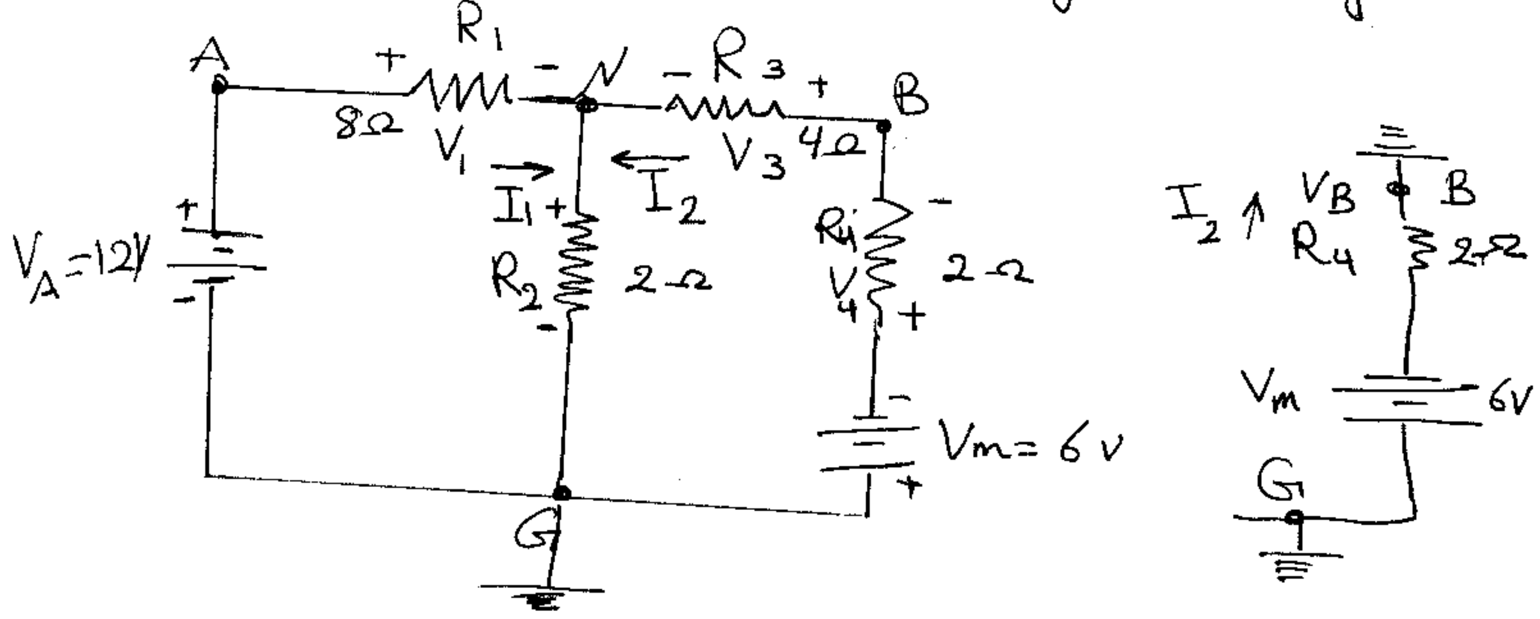
Kirchhoff's Laws

Ex

1. Find the voltage V_2 across R_2 by the method of node-voltage analysis in fig. below



ⓐ Circuit schematic ⓑ Closed path GBG

fig. Finding V_2 by the node-voltage method

step 1. Assume direction of currents shown show nodes A, B, N, G.

step 2. Apply $\Sigma I = 0$ at principal node N.

$$I_3 = I_1 + I_2 \quad (1)$$

$$I_3 = \frac{V_2}{R_2} = \frac{V_N}{2} \quad (1a)$$

$$I_1 = \frac{V_1}{R_1} = \frac{V_A - V_N}{R_1} = \frac{12 - V_N}{8} \quad (1b)$$

$$I_2 = \frac{V_3}{R_3} = \frac{V_B - V_N}{R_3} = \frac{V_B - V_N}{4} \quad (1c)$$

we are unable to determine V_B by inspection in Equation (Ic) because voltage drop V_4 is not given (fig. a). So we use KVL to find V_B by tracing the complete circuit from G to B in the direction of I_2 (fig. b). GBG is a complete path because V_B is the voltage at B with respect to ground.

