Chapter One

Introduction

1.1 Phasor representation:

The waveform of voltage at the buses of a power system can be assumed to be sinusoidal and of constant frequency.

If a voltage and a current expressed as a function of time, such as:

$$v = 141.4\cos(\omega t + 30^\circ)$$

$$i = 7.07\cos\omega t$$

The maximum values of V and I are: V_{max} = 141.4 V and I_{max} = 7.07 Amp

The magnitude value refers to root- mean- square value or the r.m.s value which equal to:

$$\text{r.m.s} = \frac{maximum\ value}{\sqrt{2}}$$

So for the above expression: |V| = 100 V, and | I | = 5 Amp

The average power expended in a resistor by a current of magnitude |I | is $|I|R^2$.

To express these quantities as phasors, employ Euler's identity must be applied

$$e^{j\theta} = \cos\theta + j\sin\theta$$

Which gives:

$$\cos\theta = \mathrm{Re}\{\varepsilon^{j\theta}\} = \mathrm{Re}\{\cos\theta + j\sin\theta\}$$

Where R means the real part. So:

$$v = \mathrm{Re}\{\sqrt{2}\,100\varepsilon^{j(\omega t + 30^\circ)}\} = \mathrm{Re}\{100\varepsilon^{j30^\circ}\sqrt{2}\,\varepsilon^{j\omega t}\}$$

$$i = \mathrm{Re}\{\sqrt{2}\,5\varepsilon^{j(\omega t + 0^\circ)}\} = \mathrm{Re}\{5\varepsilon^{j0^\circ}\sqrt{2}\,\varepsilon^{j\omega t}\}$$

If the current is the reference phasor:

$$I = 5\varepsilon^{j0^\circ} = 5\underline{/0^\circ} = 5 + j0\ \text{A}$$

And the voltage which leads the reference phasor by 30° is:

$$V = 100\varepsilon^{j30^\circ} = 100\underline{/30^\circ} = 86.6 + j50\ \text{V}$$

1.2 POWER IN SINGLE-PHASE AC CIRCUITS

The unit of power is watt. The power in Watt being absorbed by a load at any instant is the product of the instantaneous voltage drop across the load in volts and the instantaneous current across into the load in amperes.

$$V_{an} = V_{max} \cos \omega t \quad \text{and} \quad I_{an} = I_{max} \cos (\omega t - \Theta)$$

The instantaneous value of power is:

$$V_{an} \,.\, I_{an} = V_{max} \,.\, I_{max} \cos \omega t.\ \cos (\omega t - \Theta)$$

The angle in these equations is positive for current lagging the voltage and negative for leading current.

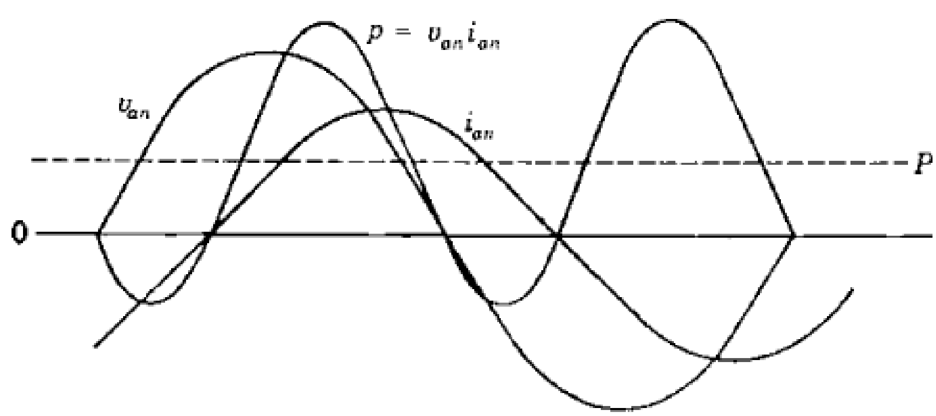


Figure (1.1) voltage, current, and power versus time

The power can expressed also as:

$$p = \frac{V_{max} I_{max}}{2} \cos\theta (1 + \cos 2\omega t) + \frac{V_{max} I_{max}}{2} \sin\theta \sin 2\omega t$$

Where $V_{max.}\, I_{max}/2$ may be replaced by the product of the r.m.s voltage and current, that is , by | V_{an} | a| I_{an} | or | V | and | I|.

Another form for the instantaneous power is to consider the component of the current in phase with V_{an} and the component 90° out of phase with V_{an}. The component of i_{an} in phase with V_{an} is i_R, and | I_R| = I_{an} cos Θ. If the maximum value of i_{an} is I_{max}, the maximum value of i_R is I_{max} cos Θ. The instantaneous current i_R must be in phase with V_{an}. For $V_{an} = V_{max}$ cos ωt.

$$i_R = \underbrace{I_{max} \cos \theta}_{\max i_R} \cos \omega t$$

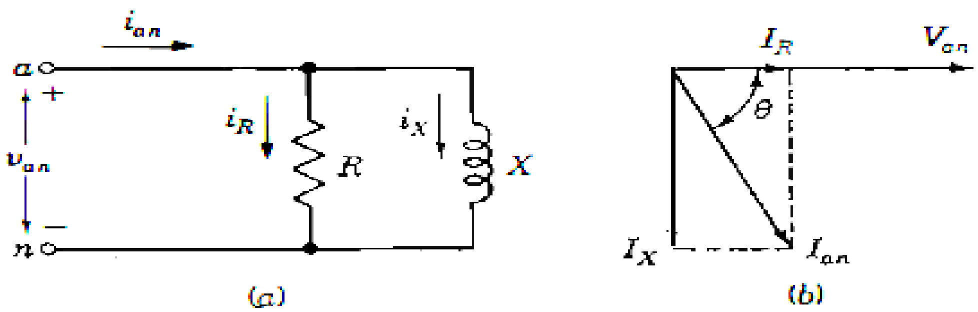


Figure (1. 2) Parallel RL circuit and it's the corresponding phasor diagram.

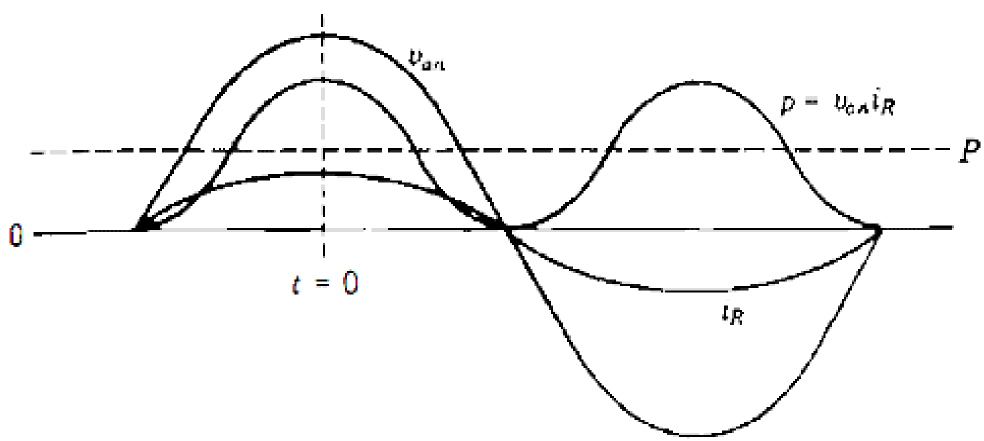


Figure (1.3) Voltage, current in phase with the voltage, and the resulting power versus time.

Also, the component of i_{an} lagging V_{an} by 90° is i_x with maximum value.

$I_{max} \sin\theta$. Since i_X must lag v_{an} by 90°,

$$i_X = \underbrace{I_{max} \sin\theta}_{\max i_X} \sin \omega t$$

Then,

$$v_{an} i_R = V_{max} I_{max} \cos\theta \cos^2 \omega t$$

$$= \frac{V_{max} I_{max}}{2} \cos\theta (1 + \cos 2\omega t)$$

$$v_{an} i_X = V_{max} I_{max} \sin\theta \sin\omega t \cos\omega t$$

$$= \frac{V_{max} I_{max}}{2} \sin\theta \sin 2\omega t$$

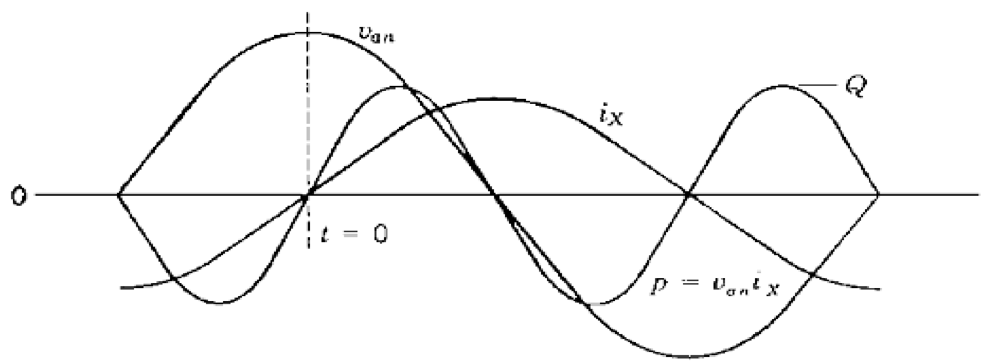


Figure (1.4) Voltage, current lagging the voltage by 90°, and their resulting power versus time.

The term containing cos Θ is always positive and has an average value of :

$$P = \frac{V_{max} I_{max}}{2} \cos\theta$$

And the voltage and current are in the r.m.s form:

$$P = |V|\ |I| \cos\theta$$

$$Q = \frac{V_{max} I_{max}}{2} \sin \theta$$

$$Q = |V| \; |I| \sin \theta$$

In a simpil series circuit where Z is equal to R + jX substituting | I| . Z in | V|

$$P = |I|^2 |Z| \cos \theta$$

$$Q = |I|^2 |Z| \sin \theta$$

$R = |Z| \cos \theta$ and $X = |Z| \sin \theta$, we then find

$$P = |I|^2 R \qquad \text{and} \qquad Q = |I|^2 X$$

$$\cos \theta = \cos\left(\tan^{-1} \frac{Q}{P}\right)$$

$$\cos \theta = \frac{P}{\sqrt{P^2 + Q^2}}$$

1.5 COMPLEX POWER:

I f the voltage across and the current into a certain load or part of a circuit are expressed by

$$V = |V| \angle \alpha \quad , \quad I = |I| \angle \beta$$

The product of voltage times the conjugate of current in polar form is

$$VI^* = |V| \varepsilon^{j\alpha} \times |I| \varepsilon^{-j\beta} = |V| \; |I| \varepsilon^{j(\alpha - \beta)} = |V| \; |I| \angle \alpha - \beta$$

This quantity, called the complex power, is usually designed by S. In rectangular form

$$S = VI^* = |V| \; |I| \cos(\alpha - \beta) + j|V| \; |I| \sin(\alpha - \beta)$$

$$S = P + jQ$$

Reactive power Q will be positive when the phase angle α - β between voltage and current is positive, that is, when α > β , which means that current is lagging the voltage. Conversely, Q will be negative for β > α, which indicates that current is leading the voltage. This agrees with the selection o f a positive sign for the reactive power of an inductive circuit and a negative sign for the reactive power of a capacitive circuit. To obtain the proper sign for Q, it is necessary to calculate S as VI * rather than V * I, which would reverse the sign for Q.

1.6 THE POWER TRIANGLE:

A power triangle can be drawn for an inductive load, as shown in the Figure (1.5)

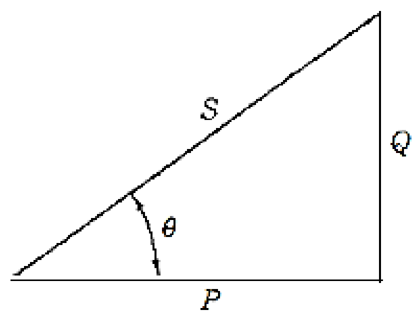


Figure (1.5) power triangle

For several loads in parallel the total P will be the sum of th e average powers of the individual loads.

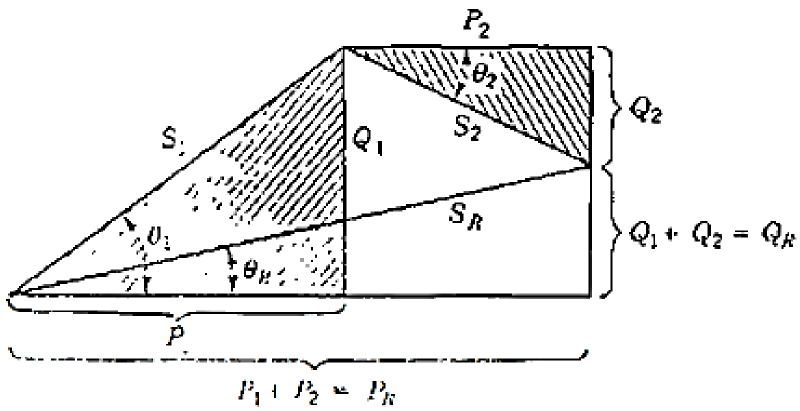


Figure (1.6) Power triangle for combined loads

1.7 DIRECT OF POWER FLOW:

In general, we can determine the P and Q absorbed or supplied by any ac circuit simply by regarding the circuit as enclosed in a box with entering current i and voltage V having the polarity shown in table (1.1) then, the numerical values of the real and imaginary parts of the product S = VI * determine the P and Q absorbed or supplied by the enclosed circuit or network. When current I lags voltage V by an angle ϴ.

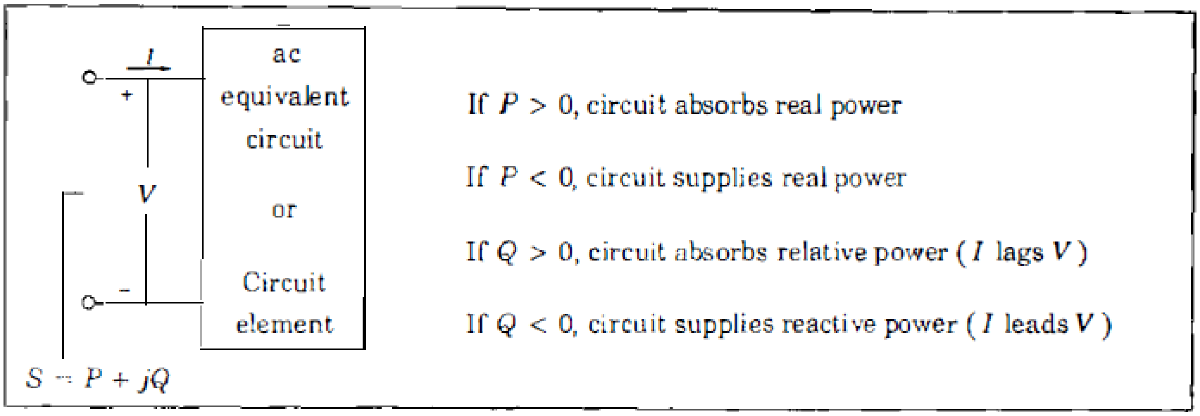


Table (1.1)

1.8 VOLTAGE AND CURRENT I N BALANCED THREE-PHASE CIRC UITS

The equivalent circuit of the three- phase generator consists of an e m f in each of the three phases, as indicated by circles on the diagram . Each emf is in series with a resistance and inductive reactance composing the impedance Zd ' Points a', b' , a nd c' are fictitious since th e generated emf can not be separated from the impedance of each phase. The terminals of the machine are the points a , b , and c

