



### Problem (1)

A plate of size 60 cm x 60 cm slides over a plane inclined to the horizontal at an angle of 30°. It is separated from the plane with a film of oil of thickness 1.5 mm. The plate weighs 25kg and slides down with a velocity of 0.25 m/s. Calculate the dynamic viscosity of oil used as lubricant. What would be its kinematic viscosity if the specific gravity of oil is 0.95.

### Solution

$$\text{Component of } W \text{ along the plane} = W \cos(60) = W \sin(30)$$

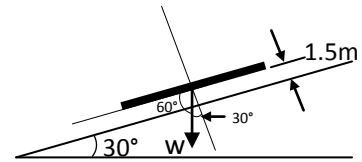
$$= 25 (0.5) = 12.5 \text{ kg}$$

$$F = 12.5 \text{ kg} (9.81 \text{ m/s}^2) = 122.625 \text{ N}$$

$$\tau = F/A = 122.625 \text{ N} / (0.6 \times 0.6) \text{ m}^2 = 340.625 \text{ Pa}$$

$$\mu = \frac{\tau}{(du/dy)} = \frac{340.625 \text{ Pa}}{(0.25/0.0015) \text{ s}^{-1}} = 2.044 \text{ Pa.s} = 20.44 \text{ poise}$$

$$\nu = \frac{\mu}{\rho} = \frac{2.044 \text{ Pa.s}}{950 \text{ kg/m}^3} = 0.00215 \text{ m}^2/\text{s} = 21.5 \text{ stoke}$$



By Dr. Salah Salman

### Problem (2)

By dimensional analysis, obtain an expression for the drag force (F) on a partially submerged body moving with a relative velocity (u) in a fluid; the other variables being the linear dimension (L), surface roughness (e), fluid density (ρ), and gravitational acceleration (g).

### Solution

Drag force (F) N

Relative velocity (u) m/s

Linear dimension (L) m

Surface roughness (e) m

Density (ρ) kg/m<sup>3</sup>

Acceleration of gravity (g) m/s<sup>2</sup>

$$\equiv [MLT^{-2}]$$

$$\equiv [LT^{-1}]$$

$$\equiv [L]$$

$$\equiv [L]$$

$$\equiv [ML^{-3}]$$

$$\equiv [LT^{-2}]$$

$$F = k (u, L, e, \rho, g)$$

$$f(F, u, L, e, \rho, g) = 0$$

$$n = 6, m = 3, \Rightarrow \Pi = n - m = 6 - 3 = 3$$

No. of repeating variables = m = 3

The selected repeating variables is (u, L, ρ)

$$\Pi_1 = u^{a_1} L^{b_1} \rho^{c_1} F \quad \text{-----(1)}$$



$$\Pi_2 = u^{a2} L^{b2} \rho^{c2} e \quad \text{-----}(2)$$

$$\Pi_3 = u^{a3} L^{b3} \rho^{c3} g \quad \text{-----}(3)$$

For  $\Pi_1$  equation (1)

$$[M^0 L^0 T^0] = [L T^{-1}]^{a1} [L]^{b1} [ML^{-3}]^{c1} [MLT^{-2}]$$

Now applied dimensional homogeneity

$$\text{For M} \quad 0 = c1 + 1 \quad \Rightarrow \quad c1 = -1$$

$$\text{For T} \quad 0 = -a1 - 2 \quad \Rightarrow \quad a1 = -2$$

$$\text{For L} \quad 0 = a1 + b1 - 3c1 + 1 \quad \Rightarrow \quad b1 = -2$$

$$\Pi_1 = u^{-2} L^{-2} \rho^{-1} F \quad \Rightarrow \quad \Pi_1 = \frac{F}{u^2 L^2 \rho}$$

For  $\Pi_2$  equation (2)

$$[M^0 L^0 T^0] = [L T^{-1}]^{a2} [L]^{b2} [ML^{-3}]^{c2} [L]$$

$$\text{For M} \quad 0 = c2 \quad \Rightarrow \quad c2 = 0$$

$$\text{For T} \quad 0 = -a2 \quad \Rightarrow \quad a2 = 0$$

$$\text{For L} \quad 0 = a2 + b2 - 3c2 + 1 \quad \Rightarrow \quad b2 = -1$$

$$\Pi_2 = L^{-1} e \quad \Rightarrow \quad \Pi_2 = \frac{e}{L}$$

For  $\Pi_3$  equation (3)

