

الجامعة التكنولوجية

قسم الهندسة الكيميائية

المرحلة الرابعة

الوحدات الصناعية

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2015-2014

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2014-2015**



Unit Operations

CHAPTER 1

Drying of Solids

1.1. Introduction.

Drying refers to the removal of water, or another solute "liquid", to reduce the content of residual liquid to an acceptably low value. The drying of materials is often the final operation in a manufacturing process, carried out immediately prior to packaging or dispatch. Drying is the operation often follows evaporation, filtration, or crystallization. In some cases, drying is an essential part of the manufacturing process, as for instance in paper making or in the seasoning of timber.

Drying is carried out for one or more of the following reasons:

- (a) To reduce the cost of transport.
- (b) To make a material more suitable for handling as, for example, with soap powders, dyestuffs and fertilizers.
- (c) To provide definite properties, such as, for example, maintaining the free-flowing nature of salt.
- (d) To remove moisture which may otherwise lead to corrosion. One example is the drying of gaseous fuels or benzene prior to chlorination.

1.2. General Principles.

The moisture content of a material is usually expressed in terms of its water "liquid" content as a percentage of the mass of the dry material, though moisture content is sometimes expressed on a wet basis. If a material is exposed to air at a given temperature and humidity, the material will either lose water "if the air have lower humidity than that corresponding to the moisture content of the solid " or gain water "if air has more humid than the solid in equilibrium with it, the solid absorbs moisture from the air" until an equilibrium condition is established.

Moisture may be present in the following forms:-

Bound Moisture.

This is the moisture "water" "contained by a substance that it exerts a vapor pressure less than that of free water at the same temperature. Bound water may exist in several conditions such as water retained in small capillaries, adsorbed on surfaces, or as a solution in cell walls and in organic substance in the physical and chemical combination.

Unbound Moisture.

This is the moisture "water" contained by a substance which exerts a vapor pressure as high as that of free water at the same temperature and is largely held in the voids of solid.

Equilibrium Moisture Content X^* .

Is the portion of the water in the wet solid which can not be removed by the inlet air.

Free Moisture X .

This is water which is in excess of the equilibrium moisture content.
Where $X = X_t - X^*$, X_t is the total moisture content.

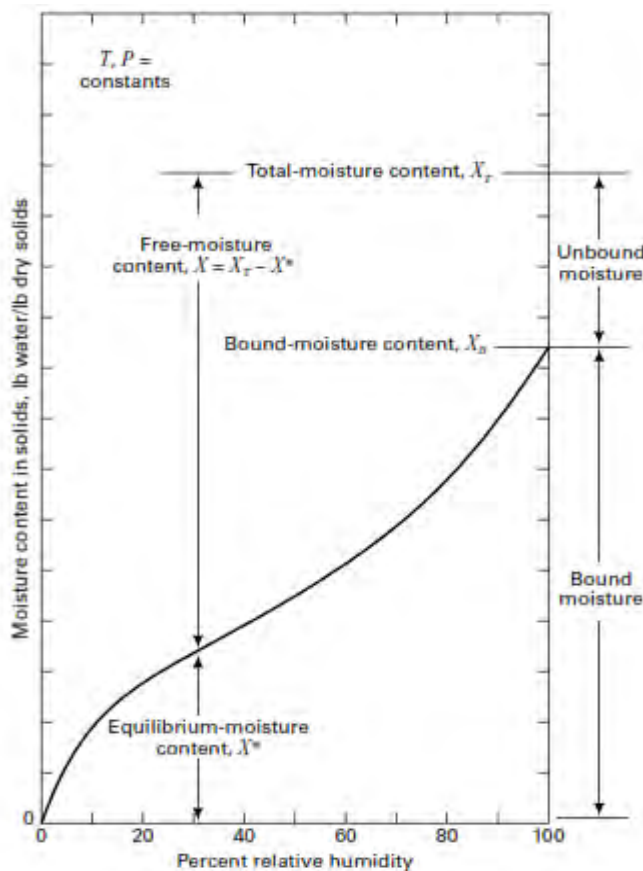


Figure (1) moisture content vs. Percentage relative humidity.

1.3. Wet Solid Group

Wet solids classified into two categories according to their drying behavior:

1. **Granular or crystalline solids** that hold moisture in open pores between particles. These are mainly inorganic materials, examples of which are crushed rocks, sand, catalysts, titanium dioxide, zinc sulfate, and sodium phosphates. During drying, the solid is unaffected by moisture removal, so selection of drying conditions and drying rate is not critical to the properties and appearance of the dried product. Materials in this category can be dried rapidly to very low moisture contents.
2. **Fibrous, amorphous, and gel-like materials** that dissolve moisture or trap moisture in fibers or very fine pores. These are mainly organic solids, including tree, plant, vegetable, and animal materials such as wood, leather, soap, eggs, glues, cereals, starch, cotton, and wool. These materials are affected by moisture removal, often shrinking when dried and swelling when wetted. With these materials, drying in the later stages can be slow. If the surface is dried too rapidly, moisture and temperature gradients can cause checking, warping, case hardening, and/or cracking. Therefore, selection of drying conditions is a critical factor. Drying to low moisture contents is possible only when using a gas of low humidity.

In a direct-heat drying process, the extent to which moisture can be removed from a solid is limited, particularly for the second category, by the equilibrium-moisture content of the solid, which depends on factors that include temperature, pressure, and moisture content of the gas. Even if the drying conditions produce a completely dry solid, subsequent exposure of the solid to a different humidity can result in an increase in moisture content.

The water removed by vaporization is generally carried away by air or hot gases, and the ability of these gases to pick up the water is determined by their temperature and humidity. In designing dryers using air, the properties of the air– water system are essential.

A non-porous insoluble solid, such as sand or china clay, has an equilibrium Moisture content approaching zero for all humidities and temperatures, although many organic materials, such as wood, textiles, and leather, show wide variations of equilibrium moisture content.

1.4. Terminology and Definitions

For the *air–water system*, the following definitions are of importance:

- *Humidity \mathcal{H}* , mass of vapor "water" per unit mass of dry air.

$$\mathcal{H} = \frac{M_A P_A}{M_B (P - P_A)}$$

P_A – Partial pressure of water vapor.

P – Total pressure.

M_A – Molecular weight of water vapor;

- **Humidity of saturated air \mathcal{H}_0** . This is the humidity of air when it is saturated with water vapor. The air then is in equilibrium with water at the given temperature and pressure.

$$\mathcal{H}_0 = \frac{M_A P_A^o}{M_B (P - P_A^o)}$$

P_A^o - Water vapor pressure

M_B –Molecular weight of air

- **Percentage relative humidity $\mathcal{RH}\%$**

$$= \frac{\text{Partial pressure of water vapour in air}}{\text{Vapour pressure of water at the same temperature}} \times 100$$

$$RH\% = \frac{P_A}{P_A^o} \times 100$$

100% humidity means saturated gas, and 0% humidity means vapor free gas.

- **Percentage humidity \mathcal{H}_ϕ**

$$= \frac{\text{Humidity of air}}{\text{Humidity of saturated air}} \times 100 = \frac{\mathcal{H}}{\mathcal{H}_0} \times 100$$

$$\begin{aligned} &= \frac{M_A P_A / M_B (P - P_A)}{M_A P_A^o / M_B (P - P_A^o)} \times 100 \\ &= \frac{P_A (P - P_A^o)}{P_A^o (P - P_A)} \times 100 = RH\% \times \frac{(P - P_A^o)}{(P - P_A)} \end{aligned}$$

- **Humid volume v_H** . Is the volume of unit mass of dry air with accompanying water vapor at a given temperature and pressure.(assume ideal gas where volume of 1 kmole of a gas at 1 atm and 0 °C = 22.4 m³).

$$v_H = \frac{22.4}{273} \left(\frac{1}{M_B} + \frac{H}{M_A} \right) \times T \quad \text{Where } T \text{ temperature in K}$$

For vapor –free gas $\mathcal{H}=0$, v_H is the specific volume of the fixed gas. For saturated gas $\mathcal{H}=\mathcal{H}_0$, v_H becomes the saturated volume.

