Chemical Engineering Department

Subject: Process Control for undereducated students

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Typical Questions & Answers

Process Control Problems

Problem (1)

A thermocouple has the following characteristics when it is immersed in a stirred bath:

Mass of thermocouple = 1 g
Heat capacity of thermocouple = 0.25 cal/g °C
Heat transfer coefficient = 20 cal/cm^2 h °C (for thermocouple and bath)
Surface area of thermocouple = 3 cm^2

(a) Derive a transfer function model for the thermocouple relating the change in its indicated output T to the change in the temperature of its surroundings T_s assuming uniform temperature (no gradients in the thermocouple bead), no conduction in the leads, constant physical properties, and conversion of the millivolt-level output directly to a °C reading by a very fast meter.

(b) If the thermocouple is initially out of the bath and at room temperature (23 °C), what is the maximum temperature that it will register if it is suddenly plunged into the bath (80 °C) and held there for 20 s?

Solution

a) Energy balance for the thermocouple,

$$mC\frac{dT}{dt} = hA(T_s - T)$$

where m is mass of thermocouple
C is heat capacity of thermocouple
h is heat transfer coefficient
A is surface area of thermocouple
t is time in sec

Substituting numerical values in (1) and noting that

$\overline{T}_s = \overline{T}$ and $\dfrac{dT}{dt} = \dfrac{dT'}{dt}$,

$$15\frac{dT'}{dt} = T_s' - T'$$

Taking Laplace transform,

$$\frac{T'(s)}{T_s'(s)} = \frac{1}{15s+1}$$

b) $T_s(t) = 23 + (80 - 23)\ S(t)$

$\overline{T}_s = \overline{T} = 23$

From $t = 0$ to $t = 20$,

$T_s'(t) = 57\ S(t)$, $T_s'(s) = \dfrac{57}{s}$

$$T'(s) = \frac{1}{15s+1} T_s'(s) = \frac{57}{s(15s+1)}$$

Applying inverse Laplace Transform,

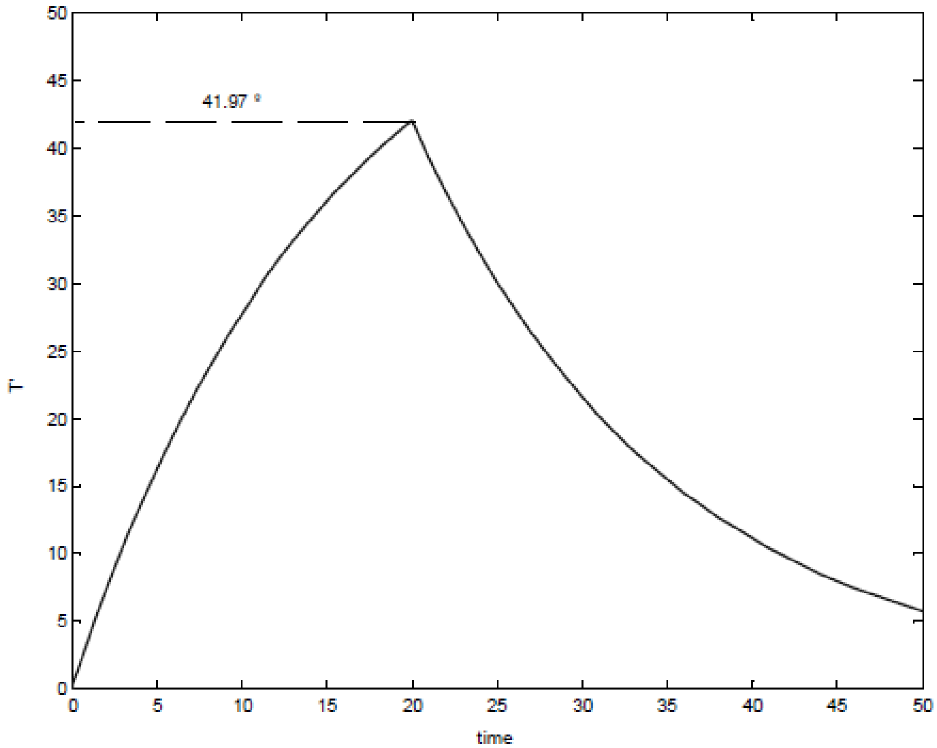
$$T'(t) = 57(1 - e^{-t/15})$$

Then

$$T(t) = T'(t) + \overline{T} = 23 + 57(1 - e^{-t/15})$$

Since $T(t)$ increases monotonically with time, maximum $T = T(20)$.

Maximum $T(t) = T(20) = 23 + 57\ (1\text{-}e^{-20/15}) = 64.97\ °C$



Problem (2)

A thermometer having a time constant of 0.2 min is placed in a temperature bath and after the thermometer comes to equilibrium with the bath, the temperature of the bath is increased linearly with time at the rate of I deg C / min what is the difference between the indicated temperature and bath temperature
(a) 0.1 min
(b) 10. min
after the change in temperature begins.
(c) What is the maximum deviation between the indicated temperature and bath temperature and when does it occurs.
(d) plot the forcing function and the response on the same graph. After the long enough time buy how many minutes does the response lag the input?

Solution

Consider thermometer to be in equilibrium with temperature bath at temperature

$X(t) = X_S + (1° / m)t, t > 0$

as it is given that the temperture varies linearly

X(t)-Xs = t

Let X(t) = X(t) - Xs = t

Y(s) = G(s).X(s)

$$Y(s) = \frac{1}{1+\tau s}\frac{1}{s^2} = \frac{A}{1+\tau s} + \frac{B}{s} + \frac{C}{s^2}$$

A = τ^2 $B = -\tau$ $C = 1$

$$Y(s) = \frac{\tau^2}{1+\tau s} - \frac{\tau}{s} + \frac{1}{s^2}$$

$$Y(t) = \tau e^{-t/\tau} - \tau + t$$

(a) the difference between the indicated temperature and bath temperature

at t = 0.1 min = X(0.1)_ Y(0.1)