$$[M^0 L^0 T^0] = [L T^{-1}]^{a3} [L]^{b3} [ML^{-3}]^{c3} [L T^{-2}]$$

$$\text{For M} \quad 0 = c3 \quad \Rightarrow \quad c3 = 0$$

$$\text{For T} \quad 0 = -a3 - 2 \quad \Rightarrow \quad a3 = -2$$

$$\text{For L} \quad 0 = a3 + b3 - 3c3 + 1 \quad \Rightarrow \quad b3 = 1$$

$$\Pi_3 = u^{-2} L g \quad \Rightarrow \quad \Pi_3 = \frac{L g}{u^2}$$

$$f_1(\Pi_1, \Pi_2, \Pi_3) = 0 \quad \Rightarrow \quad f_1\left(\frac{F}{u^2 L^2 \rho}, \frac{e}{L}, \frac{L g}{u^2}\right) = 0$$

$$\therefore F = u^2 L^2 \rho f\left(\frac{e}{L}, \frac{L g}{u^2}\right)$$



### Problem (3)

A conical vessel is connected to a U-tube having mercury and water as shown in the Figure. When the vessel is empty the manometer reads 0.25 m. find the reading in manometer, when the vessel is full of water.

### Solution

$$P_1 = P_2$$

$$P_1 = (0.25 + H) \rho_w g + P_o$$

$$P_2 = 0.25 \rho_m g + P_o$$

$$\Rightarrow (0.25 + H) \rho_w g + P_o = 0.25 \rho_m g + P_o$$

$$\Rightarrow H = 0.25 (\rho_m - \rho_w) / \rho_w$$

$$= 0.25 (12600 / 1000) = 3.15 \text{ m}$$

When the vessel is full of water, let the mercury level in the left limb go down by (x) meter and the mercury level in the right limb go up by the same amount (x) meter.

i.e. the reading manometer = (0.25 + 2x)

$$P_1 = P_2$$

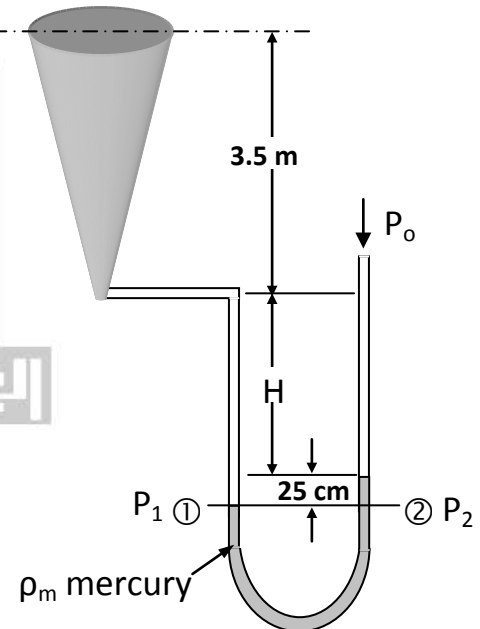
$$P_1 = (0.25 + x + H + 3.5) \rho_w g + P_o$$

$$P_2 = (0.25 + 2x) \rho_m g + P_o$$

$$\Rightarrow (0.25 + x + H + 3.5) \rho_w g + P_o = (0.25 + 2x) \rho_m g + P_o$$

$$\Rightarrow 6.9 + x = (0.25 + 2x) (\rho_m / \rho_w) \Rightarrow x = 0.1431 \text{ m}$$

$$\text{The manometer reading} = 0.25 + 2 (0.1431) = 0.536 \text{ m}$$



### Problem (4)

A pump developing a pressure of 800 kPa is used to pump water through a 150 mm pipe, 300 m long to a reservoir 60 m higher. The flow rate obtained is 0.05 m<sup>3</sup>/s. As a result of corrosion and scalling the effective absolute roughness of the pipe surface increases by a factor of 10 by what percentage is the flow rate reduced.  $\mu = 1 \text{ mPa.s}$