- **Humid heat S .** is the energy required to rise the temperature of unit mass of dry air and its associated water vapor through unit temperature difference at constant pressure .

$$S = C_a + C_w H \quad , \text{kJ/kg.K}$$

C_a and C_w is the specific heat of gas and vapor kJ/kg.K

- **Dry-Bulb temperature.** This is the temperature of air measured by a thermometer whose "bulb" is dry, i.e. not in touch with water or any other liquid. This is the true temperature of the air.
- **Wet bulb temperature.** This is the temperature attained by a small amount of evaporating water in a manner such that the sensible heat transferred from the air to the liquid is equal to the latent heat required for the evaporation. The wet bulb temperature is measured by passing air over the bulb of a thermometer which is covered with a cloth – wick saturated water.

The rate of which this temperature is reached depends on the initial temperatures and the rate of flow of gas past the liquid surface with a small area of contact between the gas stream remain virtually unchanged.

The rate of transfer of heat from the gas to the liquid is :-

$$Q = h A (T_G - T_w) \dots\dots\dots 1$$

Where Q heat transfer rate.

h heat transfer coefficient.

T_G dry bulb temperature.

T_w wet bulb temperature.

The liquid evaporating into the gas is transferred by diffusion from the interface to the gas stream as a result of a concentration difference(C_o - C).

Where C_o concentration of the vapor at the surface

C concentration in the gas stream.

C_o and C mass per unit volume.

The rate of evaporation is :- $m = h_D A (C_o - C) \dots\dots\dots 2a$

$$m = h_D A \frac{M_A}{RT} (P_A^o - P_A) \dots\dots\dots 2b$$

Where h_D is the mass transfer coefficient

P_A and P_{Ao} partial pressure of air and vapor pressure of water vapor, and can be expressed in terms of the corresponding humidities \mathcal{H} and \mathcal{H}_w .

If P_A and P_{A0} are small compared with P , $(P - P_A)$ and $(P - P_{A0})$, can be replaced by a main partial pressure of the gas P_B .

$$\mathcal{H} = \frac{M_A P_A}{M_B (P - P_A)} \quad \dots\dots \text{humidity at dry bulb temperature}$$

$$\mathcal{H}_w = \frac{M_A P_A^0}{M_B (P - P_A^0)} \quad \dots\dots \text{humidity at wet bulb temperature}$$

$$\mathcal{H}_w - \mathcal{H} = \frac{M_A P_A^0}{M_B (P - P_A^0)} - \frac{M_A P_A}{M_B (P - P_A)} = \left[\frac{P_A^0}{(P - P_A^0)} - \frac{P_A}{(P - P_A)} \right] \frac{M_A}{M_B} = \left[\frac{P_A^0 - P_A}{P_B} \right] \frac{M_A}{M_B}$$

$$P_A^0 - P_A = (\mathcal{H}_w - \mathcal{H}) P_B \frac{M_B}{M_A} \quad \dots\dots \text{sub into equ.2b}$$

$$m = h_D A \frac{M_A}{RT} (\mathcal{H}_w - \mathcal{H}) P_B \frac{M_B}{M_A}$$

$$\rho_B = \frac{P_B M_B}{RT}$$

Therefore equations (2a) and (2b) become :-

$$m = h_D A \rho_B (\mathcal{H}_w - \mathcal{H}) \quad \dots\dots\dots 3$$

The heat transfer required to maintain this rate of evaporation is :-

$$Q = m \cdot \lambda \quad \dots\dots\dots 4a$$

$$\therefore Q = h_D A \rho_B (\mathcal{H}_w - \mathcal{H}) \lambda \quad \dots\dots\dots 4b$$

Where λ is the latent heat of vaporization of liquid.

Equating equations (1) and (4b) :-

$$Q = h A (T_G - T_w) = h_D A \rho_B (\mathcal{H}_w - \mathcal{H}) \lambda$$

$$(\mathcal{H}_w - \mathcal{H}) = \frac{h}{h_D \rho \lambda} (T_G - T_w)$$

where $\frac{h}{h_D \rho} = S$ (humid heat) at moderate humidities

The above equation is known Lewis relation.

Heat and mass transport through the air film on the moist wick are illustrated in figure(2).

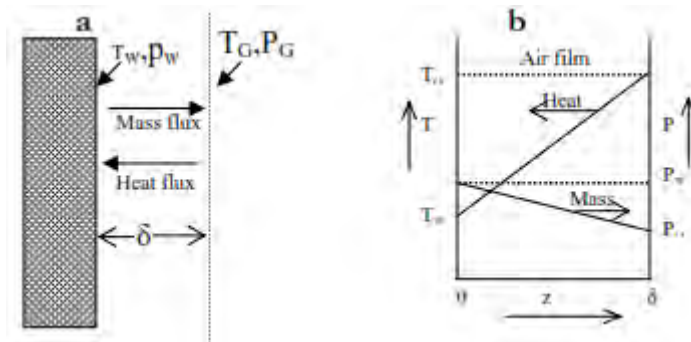


Figure (2):(a) schematic of the wick and air film;(b) temperature and partial pressure of water vapor profile in air film.

Dew point. If the vapor – gas mixture is gradually cooled at a constant pressure , the temperature at which it just becomes saturated is called its dew point. The partial pressure at water vapor in air is the same as the vapor pressure of water at this temperature. If the temperature is reduced even slightly, condensation occurs and droplets (dew) of water appear.

- **Adiabatic saturation temperature T_s .** if the gas is passed over the liquid at such a rate that the time of contact is sufficient for equilibrium to be established, the gas will become saturated and both phases will be brought to the same temperature. In a thermally insulated system, the total sensible heat falls by an amount equal to the latent heat of the liquid evaporated. As a result of continued passage of the gas ,the temperature of the liquid gradually approaches an equilibrium value which is known adiabatic saturation temperature

Make heat balance over the column,

$$\text{Enthalpy of the inlet air} = S (T_G - T_s) + H \lambda$$

$$\text{Enthalpy of exit air} = S (T_s - T_s) + H_s \lambda_s$$

$$= H_s \lambda_s$$

Where λ_s latent heat of vaporization of water at adiabatic saturation temperature.

Because water temperature dose not change , its enthalpy remain constant.

At steady state:

$$S(T_G - T_s) + H \lambda_s = H_s \lambda_s \dots\dots\dots 6$$

$$(T_G - T_s) = (H - H_s) \frac{\lambda_s}{S} \dots\dots\dots 7$$

1.5. Humidity Measurement.

The humidity of stream gas may be found by measuring either the dew point or wet bulb temperature or direct absorption method or by chemical method.

1.6. Humidity Data for Air – Water system.

Various type of humidity chart is available, based on either the temperature or enthalpy of the gas.

1.6.1. Temperature –Humidity Chart (Psychrometric Chart).

From this chart(see figure 2) the following quantities can be estimated:-

1. Humidity (absolute and relative H).
2. specific volume of dry gas.
3. saturated volume.
4. latent heat of vaporization.
5. humid heat.
6. dry and wet bulb temperature.
7. dew and saturation temperature.