= 0.1 - (0.2e$^{-0.1/0.2}$ - 0.2+0.1) since T = 0.2 given
= 0.0787 deg C

(b) t = 1.0 min

X(1) - Y(1) = 1- (0.2e$^{-1/0.}$2 - 0.2 +1) = 0.1986

(c) Deviation $D = -Y(t) + X(t)$

$$= -\tau e^{-t/T} + T = \tau\ (-e^{-t/T} + 1)$$

For maximum value $dD/dT = \tau\ (-e^{-t/T} + (\_ -1/T) = 0$

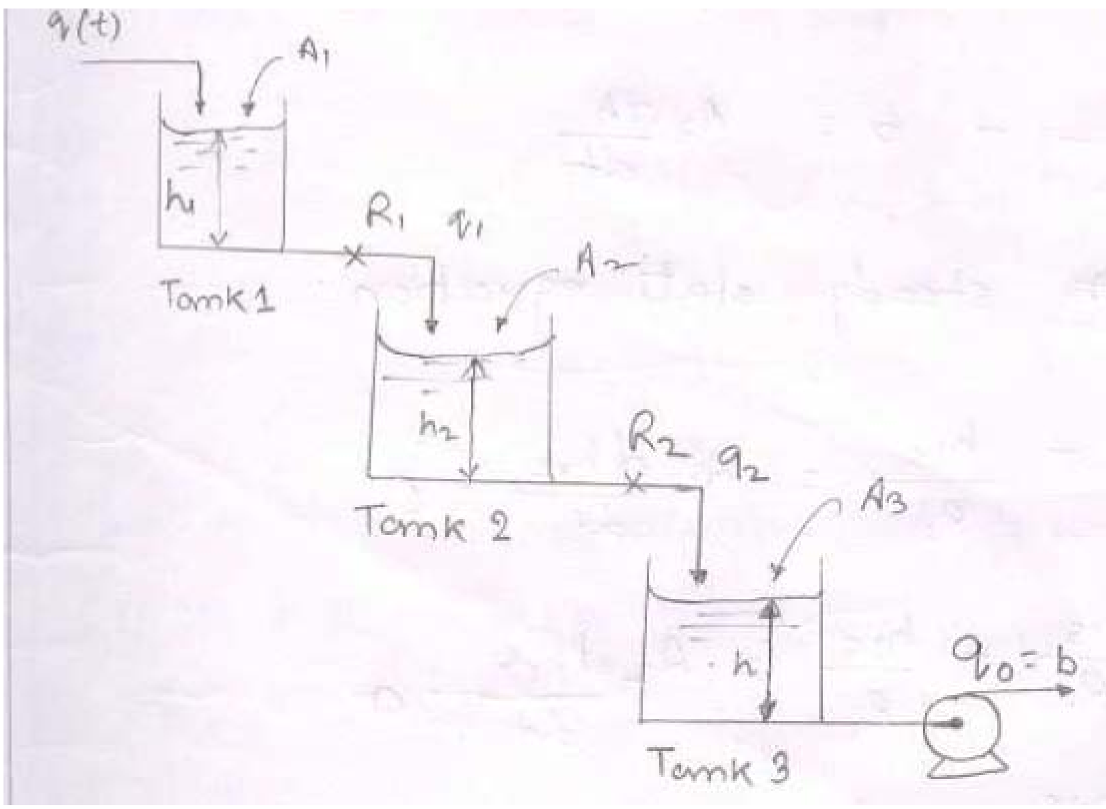
$-e^{-t/} = 0$

as t tend to infinitive

$D = \tau\ (-e^{-t/T} + (\_ -1/T) = \tau = 0.2$ deg C

Problem (3)

Determine the transfer function H(s)/Q(s) for the liquid level shown in figure below Resistance R1 and R2 are linear. The flow rate from tank 3 is maintained constant at b by means of a pump; the flow rate from tank 3 is independent of head h. The tanks are non-interacting.



Solution

A balance on tank 1 gives

$$q - q_1 = A_1 \frac{dh_1}{dt}$$

where h1 = height of the liquid level in tank 1

similarly balance on the tank 2 gives

$$q_1 - q_2 = A_2 \frac{dh_2}{dt}$$

and balance on tank 3 gives

$$q_2 - q_0 = A_3 \frac{dh}{dt}$$

here $q_1 = \frac{h_1}{R_1} q_2 = \frac{h_2}{R_2} \quad q_0 = b$

