Environmental Engineering

1. A spherical particle having a diameter of 0.0093 in. and a specific gravity of 1.85 is placed on a horizontal screen. Air is blown through the screen vertically at a temperature of 20°c and a pressure of 1 atm. Calculate the following

1. The terminal velocity required to just lift the particle. Using k value as a guide for the region
2. The particle Reynolds number at this conditions
3. The drag coefficient C_D
4. The drag force in both engineering and cgs. units

The viscosity of air at 20°c is 1.23×10^{-5} lb/ft.s

Solution

At 20°c and 1atm

$T = 20°c$

$= 1.8(20) + 32 = 68\ °F$

$= 68 + 460 = 528\ R$

$R = 0.73 \dfrac{atm.ft^3}{lbmol.R}$

$$\mu_g = 1.23 \times 10^{-5} \quad lb/ft \cdot s$$

$$\rho_g = \frac{P \; Mwt}{R \; T}$$

$$= \frac{(1)\;(29)}{(0.73)\;(528)} = 0.0752 \; lb/ft^3$$

1. K is given by

$$K = d_p \left[\frac{g \; \rho_p \; (\rho_p - \rho_g)}{\mu_g^2} \right]^{1/3}$$

$$d_p = 0.0093 \; in \times \frac{ft}{(12)in} = 77.5 \times 10^{-5} \; ft$$

$$\rho_p = spgr \times \rho_{H_2O}$$

$$= (1.85) \times (62.4) = 115.44 \quad lb/ft^3$$

$$K = (77.5 \times 10^{-5}) \left[\frac{(32.2)\,(0.0752)\,(115.44 - 0.0752)}{(1.23 \times 10^{-5})^2} \right]^{1/3}$$

$$= 9.51$$

$\because 3.3 < K < 43.6$. The intermedial region

Then the intermedial region is used

$$\therefore V_t = 0.153 \; \frac{g^{0.71} \; d_p^{1.14} \; (\rho_p - \rho_g)^{0.71}}{\mu_g^{0.43} \; \rho_g^{0.29}}$$

$$\therefore V_t = 0.153 \frac{(32.2)^{0.71} (77.5 \times 10^{-5})^{1.14} (115.44 - 0.0752)^{0.71}}{(1.23 \times 10^{-5})^{0.43} (0.0752)^{0.29}}$$

$$= 4.08 \text{ ft/s}$$

2. The Reynolds number is

$$Re = \frac{\rho_g V_t d_p}{\mu_g}$$

$$= \frac{(0.0752)(4.08)(77.5 \times 10^{-5})}{(1.23 \times 10^{-5})}$$

$$= 19.33$$

4. The drag coefficient C_D is

$$C_D = \frac{18.5}{Re^{0.6}}$$

$$= \frac{18.5}{(19.33)^{0.6}} = 3.13$$

5. The drag force is calculated from the following equation

$$F_D = \frac{1}{2} C_D \rho_g V_t^2 A_p$$

$$A_p = \frac{\pi}{4} d_p^2 = \frac{\pi}{4} (77.5 \times 10^{-5})^2$$

$$= 4.72 \times 10^{-7} \text{ ft}^2$$

$$\therefore F_D = \frac{1}{2}(3.13)(0.0752)(4.08)^2(4.72 \times 10^{-7})$$

$$= 9.23 \times 10^{-7} \text{ lb}$$

$$\text{lb}_f = 32.2 \text{ lb}$$

$$\therefore F_D = 9.23 \text{ lb} \frac{\text{lb}_f}{32.2 \text{ lb}} = 2.866 \times 10^{-8} \text{ lb}_f$$

$$\text{lb}_f = 4.448 \text{ N}$$
$$= 4.448 \times 10^5 \text{ dyne} \rightarrow \left(\frac{g \text{ cm}}{s^2}\right)$$

$$F_D = 2.866 \times 10^{-8} \text{ lb}_f \times \frac{4.448 \times 10^5 \text{ dyne}}{\text{lb}_f}$$

$$= 0.0127 \text{ dyn}$$

2- A salesman from Bogus, Inc. suggest a gravity settler for a charcoal dust — contaminated air stream that you must preclean. Your supervisor has provided the particle size distribution shown below. The inlet loading is 20 gr/ft^3 and the required outlet loading is 5 gr/ft^3. Will the settler that the salesman has suggested do the job?