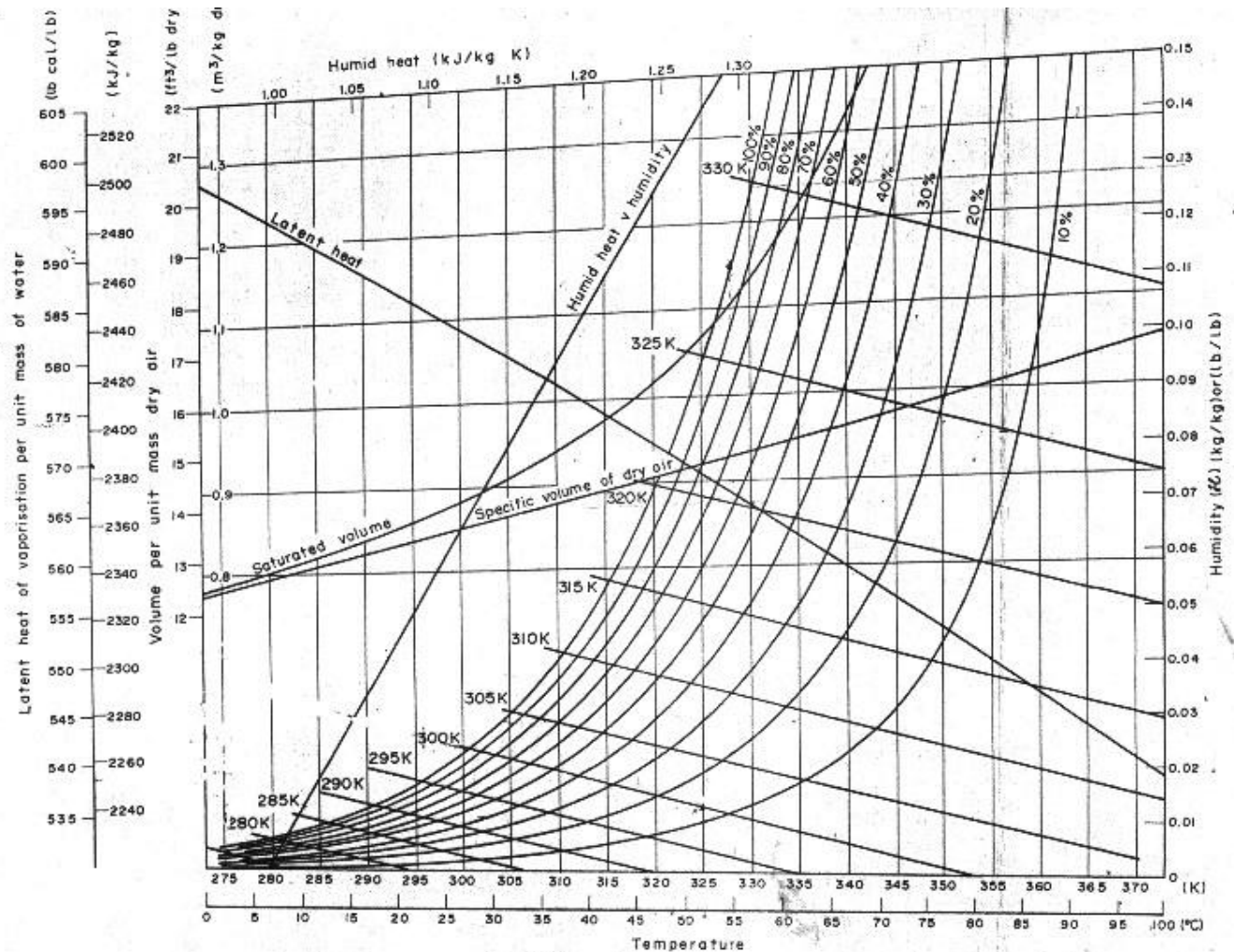


Figure (3) Temperature-Humidity Chart (Psychrometric Chart) for air water system.

If any two of the above quantities are known the others can be readily obtained. For a given wet bulb temperature equation (5) can be plotted on humidity chart (H vs. T) as a straight line having a slope of $-\frac{h}{h_D \rho \lambda}$ which known **Psychrometric line**. Also equation (7) can be plotted which gave **adiabatic cooling (saturation) line** having the slope $-\frac{S}{\lambda}$

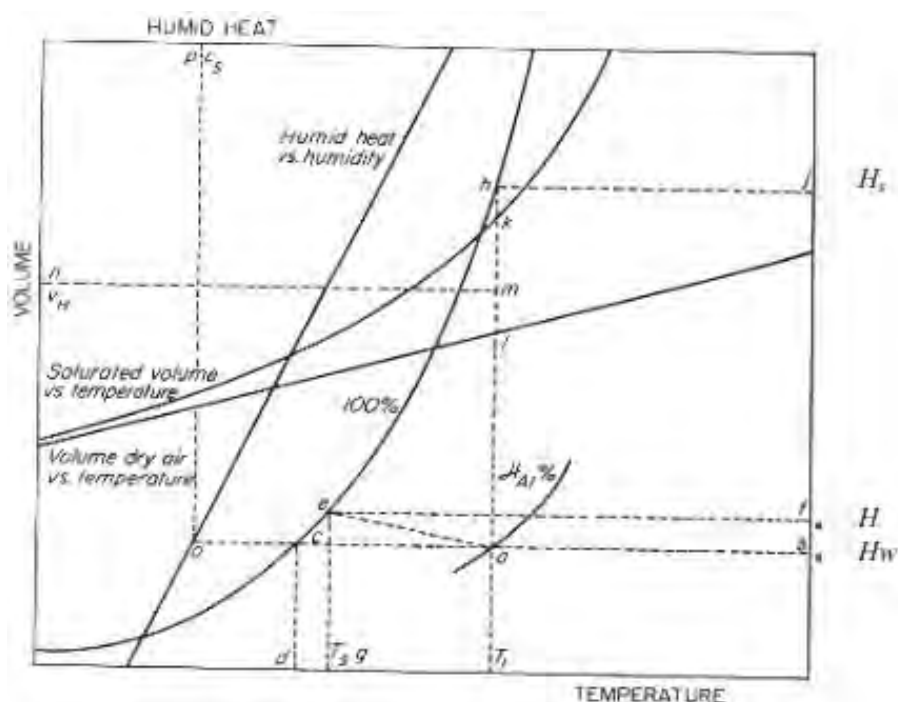
For air-water system $T_s = T_w$;therefore,

$$-\frac{S}{\lambda} = \frac{h}{h_D \rho \lambda} \dots\dots\dots 8$$

So the **adiabatic cooling (saturation) line** and **Psychrometric line** become essentially the same.

1.6.2. Uses of Humidity Chart.

1. The air dry bulb temperature T_G and its humidity H , is known and denoted in the figure(4) by the point **a**. The humidity of air is denoted by point **b** from the humidity coordinate .
2. The dew point T_d , is found by the point **d** that can be reached by moving horizontally from the point **a** to the RH=100% line at **c** and then moving vertically down to the temperature axis **d**.
3. The adiabatic saturation line through **a** is **ae**(these are a series of almost parallel lines as shown in figures (2 and 3). The adiabatic saturation temperature is obtained by drawing the vertical line from **e** then down to the temperature axis **g**. For air -water system the wet bulb temperature is the same as adiabatic saturation temperature. The humidity of the adiabatically saturated air is given by the point **f**. Interpolation between the adiabatic lines may be necessary.
4. If the air originally is subsequent saturated at constant temperature, the humidity is found by following the temperature line through point **a** to point **h** on the RH=100% line and reading the humidity at point **j**.
5. The humid volume of saturated air at temperature T_G corresponds to the point **k**, and that of dry air at T_G is given by the point **I** .The point **m** gives the humid volume if the humidity is H . It reached by interpolation between **I** and **k**.
6. The humid heat of the air is found by locating point **O** ,the intersection of the humidity line through point **a** and the humid heat line ,and reading the humid heat at point **p** on the scale at the top.



Figure(4): Uses Temperature –Humidity Chart (Psychrometric Chart) for air-water system.