So we get

$$q - \frac{h_1}{R_1} = A_1 \frac{dh_1}{dt}$$

$$\frac{h_1}{R_1} - \frac{h_2}{R_2} = A_2 \frac{dh_2}{dt}$$

$$\frac{h_2}{R_2} - b = A_3 \frac{dh}{dt}$$

writing the steady state equation

$$q_s - \frac{h_{1s}}{R_1} = A_1 \frac{dh_{1s}}{dt} = 0$$

$$\frac{h_{1s}}{R_1} - \frac{h_{2s}}{R_2} = A_2 \frac{dh_{2s}}{dt}$$

$$\frac{h_{2s}}{R_2} - b = 0$$

Subtracting and writing in terms of deviation

$$Q - \frac{H}{R_1} = A_1 \frac{dH_1}{dt}$$

$$\frac{H_1}{R_1} - \frac{H_2}{R_2} = A_1 \frac{dH_2}{dt}$$

$$\frac{H_2}{R_2} = A_3 \frac{dH}{dt}$$

where $Q = q - q_S$

$H_1 = h_1 - h_{1S}$

$H_1 = h_2 - h_{2S}$

$H = h - h_S$

Taking Laplace transforms

$$Q(s) - \frac{H_1(s)}{R_1} = A_1 s \ H_1(s) \text{---------}(1)$$

$$\frac{H_1(s)}{R_1} - \frac{H_2(s)}{R_2} = A_2 s \, H_2(s) \text{ --------}(2)$$

$$\frac{H_2(s)}{R_2} = A_3 \ s \, H(s) \text{----------}(3)$$

We have three equations and 4 unknowns (Q(s),H(s),H1(s) and H2(s). So we can express one in terms of other.

