



Note : Answer 7 questions only , 10 marks for each

Q1) Prove by induction: $1^2 - 2^2 + 3^2 - 4^2 + \dots (-1)^{n-1} n^2 = (-1)^{n-1} \cdot \frac{n(n+1)}{2}$

Q2) Sketch the coordinate graph of : $g(x) = x^3 - 3x^2 - x + 3$
Determine if $g(x)$ is : 1) function?

2) One-to-one?

3) Onto? (mention the reason)

Q3) Suppose that the input symbols = the output symbols = $\{0,1\}$, design a finite state machine that recognizes the sequence pattern 0101 in the input string but only when the last 1 in the sequence pattern 0101 occurs at a position that is a multiple of 3. As in the following example:

(e.g : input = 1011101010111110101

output = 00000000100000000000

Q4) In a survey of 1000 college students, the following data were obtained:

595 had taken a Mathematics (M) course..

595 had taken a Computer science (C) course .

550 had taken a Business (B) course.

395 had taken both a Mathematics (M) and a Business course..

350 had taken both a Mathematics (M) and a Computer science (C) course

400 had taken both a Computer science (C) and a Business (B) course.

250 had taken all three courses.

(a) How many students were surveyed who had taken none of the three courses?

(b) Fill in the correct number of students in each of the eight regions of the Venn diagram.

Q5) Test the validity of the following argument:

If I study, then I will not fail mathematics.

If I do not play basketball, then I will study.

But I failed mathematics.

Therefore I must have played basketball.

Q6) Consider the algebraic expression: $(((2 + 1) * 3) - (3 + (2 - (8 \setminus 4)))) ^ 2$

1- Draw the corresponding ORT

2- Rewrite the expression into prefix polish notation form

3- Evaluate the value of the expression in prefix polish notation form.

Q7) a- Let $A = \{1,2,3,4\}$, Given any example of relation $R \subseteq A \times A$ which is:

1- Reflexive, not anti-symmetric, not transitive.

2- Symmetric and transitive .

b- Let R be a binary relation on the set of all positive integers such that

$R = \{(x,y) \mid x+y < 6\}$

1- Write R as a set of ordered pairs?

2- Find: (a) reflexive(R); (c) transitive(R).

3- Is R Symmetric? Antisymmetric?

Q8)) Construct a deterministic finite automata which will recognize the input string $1(01)^* \text{ and } 11(10)^*0 (0+1)$ and nothing else, then decide which of the following strings are accepted by this automata: 111 , 1100