1.7. Rate of Drying.

The time required for drying of a moist solid to final moisture content can be determined from a knowledge of the rate of drying under a given set of conditions. The drying rate of a solid is a function of temperature, humidity, flow rate and transport properties (in terms of Reynolds number and Schmidt number) of the drying gas.

In drying, it is necessary to remove free moisture from the surface and also moisture from the interior of the material. If the change in moisture content for a material is determined as a function of time (see figure (5)), a smooth curve is obtained from which the rate of drying at any given moisture content may be evaluated. The form of the drying rate curve varies with the structure and type of material.

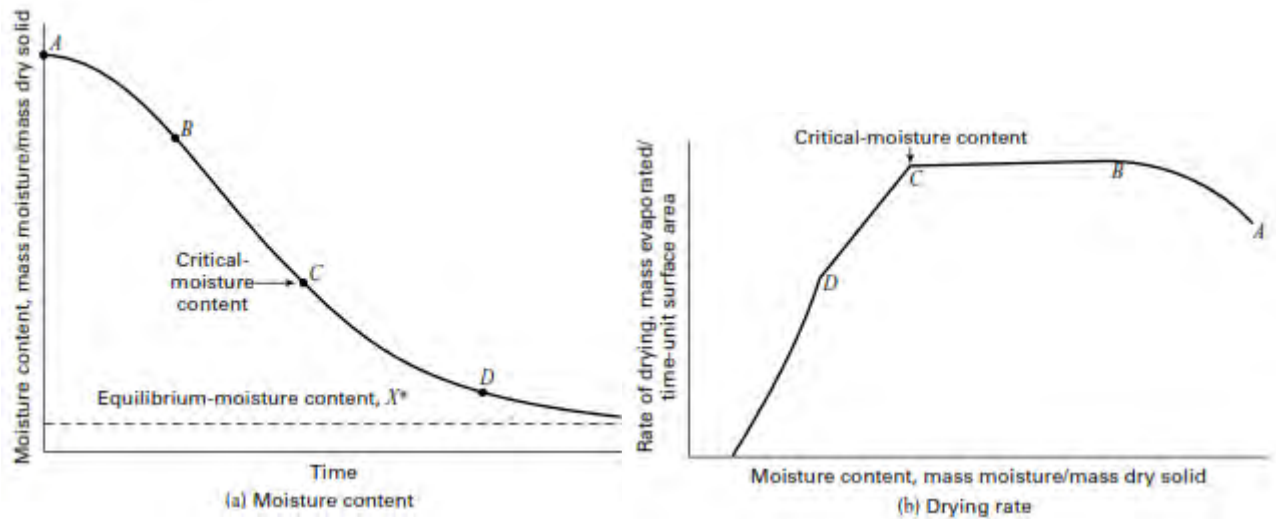


Figure (5) Rate of drying of a granular material.

Wet materials and/or some hot-gas conditions, drying periods are observed in figure 5. From figure 5 A to B, the wet solid is being preheated to an exposed-surface temperature equal to the wet-bulb gas temperature, while moisture is evaporated at an increasing rate. At the end of the preheat period, if the wet solid is of the granular, first category, a cross section has the appearance of Figures 5 and 6a, where the exposed surface is still covered by a film of moisture.

A wet solid of the second category is covered on the exposed surface by free moisture. The drying rate now becomes constant during the period from B to C, which prevails as long as free moisture covers the exposed surface. This surface moisture may be part of the original moisture that covered the surface, or it may be moisture brought to the surface by capillary action in the case of wet solids of the first category or by liquid diffusion in the case of wet solids of the second category.

In either case, the rate of drying is controlled by external mass and heat transfer between the exposed surface of the wet solid and the bulk gas.

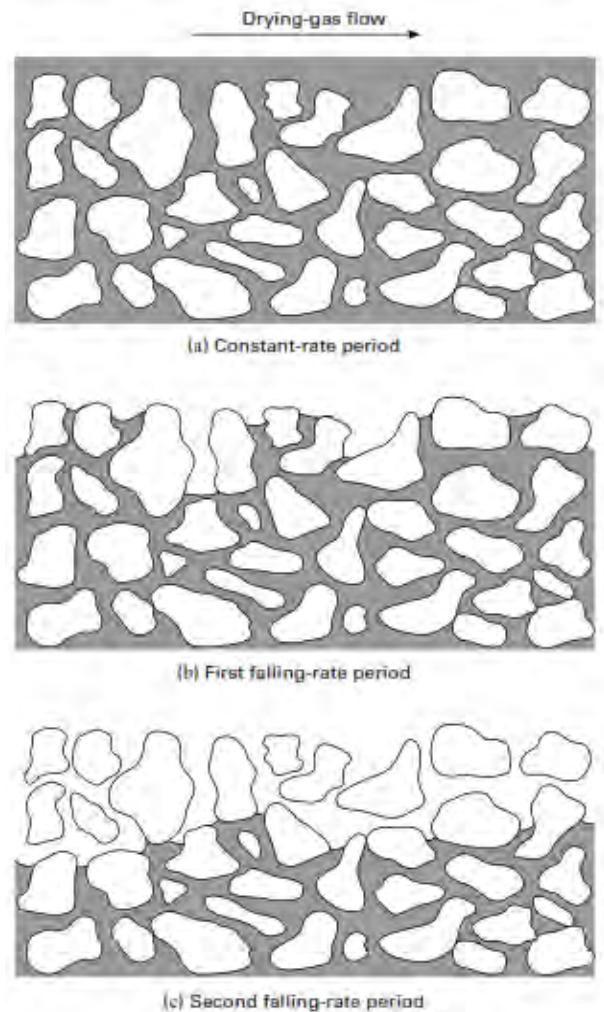


Figure 6 Drying stages for granular solids.

Migration of moisture from the interior of the wet solid to the exposed surface is not a rate-affecting factor. This period, **the constant-rate drying period**, terminates at point C, the **critical moisture content**. When drying wet solids of the first category under agitated conditions as in a direct-heat rotary dryer, fluidized-bed dryer, flash dryer, or agitated batch dryer such that all particle surfaces are in direct contact with the gas, the constant-rate drying period may extend all the way to X

At C, the moisture just barely covers the exposed surface; and then until point D is reached, as shown in Figures 5 and 6b, the surface tends to a dry state because the rate of liquid travel by diffusion or capillary action to the exposed surface is not sufficiently fast. In this period, the exposed-surface temperature remains at the wet-bulb temperature if heat conduction is adequate, but the wetted exposed area for mass transfer decreases. Consequently, the rate of drying decreases linearly with decreasing average moisture content. This is the first falling-rate drying period. It is not always observed with wet solids of the second category.

During the period from C to D, the liquid in the pores of wet solids of the first category begins to recede from the exposed surface. In the final period from D to E, as shown in Figures 5 and 6c, evaporation occurs from liquid surfaces in the pores, where the wet-bulb temperature prevails. However, the temperature of the exposed surface of the solid rises to approach the dry-bulb temperature of the gas. During this period, the **second falling-rate drying period**, the rate of drying may be controlled by vapor diffusion for wet solids of the first category and by liquid diffusion for wet solids of the second category. The rate falls exponentially with decreasing moisture content.

1.8 Calculation method of Drying Rate and Time.

1.8.1 Drying Rate.

* Method used Experimental Drying Curve.

Data obtained from batch drying experiments are usually obtained as W (total weight of the wet solid "dry + moisture") at different times t (h) in the drying period.

$$X_t(\text{dry basis}) = \frac{W - L_s}{L_s} \dots\dots\dots 9$$

Where L_s weight of dry solid.

For a given constant drying condition, the equilibrium moisture content X^* is determined then the free moisture is calculated for each value of X_t .

$$X = X_t - X^*$$

Plot X vs. t, to obtain the rate of drying (R) from this plot, the slope of the tangents drawn can be measured which give the value dX/dt at a given value of t. The rate R for each point is:-

$$R = - \frac{L_s}{A} \frac{dX}{dt} \dots\dots\dots 10$$

Where R rate of drying $\text{Kg}_{\text{H}_2\text{O}}/\text{h.m}^2$.