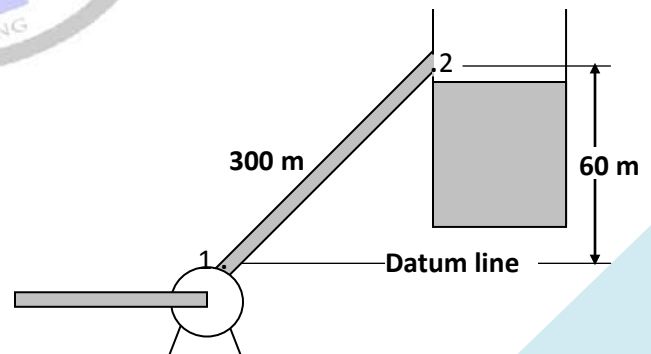
### Solution

$$\text{The total head of pump developing} = (\Delta P / \rho g) = 800,000 / (1000 \times 9.81) = 81.55 \text{ mH}_2\text{O}$$

$$\text{The head of potential energy} = 60 \text{ m}$$

$$\text{Neglecting the kinetic energy (same diameter)}$$

$$\frac{\Delta P}{\rho g} + \Delta z + \frac{\Delta u^2}{2\alpha g} - \frac{\eta W}{g} s + h_F = 0$$





$$\Rightarrow \Delta P / \rho g + \Delta z + h_F = 0$$

$$\Rightarrow h_F = -\Delta P / \rho g - \Delta z = 81.55 - 60 = 21.55 \text{ m}$$

$$u = Q/A = (0.05 \text{ m}^3/\text{s}) / (\pi/4 \cdot 0.15^2) = 2.83 \text{ m/s}$$

$$h_{Fs} = (-\Delta P_{fs} / \rho g) = 4f (L/d) (u^2 / 2g)$$

$$\Rightarrow f = h_{Fs} d 2g / (4Lu^2) = (21.55) (0.15)(9.81) / (2 \times 300 \times 2.83^2) = 0.0066$$

$$\Phi = 0.0033, \text{Re} = (1000 \times 2.83 \times 0.15) / 0.001 = 4.23 \times 10^5$$

From Figure (3.7)  $e/d = 0.003$

Due to corrosion and scalling the roughness increase by factor 10

$$\text{i.e. } (e/d)_{\text{new}} = 10 (e/d)_{\text{old}} = 0.03$$

The pump head that supplied is the same

$$(-\Delta P_{fs}) = h_{Fs} \rho g = 21.55 (1000) 9.81 = 211.41 \text{ kPa}$$

$$\Phi \text{Re}^2 = (-\Delta P_{fs} / L) (\rho d^3 / 4\mu^2) = [(211410) / (300)] [(1000)(0.15)^3 / (4)(0.01)^2] = 6 \times 10^8$$

$$\text{From Figure (3.8) } \text{Re} = 2.95 \times 10^5 \Rightarrow u = 1.97 \text{ m/s}$$

$$\text{The percentage reduced in flow rate} = (2.83 - 1.97) / 2.83 \times 100 \% = 30.1 \%$$

### Problem (5)

It is required to pump cooling water from storage pond to a condenser in a process plant situated 10 m above the level of the pond. 200 m of 74.2 mm i.d. pipe is available and the pump has the characteristics given below. The head loss in the condenser is equivalent to 16 velocity heads based on the flow in the 74.2 mm pipe. If the friction factor  $\Phi = 0.003$ , estimate the rate of flow and the power to be supplied to the pump assuming  $\eta = 0.5$

|                       |        |        |       |        |        |
|-----------------------|--------|--------|-------|--------|--------|
| Q (m <sup>3</sup> /s) | 0.0028 | 0.0039 | 0.005 | 0.0056 | 0.0059 |
| $\Delta h$ (m)        | 23.2   | 21.3   | 18.9  | 15.2   | 11.0   |

### Solution

[Usually the pump done its exact duty and return the liquid at the suction pressure i.e the pump give the minimum limit of energy required to arrive the liquid to point 2 or d]

$$\Delta h = \Delta z + \frac{\Delta P}{\rho g} + \frac{\Delta u^2}{2g\alpha} + [(h_F)_d + (h_F)_s + (h_F)_{\text{condenser}}]$$

$$(h_F)_{d+s} = 4f \frac{L}{d} \frac{u^2}{2g} = 4(0.006)(200/0.0742)(u^2/2g) = 3.3 u^2$$

$$(h_F)_{\text{condenser}} = 16 \frac{u^2}{2g} = 0.815 u^2$$

$$u = Q/A = 321.26 Q$$



$$\Rightarrow \Delta h = 10 + (0.815 + 3.3)(321.26 Q)^2 = 10 + 2.2 \times 10^5 Q^2$$