size Rang (µm) d_p	Weight percent w_i
0 - 10	5
10 - 20	11
20 - 40	10
40 - 60	9
60 - 90	22
90 - 125	23
125 - 150	10
150 +	10

Use the critical diameter ($d_p^* = 80$ µm) to calculate the size-efficiency data from the equation

$$E = k\, d_p^2$$

Solution

First calculate k

using $E = 100$ & $d_p^* = 80$

$$\therefore\ k = \frac{E}{d_p^2} = \frac{100}{(80)^2}$$

$$= 0.01563$$

$$\therefore \boxed{E = 0.01563 \; d_p^2}$$

Thus for $d_p = 5\ \mu m$ (average diameter for the first size range)

$$E = 0.01563\ (5)^2 = 0.39\ \%$$

The following table may be generated using the above approach:

Size Range μm	d_p average μm	W_i %	E_i %	$W_i E_i$ (%)
0 – 10	5	5	0.39	—
10 – 20	15	11	3.5	0.4
20 – 40	30	10	14	1.4
40 – 60	50	9	39	3.5
60 – 90	75	22	88	19.4
90 – 125	107.5	23	100	23
125 – 150	137.5	10	100	10
150 +	150 +	10	100	10
				$\Sigma = 67.7$

Therefore $E = 67.7\ \%$

The required efficiency E_{req} is

$$E_{req} = (\text{Inlet load} - \text{outlet load}) / \text{Inlet load}$$
$$= (I - O) / I$$
$$= (20 - 5) / 20 = 75\ \%$$

Since $67.7 < 75\ \%$ the gravity settler will not do the job.

3. A particulate sample in an air stream has the following weight percent distribution:

Particle size μm	1	10	50	100	200
W.%	10	20	40	20	10

A cyclone separator is employed on the basis of the following data:
Inlet width = 12 in; effective time = 5; Inlet gas velocity = 20 ft/s; Particle density = 1.6 g/cm³, gas density = 0.074 lb/ft³; gas viscosity = 0.045 lb/ft-hr.

1- Estimate the over-all collection efficiency of the cyclone separators.

2- Estimate the overall collection efficiency of the cyclone separator when

a- The number of effective turns in the cyclone be 4 instead of 5.

b- The velocity of the gas flow is doubled.

c- The gas viscosity is 0.09 lb/ft-hr.

consider that all other conditions remain the same with the exception of the one specified

Solution

1- Estimate the overall collection efficiency of the cyclone separator

$$b \text{ (inlet width)} = 12 \text{ in} * \frac{\text{ft}}{12 \text{ in}} = 1 \text{ ft}$$

$N_e = 5$ (effective time)

inlet velocity $V_g = 20 \text{ ft/s}$

particle density $\rho_p = 1.6 \frac{g}{cm^3} * \frac{30.48 \text{ cm}^3}{ft^3} * \frac{lb}{453.6 g}$

$$= 99.88 \text{ lb/ft}^3$$

gas density $\rho_g = 0.074 \text{ lb/ft}^3$

gas viscosity $\mu_g = 0.045 \frac{lb}{ft\text{-}hr} * \frac{hr}{3600 s}$

$$= 1.25 \times 10^{-5} \text{ lb/ft-s}$$

Then calculat the cut diameter d_{p50}

$$d_{p50} = \left[\frac{9 \mu_g b}{2\pi N_e V_g (\rho_p - \rho_g)} \right]^{1/2}$$

$$= \left[\frac{9 (1.25 \times 10^{-5}) (1)}{2\pi (5)(20)(99.88 - 0.074)} \right]^{1/2}$$

$$= 42.35 \times 10^{-6} \text{ ft} * \frac{0.3048 \text{ m}}{ft}$$

$$= 12.9 \times 10^{-6} \text{ m}$$

$$= 12.9 \ \mu m$$

The following table is generated using Fig. 1
Eeach particle size in the table devided by the cut diameter