From (3)

$$H_2(s) = \frac{H_2(s)}{R_1 A_3 s} \text{--------------}(4)$$

$$H_2(s) = \frac{R_2 H_1(s)}{R_1(\tau_2 s + 1)} \quad \text{where} \quad \tau_2 = R_2 A_2 \text{------------}(5)$$

From (1)

$$H_1(s) = \frac{R_1 Q(s)}{(\tau_1 s + 1)}, \qquad \tau_1 = R_1 A_1 \text{---------}(6)$$

Combining equation 4,5,6

$$H \ (s) = \frac{Q(s)}{(A_3 s)(\tau_1 s + 1)(\tau_2 s + 1)}$$

$$\frac{H(s)}{Q(s)} = \frac{1}{(A_3 s)(\tau_1 s + 1)(\tau_2 s + 1)}$$

Problem (4)

Determine Y (4) for the system response expressed by

$$Y(s) = \frac{2}{s}\frac{(2s+4)}{(4s^2+0.8s+1)}$$

$$Y(s) = 4\left(1+\frac{2}{s}\right)\frac{1}{(4s^2+0.8s+1)}$$

$$Y(s) = \frac{8}{s(4s^2+0.8s+1)} + \frac{4}{(4s^2+0.8s+1)}$$

$$= \text{(step response)} + \text{(impulse response)}$$

$$Now, \tau = \sqrt{4} = 2\,;\, 2\xi\tau = 0.8$$

$$\xi = 0.2$$

$$\text{also, } \frac{t}{\tau} = \frac{4}{2} = 2$$

impulse response $\tau Y(t) =$ 4*0.63 = 2.52 (from figure)

step response = 8*1.15 = 9.2 (from figure)

Y(4) = 1.26+9.2

Y(4) =10.46

Problem (5)

Heat transfer equipment shown in fig. consists of tow tanks, one nested inside the other. Heat is transferred by convection through the wall of inner tank.

1. Hold up volume of each tank is 1 ft3

2. The cross sectional area for heat transfer is 1 ft2

3. The overall heat transfer coefficient for the flow of heat between the tanks is 10 Btu/(hr)(ft2)(oF)

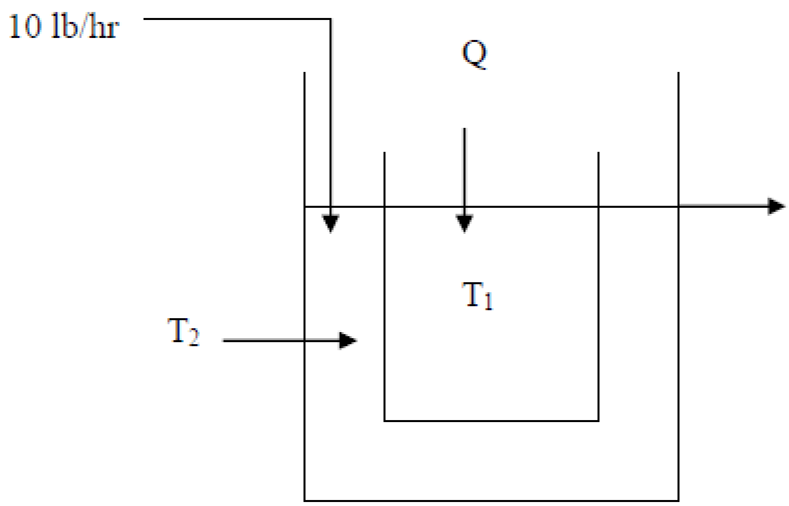
4. Heat capacity of fluid in each tank is 2 Btu/(lb)(oF)

5. Density of each fluid is 50 lb/ft3

Initially the temp of feed stream to the outer tank and the contents of the outer tank are equal to 100 oF. Contents of inner tank are initially at 100 oF. the flow of heat to the inner tank (Q) changed according to a step change from 0 to 500 Btu/hr.