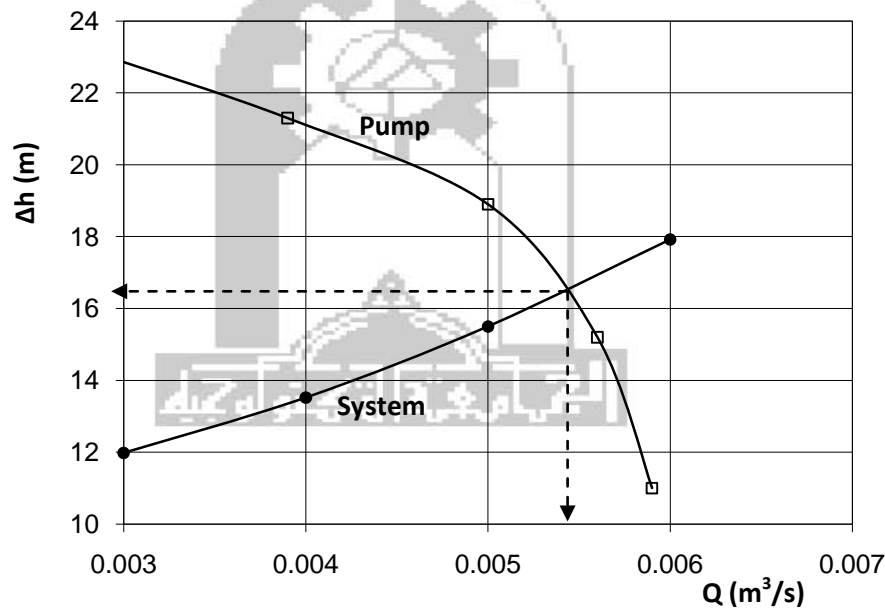
To draw the system curve

|                       |       |       |       |       |
|-----------------------|-------|-------|-------|-------|
| Q (m <sup>3</sup> /s) | 0.003 | 0.004 | 0.005 | 0.006 |
| Δh (m)                | 11.98 | 13.52 | 15.5  | 17.92 |

From Figure

$$Q = 0.0054 \text{ m}^3/\text{s}$$

$$\Delta h = 16.4 \text{ m}$$



$$\text{Power required for pump} = \frac{Q \Delta h \rho g}{\eta} = (0.0054)(16.4)(1000)(9.81)/0.5$$

$$= 17.375 \text{ kW}$$

$$u = 1.25 \text{ m/s} \Rightarrow \frac{\Delta u^2}{2g} = 0.08 \text{ m} ; \Delta z = 10 \text{ m}; h_f = 6.415 \text{ m}$$

### Problem (6)

A Power-law liquid of density  $961 \text{ kg/m}^3$  flows in steady state with an average velocity of  $1.523 \text{ m/s}$  through a tube  $2.67 \text{ m}$  length with an inside diameter of  $0.0762 \text{ m}$ . For a pipe consistency coefficient of  $4.46 \text{ Pa}\cdot\text{s}^n$  [or  $4.46 (\text{kg} / \text{m}\cdot\text{s}^2) \text{ s}^n$ ], calculate the values of the apparent viscosity for pipe flow  $(\mu_a)_P$  in  $\text{Pa}\cdot\text{s}$ , the Reynolds number  $Re$ , and the pressure drop across the tube for power-law indices  $n = 0.3, 0.7, 1.0$ , and  $1.5$  respectively.

### Solution

$$\text{Apparent viscosity } (\mu_a)_P = Kp\left(\frac{8u}{d}\right)^{n-1}$$

$$= 4.46 (\text{kg/m}) \text{ s}^{n-2} [8 (1.523)/0.0762]^{n-1} \text{ s}^{n-1}$$



$$\Rightarrow (\mu_a)_p = 4.46 (159.9)^{n-1} \quad (\text{Pa.s}) \text{-----(1)}$$

$$\text{Re} = \frac{\rho u d}{(\mu_a)_p} = \frac{\rho u d}{4.46 (159.9)^{n-1}} = 961 (1.523)(0.0762) / 4.46 (159.9)^{n-1}$$

$$\Rightarrow \text{Re} = 25.006 / (159.9)^{n-1} \text{-----(2)}$$

$$-\Delta P_{fs} = 4f(L/d)(\rho u^2/2) = 4(16/\text{Re})(2.67 / 0.0762)[961(1.523)^2/2] \text{ for laminar}$$

$$\Rightarrow -\Delta P_{fs} = 99950.56 (159.9)^{n-1} (\text{Pa}) \text{-----(3)}$$