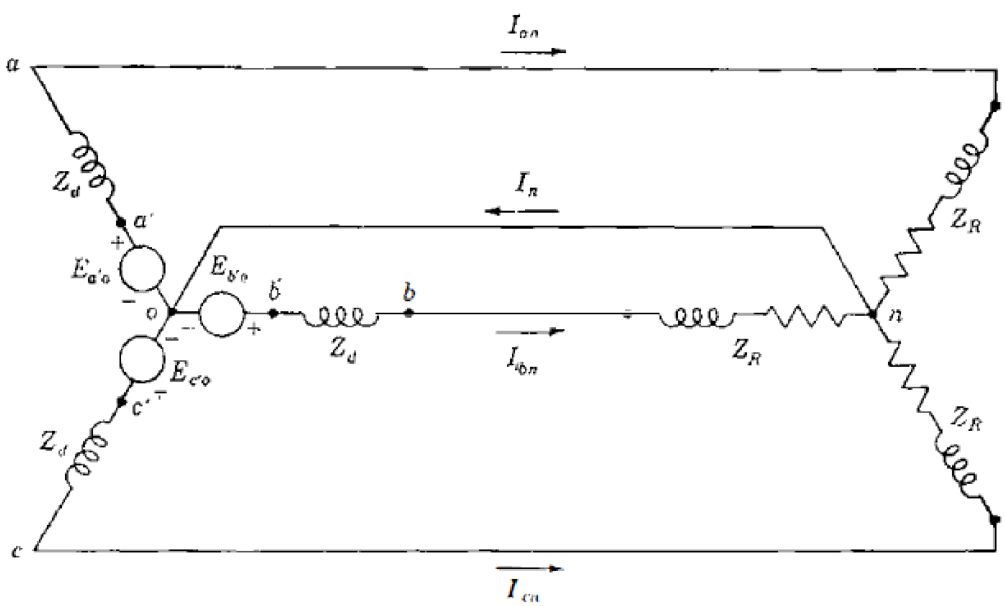


Figure (1.7) Circuit diagram of Y- connected generator

In the generator the emfs $E_{a'o}$ and $E_{b'o}$ and $E_{c'o}$ are equal in magnitude and displaced from each other 120° in phase. If the magnitude of each is 100V with $E_{a'o}$ as reference,

$$E_{a'o} = 100\angle 0^\circ \text{ V} \qquad E_{b'o} = 100\angle 240^\circ \text{ V} \qquad E_{c'o} = 100\angle 120^\circ \text{ V}$$

At the generator terminals (and at the load in this case) the terminal voltages to neutral are

$$V_{ao} = E_{a'o} - I_{an}Z_d$$

$$V_{bo} = E_{b'o} - I_{bn}Z_d$$

$$V_{co} = E_{c'o} - I_{cn}Z_d$$

The line currents (which are also the phase currents for a Y connection) are:

$$I_{an} = \frac{E_{a'o}}{Z_d + Z_R} = \frac{V_{an}}{Z_R}$$

$$I_{bn} = \frac{E_{b'o}}{Z_d + Z_R} = \frac{V_{bn}}{Z_R}$$

$$I_{cn} = \frac{E_{c'o}}{Z_d + Z_R} = \frac{V_{cn}}{Z_R}$$

The currents will also be equal in magnitude and displaced 120° from each other in phase.
Because of the phase displacement of t h e voltages and currents in a balanced three-phase system, it is convenient to have a shorthand method of indicating the rotation of a phasor through 120°.
The complex number of unit magnitude e and associated angle ϴ is an operator that rotates the phasor on which it operates through the angle ϴ.
The letter a is commonly used to designate the operator that causes a rotation of 120^0 in the counterclockwise direction. Such an operator is acom plex number of unit magnitude with an angle of 1 20^0 and is defined by

$$a = 1\angle 120^\circ = 1\varepsilon^{j2\pi/3} = -0.5 + j0.866$$

$$a^2 = 1\angle 240^\circ = 1\varepsilon^{j4\pi/3} = -0.5 - j0.866$$

$$a^3 = 1\angle 360^\circ = 1\varepsilon^{j2\pi} = 1\angle 0^\circ = 1$$

$$1 + a + a^2 = 0.$$

Figure (1.9) shows various power and function of a.

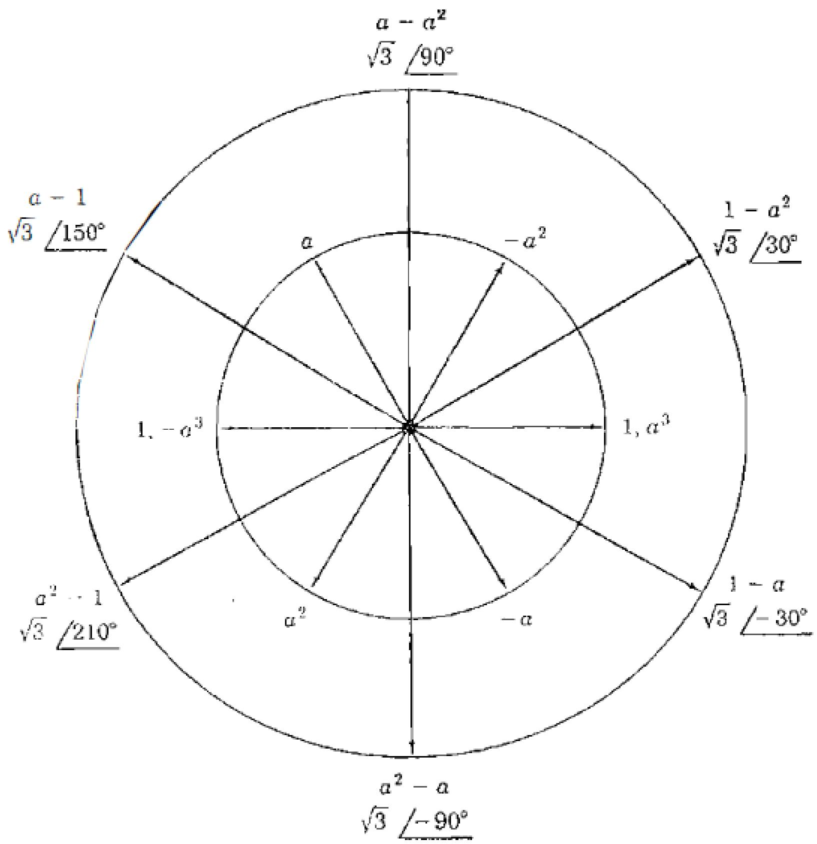


Figure (1.9) various power and function of a

The line-to-line voltages in the circuit of figure(1.7) are Vab , Vbc and Vca . Tracing a path from a to b through n yields

$$V_{ab} = V_{an} + V_{nb} = V_{an} - V_{bn}$$

$$V_{ab} = V_{an} - a^2 V_{bn} = V_{an}(1 - a^2)$$

$$V_{ab} = \sqrt{3}\, V_{an}\varepsilon^{j30^\circ} = \sqrt{3}\, V_{an}\angle 30^\circ$$

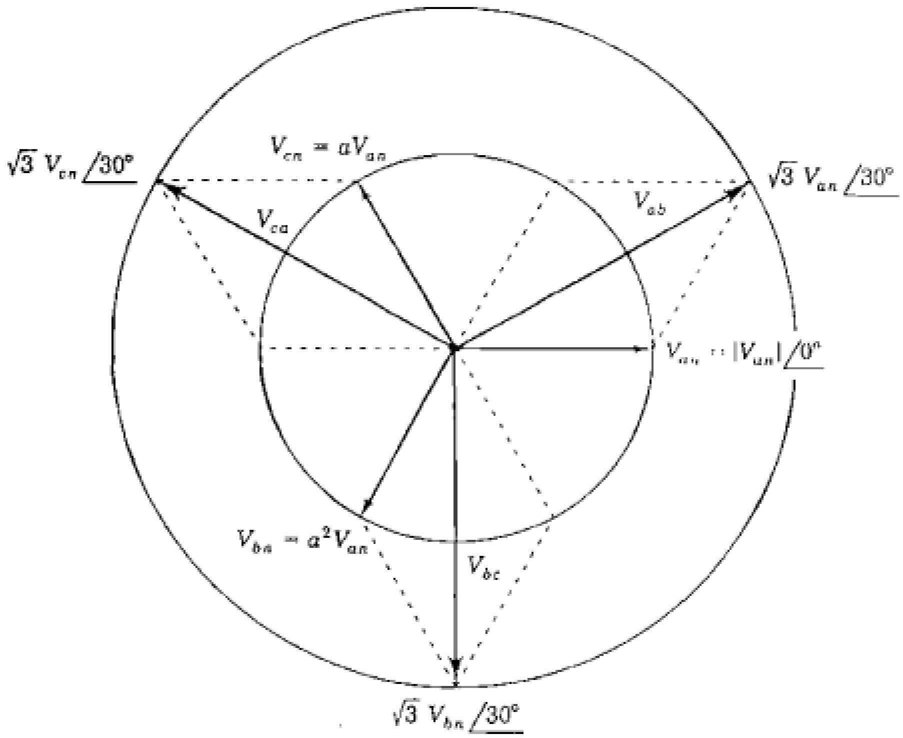


Figure (1.10)Phasor diagram of line to line voltages in relation to line to niutral voltages in balanced three phase circuit

Balance d loads are often connected in Δ as shown in figure (1.11). it is left to t h e reader using the prope rties of the operator a to show that the magnitude of a line current such as I_a is equal to √3 times the magnitude of a-phase current I_{ab} and that I_a lags I_{ab} by 30° when the phase sequence is abc. Figure (1.12) shows the current relationships when I_{ab} is chosen as reference.

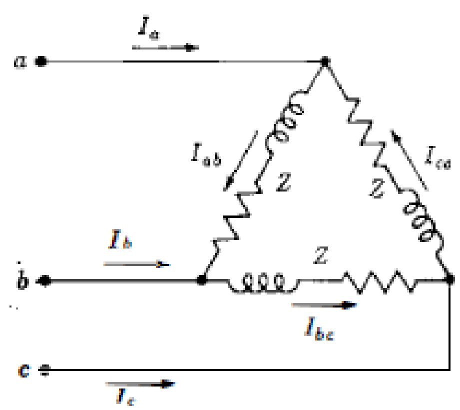


Figure (1.11) Δ connected three phase load

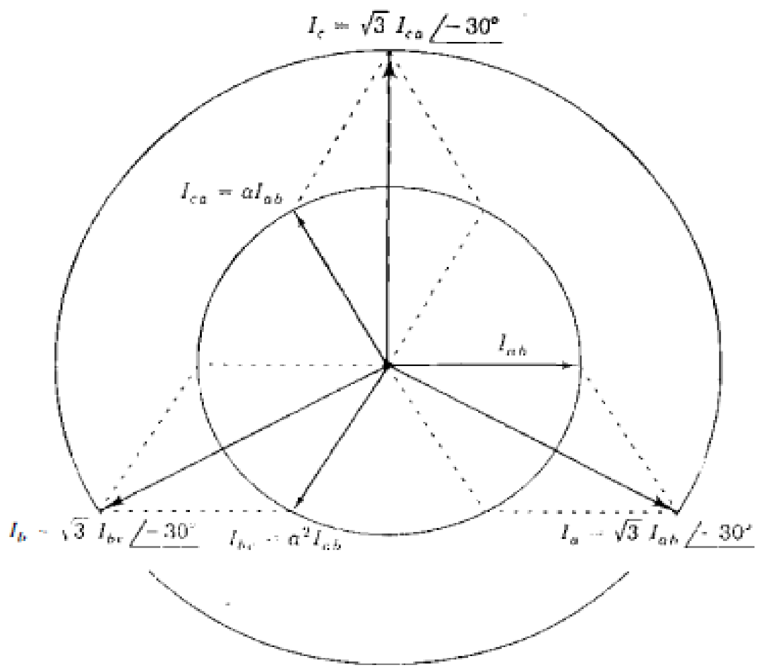


Figure (1.12) phasor diagram of the line current in relation of the phase cureent

When solving balanced three-phase circuits, it is not necessary to work with the entire three-phase circuit diagram. To solve the circuit a neutral connection of zero impedance is assumed to be present and to carry the sum of the three phase currents, which is zero for balanced conditions. The circuit is solved by applying Kirchhoff's voltage law around a closed path which includes one phase and neutral. Such a closed path is shown i n Figure(1.13) This circuit is the per-phase or single phase equivalent.

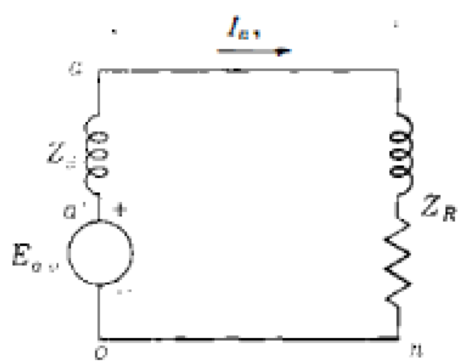


Figure (1.13) one phase of the circuit

1.9 THE PER UNITS QUANTITIES:

In many engineering situations, it is useful to scale or normalize quantities. This is done to simplify numerical calculations. The slandered method used is referred to as per unit system.

$$\text{Base current, A} = \frac{\text{base kVA}_{1\phi}}{\text{base voltage, kV}_{LN}}$$

$$\text{Base impedance, } \Omega = \frac{\text{base voltage, V}_{LN}}{\text{base current, A}}$$

$$\text{Base impedance, } \Omega = \frac{(\text{base voltage, kV}_{LN})^2 \times 1000}{\text{base kVA}_{1\phi}}$$

$$\text{Base impedance, } \Omega = \frac{(\text{base voltage, kV}_{LN})^2}{\text{MVA}_{1\phi}}$$

$$\text{Base power, kW}_{1\phi} = \text{base kVA}_{1\phi}$$

$$\text{Base power, MW}_{1\phi} = \text{base MVA}_{1\phi}$$

$$\text{Per-unit impedance of an element} = \frac{\text{actual impedance, } \Omega}{\text{base impedance, } \Omega}$$

Base impedance and base current can be computed directly from three- phase values of base kilo volts and a kilovolt amperes.

$$\text{Base current, A} = \frac{\text{base kVA}_{3\phi}}{\sqrt{3} \times \text{base voltage, kV}_{LL}}$$

$$\text{Base impedance} = \frac{\left(\text{base voltage, kV}_{LL}/\sqrt{3}\right)^2 \times 1000}{\text{base kVA}_{3\phi}/3}$$

$$\text{Base impedance} = \frac{(\text{base voltage, kV}_{LL})^2 \times 1000}{\text{base kVA}_{3\phi}}$$

$$\text{Base impedance} = \frac{(\text{base voltage, kV}_{LL})^2}{\text{base MVA}_{3\phi}}$$

Nots:

- Use line-to- line kilovolts with three-phase kilovoltamperes or megavolt amperes, and
- Use line-to-neutral kilovolts with kilovoltamperes or megavoltamperes per phase.

1.8 CHANGING THE BASE OF PER-U NIT QUANTITIES:

$$\text{Per-unit impedance} = \frac{(\text{actual impedance, }\Omega) \times (\text{base kVA})}{(\text{base voltage, kV})^2 \times 1000}$$

$$\text{Per-unit } Z_{\text{new}} = \text{per-unit } Z_{\text{given}} \left(\frac{\text{base kV}_{\text{given}}}{\text{base kV}_{\text{new}}}\right)^2 \left(\frac{\text{base kVA}_{\text{new}}}{\text{base kVA}_{\text{given}}}\right)$$

1.9 THE SINGLE-LINE O R ONE-LINE DIAGRAM

The purpose of the one-line diagram is to supply in concise form the significant information about the system. The importance of different features of a system varies with the problem under consideration, and the amount of information included on the diagram depends on the purpose for which the diagram is intended. For example the location of circuit breakers and relays is unimportant in making a load study, while they are very important in stability studies.

The American National Standards Institute (ANSI) and the Institute of Electrical and Electronics Engineers (IEEE) have published a set of standard symbols for electrical diagrams as shown in figure (1.13).

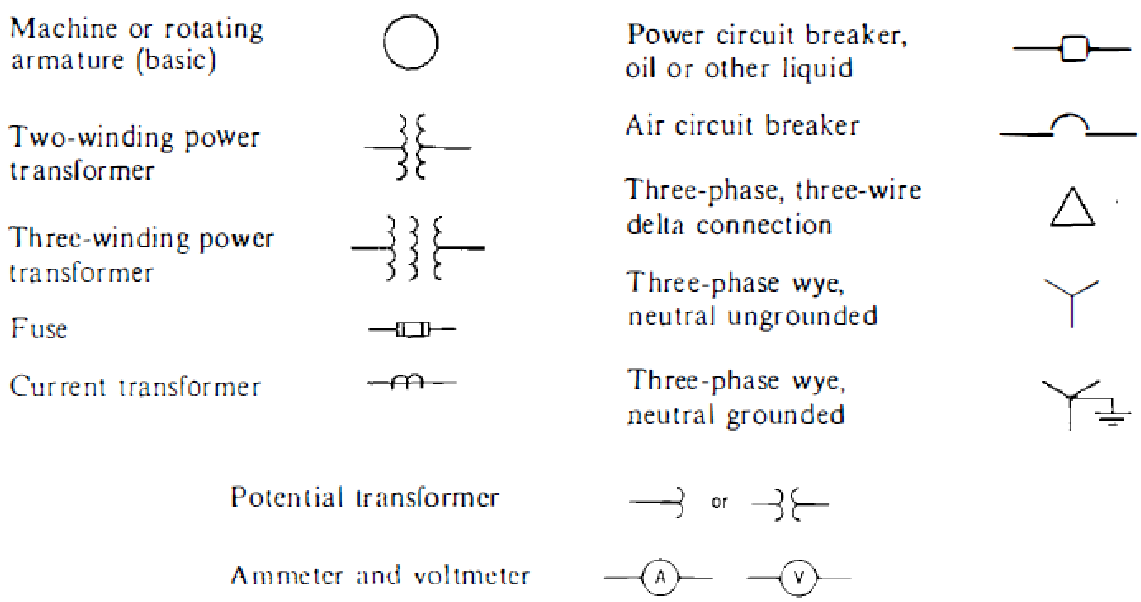


Figure (1.13)

It is important to know the location of points where a system is connected to ground in order to calculate the amount of current flowing when an unsymmetrical fault involving ground occurs. Figure (1.14) shows a single – line diagram of a simple power system

Figure (1.14)

1.10 IMPEDANCE AND REACTANCE DIAGRAMS

In order to calculate the performance of a system under load condition or the occurrence of fault, the one- line diagram is used to draw the single- phase or per phase equivalent circuit of the system.

