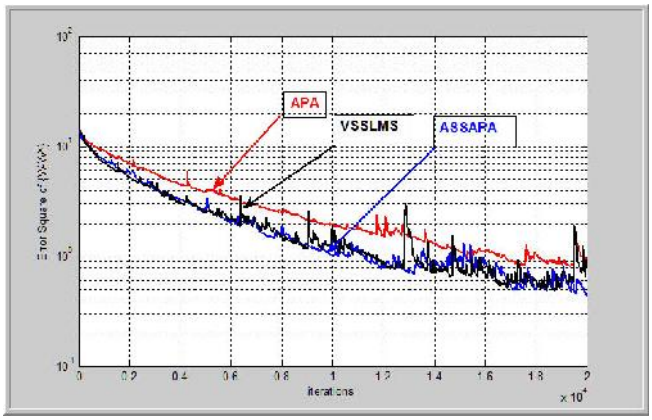

Figure 6. Squared estimation error (W-W^) for APA , VSSLMS, and ASSAPA algorithms when the projection order (p) is equal to 16.

5. Conclusions

This paper presents a new proposed adjusted step size affine projection algorithm (ASSAPA). This algorithm used an appropriate time varying value of the maximum step size μ_{MAX} that is calculated based upon inversely proportional to the instantaneous energy of the input signal vector. This method is the favourite choice because the time varying μ_{MAX} will track any change through the input signal power. Then this time varying μ_{MAX} is used to guarantee the stability of adjusted step size, which is a recursively adjusted based on a rough estimate of the performance surface gradient square.

The simulation result shows that the proposed algorithm has a faster convergence time and low level of misadjustment than the APA and VSSLMS algorithms. It is shown that the convergence rate is exponential, and that it improves as the number of input signal vectors used for adaptation is 8 and 16 respectively (i.e. higher projection order (p)).

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