( $\eta = 0.6$ )

| n   | $(\mu_a)_p$<br>Eq.(1) | Re<br>Eq.(2) | $-\Delta P_{fs}$<br>Eq.(3) | $(-\Delta P_{fs})_{\text{non-New}} / (-\Delta P_{fs})_{\text{New}}$ | Power<br>(W) |
|-----|-----------------------|--------------|----------------------------|---------------------------------------------------------------------|--------------|
| 0.3 | 0.1278                | 872.44       | 2,865                      | 0.0287                                                              | 33           |
| 0.7 | 0.9732                | 114.6        | 21,809                     | 0.218                                                               | 252.5        |
| 1.0 | 4.46                  | 25.006       | 99,950.56                  | 1.0                                                                 | 1157         |
| 1.5 | 56.4                  | 1.9776       | 1,263,890.541              | 12.7                                                                | 14630        |

### Problem (7)

A (30cm x 15cm) Venturi meter is provided in a vertical pipe-line carrying oil of sp.gr. = 0.9. The flow being upwards and the difference in elevations of throat section and entrance section of the venturi meter is 30 cm. The differential U-tube mercury manometer shows a gauge deflection of 25 cm. Take  $C_d = 0.98$  and calculate: -

- The discharge of oil
- The pressure difference between the entrance and throat sections.

### Solution

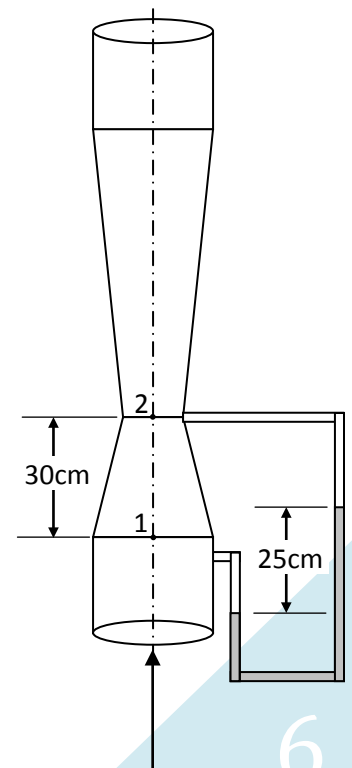
$$\begin{aligned} \text{i- } Q &= u_2 A_2 = C_d \sqrt{\frac{2R(\rho_m - \rho)g}{\rho}} \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \\ &= 0.98 \sqrt{\frac{2(0.25)(12700)9.81}{900}} \left[ \frac{0.3^2 [\pi/4(0.15)^2]}{\sqrt{0.3^4 - 0.15^4}} \right] \\ &= 0.1488 \text{ m}^3/\text{s} \end{aligned}$$

- Applying Bernoulli's equation at points 1 and 2

$$\frac{P_1}{\rho g} + \frac{u_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{u_2^2}{2g} + z_2$$

$$\frac{P_1 - P_2}{\rho g} = z_2 + \frac{u_2^2 - u_1^2}{2g}$$

$$u_1 = 0.1488 / (\pi/4 \cdot 0.3^2) = 2.1 \text{ m/s}, u_2 = 0.1488 / (\pi/4 \cdot 0.15^2) = 8.42 \text{ m/s}$$







$$\Rightarrow P_1 - P_2 = 900 (9.81) [0.3 + (8.42^2 - 2.1^2)/2(9.81)] \\ = 32.5675 \text{ kPa}$$

$$\text{but } P_1 - P_2 = 0.25 (13600 - 900)(9.81) = 31.1467 \text{ kPa}$$

$$\% \text{ error} = 4.36 \%$$

### Problem (8)

A vacuum distillation plant operating at 7 kPa pressure at top has a boil-up rate of 0.125 kg/s of xylene. Calculate the pressure drop along a 150 mm bore vapor pipe used to connect the column to the condenser. And also calculate the maximum flow rate if  $L = 6 \text{ m}$ ,  $e = 0.0003 \text{ m}$ ,  $M_{wt} = 106 \text{ kg/kmol}$ ,  $T = 338 \text{ K}$ ,  $\mu = 0.01 \text{ mPa.s}$ .