A Exposed surface area for drying m^2 .

The drying rate curve is then obtained by plotting R vs. the moisture content.

Another method to obtain the value of drying curve is to calculate the weight loss ΔX for a Δt time. For example; if $X_1=0.35$ at $t_1=1.68\text{h}$ and $X_2=0.325$ at $t_2=2.04\text{h}$, then:-

$$\frac{\Delta X}{\Delta t} = \frac{0.35 - 0.325}{2.04 - 1.68}$$

and If $\frac{L_s}{A} = 2.15$; Therefore $R = \frac{L_s \Delta X}{A \Delta t} = 1.493$

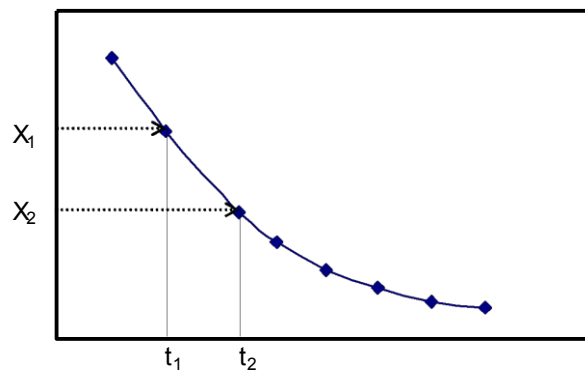
This rate is the average over the period 1.68h to 2.04h and should be plotted at the average concentration.

1.8.2 Drying Time.

1. Constant Rate Drying Period.

• Method used Drying Curve.

To estimate the time of drying for a given batch of material ,based on actual experimental data. The time required is determined directly from the drying curve of free moisture content vs. time.



Where $t = t_2 - t_1$ and $X_1, X_2 > X_c$ (critical moisture content).

• Method used Drying Rate Equation.

$$R = -\frac{L_s}{A} \frac{dX}{dt} \dots\dots\dots 10$$

Rearranged (10) and integrated over the time interval to dry from X_1 at $t_1=0$ to X_2 at $t_2=t$

$$t = \int_{t_1=0}^{t_2=t} dt = -\frac{L_s}{A} \int_{X_1}^{X_2} \frac{dX}{R} \dots\dots\dots 11$$

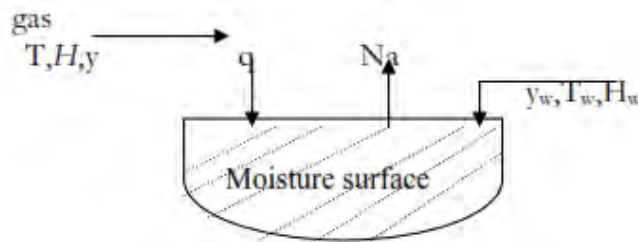
Drying takes place within the constant rate period ,then $R=\text{constant}=R_c$.

$$t = \frac{L_s}{A} (X_1 - X_2) \dots\dots\dots 12$$

In the constant rate period of drying, the surface of grains of solid in contact with drying air flow remain completely wetted. Drying of material occurs by mass transfer vapor from the saturated surface of the material through the air film to the bulk gas phase .The rate of moisture movement within the solid is sufficient to keep the surface saturated. The rate removal of water vapor (drying) is controlled by the rate of heat transfer to the evaporating surface which furnishes the latent heat of evaporation for the liquid. At steady state the rate of mass transfer balances the rate of heat transfer.

Assuming only heat transfer to the solid surface by convection from the hot gas to the surface of the solid and mass transfer from the surface to the hot gas.

- ❖ No heat transfer by conduction from metal pans or surface , neglect heat transfer by radiation.



The rate of heat transfer by convection (q) from the gas at T to the surface of the solid at T_w is :-

$$q = hA(T - T_w) \dots\dots\dots 13$$

The flux of water vapor from the surface is :-

$$Na = k_y(y_w - y) \dots\dots\dots 14a$$

$$Na = k_y \frac{M_B}{M_A} (\mathcal{H}_w - \mathcal{H}) \dots\dots\dots 14b$$

$$y = \frac{H/M_A}{1/M_B + H/M_A}, \text{ the term } (H/M_A) \text{ neglected due } H \text{ very small ...so:-}$$

$$y = \frac{M_B H}{M_A}, \quad y_w = \frac{M_B H_w}{M_A}$$

$$\text{The amount of heat needed } q = m \cdot \lambda_w = Na \cdot M_A \cdot A \cdot \lambda_w \dots\dots\dots 15$$

Where λ_w latent heat of vaporization at T_w . Equating equation(13) and(15) and sub into(14b):

$$R_C = \frac{q}{A \lambda_w} = \frac{h(T - T_w)}{\lambda_w} = k_y M_B (\mathcal{H}_w - \mathcal{H}) \dots\dots\dots 16$$

Therefore equation 12 become:-

$$t = \frac{L_s \lambda_w}{A h (T - T_w)} (x_1 - x_2) \dots\dots\dots 17$$

$$t = \frac{L_s \lambda_w}{A k_y M_B (H_w - H)} (x_1 - x_2) \dots\dots\dots 18$$

To predict R_c , the heat transfer coefficient h must be known.

- if air flowing parallel to the drying surface:-

$$h = 0.0204 G^{0.8} \quad SI \text{ unit}$$

$$h = 0.0128 G^{0.8} \quad English \text{ unit}$$

at $T_{air} 45-150\text{ }^{\circ}C$, $G=2450-2950\text{ kg/m}^2.h$, $U_g=0.61-7.6\text{ m/s}$

- if air flowing perpendicular to the drying surface :-

$$h = 1.17 G^{0.37} \quad SI \text{ unit}$$

$$h = 0.37 G^{0.37} \quad English \text{ unit}$$

$G=3900-19500\text{ kg/m}^2.h$, $U_g=0.9-4.6\text{ m/s}$

Where gas mass velocity $G=\rho_{air} \cdot v_{air}$

On the basis of the above equations, effects of some important parameters on the rate of drying can be determined as:-

Effect of gas velocity: If conduction through the solid and radiation are neglected, the drying rate becomes proportional to $G^{0.71}$ for parallel flow and to $G^{0.37}$ for perpendicular flow. If conduction and radiation are present, effect of air rate will be less important.

Effect of gas temperature : Increased air temperature increases the quantity $(T - T_w)$ increases drying rate. Neglecting radiation and, R_c becomes directly proportional to $(T - T_w)$.

Effect of gas humidity: At moderate temperatures the rate of drying varies directly as $y_w - y$ hence increasing gas humidity reduces drying rate.

Effect of thickness of drying solid: If conduction through solid occurs, R_c decreases as solid thickness increases, but rate of drying may sometimes increase due to higher conduction of heat through the edges.

2. Falling Rate Drying Period.

• Graphical Integration Method.

In falling rate period, the rate of drying R is not constant but decreases when drying proceeds past the critical free moisture content X_c . The time for drying for any region between X_1 and X_2 given by :-

$$t = -\frac{L_s}{A} \int_{X_1}^{X_2} \frac{dX}{R} \dots\dots\dots 11$$

In falling rate period, R varies for any shape of falling rate drying curve, equation(8) can be integrated graphically by plotting $1/R$ vs. X and determine the area under the curve.

• Calculation Method for special cases in Falling Rate Period.