The first $d_p = 1$

$$\therefore \frac{d_p}{d_{p50}} = \frac{1}{12.9} = 0.08$$

and the same with the other particle d_p

d_p (μm)	w_i	d_p/d_{p50}	E_i (%)	$w_i E_i$ (%)
1	10	0.08	0	0
10	20	0.8	39	7.8
50	40	3.9	94	37.6
100	20	7.8	99	19.8
200	10	15.5	100	10
			Σ	75.2

Calculate the overall collection efficiency E

$$E = \Sigma w_i E_i = 0 + 7.8 + 37.6 + 19.8 + 10$$
$$= 75.2\ \% = 0.752$$

2- Estimate the overall collection efficiency of the cyclone separator when

a- $N_e = 4$ instead of 5
(the other condition remain the same)

$$\therefore d_{p50} = \left[\frac{9\,(1.25\times10^{-5})(1)}{2\pi\,(4)(20)\,(99.88-0.074)} \right]^{1/2}$$

$$= 47.35\times10^{-6}\ ft \times \frac{0.3048\ m}{ft}$$

$$= 14.4\times10^{-6}\ m$$

$$= 14.4\ \mu m$$

The following table is generated using Fig.1

$$\frac{d_p}{d_{p50}} = \frac{1}{14.4} = 0.07$$

d_p (μm)	W_i	d_p/d_{p50}	E (%)	$W_i E_i$ (%)
1	10	0.07	0	0
10	20	0.70	34	6.8
50	40	3.50	98	37.2
100	20	7.0	99	19.8
200	10	14.0	100	10
			Σ	73.8

The overall collection efficiency E is caculated as

$$E = \sum W_i E_i = 0 + 6.8 + 37.2 + 19.8 + 10$$
$$= 73.8\%$$

b. The Velocity of the gas flow is doubled (the other conditions remain the same)

$$V_g = 2V_{g1} = 2(20) = 40 \text{ ft/s}$$

Calculat the cut diameter

$$d_{p50} = \left[\frac{9\,(1.25 \times 10^{-5})\,(1)}{2\pi(5)\,(40)\,(99.88 - 0.074)} \right]^{1/2}$$

$$= 29.95 \times 10^{-6} \text{ ft}$$
$$= 9.13 \times 10^{-6} \text{ m}$$
$$= 9.13 \text{ μm}$$

The following table is generated using Fig. 1

$$d_p/d_{p50} = 1/9.13 =$$

d_p (μm)	W_i	d_p/d_{p50}	E_i (%)	W_iE_i (%)
1	10	0.1	0	0
10	20	1.1	50	10
50	40	5.5	96	38.4
100	20	11	100	20
200	10	21.9	100	10
			Σ	78.4

The overall collection efficiency E

$$E = \Sigma W_iE_i = 0 + 10 + 38.4 + 20 + 10$$
$$= 78.4\% = 0.784$$

c. The gas viscosity is 0.09 lb/ft-hr
(The other condition remain the same)

$$\mu_g = 0.09 \frac{lb}{ft \cdot hr} \times \frac{hr}{3600 s} = 2.5 \times 10^{-5} \frac{lb}{ft \cdot s}$$

$$d_{p50} = \left[\frac{9\,(2.5 \times 10^{-5})\,(1)}{2\,\pi\,(5)\,(20)\,(99.88 - 0.074)} \right]^{1/2}$$

$$= 59.89 \times 10^{-6} \text{ ft}$$
$$= 18.3 \times 10^{-6} \text{ m}$$

$$= 18.3 \ \mu m$$

$$d_p/d_{p50} = 1 / 18.3 = 0.05$$

The following table is generated using Fig. 1

d_p (μm)	w_i	d_p/d_{p50}	E_i (%)	$w_i E_i$ (%)
1	10	0.05	0	0
10	20	0.5	20	4
50	40	2.7	88	35.2
100	20	5.5	98	19.6
200	10	10.9	100	10
			Σ	68.8