### Solution

$$G^2 \ln\left(\frac{P_1}{P_2}\right) + \frac{(P_2^2 - P_1^2)}{2P_1 v_1} + 4\phi \frac{L}{d} G^2 = 0$$

$$G = 0.125 / [\pi/4 (0.15)^2] = 7.074 \text{ kg/m}^2 \cdot \text{s}$$

$$P_1 = 7 \text{ kPa}, \quad P_2 = \text{Pressure at condenser}$$

$$P_1 v_1 = \frac{RT}{M_{wt}} = \frac{8314 (\text{Pa} \cdot \text{m}^3 / \text{kmol} \cdot \text{K}) 338 \text{ K}}{106 \text{ kg/kmol}} \\ = 26510.68 \quad (\text{J/kg} \equiv \text{m}^2 / \text{s}^2)$$

$$\text{Re} = G d / \mu = 7.074(0.15)/0.01 \times 10^{-3} = 1.06 \times 10^5$$

$$, e/d = 0.002 \Rightarrow \Phi = 0.003 \text{ (Figure 3.7)}$$

$$\Rightarrow \ln\left(\frac{7 \times 10^3}{P_2}\right) + 3.769 \times 10^{-7} [P_2^2 - (7 \times 10^3)^2] + 4(0.003) \frac{6}{0.15} = 0$$

$$\Rightarrow P_2^2 = (7 \times 10^3)^2 - \frac{\ln(7 \times 10^3 / P_2) + 0.48}{3.769 \times 10^{-7}} \Rightarrow P_2 = \sqrt{(7 \times 10^3)^2 - \frac{\ln(7 \times 10^3 / P_2) + 0.48}{3.769 \times 10^{-7}}}$$

Solution by trial and error

|                  |                      |                      |                      |                      |
|------------------|----------------------|----------------------|----------------------|----------------------|
| $P_2$ Assumed    | $5 \times 10^3$      | $6.8435 \times 10^3$ | $6.904 \times 10^3$  | $6.9057 \times 10^3$ |
| $P_2$ Calculated | $6.8435 \times 10^3$ | $6.904 \times 10^3$  | $6.9057 \times 10^3$ | $6.9058 \times 10^3$ |

$$\Rightarrow P_2 = 6.9058 \times 10^3 \text{ Pa}$$

$$-\Delta P = P_1 - P_2 = (7 - 6.9058) \times 10^3 = 94.2 \text{ Pa}$$

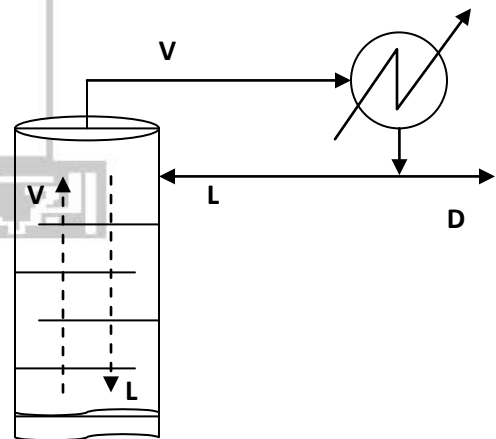
$$[(P_1 - P_2) / P_1] \% = 0.665 \% \quad \text{we can neglect the K.E. term in this problem}$$

For maximum flow rate calculations

$$\dot{m}_{\max} = A P_w \sqrt{1/P_1 v_1} \Rightarrow G_{\max} = P_w \sqrt{1/P_1 v_1}$$

$$\text{To estimate } P_w \quad \ln\left(\frac{P_1}{P_w}\right)^2 + 1 - \left(\frac{P_1}{P_w}\right)^2 + 8\phi \frac{L}{d} = 0$$

$$\text{Let } X \equiv (P_1/P_w)^2$$





$$\Rightarrow \ln(X) + 1 - X + 8 \Phi L/d = 0 \Rightarrow X = 1.96 + \ln(X)$$

Solution by trial and error

|              |      |      |      |       |       |       |       |
|--------------|------|------|------|-------|-------|-------|-------|
| X Assumed    | 1.2  | 2.14 | 2.72 | 2.96  | 3.074 | 3.086 | 3.087 |
| X Calculated | 2.14 | 2.72 | 2.96 | 3.074 | 3.086 | 3.087 | 3.087 |

$$\Rightarrow X = 3.087 = (P_1/P_w)^2 \Rightarrow P_w = P_1/(3.087)^{0.5} = 3984 \text{ Pa}$$

$\therefore$  This system did not reached maximum velocity (**H.W. explain**)

$$\Rightarrow G_{\max} = 3984 / (26510.68)^{0.5} = 24.47 \text{ kg/m}^2 \cdot \text{s}$$

### Problem (9)

Calculate the theoretical power in Watt for a 0.1 m diameter, 6-blade flat blade turbine agitator running at 16 rev/s in a tank system without baffles and conforming to the standard tank configuration. The liquid in the tank has a dynamic viscosity of 0.08 Pa.s, and a liquid density of 900 kg/m<sup>3</sup>.