(a) Obtain an expression for the laplace transform of the temperature of inner tank T(s).

(b) Invert T(s) and obtain T for t= 0,5,10, U



Solution

(a)For outer tank

$$WC(Ti - T_o) + hA\,(T_1 - T_2) - WC(T_2 - T_o) = \rho\, C\, V_2 \frac{dT_2}{dt} \text{ ---------------------------- (1)}$$

At steady state

$$WC(Tis - T_o) + hA\,(T_{1s} - T_{2s}) - WC(T_{2s} - T_o) = 0 \text{ ------------------------------------ (2)}$$

(1) – (2) gives

$$WCTi' + hA\,(T_1' - T_2') - WCT_2' = \rho\, C\, V_2 \frac{dT_2'}{dt}$$

Substituting numerical values

$$10\ Ti' + 10\ (T_1' - T_2') - 10\ T_2' = 50\frac{dT_2'}{dt}$$

Taking L.T.

$$Ti(s) + T_1(s) - 2T_2(s) = 5\ s\ T_2(s)$$

Now Ti(s) = 0, since there is no change in temp of feed stream to outer tank. Which gives

$$\frac{T_2(s)}{T_1(s)} = \frac{1}{2+5s}$$

For inner tank

$$Q - hA\ (T_1 - T_2) = \rho\ C\ V_1\ \frac{dT_1}{dt} \text{ --------------------- } (3)$$

$$Qs - hA\ (T_{1s} - T_{2s}) = 0 \text{ ------------------------------ } (4)$$

(3) – (4) gives

$$Q' - hA\ (T_1' - T_2') = \rho\ C\ V_1\ \frac{dT_1'}{dt}$$

Taking L.T and putting numerical values

$$Q(s) - 10\ T_1(s) + 10\ T_2(s) = 50\ s\ T_1(s)$$

$$Q(s) = 500/s \quad \text{and} \quad T_2(s) = T_1(s)\ /\ (2+5s)$$

$$\frac{500}{s} - 10T_1(s) + \frac{10T_1(s)}{2+5s} = 50sT_1(s)$$

$$\frac{50}{s} = T_1(s)\left[5s - \frac{1}{2+5s} + 1\right]$$

$$T_1(s) = \frac{50(2+5s)}{s(25s^2+15s+1)}$$

$$T_1(s) = \frac{2(2+5s)}{s\left(s+3.82/50\right)\left(s+26.18/50\right)}$$

$$T_1(s) = \frac{100}{s} - \frac{94.71}{\left(s+3.82/50\right)} - \frac{5.29}{\left(s+26.18/50\right)}$$