$$-6-2I_2-V_B=0$$

$$V_B=-6-2I_2$$

Substitute experssion for V_B into Eq. (1c)

$$I_2=\frac{-6-2I_2-V_N}{4}$$

from which we obtain

$$I_2=\frac{-6-V_N}{6}$$

Substitute the three expressions for current into equation (1)

$$\frac{V_N}{2}=\frac{12-V_N}{8}+\frac{-6-V_N}{6} \quad \text{---} \quad (2)$$

Now equation (2) has one unknown, V_N.

step 3. find $V_2 (V_2=V_N)$. Multiply each member of equation (2) by 24

$$12 V_N = (36 - 3V_N) + (-24 - 4V_N)$$

$$19 V_N = 12$$

$$V_N = \frac{12}{19} = 0.632 \text{ V}$$

$$V_2 = V_N = 0.632 \text{ V} \quad \text{Ans.}$$

2. Write the mesh equations for the three-mesh circuit in fig. below. Do not Solve.

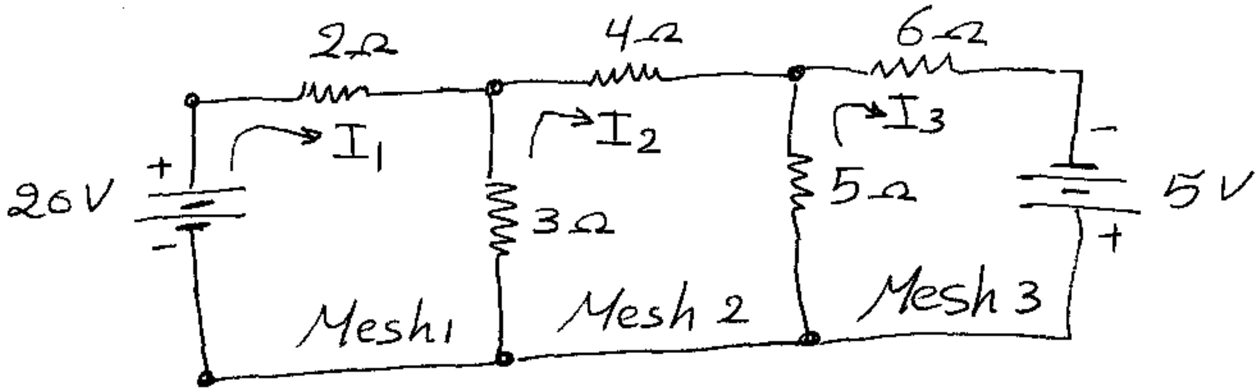


figure a three-mesh circuit

Show mesh current in clockwise direction.
Trace loops in assumed direction of current, using KVL, $\Sigma V = 0$

Mesh 1: $20 - 2I_1 - 3I_1 + 3I_2 = 0$ --- (1)

Mesh 2: $-4I_2 - 5I_2 + 5I_3 - 3I_2 + 3I_1 = 0$ -- (2)

Mesh 3: $-6I_3 + 5 - 5I_3 + 5I_2 = 0$ --- (3)

Combine and rearrange terms in each equation

Mesh 1: $20 = 5I_1 - 3I_2$ Ans.

Mesh 2: $0 = -3I_1 + 12I_2 - 5I_3$ Ans.

Mesh 3: $5 = -5I_2 + 11I_3$ Ans.

A set with any number of simultaneous equations for any number of meshes, can be solved by using determinants. Solution by determinants is not shown, here.

Waveforms and Time constant

Ex 1: A series circuit contains a resistance of 20 Ω and an inductance of 10 H connected across a voltage source of 110 V (a) what is the current after 1 sec is the circuit is closed? (b) What are V_R and V_C at this time?