To form the per-phase impedance diagram of the system in figure (1.14) when a load study is to be made, the lagging loads A and B are represented by resistance and inductive reactance in series. The impedance diagram does not include the current limiting impedances shown in the one-line diagram between the neutrals of the generators and ground because no current flows in the ground under balanced conditions and the neutrals of the generators are at the potential of the neutral of the system. Since the shunt current of a transformer is usually insignificant compared with the full-load current, the shunt admittance is usually omitted in the equivalent circuit of the transformer.

Resistance is often omitted when making fault calculations. Loads which donot involve rotating machinery have little effect on the total line current during a fault and are usually omitted. Synchronous motor loads, however, are always included in making fault calculations. Figure (1.15) shows the per-phase impedance diagram corresponding to the single-line diagram. While figure (1.16) shows the per -phase diagram after omitting all load resistances and shunt admittances.

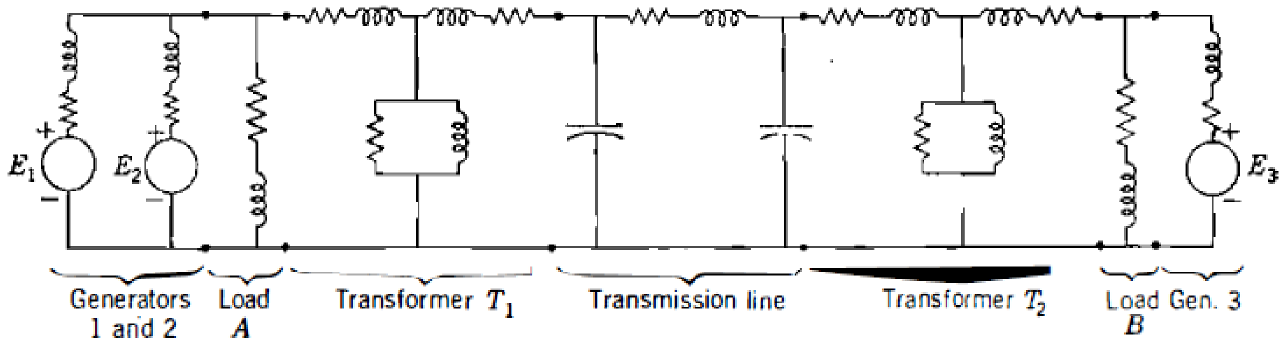


Figure (1.15)

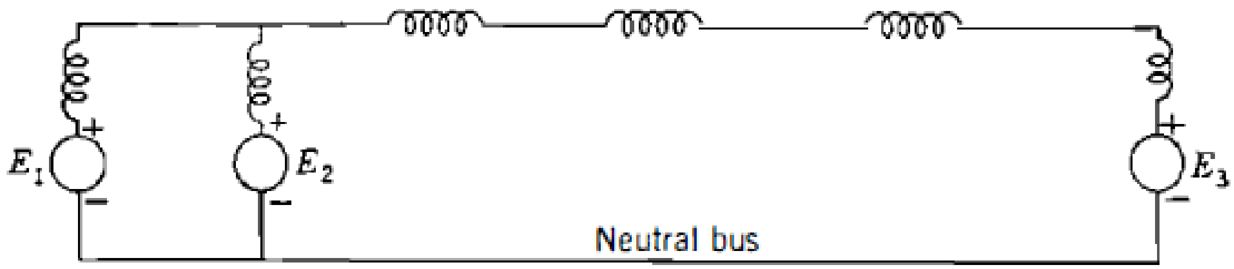


Figure (1.15)

Chapter two / Modeling of Power system elements

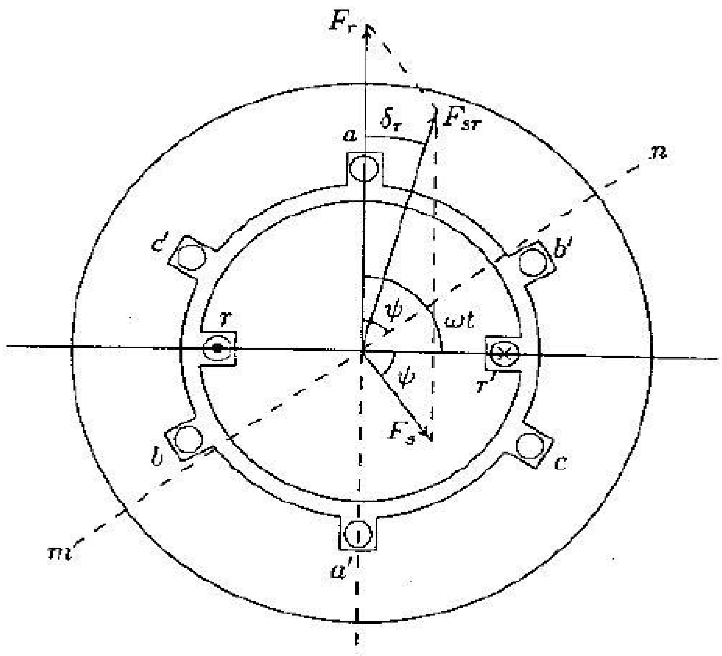
2.1 Introduction:

Before the power systems network can solved, it must first modeled. So a simple models of generators, transformers and transmission lines will be represented in this section. There are two types of synchronous generators those are cylindrical and salient pole generators.

2.2 Generators modeling:

There are two types of synchronous generators, cylindrical type of rotor and salient type of rotor.

2.2.1 Cylindrical synchronous generators:



Figure(2.1)two pole three phase synchronous generator

The flux linkage with coil a is

$$\lambda_a = N\phi \cos \omega t$$

The voltage induced in coil aa\ is obtained from Fraday's law is

$$
\begin{aligned}
e_a = -\frac{d\lambda}{dt} &= \omega N \phi \sin \omega t \\
&= E_{max} \sin \omega t \\
&= E_{max} \cos(\omega t - \frac{\pi}{2})
\end{aligned}
$$

The r.m.s of the generated voltage is

$$E = 4.44 f N \phi$$

Since the induced e.m.f in different slots are not in phase, their phasor sum is less than their numerical sum, thus, a reduction factor K_w , called the winding factor must be applied

$$E = 4.44 K_w f N \phi$$

The frequency of the induced armature voltages depends on the speed at which the rotor runs and on the number of poles for which the machine is wound. The frequency of the armature voltage is given by

$$f = \frac{P}{2} \frac{n}{60}$$

Where n is the rotor speed known as synchronous speed. During normal conditions, the generators operate synchronously producing three- phase balanced currents in the armature.

$$
\begin{aligned}
i_a &= I_{max} \sin(\omega t - \psi) \\
i_b &= I_{max} \sin(\omega t - \psi - \frac{2\pi}{3}) \\
i_c &= I_{max} \sin(\omega t - \psi - \frac{4\pi}{3})
\end{aligned}
$$

At any instants of time, these currents will produce a sinusoidal distributed m.m.f waves

$$F_a = Ki_a = KI_{max}\sin(\omega t - \psi) = F_m\sin(\omega t - \psi)$$
$$F_b = Ki_b = KI_{max}\sin(\omega t - \psi - \frac{2\pi}{3}) = F_m\sin(\omega t - \psi - \frac{2\pi}{3})$$
$$F_c = Ki_c = KI_{max}\sin(\omega t - \psi - \frac{4\pi}{3}) = F_m\sin(\omega t - \psi - \frac{4\pi}{3})$$

Where K is proportional to the numbers of armature turns per phase and is a function of winding type the resultant m.m.f is the vector sum of the above m.m.fs

When the armature is carrying balanced three phase currents, F_s is produced perpendicular to line mn. The interaction of armature mmf and field mmf, known as armature reaction, gives rise to the resultant air gap mmf F_{sr} which is the vecyor sum of field mmf F_r and the armature mmf F_s . the F_{sr} is responsible for the resultant air gap flux φ_{sr} that induces the generated emf on the load, E_{sr}. the F_s induces E_{ar} known as armature reaction voltage, which is perpendicular to F_s . the voltage E_{ar} lead I_a by 90^o and thus can be represented by a voltage drop across a reactance X_{ar} due to the current I_a. X_{ar} is known as the reactance then the on line generated emf is

$$E = E_{sr} + jX_{ar}I_a$$

The terminal voltage

$$E = V + [R_a + j(X_l + X_{ar})]I_a$$

$$E = V + [R_a + jX_s]I_a$$

Where X_s = (X_l + X_{ar}) and known as the synchronous reactance. A simple per-phase model for a cylindrical rotor generator is shown in figure (2.2) where

$$E = V + jX_sI_a$$

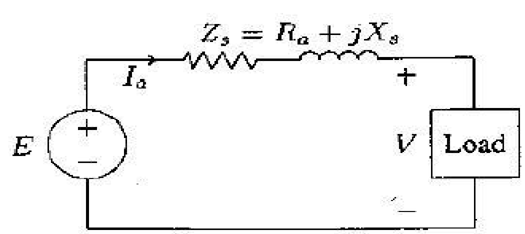


Figure (2.2)

Figure (2.3) shows the phasor diagramof the generator with terminal voltage as reference for excitations corresponding to lagging, unity, leading power factors.

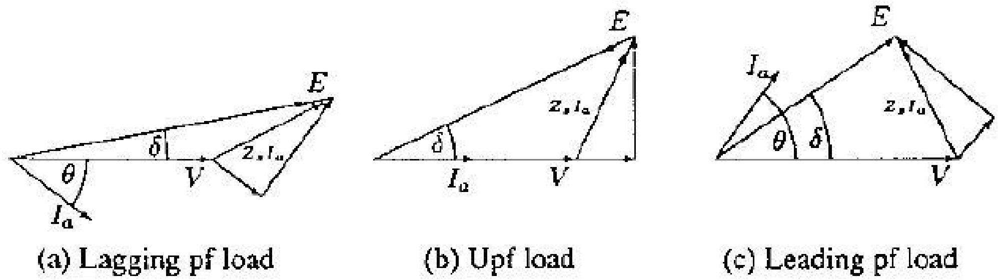


Figure (2.3)

Consider the circuit in figure (2.2)

$$S_{3\phi} = 3VI_a^*$$

$$I_a = \frac{|E|\angle\delta \; - |V|\angle 0}{|Z_s|\angle\gamma}$$

$$S_{3\phi} = 3\frac{|E||V|}{|Z_s|}\angle\gamma - \delta \; - 3\frac{|V|^2}{|Z_s|}\angle\gamma$$

$$P_{3\phi} = 3\frac{|E||V|}{|Z_s|}\cos(\gamma - \delta) - 3\frac{|V|^2}{|Z_s|}\cos\gamma$$
$$Q_{3\phi} = 3\frac{|E||V|}{|Z_s|}\sin(\gamma - \delta) - 3\frac{|V|^2}{|Z_s|}\sin\gamma$$

$$P_{3\phi} = 3\frac{|E||V|}{X_s}\sin\delta$$
$$Q_{3\phi} = 3\frac{|V|}{X_s}(|E|\cos\delta - |V|)$$

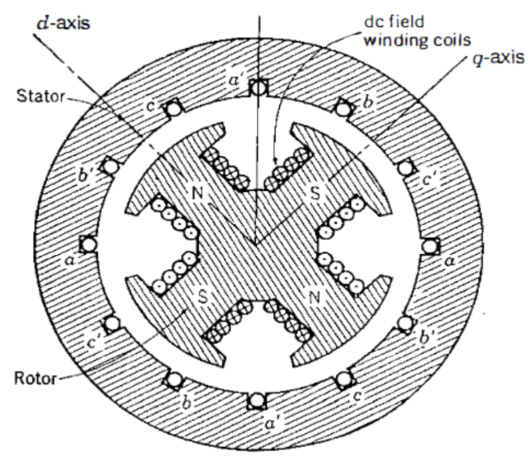
$$P_{max(3\phi)} = 3\frac{|E||V|}{X_s}$$

$$Q_{3\phi} \simeq 3\frac{|V|}{X_s}(|E| - |V|)$$

The voltage regulation of an alternator is defined as the percentage change in terminal voltage from no –load to rated load. This gives an indication of the change in field current required to maintain system voltage when going no-load to rated load at some specific power factor.

$$VR = \frac{|V_{nl}| - |V_{rated}|}{|V_{rated}|} \times 100 = \frac{|E| - |V_{rated}|}{|V_{rated}|} \times 100$$

2.2.2 Salient pole synchronous generator:



The salient –pole rotor results in nonuniformity of the magnetic reluctance of the air gap. The reluctance along the polar axis, commonly referred to as rotor direct axis, is less than that along the interpolar axis commonly referred to as quadrature axis. Therefore X_d (reluctance along the direct axis) is high and X_q (along the quadrature axis) is low. There reactances produce voltage drop in the armature and can be taken into account by resolving I_a into I_d and I_q figure (2.4)shows the phasor diagram with armature resistance neglected

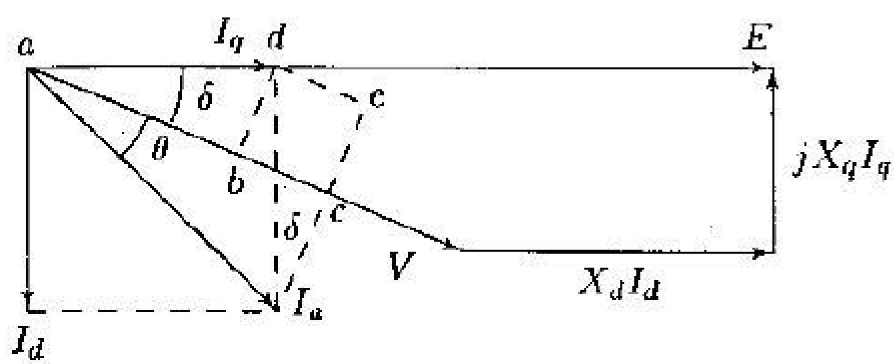


Figure (2.4)

From the figure :

$$|E| = |V|\cos\delta + X_d I_d$$

The three phase real power at the generator terminal

$$P = 3|V||I_a|\cos\theta$$

Since

$$\begin{aligned} |I_a|\cos\theta &= ab + de \\ &= I_q\cos\delta + I_d\sin\delta \end{aligned}$$

$$P = 3|V|(I_q\cos\delta + I_d\sin\delta)$$

$$|V|\sin\delta = X_q I_q$$

$$I_q = \frac{|V|\sin\delta}{X_q}$$

$$I_d = \frac{|E| - |V|\cos\delta}{X_d}$$

$$P_{3\phi} = 3\frac{|E||V|}{X_d}\sin\delta + 3|V|^2\frac{X_d - X_q}{2X_d X_q}\sin 2\delta$$

2.3 TRANSFORMARS

Transformers are the link between the generators of the power system and the transmission lines, and between lines of different voltage levels. Transmission lines operate at nominal voltages up to 7 6 5 kV line to line.

2.3.1 THE IDEAL TRANSFORMER

Transformers consist of two or more coils placed so that they are l inked by the same magnetic flux. Figure (2.5) represents a single phase transformer shell type.