$$T_1'(t) = 100 - 94.71\,e^{-3.82t/50} - 5.29\,e^{-26.18t/50}$$

and

$$T_1(t) = 200 - 94.71\,e^{-3.82t/50} - 5.29\,e^{-26.18t/50}$$

For t=0,5,10 and ∞

T(0) = 100 °F

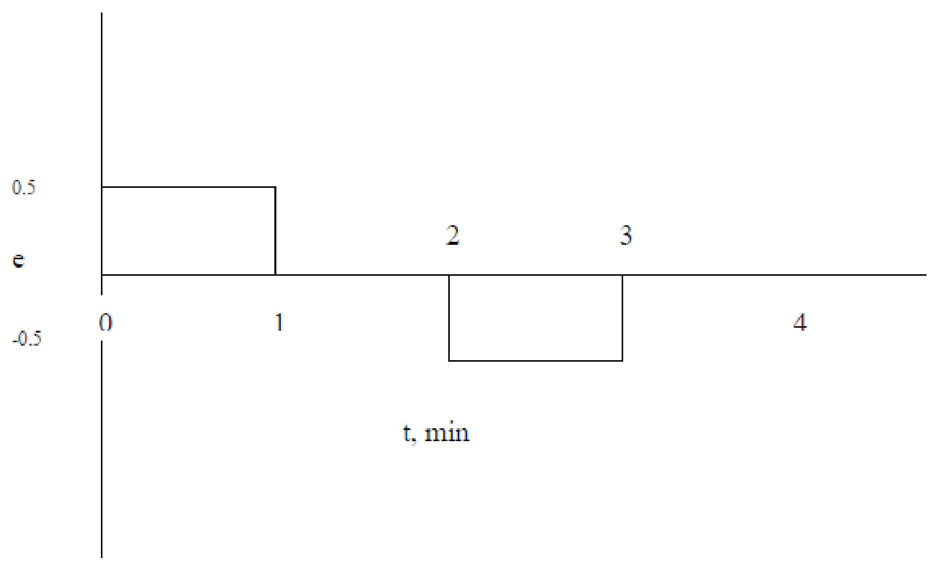
T(5) = 134.975 °F

T(10) = 155.856 °F

T(∞) = 200 °F

Problem (6)

The input (e) to a PI controller is shown in the fig. Plot the output of the controller if KC = 2 and XI = 0.5 min



Solution

$$e(t) = 0.5\ (\ u(t) - u(t-1) - u(t-2) + u(t-3)\ )$$

$$e(s) = (0.5/s)\ (\ 1 - e^{-s} - e^{-2s} + e^{-3s}\ )$$

$$P(s)/e(s) = K_C\ (\ 1 + (1\ /\ \tau_I\ s)\ ) = \ 2\ (\ 1 + 2/s\ )$$

$$P(s) = (\ 1/s + 2/s^2\ )\ (1 - e^{-s} - e^{-2s} + e^{-3s}\ )$$

$$P(t) = 1 + 2t \qquad 0 \le t < 1$$

$$= 2 \qquad 1 \le t < 2$$

$$= 5 - 2t \qquad 2 \le t < 3$$

$$= 0 \qquad 3 \le t < \infty$$

Problem (7)

The thermal system shown in fig P 13.6 is controlled by PD controller.

Data; w = 250 lb/min; ρ= 62.5 lb/ft3;

V1 = 4 ft3, V2=5 ft3; V3=6ft3;

C = 1 Btu/(lb)(°F)

Change of 1 psi from the controller changes the flow rate of heat of by 500 Btu/min. the temperature of the inlet stream may vary. There is no lag in the measuring element.

(a) Draw a block diagram of the control system with the appropriate transfer function in each block. Each transfer function should contain a numerical values of the parameters.

(b) From the block diagram, determine the overall transfer function relating the temperature in tank 3 to a change in set point.

(c) Find the offset for a unit steo change in inlet temperature if the controller gain KC is 3psi/°F of temperature error and the derivative time is 0.5 min.

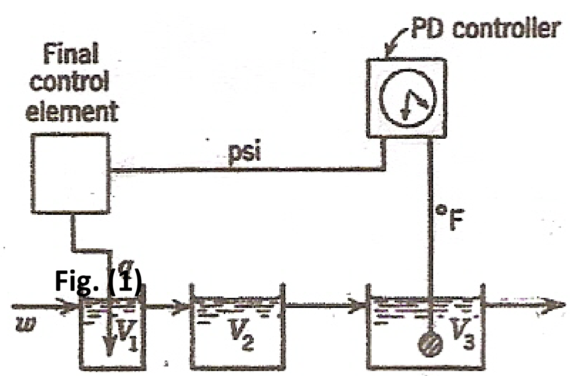


Fig. (1)