#### Solution

$$(Re)_m = \rho N D_A^2 / \mu = (900) (16) (0.1)^2 / (0.08) = 1,800$$

From Power curve Figure (2)  $\Phi = 2.2$

$$\text{The theoretical power for mixing } P_A = \Phi [(Fr)_m]^y \rho N^3 D_A^5$$

$$y = \frac{\alpha - \log(Re)_m}{\beta} \Rightarrow y = \frac{1 - \log(1800)}{40} = -0.05638$$

$$(Fr)_m = N^2 D_A / g = (16)^2 (0.1) / 9.81 = 2.61$$

$$[(Fr)_m]^y = [2.61]^{-0.05638} = 0.9479$$

$$\Rightarrow P_A = 2.2 (0.9479) (900) (16)^3 (0.1)^5 = 76.88 \text{ W}$$

### Problem (10)

In contact sulphuric acid plant the secondary converter is a tray type converter 2.3 m I.D. with the catalyst arranged in three layers, each 0.45 m thickness. The catalyst is in form of cylindrical pellets 9.5 mm I.D. and 9.5 mm long. The void fraction is 0.35. The gas enters the converter at 675 K and leaves at 720 K. Its inlet and outlet compositions are:-

| Gas       | SO <sub>3</sub> | SO <sub>2</sub> | O <sub>2</sub> | N <sub>2</sub> |
|-----------|-----------------|-----------------|----------------|----------------|
| mol % In  | 6.6             | 1.7             | 10             | 81.7           |
| mol % Out | 8.2             | 0.2             | 9.3            | 82.3           |

The gas flow rate is 0.68 kg/m<sup>2</sup>.s. Calculate the pressure drop through the converter. Taken that the dynamic viscosity = 0.032 mPa.s.

#### Solution





$$\Phi' = J'_f = \frac{(-\Delta P)}{L} \frac{e^3}{S(1-e)\rho u^2} = \frac{5}{\text{Re}'} + \frac{0.4}{\text{Re}'^{0.1}} \text{ and } \text{Re}' = \frac{\rho u}{\mu_s(1-e)}$$

$$S = \frac{A_p}{V_p} = \frac{\text{Surface area of particle}}{\text{Volume of particle}} = \frac{2(\pi/4 d_p^2) + (\pi d_p L_p)}{\pi/4 d_p^2 L_p} = \frac{\pi/2 d_p^2 + \pi d_p^2}{\pi/4 d_p^3} = \frac{6}{d_p}$$

$$\Rightarrow \text{Re}' = \frac{\rho u d_p}{6(1-e)\mu} = \frac{G d_p}{6(1-e)\mu}$$

$$\rho_{\text{gas}} = \frac{(Mwt)_{\text{avg}} P}{RT_{\text{avg}}}, \quad T_{\text{avg}} = \frac{T_{\text{in}} + T_{\text{out}}}{2}, \quad (Mwt)_{\text{avg}} = \frac{Mwt_{\text{in}} + Mwt_{\text{out}}}{2}, \quad (Mwt) = \sum_{i=1}^n x_i Mwt_i$$

$$(Mwt)_{\text{in}} = 0.066(80) + 0.017(64) + 0.1(32) + 0.817(28) = 32.44 \text{ kg/kmol}$$

$$(Mwt)_{\text{out}} = 0.082(80) + 0.002(64) + 0.093(32) + 0.823(28) = 32.71 \text{ kg/kmol}$$

$$(Mwt)_{\text{avg}} = 32.58 \text{ kg/kmol}; \quad T_{\text{avg}} = 697.5 \text{ K}$$

$$\rho_{\text{gas}} = \frac{32.58(1.01325 \times 10^5)}{8314(697.5)} = 0.569 \text{ kg/m}^3$$

$$\Rightarrow \text{Re}' = \frac{G d_p}{6(1-e)\mu} = \frac{0.68(9.5 \times 10^{-3})}{6(1-0.35)0.032 \times 10^{-3}} = 51.76 \quad \Rightarrow \Phi' = \frac{5}{51.76} + \frac{0.4}{(51.76)^{0.1}} = 0.366$$

$$\Rightarrow -\Delta P = L \Phi' \frac{S(1-e)\rho u^2}{e^3} = \frac{\Phi' L 6(1-e) G^2}{d_p e^3 \rho} = \frac{0.366(3 \times 0.45) 6(1-0.35)(0.68)^2}{9.5 \times 10^{-3} (0.35)^3 0.569} = 3844.65 \text{ Pa}$$