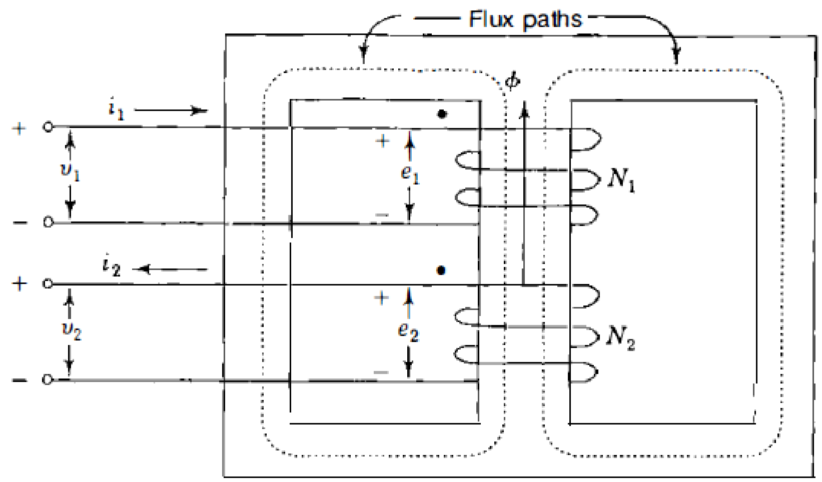


Figure (2.5)

Since the transformer is ideal, then: (1) the permeability μ of the core is infinite, (2) all of the flux is confined to the core and therefore links all of the turns of both windings, and (3) core losses and winding resistances are zero.

$$v_1 = e_1 = N_1 \frac{d\phi}{dt}$$

$$v_2 = e_2 = N_2 \frac{d\phi}{dt}$$

$$\frac{V_1}{V_2} = \frac{E_1}{E_2} = \frac{N_1}{N_2}$$

Voltage drops from dotted to unmarked terminals of all windings are in phase. Figure (2.6) shows a schematic diagram for a single phase transformer.

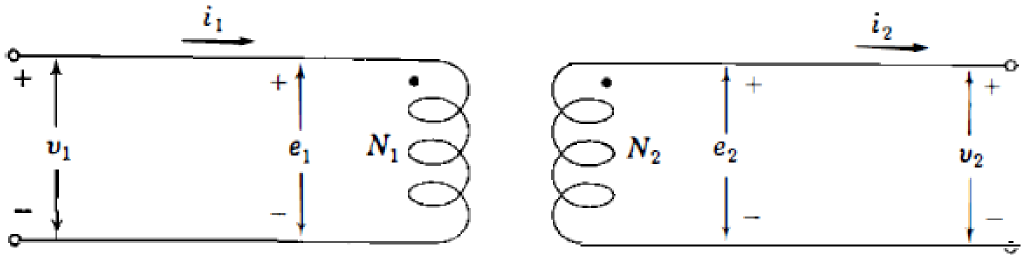


Figure (2.7)

To find the relation between the currents i_1 and i_2, apply ampere's law

$$\oint H \cdot ds = i$$

$$\oint H \cdot ds = N_1 i_1 - N_2 i_2$$

$$N_1 I_1 - N_2 I_2 = 0$$

$$\frac{I_1}{I_2} = \frac{N_2}{N_1}$$

$$I_1 = \frac{N_2}{N_1} I_2$$

If an impedance Z_2 is connected across winding 2, then

$$Z_2 = \frac{V_2}{I_2}$$

$$Z_2 = \frac{(N_2/N_1)V_1}{(N_1/N_2)I_1}$$

$$Z_2' = \frac{V_1}{I_1} = \left(\frac{N_1}{N_2}\right)^2 Z_2$$

$$V_1 I_1^* = \frac{N_1}{N_2} V_2 \times \frac{N_2}{N_1} I_2^* = V_2 I_2^*$$

$$S_1 = S_2$$

2.3.2 PRACTICAL TRANSFORMER

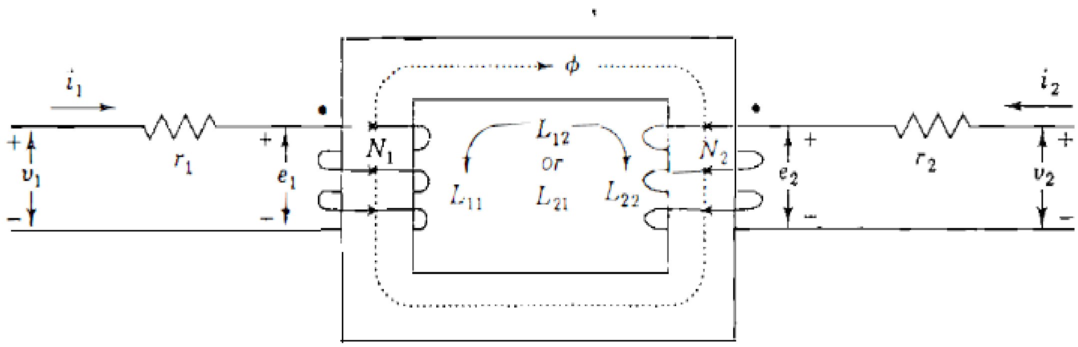


Figure (2.8)

From the figure:

$$\lambda_{11} = N_1 \phi_{11} = L_{11} i_1$$

$$\lambda_{22} = N_2 \phi_{22} = L_{22} i_2$$

$$\lambda_{21} = N_2 \phi_{21} = L_{21} i_1$$

$$\lambda_{12} = N_1 \phi_{12} = L_{12} i_2$$

$$\lambda_1 = \lambda_{11} + \lambda_{12} = L_{11} i_1 + L_{12} i_2$$

$$\lambda_2 = \lambda_{21} + \lambda_{22} = L_{21} i_1 + L_{22} i_2$$

$$v_1 = r_1 i_1 + \frac{d\lambda_1}{dt} = r_1 i_1 + L_{11}\frac{di_1}{dt} + L_{12}\frac{di_2}{dt}$$

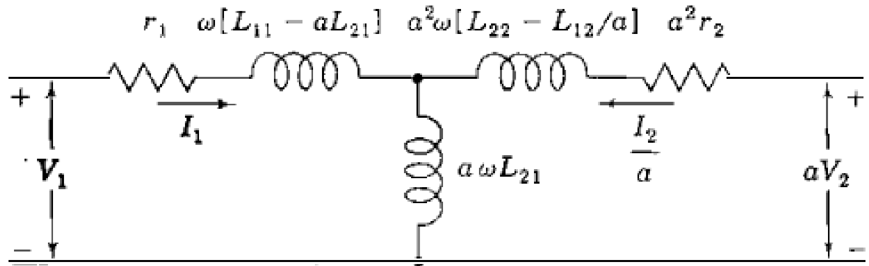
$$v_2 = r_2 i_2 + \frac{d\lambda_2}{dt} = r_2 i_2 + L_{21}\frac{di_1}{dt} + L_{22}\frac{di_2}{dt}$$

$$v_2' = -v_2 = -r_2 i_2 - \frac{d\lambda_2}{dt} = -r_2 i_2 - L_{21}\frac{di_1}{dt} - L_{22}\frac{di_2}{dt}$$

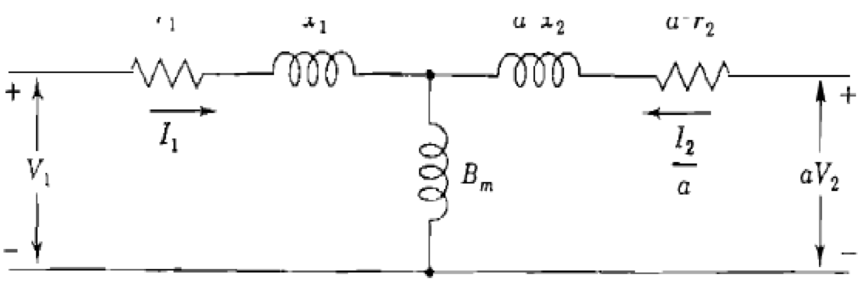
$$V_1 = \underbrace{(r_1 + j\omega L_{11})}_{z_{11}} I_1 + \underbrace{(j\omega L_{12})}_{z_{12}} I_2$$

$$V_2 = \underbrace{(j\omega L_{21})}_{z_{21}} I_1 + \underbrace{(r_2 + j\omega L_{22})}_{z_{22}} I_2$$

From the above the equivalent circuit of the transformer can be expressed as in the figure



(a)



(b)

Figure (2.9) transformer equivalent circuit

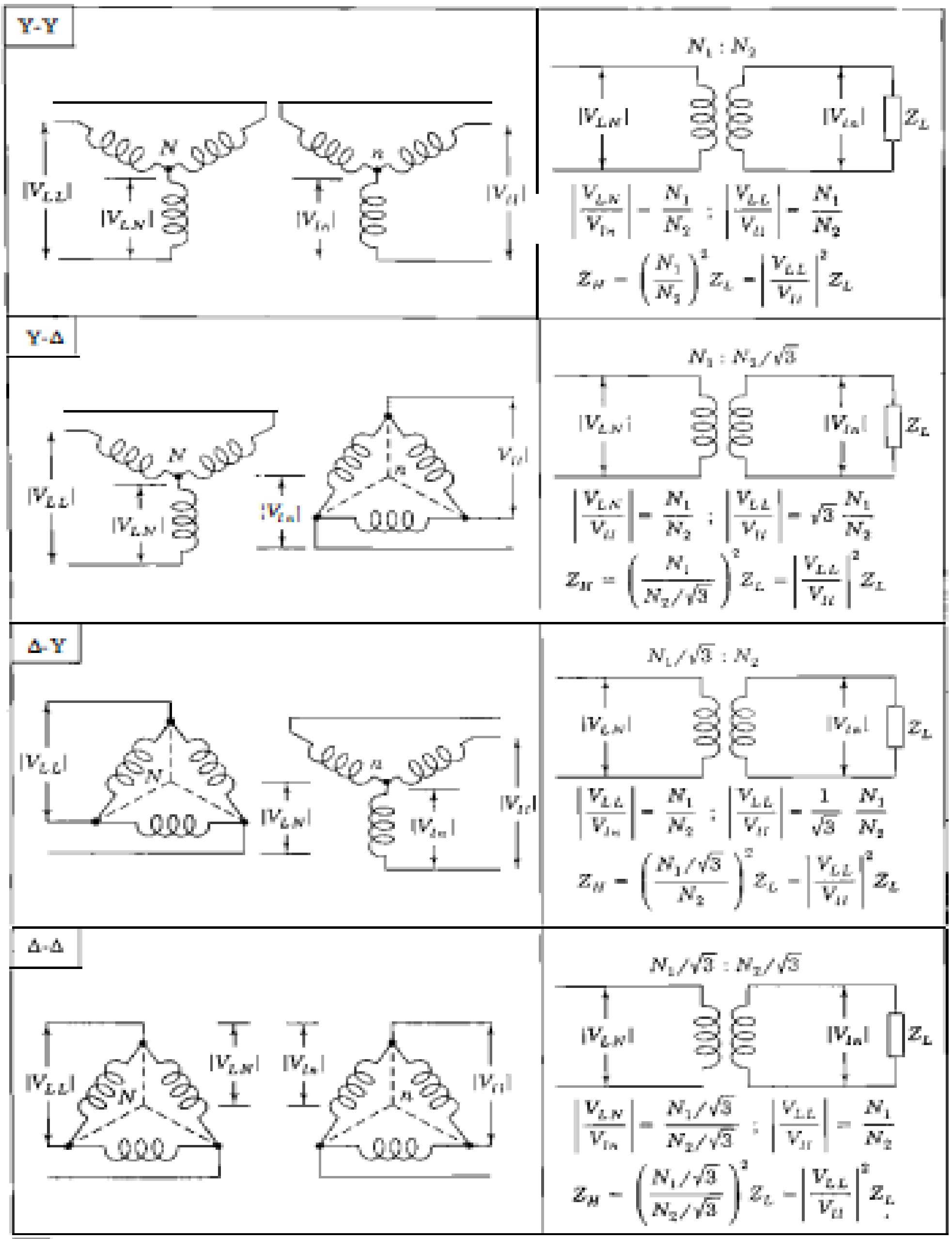
Voltage regulation is defined as the difference between the voltage magnitude at the at the load terminal of the transformer at full load and at no load in the percent of full load voltage with constant in put voltage.

$$\text{Percent regulation} = \frac{|V_{2,\,NL}| - |V_{2,\,FL}|}{|V_{2,\,FL}|} \times 100$$

2.3.3 THREE PHASE TRANSFORMER

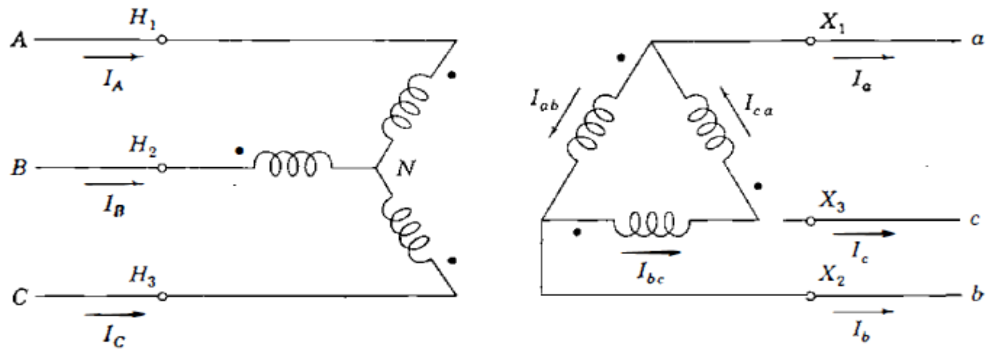
There are the four types of connection for the three phase transformers those are Y-Y, Y-Δ, Δ-Y, and Δ-Δ.

Depending on the relations of voltage and currents in the Y and the Δ connection. Table (2.1) shows transferring ohmic values of per-phase impedances from one side of a three-phase transformer to another

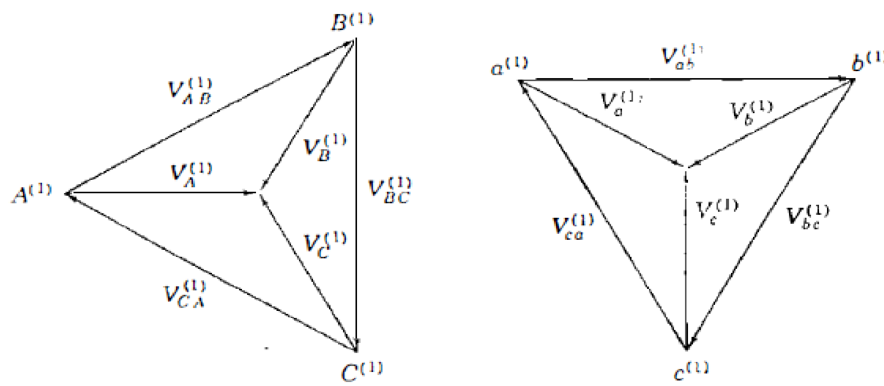


2.3.4 **THREE-PHASE TRANSFORMERS: PHASE SHIFT AND EQUIVALENT CIRCUITS**

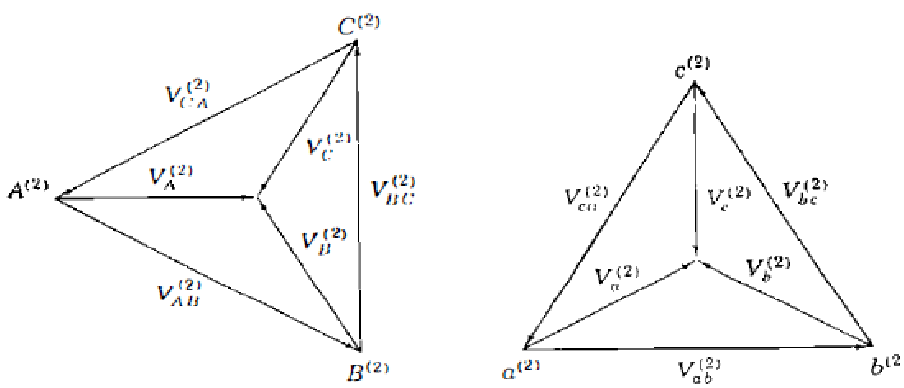
The phase shift occ**ur in** Y-Δ and Δ-Y only. Figure (2.10) is the schematic wiring diagram of a Y-Δ transformer, where the Y side is the high-voltage side.