The overall collection efficiency E

$$E = \sum w_i E_i = 0 + 4 + 35.2 + 19.6 + 10$$

$$= 68.8\% = 0.688$$

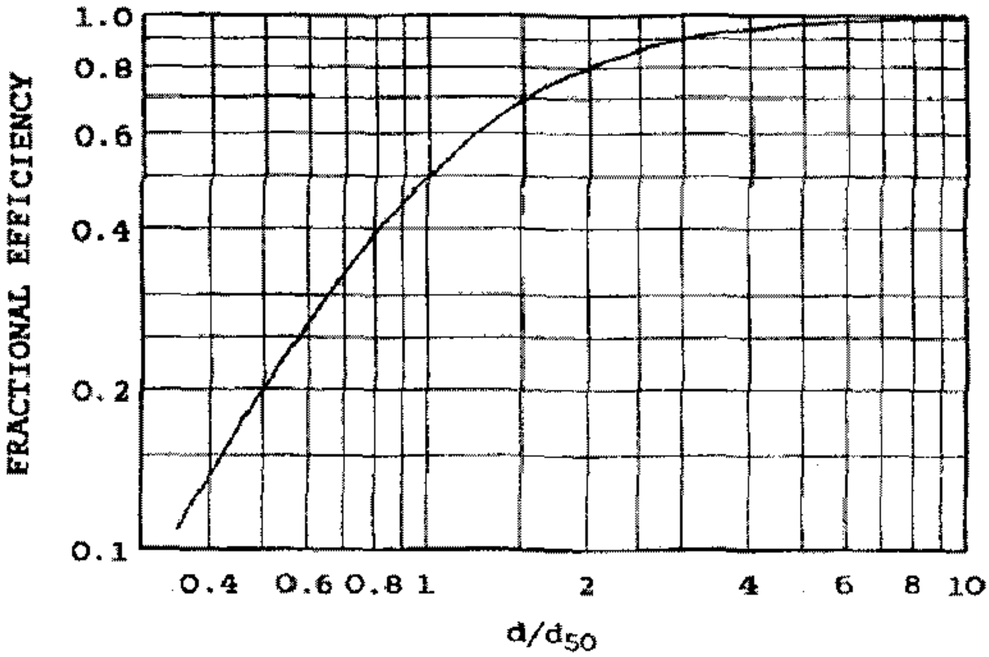

Fig.1 Cyclone efficiency versus particle-size ratio

4- A conventional cyclone with diameter of 1.0 m is to be used to separate particulate matter of density 1500 kg/m³ from standard air at a flow rate of 2 m³/s. A analysis of the particles yielded the following size distribution

Size (μm)	3	6	10	20	40	80	140	200
Weigh Fraction Less than Stated Size	0.03	0.08	0.15	0.33	0.60	0.85	0.96	0.99

The air viscosity is 1.81×10^{-5} kg/m-s

Using $N_e = 6$ estimate the overall separation efficiency.

For conventional Cyclone the inlet width $b = 0.25$ m and the inlet Length $a = 0.5$ m, where is the cyclone diameter.

Solution

1. It is required to calculate the cut diameter dp_{50}

$$dp_{50} = \left[\frac{9 \mu_g b}{2 \pi N_e V_g (\rho_p - \rho_g)} \right]^{1/2}$$

The air flow rate $Q = 2$ m³/s

The inlet gas velocity $= V_g$ m/s

$$V_g = \frac{Q}{ab}$$

$$= \frac{2}{(0.5)(0.25)} = 16 \text{ m/s}$$

$N_e = 6$

$\rho_p = 1500$ kg/m³

$\mu_g = 1.81 \times 10^{-5}$ kg/m·s

$b = 0.25$

$$d_{p50} = \left[\frac{9\,(1.81 \times 10^{-5})\,(0.25)}{2\pi\,(6)\,(16)\,(1500)}\right]^{1/2}$$

$$= 6.7 \times 10^{-6} \text{ m} = 6.7\,\mu m$$

Using d_{p50} and Fig., the following table is generated

in this table the average diameter of each particle devided by the cut dyameter

$$\frac{d_p}{d_{p50}} = \frac{1.5}{6.7} = 0.22$$

size rang μm	W_i in rang	dp (μm) averg	d_p/d_{p50}	E_i(%)	$W_i E_i$(%)
0 – 3	0.03	1.5	0.22	0	0
3 – 6	0.05	4.5	0.7	33	1.65
6 – 10	0.07	8	1.2	58	4.06
10 – 20	0.18	15	2.2	83	14.94
20 – 40	0.27	30	4.5	95	25.65
40 – 80	0.25	60	9.0	99.5	24.88
80 – 140	0.11	110	15.4	100	11
140 – 200	0.03	170	25.4	100	3
> 200	0.01	200	29.9	100	1
	Σ 1				Σ 86.18