Some time after the current reaches a steady value, the switch is opened. When the series circuit is open, the RL circuit opposes the decay of current toward the steady-state value of zero (c) what is the current 2 sec. after the circuit is opened? (d) What are V_R and V_L at this time?

Solution :-

(a) step 1. Write the formula for charging or rising current when the switch is closed.

$$i = \frac{V}{R}\left(1 - e^{-Rt/L}\right) \quad --- (1)$$

step 2. find the value of $e^{-Rt/L}$ for $t = 1$ sec

$$-\frac{Rt}{L} = \frac{-20(1)}{10} = -2 \Rightarrow e^{-2} = 0.135$$

step 3. Substitute values for $e^{-Rt/L}$, V and R in equation (1)

$$i = \frac{110}{20}(1 - 0.135) = 5.5(0.865) = 4.76\,A$$

(b) Write the formulas for v_R and v_L after the switch is closed and solve them when $t = 1sec$.

$$v_R = V\left(1 - e^{-Rt/L}\right)$$

$$= 110\,(0.865) = 95.2\ V$$

check

$$V = v_R + v_L$$

$$110 = 95.2 + 14.8 \Rightarrow$$

$$110 = 110\ V$$

(c) The steady-state value of current is.

$$I = \frac{V}{R} = \frac{110}{20} = 5.5\ A$$

Write the formula for discharging or decaying current when the switch is opened and substitute values with $t = 2Sec$.

$$i = \frac{V}{R}\, e^{-Rt/L}$$

$$-\frac{Rt}{L} = \frac{-20(2)}{10} = -4$$

$$i = 5.5\,(0.018) = 0.10\ A$$

(d) Write the formula for v_R and v_L after the switch is opened and then solve them when $t = 2Sec$.

$$v_R = V e^{-Rt/L} = 110\,(0.018) = 1.98\ V$$

$$v_L = -V e^{-Rt/L} = -110\,(0.018) = -1.98\ V$$

check

$$0 = v_R + v_L$$

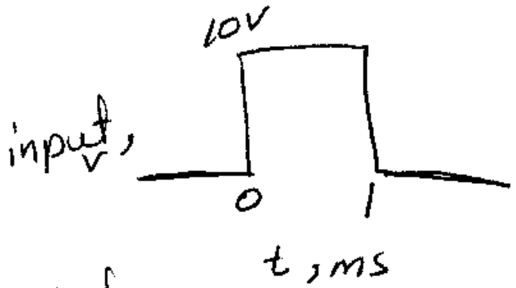
$$0 = 1.98 - 1.98$$

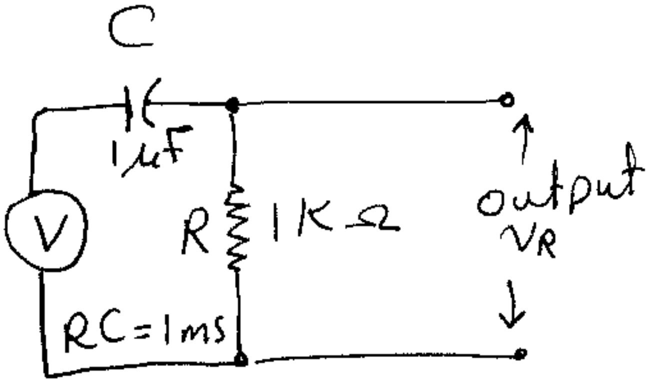
$$0 = 0\ V$$

Ex 2

Waveforms and Time Constant

An RC series circuit with a rectangular pulse input is shown in figure below. A rectangular pulse is a special case of a constant dc source because the voltage is a constant V when the pulse is on and a constant zero when the pulse is off. If a single pulse is applied to the circuit, find the voltage across the resistance at the time of 0.5, 1 and 2 ms. The pulse has a maximum voltage of 10V and lasts for 1ms





Solution:

Find the time constant of the circuit.

$$T = RC = (1\times10^{3})(1\times10^{-6}) = 10^{-3} = 1ms$$

Write the formula for v_R when the circuit is charging.