1. Rate is a linear function of X .

Both X_1 and $X_2 < X_c$ and the rate R is linear in X over this region.

$$R = a X + b \dots\dots\dots 19$$

Where a is a slope of the line and b is a constant .Differentiating equation(19) gives:-

$$dR = a dX \dots\dots\dots \text{sub into equation(11)}$$

$$t_f = -\frac{L_s}{aA} \int_{R_1}^{R_2} \frac{dR}{R}$$

$$t_f = \frac{L_s}{aA} \ln \frac{R_1}{R_2} \dots\dots\dots 20$$

$$\text{Since } R_1 = a X_1 + b \text{ and } R_2 = a X_2 + b$$

$$a = \frac{R_1 - R_2}{X_1 - X_2} \dots\dots\dots \text{sub in 20}$$

$$t_f = \frac{L_s(X_1 - X_2)}{A(R_1 - R_2)} \ln \frac{R_1}{R_2} \dots\dots\dots 21$$

2. Rate is a Linear Function Through Origin.

In some cases a straight line from the critical moisture content passing through the origin adequately represents the whole falling rate period. Then the rate of drying is directly proportional to the free moisture content.

$$R = a X \dots\dots\dots 22$$

Differentiating equation(22) , $dR = a dx$

$$dx = \frac{dR}{a} \dots\dots\dots \text{sub in 11}$$

$$t_f = -\frac{L_s}{A} \int_{R_1}^{R_2} \frac{dR}{R} = \frac{L_s}{aA} \ln \frac{R_1}{R_2} \dots\dots\dots 23$$

The slope **a** of the line is R_c/X_c and for $X_1=X_c$ at $R_1=R_c$

$$t_f = \frac{L_s X_c}{A R_c} \ln \left[\frac{R_1}{R_2} \right] \dots\dots\dots 24$$

Noting also that $R_c/R_2 = X_c/X_2$, Therefore equation (24) become:

$$t_f = \frac{L_s X_c}{A R_c} \ln \left[\frac{X_1}{X_2} \right] \dots\dots\dots 25$$

3. Rate of drying in falling rate period can be expressed as a parabolic function

$$R = ax + bx^2 \dots\dots\dots 26$$

Sub equation 26 in equation 11 , then

$$t_f = -\frac{L_s}{A} \int_{X_1}^{X_2} \frac{dX}{ax+bx^2}$$

The time for falling rate period:-

$$t_f = \frac{L_s}{aA} \ln \left[\frac{x_c(a+bx_f)}{x_f(a+bx_c)} \right] \dots\dots\dots 27a$$

$$t_f = \frac{L_s}{aA} \ln \left[\frac{x_c^2 R_f}{x_f^2 R_c} \right] \dots\dots\dots 27b$$

1.9. Mechanism of Moisture Movement in Wet Solid.

When drying takes place, moisture moves from the inner core to the external surface and evaporates. The nature of movement influences the drying during the falling rate period and the following theories have been to explain the moisture movement in solids.

Liquid diffusion

Moisture moves in liquid state through the solid on account of concentration gradient between the interior of the solid where the concentration is high and the surface where the same is low. In these cases, diffusion controlled falling rate period may start immediately after constant rate period and unsaturated surface drying may not be present. Liquid diffusion is encountered in soap, glue, gelatin, etc. as well as during last phase of drying of clay, textile, paper, wood, etc.

Capillary movement

Unbound moisture in granular or porous solids moves through the capillaries or interstices of the solid by the action of surface tension. Initially moisture moves fast enough to keep the surface wet and constant rate drying continues. As drying proceeds, a stage is reached when the surface becomes dry at some spots and falling rate period starts with unsaturated surface drying. The sub-surface water soon dries up and the surface of evaporation gradually recedes into the solid. Evaporation then takes place well within the solid and the vapour thus produced diffuses to the surface.

Vapour diffusion

If heat is supplied to one surface of the solid and drying takes place from the other, moisture may be evaporated within the solid and the Vapour diffuses to the surface.

Pressure action

The solid may shrink on drying and squeeze the moisture to the surface where drying takes place.

If internal diffusion controls drying during the entire falling rate period, the rate is independent of air velocity and the effect of humidity is limited to deciding the equilibrium moisture content.

1.9.1 Drying Time in Falling Period According Liquid Diffusion Theory.

When liquid diffusion of moisture controls the rate of drying in the falling rate period. The equation for diffusion used Fick's second law for unsteady state diffusion using the concentrations as X kg free moisture/kg dry solid instead of concentrations kg mole moisture /m³,

$$\frac{dX}{dt} = D_L \frac{d^2 X}{dx^2} \dots\dots\dots 28$$

Where D_L liquid diffusion coefficient m²/h.
 x distance in the solid m.

This type of diffusion is often characteristic of relating slow drying in no granular materials such as soap, gelatin and others. During diffusion type drying, the resistance to mass transfer of water vapor from the surface is usually very small and the diffusion in the solid controls the rate of drying. Then the moisture content at the surface is at the equilibrium value X^* . This means that the free moisture content X at the surface is zero.

Assume that initial moisture distribution is uniform at $t=0$, equation (28) integrated to give :-

$$\frac{X_t - X^*}{X_1 - X^*} = \frac{X}{X_1} = \frac{8}{\Pi^2} \left[e^{-D_L t \left(\frac{\Pi}{2x_1}\right)^2} + \frac{1}{9} e^{-9D_L t \left(\frac{\Pi}{2x_1}\right)^2} + \frac{1}{25} e^{-25D_L t \left(\frac{\Pi}{2x_1}\right)^2} + \dots \right] \dots\dots 29a$$

Where X average free moisture content at time t .

X_1 initial free moisture content at time $=0$.

X^* equilibrium free moisture content.

$x_1 = 0.5$ thickness of the slab when drying occurs from the top and the bottom parallel faces, and x_1
 $=$ total thickness of slab if drying only from the top face.

From equation (29) assume that D_L is constant, but D_L is rarely constant, it varies with moisture content, temperature, and humidity. For long drying times, only the first term in the equation (29) is significant:-

$$\frac{X}{X_1} = \frac{8}{\Pi^2} e^{-D_L t \left(\frac{\Pi}{2x_1}\right)^2} \dots\dots\dots 29b$$

Solving for the time of drying:-

$$t = \frac{4x_1^2}{\Pi^2 D_L} \ln \frac{8X_1}{\Pi^2 X} \dots\dots\dots 30$$

In this equation if the diffusion mechanism starts at $X=X_c$. Differentiating equation (30) with respect to time an rearranging ,

$$\frac{dX}{dt} = \frac{-\Pi^2 D_L X}{4 x_1^2}$$

Multiplying both sides by L_s/A :-

$$R = -\frac{L_s}{A} \frac{dX}{dt} = \frac{-\Pi^2 L_s D_L X}{4 x_1^2 A} \dots\dots\dots 31$$

Equation (30 and 31) state that when internal diffusion controls for long times, the rate of drying is directly proportional to the free moisture X and the liquid diffusivity and that the rate of drying is inversely proportional to the thickness squared.