(a) wiring diagram



(b) Positive sequence

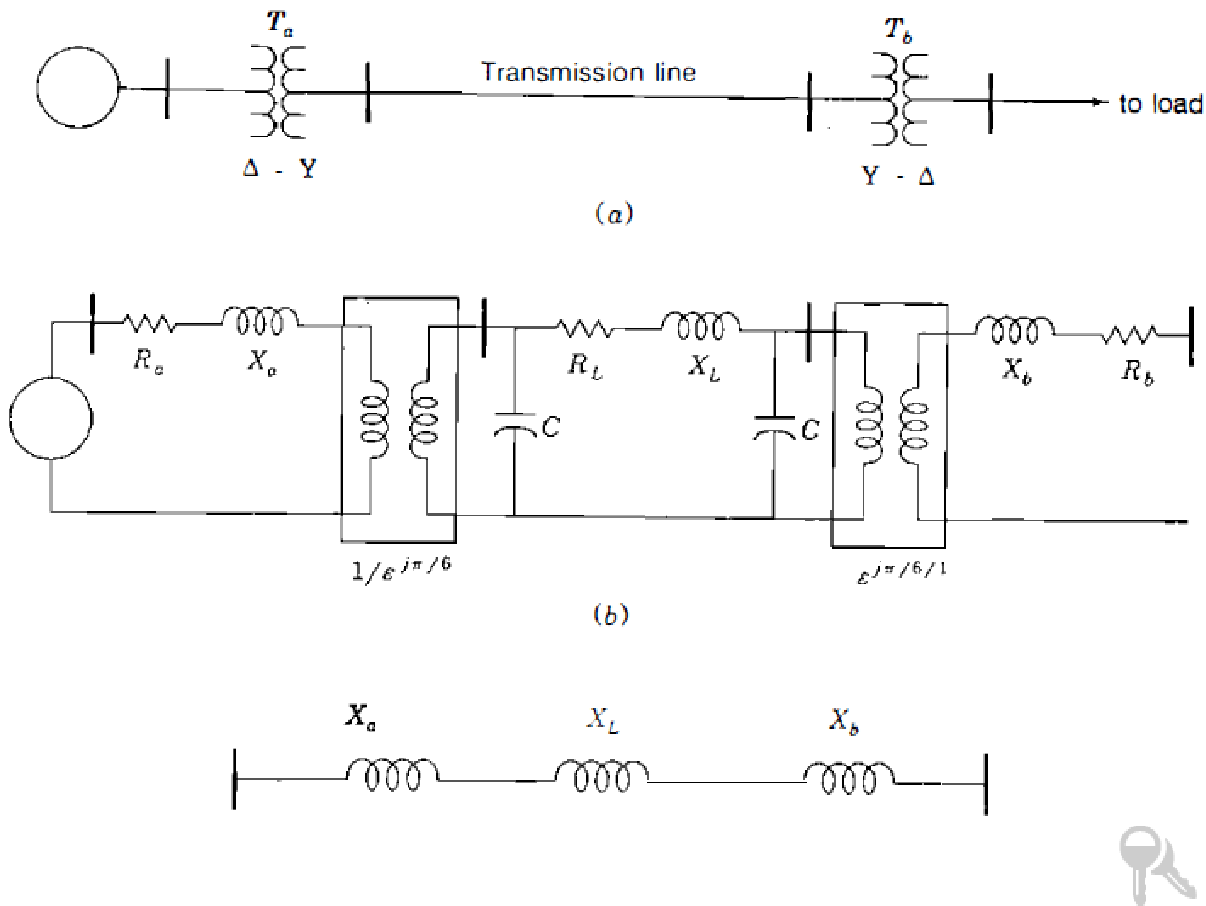


(b) Negative sequence

Figure (2.10)

$$V_A^{(1)} = \frac{N_1}{N_2}\sqrt{3}\,V_a^{(1)}\angle 30^\circ \qquad V_A^{(2)} = \frac{N_1}{N_2}\sqrt{3}\,V_a^{(2)}\angle -30^\circ$$

Figure (2.11) (a) shows the single-line diagram indicates Y -6 transformers, (b) transformer resistance and leakage reactance are in per unit, and (c) is a further simplification.



(c)

Figure (2.11)

2.2.5 THE AUTOTRANSFORMER

An autotransformer differs from the ordinary transformer in that the windings of the autotransformer are electrically connected as well as coupled by a mutual flux.

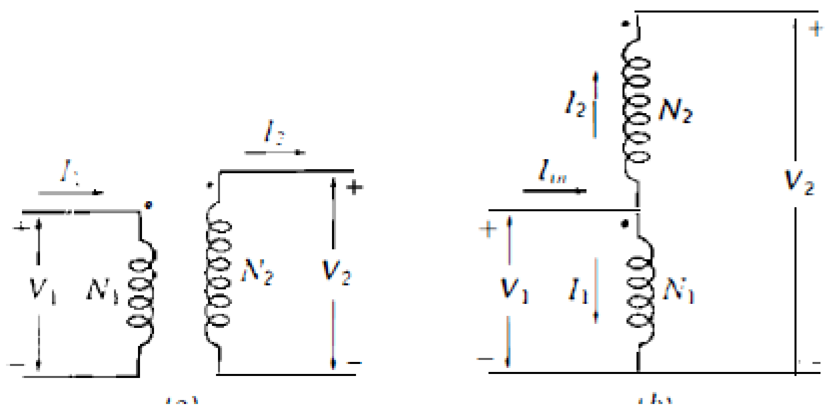


Figure (2.12) an auto transformer

If $V_c = V_1 = V_L$, $V_2 = V_H$

$V_C / V_{Ser} = N_C / N_{Ser}$

$$N_C . I_C = N_{Ser} . I_{Ser}$$

The voltage at the output of the whole transformer is equal to the sum of the voltages on the common and the series windings. The voltage of the common winding appears on both sides of the transformer and the series winding is connected in series with the common winding.

$$V_H = V_C + V_{Ser}$$

Since $V_L = V_C$

$$V_H = V_L + V_{Ser}$$

And $I_L = I_C + I_{Ser}$

Since $I_H = I_{Ser}$

$$I_L = I_C + I_H$$

2.3.5.1 Voltage and current Relations in an Autotransformer:

Since $V_H = V_C + V_{Ser}$ and $V_C / V_{Ser} = N_C / N_{Ser}$

Then $V_H = V_C + (N_{Ser} / N_C).V_C$

Noting that $V_L = V_C$ we get:

$$V_H = V_C + (N_{Ser} / N_C).V_L$$

$$V_H = (N_{Ser} + N_C / N_C) .V_L$$

$$V_L / V_H = [N_C /(N_{Ser} + N_C)] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Also $I_L = I_C + I_{Ser}$

And $I_C = (N_{Ser} / N_C) I_{Ser}$

$$I_L = (N_{Ser} / N_C) I_{Ser} + I_{Ser}$$

Noting that $I_H = I_{Ser}$

$$I_L = (N_{Ser} + N_C / N_C) I_H$$

$$I_L / I_H = [(N_{Ser} + N_C) / N_C] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots..(2)$$

2.3.5.2 Apparent power Rating Advantage of Autotransformer:

It must be noted that not all the power travelling from the primary to the secondary in the autotransformer goes through the windings. As a result, if a conventional transformer is reconnected as an autotransformer, it can handle much more power than it was originally rated for. To understand this

If the apparent input power to the autotransformer is:

$$S_{in} = V_L \,.\, I_L$$

And the output apparent power is:

$$S_{out} = V_H \,.\, I_H$$

From equations (1) and (2) we can obtain:

$S_{in} = S_{out} = S_{IO}$ where S_{IO} : input output apparent power

If the apparent power in the transformer's windings is:

$$S_W = V_C \,.\, I_C = V_{Ser} \,.\, I_{Ser}$$

$$S_W = V_C \,.\, I_C = V_L (I_L - I_H)$$

$$S_W = V_L \,.\, I_L - V_L \,.\, I_H$$

$$S_W = V_L \,.\, I_L - V_L \,.\, I_L [N_C / (N_{Ser} + N_C)]$$

$$S_W = V_L \; I_L \,.\, [(N_{Ser} + N_C - N_C) / (N_{Ser} + N_C)]$$

$$S_W = S_{IO} \,.\, [N_{Ser} / (N_{Ser} + N_C)]$$

$$S_{IO} / S_W = [(N_{Ser} + N_C) / N_{Ser}] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

3.4 PER UNIT IMPEDANCES OF THREE-WINDING TRANSFORMERS

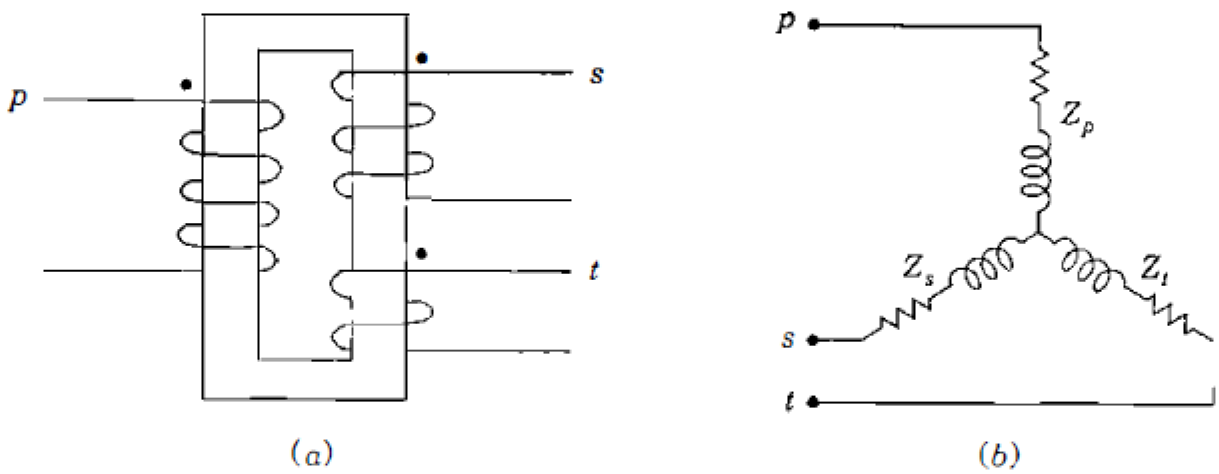


Figure (2. 13) a single-phase three-winding transformer

Z_{ps}	leakage impedance measured in primary with secondary short-circuited and tertiary open
Z_{pt}	leakage impedance measured in primary with tertiary short-circuited and secondary open
Z_{st}	leakage impedance measured in secondary with tertiary short-circuited and primary open

$$Z_{ps} = Z_p + Z_s$$

$$Z_{pt} = Z_p + Z_t$$

$$Z_{st} = Z_s + Z_t$$

$$Z_p = \tfrac{1}{2}(Z_{ps} + Z_{pt} - Z_{st})$$

$$Z_s = \tfrac{1}{2}(Z_{ps} + Z_{st} - Z_{pt})$$

$$Z_t = \tfrac{1}{2}(Z_{pt} + Z_{st} - Z_{ps})$$

2.5 TAP-CHANGING AND REGULATING TRANSFORM ERS

Transformers which provide a small adjustment of voltage magnitude, usually in the range of $\pm$ 10%, and others which shift the phase angle of the line voltages are important components of a power system. Some transformers regulate both the magnitude and phase angle.

Almost all transformers provide taps on windings to adjust the ratio of transformation by changing taps when the transformer is deenergized. A change in tap can be made while the transformer is energized, and such transformers are called load-tap-changing (LTC) transformers or tap-changing-under-load (TCU L) transformers. The tap changing is automatic and operated by motor

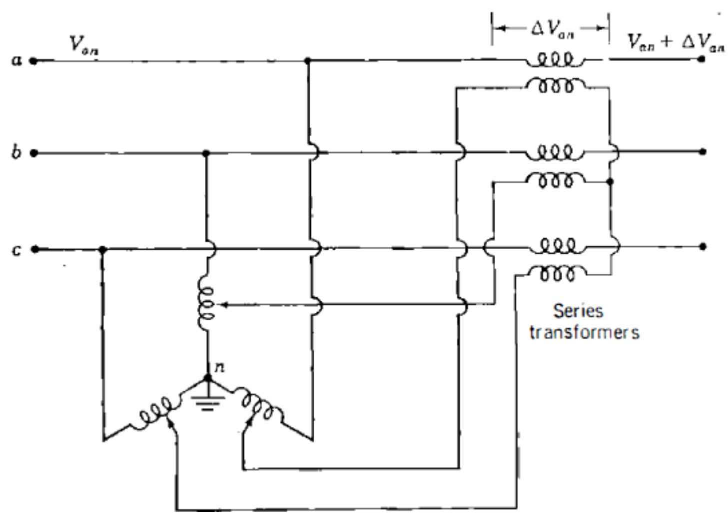


Figure (2.14) Regulating transformer for control of voltage magnitude.

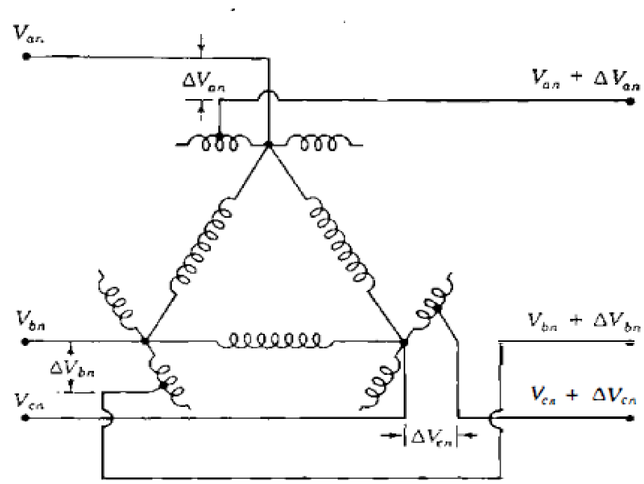


Figure (2.15) Regulating transformer for control of phase angle.

Chapter Three

3.1 BRANCH AND NODE ADMITTANCES

The single-line diagram of a small power system is shown in Figure (3.1) The corresponding reactance diagram, with reactances specified in per unit, is shown in Figure (3.2) A generator with emf equal to 1 .25 $\angle 0^{o}$ per unit is connected through a transformer to high-voltage node 3 , while a motor with internal voltage equal to 0.85 $\angle -45^{\circ}$ is similarly connected to node 4 · Develop the nodal admittance matrix for each of the network branches and then write the nodal admittance equations of the system.

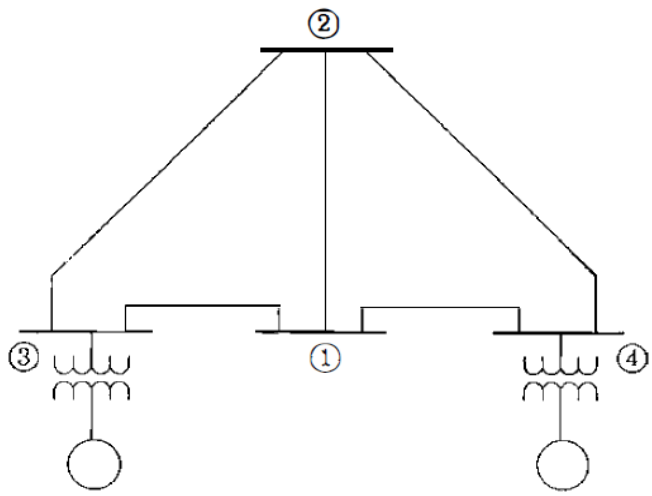


Figure (3.1)

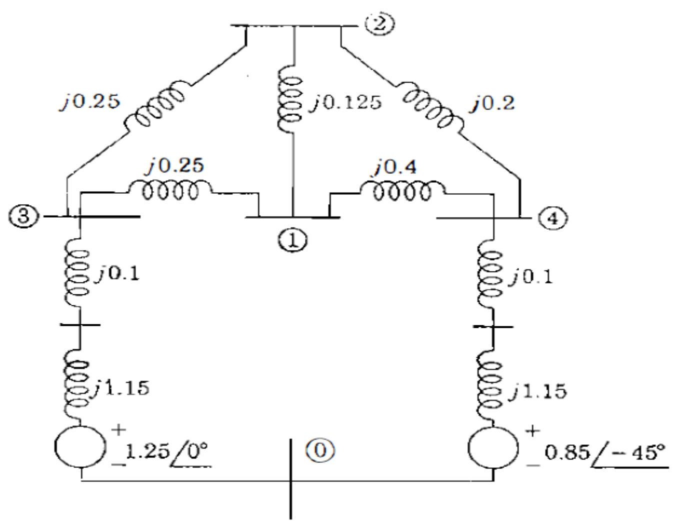
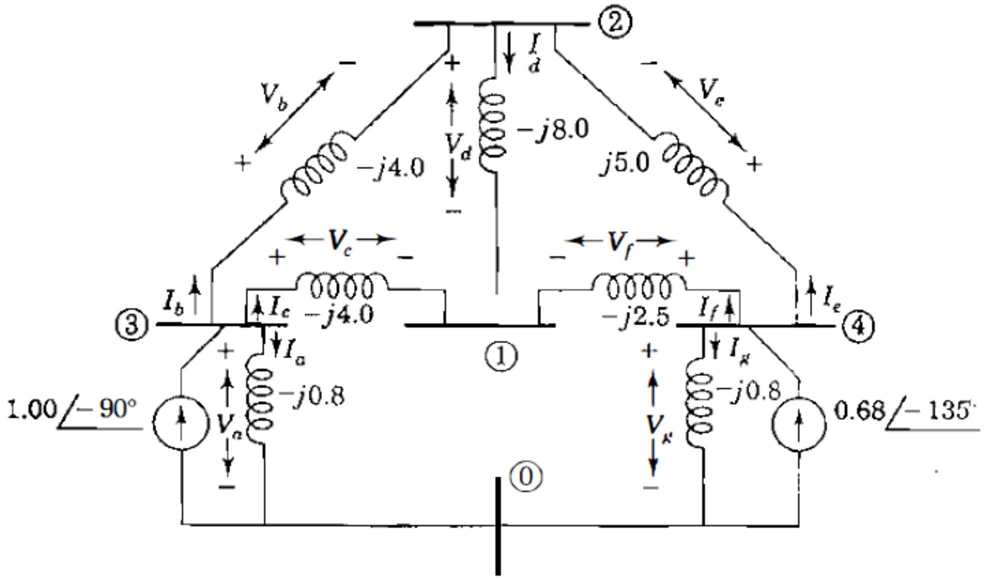