$$v_R = V e^{-t/RC} \quad \text{when } t = 0.5\,ms \Rightarrow$$

$$v_R = 10 e^{-0.5/1} = 10 e^{-0.5} = 10(0.607)$$

$$= 6.07\,V$$

When $t = 1 msec$

$$v_R = 10 e^{-1/1} = 10 e^{-1} = 10(0.368)$$
$$= 3.68\ V$$

At $t = 1ms$, the pulse is Turened off. At that instante The source voltage is zero, so.

$$v_R + v_C = 0$$
$$v_R = -v_C$$

Since the voltage across the capacitor v_C cannot change instantly, the voltage across the resistor becomes $-v_C$. Now at an instant befor $t = 1ms$,

$$v_R + v_C = 10$$
$$v_C = 10 - v_R = 10 - 3.68 = 6.32V$$

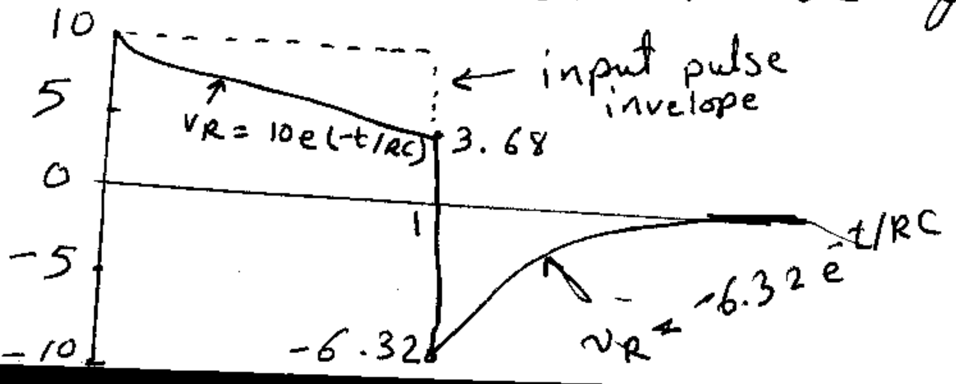
Therefore, at $t = 1ms$, $v_R = -v_C = -6.32V$ and then v_R decays to zero, so the formula for v_R at $t = 1ms$ is

$$v_R = -6.32\, e^{-t/RC}$$

Then in the next 1ms or when $t = 2ms$ measured from the origin,

$$v_R = -6.32\, e^{-1} = -6.32(0.368) = -2.33$$

The plot for v_R is shown in this figure



(8)

EX

A sinusoidal alternating current of frequency 25 HZ has a maximum value of 100A. How long will it take for the current to attain values of 20, 50, 100 A ?

Solution

$$i = I_{max} \sin(\omega t)$$
$$= I_{max} \sin(2\pi f t)$$

when $I_m = 100$ A
$f = 25$ HZ

∴ instantaneous of current

$$i = 100 \sin(2\pi \times 25 \times t)$$

(a) when $i = 20$

∴ $20 = 100 \sin(50\pi t)$

∴ $t = 0.001285$ sec

(b) $i = 50$ A

$50 = 100 \sin(50\pi t)$

$t = 0.0033$ s

(c) $i = 100$ A

∴ $100 = 100 \sin(50\pi t)$

$t = \frac{90}{50 \times 180} = 0.01$ sec

EX: Find the current that will flow through a coil of negligible resistance and inductance of 60 mH, when connected to 230 V, 50 Hz single phase supply.
What will be the current if the frequency is (a) decreased to 20 Hz
(b) increased to 60 Hz

Solution:

