Computer Sciences
University of
Technology

Final Exam. 2011-2012
Term

Date: **26** / 5 / 2012
Time: 3 hours
Lecturer: Dr. Hana'aM.A.Salman

Subject: Number Theory
Class: 2nd
Branch: security Systems

Not: Choose only 4questions (15 mark for each)

1.) Let, n = 16

1. **Find all the residue classes of n**
2. **Find all the prime residue classes of n**
3. **Find all the non-negative residue classes of n**
4. **Find all the odd residue classes of n**
5. **Find a system**

b) Solve the bellow

1. $[7]_{12} +_{12} [8]_{12} =$
2. $[7]_{5} -_{5} [8]_{5} =$
3. $[7]_{11} *_{11} [8]_{11} =$

2. Find all the solutions to

$X \equiv 1 \bmod 2$

$X \equiv 2 \bmod 3$

$X \equiv 3 \bmod 5$

$X \equiv 4 \bmod 11$

3. a)Define prime number and show if the bellow integers are prime or not
97, 47, -43, -415

b) Find the remainder of

1. **23 mod 7**
2. **-4 mod 5**
3. **121 mod 0**
4. **-333 mod -10331**

4. Expand the rational numbers $\frac{239}{51}$ as simple continued fractions.

5. Let a=13, b=2222, find

a) gcd(a,b), gcd(-a,b), gcd(a,b-) , gcd(-a,b-) , gcd(b,a)

b) lcm(a,b), lcm(-a,b), lcm(a,b-) , lcm(-a,b-) , lcm(b,a)

1. (a)

1) {$[0]_{16}$,$[1]_{16}$,$[2]_{16}$,$[3]_{16}$,$[4]_{16}$,$[5]_{16}$,$[6]_{16}$,$[7]_{16}$,$[8]_{16}$,$[9]_{16}$,$[10]_{16}$,$[11]_{16}$,$[12]_{16}$,$[13]_{16}$,$[14]_{16}$,$[15]_{16}$}

2) {$[2]_{16}$,$[3]_{16}$,$[5]_{16}$,$[7]_{16}$,$[11]_{16}$,$[13]_{16}$}

3) {$[0]_{16}$,$[1]_{16}$,$[2]_{16}$,$[3]_{16}$,$[4]_{16}$,$[5]_{16}$,$[6]_{16}$,$[7]_{16}$,$[8]_{16}$,$[9]_{16}$,$[10]_{16}$,$[11]_{16}$,$[12]_{16}$,$[13]_{16}$,$[14]_{16}$,$[15]_{16}$}

4) {$[1]_{16}$,$[3]_{16}$,$[5]_{16}$,$[7]_{16}$,$[9]_{16}$,$[11]_{16}$,$[13]_{16}$,$15]_{16}$}

5) {0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15}

1. (b)

1) $[3]_{12}$

2)$[4]_{5}$

3) $[1]_{11}$

2. (a)

M=$\prod_{i=1}^{m} m_i$=$2 \times 3 \times 5 \times 11 = 330$

$$M_i = {}^{m}/_{m_i} \quad \rightarrow \quad \begin{aligned} M_1 &= {}^{330}/_{2} = 165 \\ M_2 &= {}^{330}/_{3} = 110 \\ M_3 &= {}^{330}/_{5} = 66 \\ M_4 &= {}^{330}/_{11} = 30 \end{aligned}$$

$M_1 \quad S_1 \equiv 1\ mod\ m_1$ S_1=$165^{-1} mod 2 = 1$ $\rightarrow$ $S_1 = 1$	$M_3 \quad S_3 \equiv 1\ mod\ m_3$ S_3=$66^{-1} mod 5 = 1$ $\rightarrow$ $S_3 = 1$
$M_2 \quad S_2 \equiv 1\ mod\ m_2$ S_2=$110^{-1} mod 3 = 2$ $\rightarrow$ $S_2 = 2$	$M_4 \quad S_4 \equiv 1\ mod\ m_4$ S_4=$30^{-1} mod 11 = 7$ $\rightarrow$ $S_4 = 7$

$$X = 1 \cdot 165 \cdot 1 + 2 \cdot 110 \cdot 2 + 3 \cdot 66 \cdot 1 + 4 \cdot 30.7 (mod\ 330)$$

= 1643 mod 330

3. (a)

Prime number: an integer p>1 is a prime if it has no positive divisor other than 1 and it self.

- 97

  $\sqrt{97 < \sqrt{100}} = 10$

  Prime less than 10 are {2,3,5,7} and none of these divides 97 and so 97 is a prime

- 47

$\sqrt{47 < \sqrt{49}} = 7$

Prime less than 10 are {2,3,5} and none of these divides 47 and so 47 is a prime

- -43

  By definition -47 is not prime

- **-415**
  By definition -415 is not prime

3. (b)
1) since 23=7·3+2→23mod7=2
2) since -4=5$(-1)+12\rightarrow -4$mod5=1
3) not passible since b=0
4) not passible by the definition of remainder "for b>0 define a and b=r, where r is the remainder a is divided by b

4.
239=51$\times 4+35$
51$= 35\times 1+16$
35$= 16\times 2+3$
16=3$\times 5+1$
3=3$\times 1+0$

$$\frac{239}{51}=\{4,1,2,5,3\}$$

5. (a) : a=13, b=2222
Since 13, and 2222 relatively prime then gcd(13,2222)=1
gcd(13,222)= gcd(-13,222)=gcd(13,-222)=gcd(-13,-222)=1

5.(b): a=13, b=222
Since 13, and 2222 relatively prime then gcd(13,2222)=1
lcm(13,2222)=$\frac{13\times 2222}{gcd(13,2222)}$=28886
lcm(13,222)= lcm (-13,222)= lcm (13,-222)= lcm (-13,-222)=28886