Figure (3.2)



$$
\begin{array}{c|c}
 & \begin{array}{cccc} ① & ② & ③ & ④ \end{array} \\
\begin{array}{c} ① \\ ② \\ ③ \\ ④ \end{array} &
\begin{bmatrix}
(Y_c + Y_d + Y_f) & -Y_d & -Y_c & -Y_f \\
-Y_d & (Y_b + Y_d + Y_e) & -Y_b & -Y_e \\
-Y_c & -Y_b & (Y_a + Y_b + Y_c) & 0 \\
-Y_f & -Y_e & 0 & (Y_e + Y_f + Y_g)
\end{bmatrix}
\end{array}
$$

$$
\begin{bmatrix}
-j14.5 & j8.0 & j4.0 & j2.5 \\
j8.0 & -j17.0 & j4.0 & j5.0 \\
j4.0 & j4.0 & -j8.8 & 0.0 \\
j2.5 & j5.0 & 0.0 & -j8.3
\end{bmatrix}
\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}
=
\begin{bmatrix} 0 \\ 0 \\ 1.00\angle -90° \\ 0.68\angle -135° \end{bmatrix}
$$

3.2 THE METHOD OF SUCCESSIVE ELIMINATION:

The Y bus matrices for large networks of thousands of nodes have associated systems of nodal equations to be solved for a correspondingly large number of unknown bus voltages. For such solutions computer-based numerical techniques are required to avoid direct matrix inversion, thereby minimizing computational effort and computer storage.

3.2.1 Gaussian elimination:

Using the nodal equations of the four-bus system:

$$Y_{11}V_1 + Y_{12}V_2 + Y_{13}V_3 + Y_{14}V_4 = I_1$$

$$Y_{21}V_1 + Y_{22}V_2 + Y_{23}V_3 + Y_{24}V_4 = I_2$$

$$Y_{31}V_1 + Y_{32}V_2 + Y_{33}V_3 + Y_{34}V_4 = I_3$$

$$Y_{41}V_1 + Y_{42}V_2 + Y_{43}V_3 + Y_{44}V_4 = I_4$$

Gaussian elimination consists of reducing this system of four equations to a system of three equations then two then one equation. The final equation yields a value for the corresponding unknown, which is then substituted back in the reduced sets of equations to calculate the remaining unknowns. The successive elimination of unknowns in the forward direction is called forward elimination and the substitution process using the latest calculated values is called back substitution.

- Step one : eliminate V_1 from the first equation:

1. Divide the first equation by the pivot Y_{11} to obtain

$$V_1 + \frac{Y_{12}}{Y_{11}}V_2 + \frac{Y_{13}}{Y_{11}}V_3 + \frac{Y_{14}}{Y_{11}}V_4 = \frac{1}{Y_{11}}I_1$$

2. Multiply the last equation by Y_{21} , Y_{31} , Y_{41} respectively then subtract the results equation from the first equation, to get

$$\left(Y_{22} - \frac{Y_{21}Y_{12}}{Y_{11}}\right)V_2 + \left(Y_{23} - \frac{Y_{21}Y_{13}}{Y_{11}}\right)V_3 + \left(Y_{24} - \frac{Y_{21}Y_{14}}{Y_{11}}\right)V_4 = I_2 - \frac{Y_{21}}{Y_{11}}I_1 \tag{7.44}$$

$$\left(Y_{32} - \frac{Y_{31}Y_{12}}{Y_{11}}\right)V_2 + \left(Y_{33} - \frac{Y_{31}Y_{13}}{Y_{11}}\right)V_3 + \left(Y_{34} - \frac{Y_{31}Y_{14}}{Y_{11}}\right)V_4 = I_3 - \frac{Y_{31}}{Y_{11}}I_1 \tag{7.45}$$

$$\left(Y_{42} - \frac{Y_{41}Y_{12}}{Y_{11}}\right)V_2 + \left(Y_{43} - \frac{Y_{41}Y_{13}}{Y_{11}}\right)V_3 + \left(Y_{44} - \frac{Y_{41}Y_{14}}{Y_{11}}\right)V_4 = I_4 - \frac{Y_{41}}{Y_{11}}I_1$$

Or

$$V_1 + \frac{Y_{12}}{Y_{11}} V_2 + \frac{Y_{13}}{Y_{11}} V_3 + \frac{Y_{14}}{Y_{11}} V_4 = \frac{1}{Y_{11}} I_1$$

$$Y_{22}^{(1)} V_2 + Y_{23}^{(1)} V_3 + Y_{24}^{(1)} V_4 = I_2^{(1)}$$

$$Y_{32}^{(1)} V_2 + Y_{33}^{(1)} V_3 + Y_{34}^{(1)} V_4 = I_3^{(1)}$$

$$Y_{42}^{(1)} V_2 + Y_{43}^{(1)} V_3 + Y_{44}^{(1)} V_4 = I_4^{(1)}$$

Where the superscript denotes the Step 1 set of derived coefficients

$$Y_{jk}^{(1)} = Y_{jk} - \frac{Y_{j1} Y_{1k}}{Y_{11}} \qquad \text{for } j \text{ and } k = 2, 3, 4$$

And the modified right-hand side expressions

$$I_j^{(1)} = I_j - \frac{Y_{j1}}{Y_{11}} I_1 \qquad \text{for } j = 2, 3, 4$$

- Step two eliminate V_2 :

$$V_2 + \frac{Y_{23}^{(1)}}{Y_{22}^{(1)}} V_3 + \frac{Y_{24}^{(1)}}{Y_{22}^{(1)}} V_4 = \frac{1}{Y_{22}^{(1)}} I_2^{(1)}$$

$$\left(Y_{33}^{(1)} - \frac{Y_{32}^{(1)} Y_{23}^{(1)}}{Y_{22}^{(1)}} \right) V_3 + \left(Y_{34}^{(1)} - \frac{Y_{32}^{(1)} Y_{24}^{(1)}}{Y_{22}^{(1)}} \right) V_4 = I_3^{(1)} - \frac{Y_{32}^{(1)}}{Y_{22}^{(1)}} I_2^{(1)}$$

$$\left(Y_{43}^{(1)} - \frac{Y_{42}^{(1)} Y_{23}^{(1)}}{Y_{22}^{(1)}} \right) V_3 + \left(Y_{44}^{(1)} - \frac{Y_{42}^{(1)} Y_{24}^{(1)}}{Y_{22}^{(1)}} \right) V_4 = I_4^{(1)} - \frac{Y_{42}^{(1)}}{Y_{22}^{(1)}} I_2^{(1)}$$

$$V_2 + \frac{Y_{23}^{(1)}}{Y_{22}^{(1)}} V_3 + \frac{Y_{24}^{(1)}}{Y_{22}^{(1)}} V_4 = \frac{1}{Y_{22}^{(1)}} I_2^{(1)}$$

$$Y_{33}^{(2)} V_3 + Y_{34}^{(2)} V_4 = I_3^{(2)}$$

$$Y_{43}^{(2)} V_3 + Y_{44}^{(2)} V_4 = I_4^{(2)}$$

$$Y_{jk}^{(2)} = Y_{jk}^{(1)} - \frac{Y_{j2}^{(1)} Y_{2k}^{(1)}}{Y_{22}^{(1)}} \quad \text{for } j \text{ and } k = 3, 4$$

- Step three eliminate V_3 :

$$V_3 + \frac{Y_{34}^{(2)}}{Y_{33}^{(2)}} V_4 = \frac{1}{Y_{33}^{(2)}} I_3^{(2)}$$

$$Y_{44}^{(3)} V_4 = I_4^{(3)}$$

$$Y_{44}^{(3)} = Y_{44}^{(2)} - \frac{Y_{43}^{(2)} Y_{34}^{(2)}}{Y_{33}^{(2)}} \quad \text{and} \quad I_4^{(3)} = I_4^{(2)} - \frac{Y_{43}^{(2)}}{Y_{33}^{(2)}} I_3^{(2)}$$

- Step four eliminate V_4 :

$$V_4 = \frac{1}{Y_{44}^{(3)}} I_4^{(3)}$$

Example 1: Using gaussian elemination , solve the nodal equation bellow to find the bus voltages. At each step of elimination find the equivalent circuit of the reduced matrix.

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} \;\;1\;\; & \;\;\;\;2\;\;\;\; & \;\;\;\;3\;\;\;\; & \;\;\;\;4\;\;\;\; \end{array} \\ \begin{bmatrix} -j16.75 & j11.75 & \boxed{j2.50} & j2.50 \\ \boxed{j11.75} & -j19.25 & j2.50 & j5.00 \\ j2.50 & j2.50 & -j5.80 & 0.00 \\ j2.50 & j5.00 & 0.00 & -j8.30 \end{bmatrix} \end{array} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1.00\angle -90^\circ \\ 0.68\angle -135^\circ \end{bmatrix}$$

3.2.2 NODE ELIMINATION (KRON REDUCTION)

Current injection is always zero at those buses of the network to which there is no external load or generating source connected. At such buses it is usually not necessary to calculate the voltages explicitly, and so we may eliminate them from our representation. For example, when $I_1 = 0$ in the four-bus system, we may write nodal admittance' equations in the form

$$\begin{array}{c|cccc} & ① & ② & ③ & ④ \\ \hline ① & Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ ② & Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ ③ & Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ ④ & Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{array} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 0 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}$$

By the elimination of node 1 , we obtain the 3 X 3 system

$$\begin{array}{c|ccc} & ② & ③ & ④ \\ \hline ② & Y_{22}^{(1)} & Y_{23}^{(1)} & Y_{24}^{(1)} \\ ③ & Y_{32}^{(1)} & Y_{33}^{(1)} & Y_{34}^{(1)} \\ ④ & Y_{42}^{(1)} & Y_{43}^{(1)} & Y_{44}^{(1)} \end{array} \begin{bmatrix} V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} I_2 \\ I_3 \\ I_4 \end{bmatrix}$$

By choosing Y_{pp} as the pivot and by eliminating bus p using the formula

$$Y_{jk(\text{new})} = Y_{jk} - \frac{Y_{jp} Y_{pk}}{Y_{pp}}$$

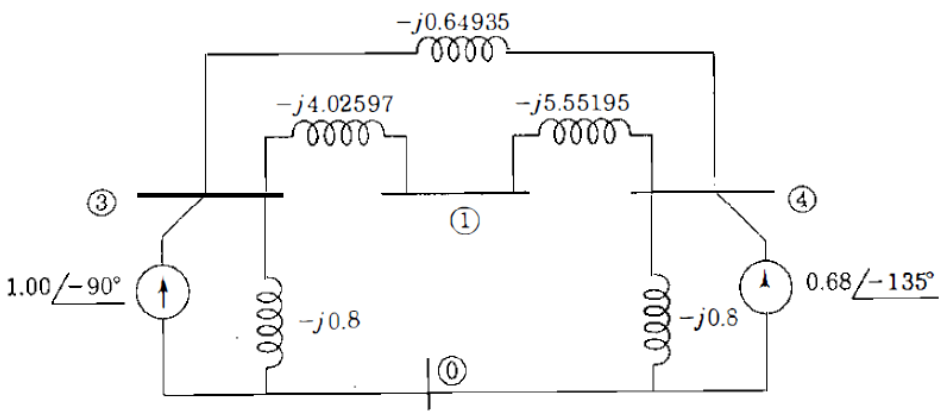
Example 2: Using Y_{22} as initial pivot, eliminate node 2 and V_2 from the 4 x 4 system of example 1

$$Y_{11(new)} = Y_{11} - \frac{Y_{12}Y_{21}}{Y_{22}} = -j16.75 - \frac{(j11.75)(j11.75)}{-j19.25} = -j9.57792$$

$$Y_{13(new)} = Y_{13} - \frac{Y_{12}Y_{23}}{Y_{22}} = j2.50 - \frac{(j11.75)(j2.50)}{-j19.25} = j4.02597$$

$$Y_{14(new)} = Y_{14} - \frac{Y_{12}Y_{24}}{Y_{22}} = j2.50 - \frac{(j11.75)(j5.00)}{-j19.25} = j5.55195$$

$$\begin{array}{c} \\ ① \\ ③ \\ ④ \end{array} \begin{array}{c} \begin{array}{ccc} ① & ③ & ④ \end{array} \\ \begin{bmatrix} -j9.57791 & j4.02597 & j5.55195 \\ j4.02597 & -j5.47532 & j0.64935 \\ j5.55195 & j0.64935 & -j7.00130 \end{bmatrix} \end{array} \begin{bmatrix} V_1 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 1.00\angle -90^\circ \\ 0.68\angle -135^\circ \end{bmatrix}$$



3.2.3 TRIANGULAR FACTORIZATION:

Representing Y bus as the product of two matrices L and U defined for the four-bus system. The matrices L and U are called the lower- and upper-triangular factors of Y bus

$$\mathbf{LU} = \mathbf{Y}_{bus}$$

For the 4 x4 matrix

$$
\mathbf{Y}_{bus} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}
\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} \end{array}
$$

1. Y_{11}, Y_{21}, Y_{31}, Y_{41} are Eliminated from the first column.
2. New coefficients 1, Y_{12}/Y_{11} , Y_{13}/ Y_{11} , and Y_{14}/ Y_1 are generated to replace those in the first row.
3. The other elements should be calculated by kroon reduction. Then the second row will be divided by $Y_{22}{}^{(2)}$ and so on.

$$
\mathbf{L} = \begin{bmatrix} Y_{11} & \cdot & \cdot & \cdot \\ Y_{21} & Y_{22}^{(1)} & \cdot & \cdot \\ Y_{31} & Y_{32}^{(1)} & Y_{33}^{(2)} & \cdot \\ Y_{41} & Y_{42}^{(1)} & Y_{43}^{(2)} & Y_{44}^{(3)} \end{bmatrix}
\qquad
\mathbf{U} = \begin{bmatrix} 1 & \dfrac{Y_{12}}{Y_{11}} & \dfrac{Y_{13}}{Y_{11}} & \dfrac{Y_{14}}{Y_{11}} \\ \cdot & 1 & \dfrac{Y_{23}^{(1)}}{Y_{22}^{(1)}} & \dfrac{Y_{24}^{(1)}}{Y_{22}^{(1)}} \\ \cdot & \cdot & 1 & \dfrac{Y_{34}^{(2)}}{Y_{33}^{(2)}} \\ \cdot & \cdot & \cdot & 1 \end{bmatrix}
$$

We may use the triangular factors to solve the original system of equations by substituting the product LU for Y bus to obtain

$$
\underbrace{\mathbf{L}}_{N \times N} \underbrace{\mathbf{U}}_{N \times N} \underbrace{\mathbf{V}}_{N \times 1} = \underbrace{\mathbf{I}}_{N \times 1}
$$

$$
\underbrace{\mathbf{L}}_{N \times N} \underbrace{\mathbf{V}'}_{N \times 1} = \underbrace{\mathbf{I}}_{N \times 1} \quad \text{and} \quad \underbrace{\mathbf{U}}_{N \times N} \underbrace{\mathbf{V}}_{N \times 1} = \underbrace{\mathbf{V}'}_{N \times 1}
$$

$$\begin{bmatrix} Y_{11} & \cdot & \cdot & \cdot \\ Y_{21} & Y_{22}^{(1)} & \cdot & \cdot \\ Y_{31} & Y_{32}^{(1)} & Y_{33}^{(2)} & \cdot \\ Y_{41} & Y_{42}^{(1)} & Y_{43}^{(2)} & Y_{44}^{(3)} \end{bmatrix} \begin{bmatrix} V_1' \\ V_2' \\ V_3' \\ V_4' \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & \frac{Y_{12}}{Y_{11}} & \frac{Y_{13}}{Y_{11}} & \frac{Y_{14}}{Y_{11}} \\ \cdot & 1 & \frac{Y_{23}^{(1)}}{Y_{22}^{(1)}} & \frac{Y_{24}^{(1)}}{Y_{22}^{(1)}} \\ \cdot & \cdot & 1 & \frac{Y_{34}^{(2)}}{Y_{33}^{(2)}} \\ \cdot & \cdot & \cdot & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} V_1' \\ V_2' \\ V_3' \\ V_4' \end{bmatrix}$$