Inductance of the coil $L = 60\ mH$

$= 60 \times 10^{-3}\ H$

$f = 50\ Hz$

$\therefore$ inductive reactance, $X_L = 2\pi f L$

$= 2\pi \times 50 \times 60 \times 10^{-3}$

$= 18.86\ \Omega$

Current flow through the coil:

$$I = \frac{V}{X_L} = \frac{230}{18.86} = 12.2\ A$$

a) The frequency is decreased to 20 Hz

$\therefore X_L = 2\pi \times \times 20 \times 60 \times 10^{-3}$

$= 7.54\ \Omega$

Current flow through the coil $= \dfrac{230}{7.5413}$

$= 30.49\ A$

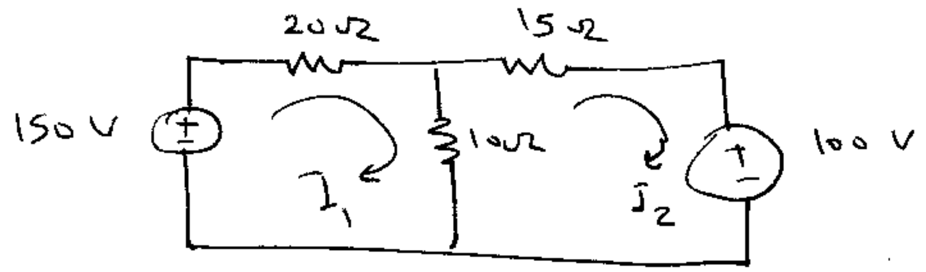
b) at 60 Hz

$X_L = 2\pi \times 60 \times 60 \times 10^{-3} = 22.63\ \Omega$

$I = 230/22.63 = 10.16\ A$

Mesh analysis

EX: solve for the current in each element of the Circuit shown in figure



mesh-1 :

$$20 i_1 + 10(i_1 - i_2) - 150 = 0 \quad \text{—— (1)}$$

Mesh 2 :

$$10(i_2 - i_1) + 15 i_2 + 100 = 0 \quad \text{—— (2)}$$

Rearranged eqn 1 and

$$30 i_1 - 10 i_2 = 150 \quad \text{—— (3)}$$

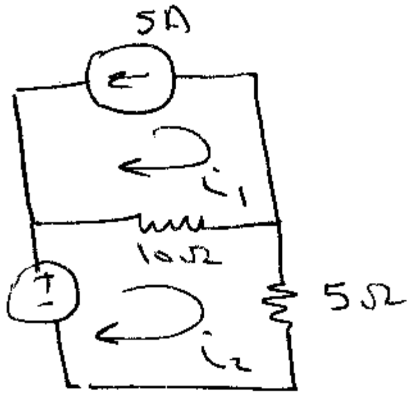
$$-10 i_1 + 25 i_2 = -100 \quad \text{—— (4)}$$