1.9.2 Drying Time in Falling Period According Capillary Theory.

Water can flow from region of high concentrations to those of low concentrations as a results of capillary action rather than by diffusion if the pore size of granular materials are suitable. The capillary theory assumes that a packed bed of nonporous spheres contains a void space between the spheres called pores. As water is evaporated, capillary forces are set up by the interfacial tension between the water and solid. These forces provide the driving force for moving the water through the pores to the drying surface. If the moisture movement follows the capillary theory rate of drying R will vary linearly with X :-

$$R = -\frac{L_s}{A} \frac{dX}{dt} \dots\dots\dots 10$$

For the rate R varying linearly with X ; then $R = R_c(X / X_c)$; therefore;-

$$t_f = \frac{L_s X_c}{A R_c} \ln \left[\frac{X_1}{X_2} \right] \dots\dots\dots 25$$

Where $L_s = x_1.A.\rho_s$ kg dry solid

ρ_s Solid density kg/m³

x_1 Thickness m

A Exposed surface area m²

Equation 25 become :-
$$t_f = \frac{x_1 \rho_s X_c}{R_c} \ln \left[\frac{X_1}{X_2} \right] \dots\dots\dots 26$$

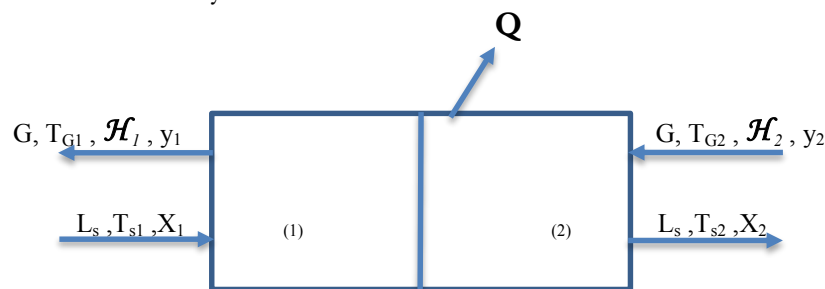
Where R_c can be calculated from equation (16) :-

$$R_C = \frac{q}{A\lambda_w} = \frac{h(T-T_w)}{\lambda_w} = k_y M_B (\mathcal{H}_w - \mathcal{H}) \dots\dots\dots 16$$

The time of drying is directly proportional to the thickness, while the rate of drying inversely proportional to the thickness when capillary controls the falling rate period.

1.10 Material and Heat Balance for Continuous Dryers.

Figure below show a flow diagram for a continuous type dryer where the drying gas flows counter currently to the solids flow.



The solid enters at a rate of L_s kg_{dry solid}/h, having a free moisture content X_1 and a temperature T_{s1} . It leaves at X_2 and T_{s2} . The gas enters at a rate G kg_{dry air}/h, having a humidity \mathcal{H}_2 kg water vapor/ kg_{dry air} and a temperature of T_{G2} . The gas leaves at T_{G1} and \mathcal{H}_1 .

For **material balance** on the moisture :-

$$G \mathcal{H}_2 + L_s X_1 = G \mathcal{H}_1 + L_s X_2 \dots\dots\dots 32$$

For **heat balance** :- the enthalpy of wet solid is composed of the enthalpy of the dry solid plus that of free liquid as free moisture. The heat of wetting is usually neglected .

The enthalpy of **gas H_G** in kJ/kg_{dry air} is:-

$$H_G = S (T_G - T_0) + \lambda_0 \mathcal{H} \dots\dots\dots 33$$

$$S = C_a + C_H \mathcal{H} \text{ ,kJ/kg.K.} \dots\dots\dots 34$$

Where

λ_0 latent heat of vaporization of water (liquid)at reference temperature=2501 kJ/kg.

S humid heat kJ/kg.k.

C_a and C_H specific heat of gas and vapor kJ/kg.K.

The enthalpy of **wet solid H_s** in kJ/kg_{dry solid} is :-

$$H_s = C_{ps} (T_s - T_0) + X C_{pA} (T_s - T_0) \dots\dots\dots 3$$

Where

C_{ps} Specific heat of dry solid kJ/kg dry solid.k.

C_{pA} Specific heat of liquid moisture kJ/kg water.k.

The heat of wetting or adsorption is neglected.

So the heat balance on the dryer is :-

$$GH_{G2} + L_s H_{s1} = G H_{G1} + L_s H_{s2} + Q \dots\dots\dots 36$$

Where Q is the heat loss in the dryer kJ/kg. For adiabatic process $Q=0$, and if heat is added Q is negative

1.11 Rate of Drying for continuous Direct Heat Driers.

During high temperature drying, humidity of air has only minor effect on the rate of drying and calculations are generally based on rate of heat transfer. At low temperatures, mass transfer driving force is considered. These calculations, however, give only rough estimates and experiments should be conducted for obtaining reliable design data.

1.11.1 Drying at High Temperature.

In continuous driers, where the solid moves from one end of the equipment to the other end as in rotary driers, the drying gas undergoes a more or less uniform change in temperature. In case of the solids, on the other hand, three different trends in temperature change can be recognized and accordingly the drier is divided into three distinct zones as shown in Figure(7)

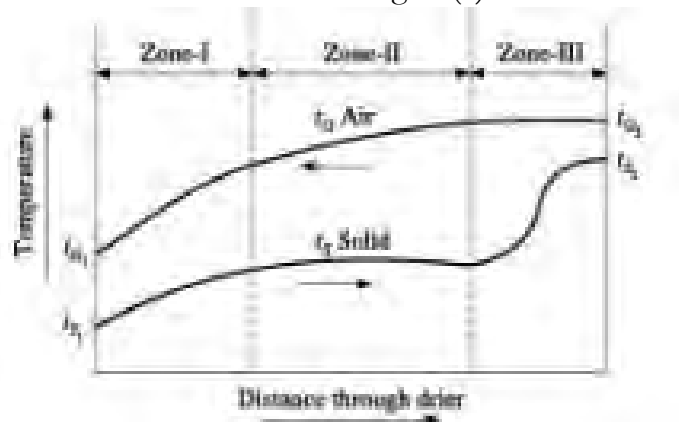


Figure (7) Three zones in a continuous high temperature drier.

Zone-I is the pre-heat zone where the solid is heated by sensible heat transfer from the gas until there is a dynamic balance between the heat required for evaporation and that supplied by the gas. Very little evaporation takes place here.

Zone-II, surface moisture and unbound moisture are removed and the temperature of the solid remains more or less constant.

Zone-III, unsaturated surface drying and evaporation of bound moisture take place. The temperature of the solid increases considerably in this zone and it leaves the drier at a temperature very close to that of inlet gas. This zone constitutes the major part of many driers.

Considering heat transfer only between gas and solid, an enthalpy balance for a differential length dz of the drier can be written as:-

$$dq_G = dq + dQ \dots\dots\dots 37$$

Where,

q_G = heat lost by the gas.

q = heat actually transferred to the solid.

Q = heat loss to the surrounding.

Rearranging equation (37), we obtain

$$dq = (dq_G - dQ) = U dS (T_G - T_w) = U a (T_G - T_w) dz \dots\dots 38$$

Where,

U = overall heat transfer coefficient between gas and solid

$(T_G - T_w)$ = temperature difference for heat transfer

S = interfacial surface per unit drier cross section

a = interfacial area per unit drier volume

Equation (38) may therefore be written as

$$dq = G C_s dT_G = U a (T_G - T_w) dz \dots\dots\dots 39$$

Where,

dT_G is the temperature drop in the gas due to heat transfer to the solid alone, C_s is the humid heat of the gas.

$$dN_{toG} = \frac{dT_G}{(T_G - T_w)_m} = \left(\frac{Ua dz}{GC_s} \right) \dots\dots\dots 40$$

If the heat transfer is constant ,

$$N_{toG} = \frac{z}{H_{toG}} = \frac{dT_G}{(T_G - T_w)} \dots\dots\dots 41$$

$$H_{toG} = \frac{GC_s}{Ua} \dots\dots\dots 42$$

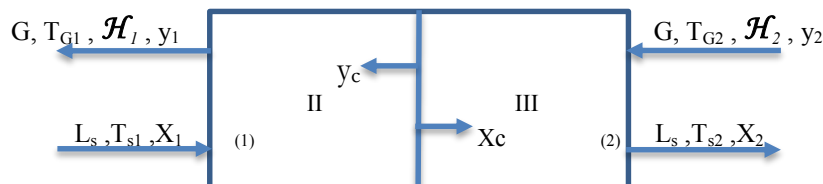
N_{toG} and H_{toG} = the overall number and length of heat transfer units
 $(T_G - T_w)_m$ = change in gas temperature due to heat transfer to solid only
 dT_G = approximate average temperature difference between gas and solid

Assuming the temperature profile to be straight line for each of the three zones separately, $(T_G - T_w)_m$ becomes the logarithmic mean of the terminal temperature differences for each zone and N_{toG} becomes the corresponding number of transfer units.