### Problem (11)

What will be the settling velocity of a spherical steel particle, 0.4 mm diameter, in an oil of sp.gr 0.82 and viscosity 10 mPa.s? The sp.gr. of steel is 7.87.

### Solution

For a sphere

$$C_D (\text{Re})_p^2 = \frac{4}{3} Ga, \quad Ga = \frac{\rho(\rho_p - \rho) g d_p^3}{\mu^2} = 36.29$$

$$\Rightarrow C_D (\text{Re})_p^2 = 48.34 \quad \Rightarrow \frac{C_D}{2} (\text{Re})_p^2 = 24.2$$

From Figure at  $\frac{C_D}{2} (\text{Re})_p^2 = 24.2$ ,  $(\text{Re})_p = 1.667$

$$\Rightarrow u_p = \frac{(\text{Re})_p \mu}{\rho d_p} = \frac{1.667(10 \times 10^{-3})}{820(0.0004)} = 0.051 \text{ m/s}$$



### Problem (12)

A mixture of gas and liquid flows through a tube of ID 0.02m at a steady total flow rate of 0.2 kg/s. Calculate the pressure gradient in the pipe using the Lockhart-Martinelli correlation. Given that  $e = 0.0015$  mm,  $\mu_G = 0.01$  and  $\mu_L = 2$  mPa.s,  $\rho_G = 60$  and  $\rho_L = 1000$  kg/m<sup>3</sup>, and the quality  $w = 0.149$ .

#### Solution

$$G = 0.2 / (\pi/4 \times 0.02^2) = 636.62 \text{ kg/m}^2\text{s}$$

$$Re_L = [636.62 (1-0.149) \times 0.02] / 2 \times 10^{-3} = 5.417 \times 10^3 \text{ turbulent}$$

$$Re_G = [636.62 (0.149) \times 0.02] / 0.01 \times 10^{-3} = 1.897 \times 10^5 \text{ turbulent}$$

$$X_{tt} = \left( \frac{1-w}{w} \right)^{0.9} \left( \frac{\mu_L}{\mu_G} \right)^{0.1} \left( \frac{\rho_G}{\rho_L} \right)^{0.5} = 1.996$$

$$\Phi_L^2 = 1 + \frac{C}{X} + \frac{1}{X^2}, \text{ for } C = 20 \Rightarrow \Phi_L^2 = 11.27$$

$$Re_L = 5.417 \times 10^3, e/d = 0.000075 \Rightarrow f = 0.009$$

$$\left( \frac{dP}{d\ell} \right)_L = 4f \frac{1}{d} \frac{G^2 (1-w)^2}{2\rho_L} = 4(0.009) \frac{1}{0.02} \frac{636.62^2 (1-0.149)^2}{2(1000)} = 264.157 \text{ Pa/m}$$

$$\left( \frac{dP}{d\ell} \right)_T = \Phi_L^2 \left( \frac{dP}{d\ell} \right)_L = 11.27(264.157) = 2977.05 \text{ Pa/m}$$

### Problem (12)

A cylindrical tank 0.9 m ID and 2 m high open at top is filled with water to a depth of 1.5 m. it is rotated about its vertical axis at N rpm. Determine the value of N which will raise water level even with the brim.

#### Solution

The water level even with brim  $P = P_o, z = 2$  m

The rise of liquid at wall  $y_r = 2 - 1.5 = 0.5$  m

The fall of liquid at wall  $y_f = y_r = 0.5$  m

$z_o = H - y_f = 1$  m,  $z = 2$  m at  $r = R$

$$z = z_o + \frac{\omega^2}{2g} r^2$$

$$\Rightarrow 2 = 1 + \frac{\omega^2}{2(9.81)} 0.45^2$$

$$\omega = 9.843 \text{ rad/s}$$

$$\omega = \frac{2\pi N}{60} \Rightarrow N = \frac{60(9.843)}{2\pi} \approx 94 \text{ rpm}$$