$\therefore E = \Sigma W_i E_i = 86.18\% = 0.8618$

5. A large stream has a rate of reaeration $K_2 = 0.55$ and a rate of deoxygenation $K_1 = 0.23$ per day. The dissolved oxygen deficit of the mixture of stream water and wastewater at the point of refrence Do is 4.0 mg/l and the ultimate BOD of the waste Lu, is 75 mg/l. Calculate

1- the dissolved oxygen deficit D at a point 1, 3, 5 day distance from the point of reference
2- the critical deficit D_c and the critical time t_c
3- The location of the critical oxygen deficit, X_c, if the velocity of the stream is 0.25 m/s

Solution

1. The dissolved oxygen deficit D at a point 1, 3, 5 day

The dissolved oxygen deficit time t can be obtained using

$$D = \frac{k_1 L_u}{k_2 - k_1}\left(e^{-k_1 t} - e^{-k_2 t}\right) + D_o e^{-k_2 t}$$

$k_1 = 0.23 \ day^{-1}$
$k_2 = 0.55 \ day^{-1}$
$L_u = 75 \ mg/l$
$D_o = 4 \ mg/l$

$$\therefore D = \frac{(0.23)(75)}{0.55 - 0.2}\left(e^{-0.23t} - e^{-0.55t}\right) + 4e^{-0.55t}$$

$t = 1$

$$D = \frac{(0.23)(75)}{0.55-0.23}\left(e^{-0.23(1)} - e^{-0.55(1)}\right) + 4e^{-0.55(1)}$$

$= 14 \ mg/l$

$t = 3$

$$D = \frac{(0.23)(75)}{0.55-0.23}\left(e^{-0.23(3)} - e^{-0.55(3)}\right) + 4e^{-0.55(3)}$$

$= 17.5 \ mg/l$

$t = 5$

$$D = \frac{(0.23)(75)}{0.55-0.23}\left(e^{-0.23(5)} - e^{-0.55(5)}\right) + 4e^{-0.55(5)}$$

$= 13.7 \ mg/l$

2- The critical deficit D_c and the critical time t_c

$$t_c = \frac{1}{(k_2-k_1)} \ln\left[\frac{k_2}{k_1}\left(1 - D_o \frac{(k_2-k_1)}{k_1 L_u}\right)\right]$$

$$= \frac{1}{(0.55-0.23)} \ln\left[\frac{0.55}{0.23}\left(1 - 4\frac{(0.55-0.23)}{(0.23)(75)}\right)\right] = 2.48 \text{ days}$$

$$D_c = \frac{k_1}{k_2} L_u \exp(-k_1 t_c)$$

$$= \frac{0.23}{0.55} (75) \exp(-(0.23)(2.48))$$

$$= 17.7 \text{ mg/l}$$

3- The location of the critical oxygen deficit X_c, if the velocity of the stream is 2.5 km/hr

$$V = 2.5 \frac{\text{km}}{\text{hr}} \times \frac{24 \text{ hr}}{\text{day}} = 60 \text{ km/day}$$

$$X_c = t_c V$$

$$= (2.48)(60) = 148.8 \text{ Km}$$

6. What propotion of total BOD of a wastewater would be utilised in 5 days with k_1' values of 0.05, 0.1 0.15, 0.20 and 0.25?

Solution

The 5 day BOD is given by

$$Y_5 = L_u (1 - 10^{-5k_1'})$$

Y_5 / L_u is the proportional of Total BOD (or the ratioo of the 5 day BOD to the ultimate BOD)

$$\therefore \frac{Y_5}{L_u} = 1 - 10^{-5k_1'}$$

Then the (Y_5/L_u) for each k_1' is given

$k_1' = 0.15$

$$\therefore \frac{Y_5}{L_u} = 1 - 10^{-5(0.15)} = 0.438$$

The same calculation for each value of k_1' The following table obtained

K_i	Y_5/L_u
0.05	0.438
0.10	0.684
0.15	0.822
0.2	0.9
0.25	0.944