The approximate average temperature differences in the three zones, particularly in Zone-II are the average wet-bulb depressions in the respective zones since the surface of the solid is at the wet-bulb temperature of the drying gas.

1.11.2 Drying at Low Temperature.

Since the drying takes place at low temperature the preheating of solid is not a major factor. The preheating zone merges with zone II. In zone II unbound and surface moisture constant, X_c as in drying at high temperature. The unsaturated surface drying and evaporation of bound moisture occurs in zone III. The humidity of incoming gas increases from y_2 to y_c as it leaves zone III.



The retention time can be calculated as:

$$t = t_{II} + t_{III}$$

$$t = \frac{L_s}{A} \left[\int_{X_c}^{X_1} \frac{dX}{R} + \int_{X_2}^{X_c} \frac{dX}{R} \right] \dots\dots\dots 43$$

In zone II,

$X > X_c$, the rate of drying $R = R_c$ (is given by equation 14a, 14b and 16):

$$t_{II} = \frac{L_s}{A} \left[\int_{X_c}^{X_1} \frac{dX}{R} \right] \dots\dots\dots 44a$$

$$t_{II} = \frac{L_s}{A} \left[\int_{X_c}^{X_1} \frac{dX}{k_{ymB}(H_w - H)} \right] \dots\dots\dots 44b$$

Make material balance:-

$$G d\mathcal{H} = L_s dX \dots\dots\dots 45$$

Sub for dX in equation 44 from equation 45,

$$t_{II} = \frac{L_s}{A} \cdot \frac{G}{L_s} \left[\int_{H_1}^{H_c} \frac{dH}{k_{yMB}(H_w - H)} \right] \dots\dots\dots 46$$

Assuming y_w to be constant (which will correspond to saturation humidity at the wet bulb temperature of incoming air and there is no heat loss).

$$t_{II} = \left(\frac{L_s}{A} \right) \cdot \left(\frac{G}{L_s} \right) \cdot \left(\frac{1}{k_{yMB}} \right) \ln \left[\frac{H_w - H_c}{H_w - H_1} \right] \dots\dots\dots 47$$

In zone III,

$X < X_c$, the rate of drying R give from linear proportional ($R = aX + b$) as:-

In a specific case of drying from X_c to X^* ,

- 1- $R_c = aX_c + b$
 - 2- $R^* = 0 = aX^* + b$
- $R_c = a(X_c - X^*)$,
 $R = a(X - X^*)$, then

$$R = \frac{[R_c(X - X^*)]}{(X_c - X^*)} \dots\dots\dots 48$$

Applying $R_c = k_y(y_w - y)$ sub in equation 48

$$R = \frac{[k_{yMB}(H_w - H)(X - X^*)]}{(X_c - X^*)} \dots\dots\dots 49$$

From equation 43 time of drying in zone III,

$$t_{III} = \frac{L_s}{A} \left[\int_{X_2}^{X_c} \frac{dX}{R} \right] \dots\dots\dots 50a$$

$$t_{III} = \frac{L_s(X_c - X^*)}{A k_{yMB}} \left[\int_{X_c}^{X_1} \frac{dX}{(H_w - H)(X - X^*)} \right] \dots\dots\dots 50b$$

From material balance for moisture across any section yields,

$$G[\mathcal{H} - \mathcal{H}_2] = L_s [X - X_2] \dots\dots\dots 51$$

$$H = H_2 + (X - X_2) \left(\frac{L_s}{G} \right) \dots\dots\dots 52$$

Differentiating equation 51,

$$G .d\mathcal{H} = L_s .dX \dots\dots\dots 53$$

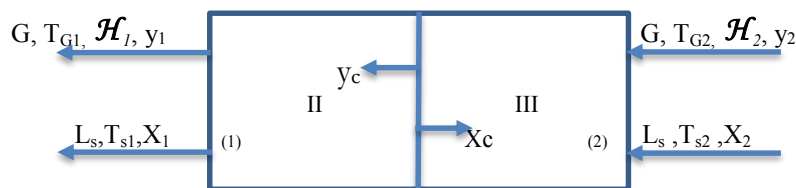
Sub equation 52 and 53 , $X^*=0$ in equation 50b

$$t_{III} = \frac{L_s}{A} \cdot \frac{X_c}{k_{yMB}} \cdot \frac{G}{L_s} \left[\int_{H_c}^{H_2} \frac{dH}{(H_w - H) \left((H - H_2) \cdot \left(\frac{G}{L_s} \right) + X_2 \right)} \right] \dots\dots\dots 54$$

$$t_{III} = \frac{L_s}{A} \cdot \frac{X_c}{k_{yMB}} \cdot \frac{G}{L_s} \cdot \frac{1}{\left((H - H_2) \cdot \left(\frac{G}{L_s} \right) + X_2 \right)} \ln \left[\frac{X_c (H_w - H_2)}{X_2 (H_w - H_c)} \right] \dots\dots 55$$

Note :- equation 55 cannot be applied for rate of drying controlled by internal diffusion

❖ In case of parallel flow of driers,



$$t_{II} = \left(\frac{L_s}{A} \right) \cdot \left(\frac{G}{L_s} \right) \cdot \left(\frac{1}{k_{yMB}} \right) \ln \left[\frac{H_w - H_1}{H_w - H_c} \right] \dots\dots\dots 56$$

$$t_{III} = \left(\frac{L_s}{A} \right) \cdot \left(\frac{X_c}{k_{yMB}} \right) \cdot \left(\frac{G}{L_s} \right) \cdot \frac{1}{\left((H - H_2) \cdot \left(\frac{G}{L_s} \right) - X_2 \right)} \ln \left[\frac{X_c (H_w - H_2)}{X_2 (H_w - H_c)} \right] \dots\dots 57$$

1.12 Drying Equipment.

Some dryers are continuous, and some operate batch wise, some agitate the solids, and some are essentially un-agitated. Operation under vacuum may be used to reduce the drying temperature. Some dryers can handle almost any kind of material, while others are severely limited in the type of feed they can accept.

A major division may be made between:-

1. Dryers in which the solid is directly exposed to a hot gas.
2. Dryers in which heat is transferred to the solid from an external medium such as condensing steam, usually through a metal surface with which the solid is not in contact.

Dryers which exposed the solids to a hot gas are called direct dryers; those in which heat is transferred from an external medium are known indirect dryers.

Solids handling in dryers, most industrial dryers handle particulate solids during part or all of the drying cycle, in adiabatic dryers the solids are exposed to the gas in the following ways :-

1. Gas is blown through a bed of coarse granular solids which are supported on a screen. This is known as through – circulation drying. As in cross-circulation drying the gas velocity is kept low to avoid any entrainment of solid particles.
2. Solids are showered downward through a slowly moving gas stream, often with some undesired entrainment of fine particles in the gas.
3. Gas is blown across the surface of the bed or slab of solids, or across one or both faces of a continuous sheet or film. This process is called cross-circulation drying.
4. Gas passes through the solids at a velocity sufficient to fluidize the bed.
5. The solids are all entrained in a high-velocity gas stream and are pneumatically conveyed from a mixing device to a mechanical separator.

In indirect dryers heating the only gas to be removed is the vaporized water or solvent ,although sometimes a small amount of "sweep gas" often air or nitrogen is passed through the unit.

1. Solids are spread over a stationary or slowly moving horizontal surface and cooked until dry. The surface may b heated electrically or by a heat transfer fluid such as steam or hot water.
2. solids are moved over a heated surface ,usually cylindrical by an agitator or a screw or paddle conveyor.
3. Solids slides by a gravity over an inclined heated surface or are carried upward with the surface for a time and then slides to a new location "Rotary dryer".