Slovi-g: we find that

$$i_1 = 4.231 \text{ A}$$

$$i_2 = -2.308 \text{ A}$$

EX : write the equations needed to solve for the mesh currents in figure



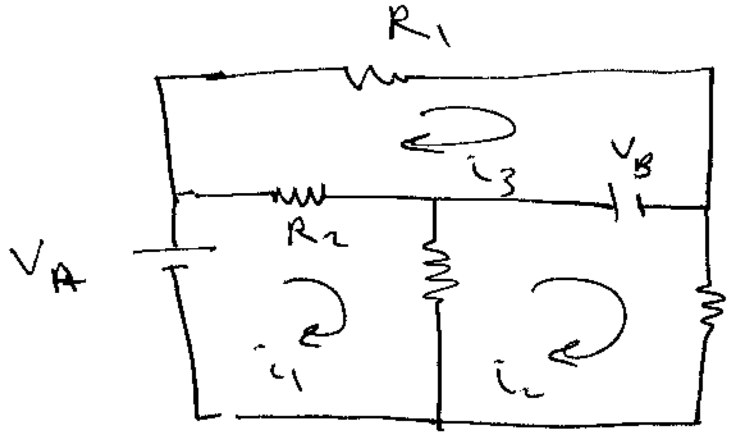
$i_1 = 5\ A$

$10(i_2 - i_1) + 5 i_2 - 100 = 0$

$10(i_2 - 5) + 5 i_2 - 100 = 0$

$i_2 = 10\ A$

EX : write the equation needed to solve for the mesh currents shown in figure



mesh-1

$$R_2(i_1 - i_3) + R_3(i_1 - i_2) - V_A = 0$$

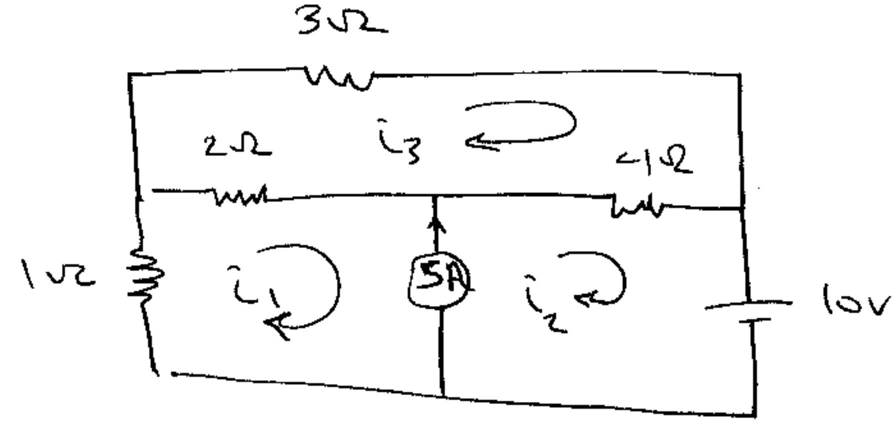
mesh-2

$$R_3(i_2 - i_1) + R_4 i_2 + V_B = 0$$

mesh-3

$$R_2(i_3 - i_1) + R_1 i_3 - V_B = 0$$

EX: Write the equation needed to solve for the mesh shown in Figure.



in mesh 1 and 2

$$i_1 + 2(i_1 - i_2) + 4(i_2 - i_1) + 10 = 0$$

for mesh - 3

$$3i_3 + 4(i_3 - i_2) + 2(i_3 - i_1) = 0$$

Current in the Current Source:

$$i_2 - i_1 = 5\ A$$

Ex A capacitor of 30 μF connected in series with a resistor of 500 Ω is suddenly connected across a 100 V dc supply. Find (a) time constant of the circuit (b) initial current (c) equation of voltage as a function of time (d) equation of current as a function of time (e) equation of charge as a function of time (f) charge on a capacitor after 0.05 sec (g) charging current ~~after~~ after 0.05 sec (h) value of current when time is equal to time constant (i) voltage across resistance after 0.05 sec.

Solution

a-) T = time constant $= RC$

$C = 30\ \mu F = 30 \times 10^{-6}\ F \qquad R = 500\ \Omega$

$\therefore T = 30 \times 10^{-6} \times 500 = 0.015\ S$

b-) $I = \frac{V}{R} = \frac{100}{500} = 0.2\ A$

c-) $v = V(1 - e^{-t/RC}) \qquad t = RC = 0.015$ sec, $V = 100\ V$

$\therefore v = 100(1 - e^{-t/0.015})$

d) $i = \frac{V}{R} e^{-t/RC}$

$i = \frac{100}{500} e^{-t/0.015} \qquad \therefore i = 0.2\, e^{-66.7t}$

e) $q = Q(1 - e^{-tRC})$ but $Q = CV = 30 \times 10^{-6} \times 100 = 0.003\ C$

$\therefore q = 0.003(1 - e^{-66.6t})$

f) charge on capacitor after 0.05 sec

$$q = 0.003\left(1 - e^{-66.67 \times 0.05}\right)$$

$$= 0.00289\ C$$

g) Charging current after 0.05 sec

$$i = 0.2\, e^{-66.67\, t}$$

$$\therefore i = 0.2\, e^{-66.67 \times 0.05}$$

$$= 9.95 \times 10^{-3}\ A$$

h) Value of current when time is equal to time constant

$$i = 0.2\, e^{-\frac{0.015}{0.015}} = 0.0735\ A$$

i) Voltage across resistor after 0.05 s

$$= I\ R$$

$$= 9.95 \times 10^{-3} \times 500 = 4.975\ V$$