The lower triangular system is readily solved by forward substitution beginning with $V_1^{\backslash}$ We then use the calculated values of $V_1^{\backslash}$, $V_2^{\backslash}$, $V_3^{\backslash}$, and $V_4^{\backslash}$ to solve the Eq. by back substitution for the actual unknowns V_1 , V_2 , V_3 and V_4 .Therefore, when changes are made in the current vector I, the solution vector V is found in two sequential steps; the first involves forward substitution using L and the second employs back substitution using U

Example 3: using the triangle factors of Y bus bellow determine the voltage at bus 3 when the current source at bus 4 is changed to $I_4 = 0.60 \angle -120^o$ per unit

$$\begin{array}{c} \\ ① \\ ② \\ ③ \\ ④ \end{array} \begin{array}{c} \begin{array}{cccc} ① & ② & ③ & ④ \end{array} \\ \begin{bmatrix} -j16.75 & j11.75 & j2.50 & j2.50 \\ j11.75 & -j19.25 & j2.50 & j5.00 \\ j2.50 & j2.50 & -j5.80 & 0 \\ j2.50 & j5.00 & 0 & -j8.30 \end{bmatrix} \end{array} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1.00\angle -90^\circ \\ 0.68\angle -135^\circ \end{bmatrix}$$

2.4THE BUS ADMITTANCE AND IMPEDANCE MATRICES

$$\mathbf{Z}_{\text{bus}} = \mathbf{Y}_{\text{bus}}^{-1}$$

And for a network of three independent nodes the standard form is

$$\mathbf{Z}_{\text{bus}} = \begin{array}{c c} & \begin{array}{ccc} ① & ② & ③ \end{array} \\ \begin{array}{c} ① \\ ② \\ ③ \end{array} & \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{21} & Z_{22} & Z_{23} \\ Z_{31} & Z_{32} & Z_{33} \end{bmatrix} \end{array}$$

The impedance elements of Z bus on the principal diagonal are called driving-point impedances of the buses, and the off-diagonal elements are called the transfer impedances of the buses.

Starting with the node equations expressed as

$$\mathbf{I} = \mathbf{Y}_{\text{bus}}\mathbf{V}$$

At node 2

$$I_2 = Y_{21}V_1 + Y_{22}V_2 + Y_{23}V_3$$

If V_1 and V_3 are reduced to zero by shorting buses 1 and 3 to the reference node, and voltage V_2 is applied at bus 1 so that current I_2 enters at bus 2, the self-admittance at bus is

$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1 = V_3 = 0}$$

At node 1

$$I_1 = Y_{11}V_1 + Y_{12}V_2 + Y_{13}V_3$$

$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1 = V_3 = 0}$$

The resultant admittance is the negative of the admittance directly connected between buses 1 and 2, so

$$\mathbf{V} = \mathbf{Z}_{bus}\mathbf{I}$$

$$V_1 = Z_{11}I_1 + Z_{12}I_2 + Z_{13}I_3$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2 + Z_{23}I_3$$

$$V_3 = Z_{31}I_1 + Z_{32}I_2 + Z_{33}I_3$$

the driving-point impedance Z_{22} is determined by open-circuiting the current sources at buses 1 and 3 and by injecting the source current I_2 at bus 2 . Then

$$Z_{22} = \frac{V_2}{I_2}\bigg|_{I_1 = I_3 = 0}$$

$$Z_{12} = \frac{V_1}{I_2}\bigg|_{I_1 = I_3 = 0}$$

$$Z_{32} = \frac{V_3}{I_2}\bigg|_{I_1 = I_3 = 0}$$

2.4.1 THEVENIN 'S THEOREM AND Z bus

Assume the bus voltages corresponding to the initial values I^o of the bus currents I by $V^o = Z_{bus}\ I^0$. The voltages V_1^o to V_n^o are the effective open -circuit voltages, which can be measured by voltmeter between the buses of the network and the reference node. When the bus currents are changed from their initial values to new values $I^o = \Delta$ I, the new bus voltages are given by the

$$\mathbf{V} = \mathbf{Z}_{bus}(\mathbf{I}^0 + \Delta\mathbf{I}) = \underbrace{\mathbf{Z}_{bus}\mathbf{I}^0}_{\mathbf{V}^0} + \underbrace{\mathbf{Z}_{bus}\,\Delta\mathbf{I}}_{\Delta\mathbf{V}}$$

The Figure below shows a large-scale system in schematic form with a representative bus k extracted along with the reference node of the system. Consider the circuit not to be energized so that the bus currents I° and voltages V^O are zero.

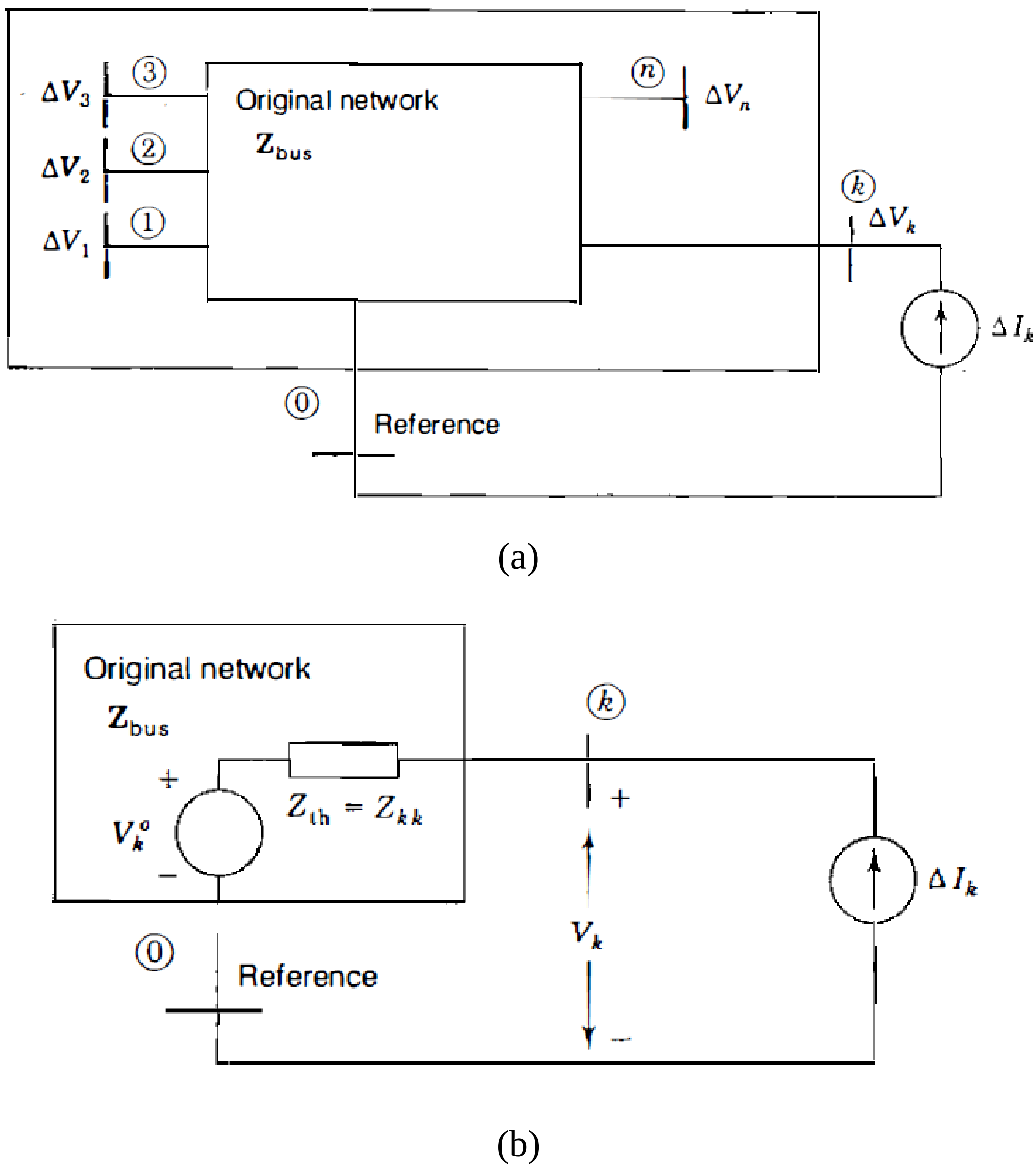


Figure (3.3) (a) Original network with bus k and reference node extracted. (b) Thevenin equivalent circuit at node k

Then, into bus k a current of ΔI_k amp is injected in to the system from a current source connected to the reference node. The resulting voltage changes at the buses indicated by the incremental quantities ΔV_1 to ΔV_n, are given by

$$
\begin{array}{c c c}
 & & \begin{array}{cccccc} (1) & (2) & & (k) & & (N) \end{array} \\
\begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ \vdots \\ \Delta V_k \\ \vdots \\ \Delta V_N \end{bmatrix} = & \begin{array}{c} (1) \\ (2) \\ \\ (k) \\ \\ (N) \end{array} & \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1k} & \cdots & Z_{1N} \\ Z_{21} & Z_{22} & \cdots & Z_{2k} & \cdots & Z_{2N} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ Z_{k1} & Z_{k2} & \cdots & Z_{kk} & \cdots & Z_{kN} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ Z_{N1} & Z_{N2} & \cdots & Z_{Nk} & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ \Delta I_k \\ \vdots \\ 0 \end{bmatrix}
\end{array}
$$

$$
\begin{array}{c c c}
 & & (k) \\
\begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ \vdots \\ \Delta V_k \\ \vdots \\ \Delta V_N \end{bmatrix} = & \begin{array}{c} (1) \\ (2) \\ \\ (k) \\ \\ (N) \end{array} & \begin{bmatrix} Z_{1k} \\ Z_{2k} \\ \vdots \\ Z_{kk} \\ \vdots \\ Z_{Nk} \end{bmatrix} \Delta I_k
\end{array}
$$

$$
V_k = V_k^{0} + Z_{kk}\,\Delta I_k
$$

$$
Z_{th} = Z_{kk}
$$

The Thevenin impedance between any two buses j and k of the network can be calculated

$$
\begin{array}{c c c}
 & & \begin{array}{cccccc} (1) & & (j) & (k) & & (N) \end{array} \\
\begin{bmatrix} \Delta V_1 \\ \vdots \\ \Delta V_j \\ \Delta V_k \\ \vdots \\ \Delta V_N \end{bmatrix} = & \begin{array}{c} (1) \\ \\ (j) \\ (k) \\ \\ (N) \end{array} & \begin{bmatrix} Z_{11} & \cdots & Z_{1j} & Z_{1k} & \cdots & Z_{1N} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ Z_{j1} & \cdots & Z_{jj} & Z_{jk} & \cdots & Z_{jN} \\ Z_{k1} & \cdots & Z_{kj} & Z_{kk} & \cdots & Z_{kN} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ Z_{N1} & \cdots & Z_{Nj} & Z_{Nk} & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} 0 \\ \vdots \\ \Delta I_j \\ \Delta I_k \\ \vdots \\ 0 \end{bmatrix}
\end{array}
$$

$$
= \begin{bmatrix} Z_{1j}\,\Delta I_j + Z_{1k}\,\Delta I_k \\ \vdots \\ Z_{jj}\,\Delta I_j + Z_{jk}\,\Delta I_k \\ Z_{kj}\,\Delta I_j + Z_{kk}\,\Delta I_k \\ \vdots \\ Z_{Nj}\,\Delta I_j + Z_{Nk}\,\Delta I_k \end{bmatrix}
$$

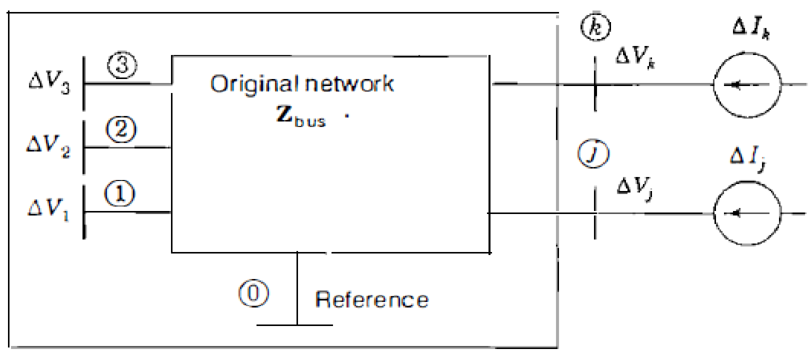
$$V_j = V_j^0 + Z_{jj}\,\Delta I_j + Z_{jk}\,\Delta I_k$$

$$V_k = V_k^0 + Z_{kj}\,\Delta I_j + Z_{kk}\,\Delta I_k$$

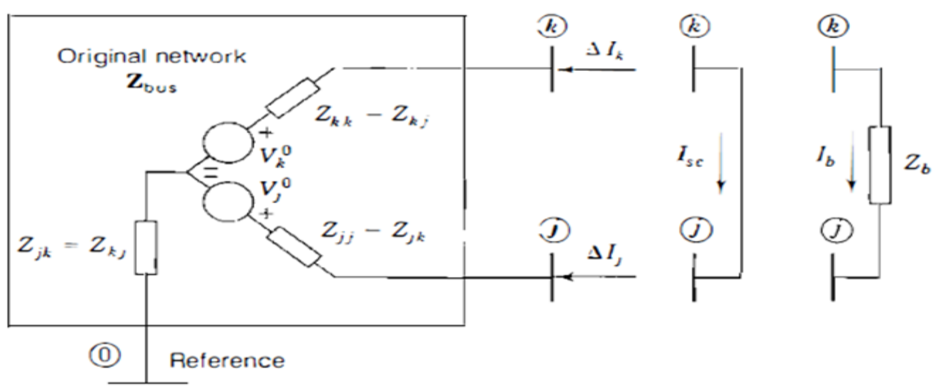
Adding and subtracting $Z_{jk}\ \Delta I_j$, and likewise, $Z_{kj}\,\Delta I_k$

$$V_j = V_j^0 + (Z_{jj} - Z_{jk})\,\Delta I_j + Z_{jk}(\Delta I_j + \Delta I_k)$$

$$V_k = V_k^0 + Z_{kj}(\Delta I_j + \Delta I_k) + (Z_{kk} - Z_{kj})\,\Delta I_k$$



(a)



(b) (c) (d)

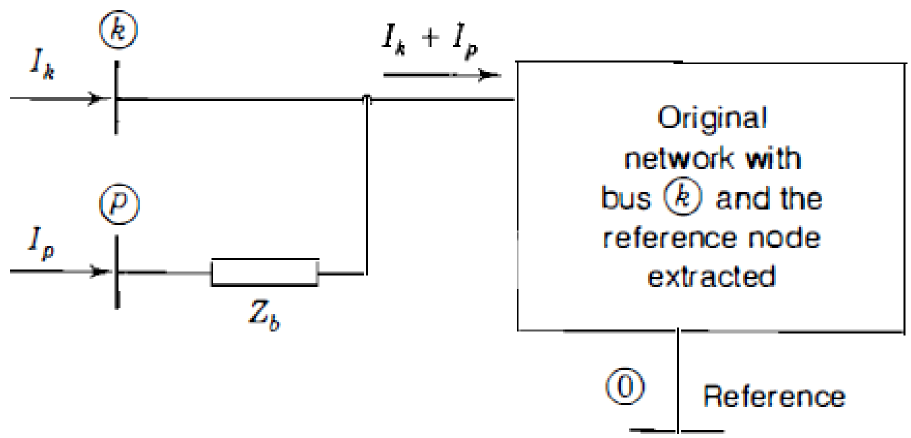
$$Z_{th,jk} = Z_{jj} + Z_{kk} - 2Z_{jk}$$

$$I_b = \frac{V_k^0 - V_j^0}{Z_{th,jk} + Z_b} = \frac{V_k - V_j}{Z_b}$$

3.4.2 MODIFICATION OF AN EXISTING Z BUS

Four cases are considered in this section:

CASE 1. Adding Z_b from a new bus P to the reference node.