$$WT_0C + q = \rho CV_1(T_1 - T_0) + WT_1C$$

$$WT_1C = \rho CV_2(T_2 - T_1) + WT_2C$$

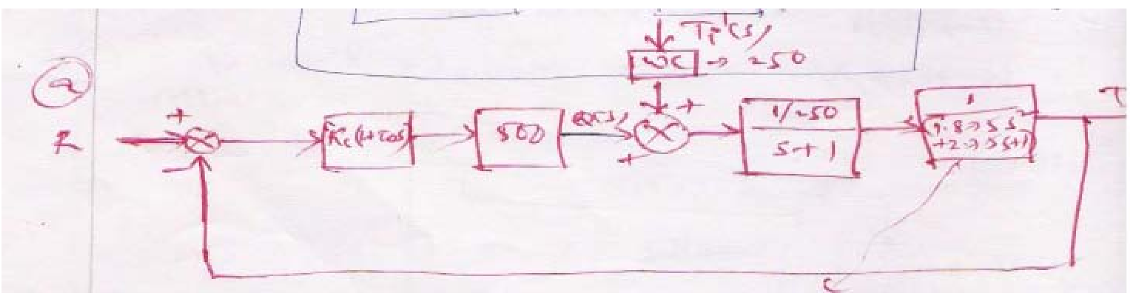
$$WT_2C = \rho CV_3(T_3 - T_2) + WT_3C$$

$$T_0(WC + \rho CV_1) + q = T_1(W\,C + \rho CV_1)$$

$$T_1 = T_0 + \frac{q}{WC + \rho CV_1}$$

$T_1 = T_2 = T_3$

$$T_3 = T_0 + \frac{q}{WC + \rho CV_1} \Rightarrow T_3(s) = \frac{q(s)}{(WC + \rho CV_1)s}$$



(b)

$$\frac{T_3'(s)}{R(s)} = \frac{k_C(1+\tau_D s)\dfrac{2}{(s+1)(1.25s+1)(1.5s+1)}}{1 + k_C(1+\tau_D s)\dfrac{2}{(s+1)(1.25s+1)(1.5s+1)}}$$

$$= \frac{T_3'(s)}{R(s)} = \frac{2k_C(1+\tau_D s)}{(s+1)(1.875s^2 + 2.75s + 1) + 2k_C(1+\tau_D s)}$$

$$\frac{T_3'(s)}{R(s)} = \frac{2k_C(1+\tau_D s)}{1.875s^3 + 4.625s^2 + (3.75 + 2k_C\tau_D)s + 2k_c + 1}$$

c) k_C=3; $\tau_D = 0.5, offset = ?, \tau_0^{'}(s) = \frac{1}{s}$

$$\frac{T_3^{'}(s)}{T_i^{'}(s)} \underset{s \to 0}{Lt} \frac{1}{1.875s^3 + 4.625s^2 + (3.75 + 2k_C\tau_D)s + 2k_C + 1}$$

$$= \frac{1}{2k_C + 1} = \frac{1}{7} = 0.143$$

Offset =0.143

Problem (8)

for the control shown, the characteristics equation is
$s^4 + 4s^3 + 6s^2 + 4s + (1 + k) = 0$
(a) Determine value of k above which the system is unstable.
(b) Determine the value of k for which the two of the roots are on the imaginary axis.

Solution

$s^4 + 4s^3 + 6s^2 + 4s + (1 + k) = 0$

$$\begin{array}{lll}
s^4 & 1 & 6(1+k) \\
s^3 & 4 & 4 \\
s^2 & 5 & 1+k \\
s & 4 - \frac{4}{5}(1+k) & \\
1 & 1+k &
\end{array}$$

For the system to be unstable

$$4\left(1-\left(\frac{1+k}{5}\right)\right)<0$$

$$1<\frac{1+k}{5}$$

$$\begin{aligned}
&k>4\\
&1+k<0\\
&k<-1\\
&k>-1
\end{aligned}$$

The system is stable at -1<k<4

(b) For two imaginary roots

$$4=\frac{4}{5}(1+k);k=4$$