Voltage V_p at the new bus is equal to $I_p Z_p$, Then

$$\begin{bmatrix} V_1^0 \\ V_2^0 \\ \vdots \\ V_N^0 \\ \hline V_p \end{bmatrix} = \begin{array}{c} \\ \\ \\ \\ (p) \end{array} \underbrace{\left[\begin{array}{cccc|c} & & & & 0 \\ & & & & 0 \\ & & \mathbf{Z}_{\text{orig}} & & \vdots \\ & & & & 0 \\ \hline 0 & 0 & \cdots & 0 & Z_b \end{array}\right]}_{\mathbf{Z}_{\text{bus(new)}}} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \\ \hline I_p \end{bmatrix}$$

CAS E 2 . Adding Z_b from a new bus P to an existing bus K

$$V_k = V_k^0 + I_p Z_{kk}$$

$$V_p = V_k^0 + I_p Z_{kk} + I_p Z_b$$

$$V_p = \underbrace{I_1 Z_{k1} + I_2 Z_{k2} + \cdots + I_N Z_{kN}}_{V_k^0} + I_p(Z_{kk} + Z_b)$$

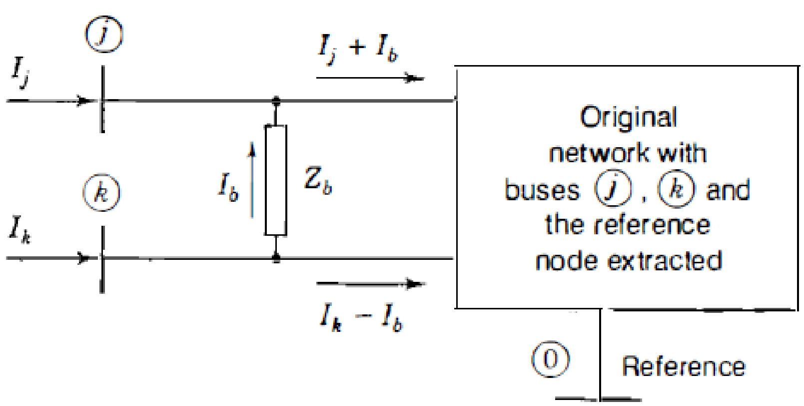
$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \\ \hline V_p \end{bmatrix} = \underbrace{\begin{array}{rc} & \hspace{6cm} p \\ p & \left[\begin{array}{c|c} \mathbf{Z}_{\text{orig}} & \begin{matrix} Z_{1k} \\ Z_{2k} \\ \vdots \\ Z_{Nk} \end{matrix} \\ \hline Z_{k1} \quad Z_{k2} \quad \cdots \quad Z_{kN} & Z_{kk} + Z_b \end{array}\right] \end{array}}_{\mathbf{Z}_{\text{bus(new)}}} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \\ \hline I_p \end{bmatrix}$$

CASE 3. Adding Z_b from existing bus K to the reference node

The matrix is the same as case 2 but V_b equal to zero

$$Z_{hi(\text{new})} = Z_{hi} - \frac{Z_{h(N+1)} Z_{(N+1)i}}{Z_{kk} + Z_b}$$

CASE 4. Adding Z_b between two existing buses j and k



$$\Delta V_h = (Z_{hj} - Z_{hk}) I_b$$

$$\begin{bmatrix} V_1 \\ \vdots \\ V_j \\ V_k \\ \vdots \\ V_N \\ \hline 0 \end{bmatrix} = \left[\begin{array}{c|c} \mathbf{Z}_{\text{orig}} & \begin{array}{c}(\text{col. } j - \text{col. } k) \\ \text{of } \mathbf{Z}_{\text{orig}}\end{array} \\ \hline (\text{row } j - \text{row } k) \text{ of } \mathbf{Z}_{\text{orig}} & Z_{bb} \end{array}\right] \begin{bmatrix} I_1 \\ \vdots \\ I_j \\ I_k \\ \vdots \\ I_N \\ \hline I_b \end{bmatrix}$$

Example 4 : Modify the bus impedance matrix below to account for the connection of a capacitor having a reactance of 5.0 per unit between bus 4 and the reference node . Then, find V4 using the impedances of the new matrix and the current sources.

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} j0.73128 & j0.69140 & j0.61323 & j0.63677 \\ j0.69140 & j0.71966 & j0.60822 & j0.64178 \\ j0.61323 & j0.60822 & j0.69890 & j0.55110 \\ j0.63677 & j0.64178 & j0.55110 & j0.69890 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1.00\angle -90^\circ \\ 0.68\angle -135^\circ \end{bmatrix}$$

$k = 4$, and $Z_b = -j5.0$ per unit

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ \hline 0 \end{bmatrix} = \left[\begin{array}{cccc|c} & & & & j0.63677 \\ & \mathbf{Z}_{\text{orig}} & & & j0.64178 \\ & & & & j0.55110 \\ & & & & j0.69890 \\ \hline j0.63677 & j0.64178 & j0.55110 & j0.69890 & -j4.30110 \end{array}\right] \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ \hline I_b \end{bmatrix}$$

The terms in the fifth row and column were obtained by repeating the fourth row and column of Z_{orig} and noting that

$$Z_{44} + Z_b = j0.69890 - j5.0 = -j4.30110$$

Then, eliminating the fifth row and column, we obtain for $Z_{bus(new)}$

$$Z_{11(new)} = j0.73128 - \frac{j0.63677 \times j0.63677}{-j4.30110} = j0.82555$$

$$Z_{24(new)} = j0.64178 - \frac{j0.69890 \times j0.64178}{-j4.30110} = j0.74606$$

$$\mathbf{Z}_{bus(new)} = \begin{bmatrix} j0.82555 & j0.78641 & j0.69482 & j0.74024 \\ j0.78641 & j0.81542 & j0.69045 & j0.74606 \\ j0.69482 & j0.69045 & j0.76951 & j0.64065 \\ j0.74024 & j0.74606 & j0.64065 & j0.81247 \end{bmatrix}$$

$$\begin{aligned} V_4 &= j0.64065(1.00\angle -90^\circ) + j0.81247(0.68\angle -135^\circ) \\ &= 1.03131 - j0.39066 \\ &= 1.10281\angle -20.7466^\circ \text{ per unit} \end{aligned}$$

3.4.3 DIRECT DETERMINATION OF Z_{bus}

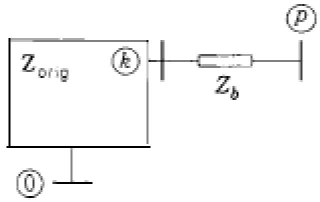
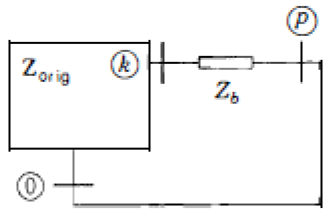
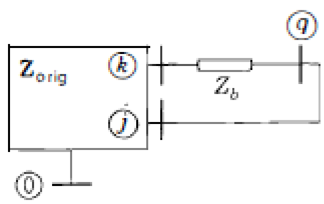
We could determine Z_{bus} by first finding Y_{bus} and then inverting it, but this is not convenient for large-scale systems as we have seen. Fortunately, formulation of Z_{bus} using a direct building algorithm is a straight forward process on the computer. At the outset we have a list of the branch impedances showing the buses to which they are connected. We start by writing the equation for one bus connected through a branch impedance Z_u to the reference as

$$\begin{array}{cc} & 1 \\ [V_1] = 1 & [Z_a][I_1] \end{array}$$

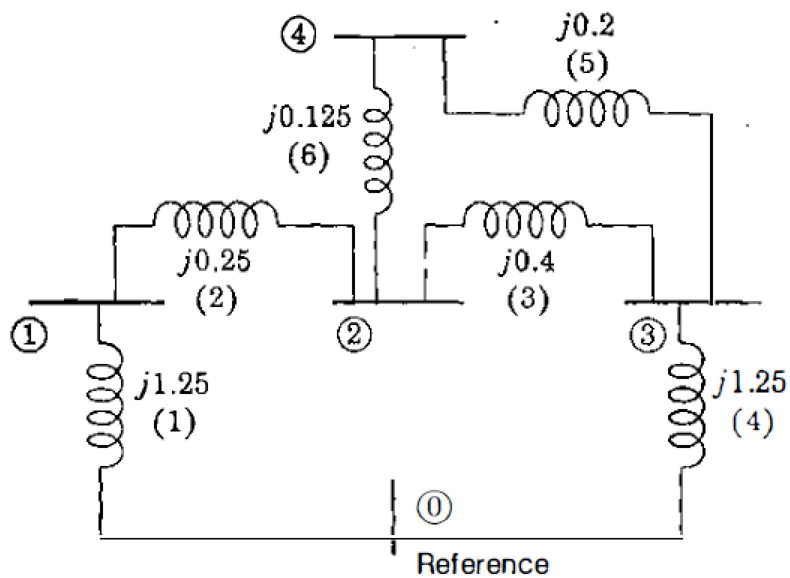
Now we might add a new bus connected to the first bus or to the reference node. For instance, if the second bus is connected to the reference node through Zb ' we have the matrix equation

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{array}{c} \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \begin{bmatrix} Z_a & 0 \\ 0 & Z_b \end{bmatrix} \end{array} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

And we proceed to modify the evolving Z_{bus} matrix by adding other buses and branches. The combination of these procedures constitutes the Z_{bus} building algorithm. The buses of a network must be renumbered internally by the computer algorithm to agree with the order in which they are to be added to Z_{bus} as it is built up.

2	Existing bus (k) to new bus (p) 	$\begin{array}{cc} & \begin{matrix} (k) & (p) \end{matrix} \\ \begin{matrix} (k) \\ (p) \end{matrix} & \left[\begin{array}{c\|c} \mathbf{Z}_{orig} & \text{col. } k \\ \hline \text{row } k & Z_{kk} + Z_b \end{array}\right] \end{array}$
3	Existing bus (k) to reference node  (Node (p) is temporary.)	• Repeat Case 2 and • Remove row p and column p by Kron reduction
4	Existing bus (j) to existing bus (k)  (Node (q) is temporary.)	• Form the matrix $\begin{array}{cc} & \begin{matrix} \phantom{\mathbf{Z}_{orig}} & (q) \end{matrix} \\ \begin{matrix} \\ (q) \end{matrix} & \left[\begin{array}{c\|c} \mathbf{Z}_{orig} & \text{col. } j - \text{col. } k \\ \hline \text{row } j - \text{row } k & Z_{th,jk} + Z_b \end{array}\right] \end{array}$ where $Z_{th,jk} = Z_{jj} + Z_{kk} - 2Z_{jk}$ and • Remove row q and column q by Kron reduction

Example 2: Determine Z_{bus} for the network shown in the Fig., where the impedances labeled 1 through 6 are shown in per unit. Preserve all buses.



Establishing bus 1 with its impedance to the reference node and write

$$[V_1] = \begin{array}{cc} & ① \\ ① & [j1.25] \end{array} \; [I_1]$$

We then have the 1 x 1 bus impedance matrix

$$\mathbf{Z}_{\text{bus, 1}} = \begin{array}{cc} & ① \\ ① & [j1.25] \end{array}$$

To establish bus 2 with its impedance to bus 1

$$\mathbf{Z}_{\text{bus,2}} = \begin{array}{cc} & \begin{array}{cc} ① & ② \end{array} \\ \begin{array}{c} ① \\ ② \end{array} & \begin{bmatrix} j1.25 & j1.25 \\ j1.25 & j1.50 \end{bmatrix} \end{array}$$

The term j1 .50 above is the sum of j l .2S and j0.2S. The elements j l.2S in the new row and column are the repetition of the elements of row 1 and column 1 of the matrix being modified.

Bus 3 with the impedance connecting it to bus 2 is established by writing

$$\mathbf{Z}_{\text{bus},3} = \begin{array}{c} \\ ① \\ ② \\ ③ \end{array} \begin{array}{c} \begin{array}{ccc} ① & ② & ③ \end{array} \\ \begin{bmatrix} j1.25 & j1.25 & j.125 \\ j1.25 & j1.50 & j1.50 \\ j1.25 & j1.50 & j1.90 \end{bmatrix} \end{array}$$

If we now decide to add the impedance Zb = j l .25 from bus 3 to the reference node.

$$\mathbf{Z}_{\text{bus},4} = \begin{array}{c} \\ ① \\ ② \\ ③ \\ ⓟ \end{array} \begin{array}{c} \begin{array}{cccc} ① & ② & ③ & ⓟ \end{array} \\ \left[\begin{array}{ccc|c} j1.25 & j1.25 & j.125 & j1.25 \\ j1.25 & j1.50 & j1.50 & j.150 \\ j1.25 & j1.50 & j1.90 & j1.90 \\ \hline j1.25 & j1.50 & j1.90 & j3.15 \end{array}\right] \end{array}$$

We now eliminate row p and column p by Kron reduction . Some of the elements of the new matrix are

$$Z_{11(\text{new})} = j1.25 - \frac{(j1.25)(j1.25)}{j3.15} = j0.75397$$

$$Z_{22(\text{new})} = j1.50 - \frac{(j1.50)(j1.50)}{j3.15} = j0.78571$$

$$Z_{23(\text{new})} = Z_{32(\text{new})} = j1.50 - \frac{(j1.50)(j1.90)}{j3.15} = j0.59524$$

When all the elements are determined, we have

$$
\mathbf{Z}_{\text{bus},5} = \begin{array}{c} \\ ① \\ ② \\ ③ \end{array}
\begin{array}{c} \begin{array}{ccc} ① & ② & ③ \end{array} \\
\begin{bmatrix} j0.75397 & j0.65476 & j0.49603 \\ j0.65476 & j0.78571 & j0.59524 \\ j0.49603 & j0.59524 & j0.75397 \end{bmatrix} \end{array}
$$

We now decide to add the impedance j0 .20 from bus 3 to establish bus 4

$$
\mathbf{Z}_{\text{bus},6} = \begin{array}{c} \\ ① \\ ② \\ ③ \\ ④ \end{array}
\begin{array}{c} \begin{array}{cccc} ① & ② & ③ & ④ \end{array} \\
\left[\begin{array}{ccc|c} j0.75397 & j0.65476 & j0.49603 & j0.49603 \\ j0.65476 & j0.78571 & j0.59524 & j0.59524 \\ j0.49603 & j0.59524 & j0.75397 & j0.75397 \\ \hline j0.49603 & j0.59524 & j0.75397 & j0.95397 \end{array}\right] \end{array}
$$

Finally, we add the impedance Zb = j0.l25 between buses 2 and 4 . If

we let j and k equal 2 and 4 , respectively, we obtain the elements for row 5 and column 5

$$Z_{15} = Z_{12} - Z_{14} = j0.65476 - j0.49603 = j0.15873$$

$$Z_{25} = Z_{22} - Z_{24} = j0.78571 - j0.59524 = j0.19047$$

$$Z_{35} = Z_{32} - Z_{34} = j0.59524 - j0.75397 = -j0.15873$$

$$Z_{45} = Z_{42} - Z_{44} = j0.59524 - j0.95397 = -j0.35873$$

$$Z_{55} = Z_{22} + Z_{44} - 2Z_{24} + Z_b$$

$$= j\{0.78571 + 0.95397 - 2(0.59524)\} + j0.125 = j0.67421$$

then

$$
\begin{array}{c}
\\
\\
\\
\\
\\
(q)
\end{array}
\left[
\begin{array}{cccc|c}
 & & & & (q) \\
 & & & & j0.15873 \\
 & & & & j0.19047 \\
 & & \mathbf{Z}_{\text{bus},6} & & -j0.15873 \\
 & & & & -j0.35873 \\
\hline
j0.15873 & j0.19047 & -j0.15873 & -j0.35873 & j0.67421
\end{array}
\right]
$$

we find by Kron reduction

$$
\mathbf{Z}_{\text{bus}} =
\begin{array}{c}
\\
(1) \\
(2) \\
(3) \\
(4)
\end{array}
\begin{array}{c}
\begin{array}{cccc}
\quad(1)\quad & \quad(2)\quad & \quad(3)\quad & \quad(4)\quad
\end{array} \\
\begin{bmatrix}
j0.71660 & j0.60992 & j0.53340 & j0.58049 \\
j0.60992 & j0.73190 & j0.64008 & j0.69659 \\
j0.53340 & j0.64008 & j0.71660 & j0.66951 \\
j0.58049 & j0.69659 & j0.66951 & j0.76310
\end{bmatrix}
\end{array}
$$