

الجامعة التكنولوجية

قسم الهندسة الكيميائية

المرحلة الثالثة

انتقال كتلة

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Distillation

① Introduction:-

Distillation is the technique of separation of the more volatile component from the less volatile ones in a feed solution by partial vaporization of the feed followed by condensation.

The vapor produced is richer in the more volatile component. The distribution of the components in the phases is governed by the vapor-liquid equilibrium relationship.

The more volatile components move from the liquid to the vapor phase, and the less volatile move from the vapor to the liquid phase.

The function of distillation may be :-

- Separation of products from a mixture, separation of aniline and nitrobenzene.
- Recovery of products, recovery of ethanol from its solution in water.
- Increasing the purity of a product, drying of benzene to remove any trace of water.

Because of its importance and wide use in separation of mixtures, distillation is called "The work horse" of chemical engineering or chemical industries.

- The main difference between distillation and evaporation is that in evaporation a solution containing a volatile solvent and a non volatile solute or a solute having a very low volatility (sugar (non volatile solute), water (volatile solvent)).

Schematic of a typical distillation column is shown in Figure below :-

There are two main types of mixtures:-

- 1- Binary mixture, consist of two components only.
- 2- Multi-component mixture, for more than two components

- Bubble point :- temp. cause first bubble on the liquid surface to appear. It is a characteristic for mixture and compounds.

- Boiling point :- temp. at which vap. press. equal to atm. press. It is a characteristic for pure substance like P, μ, \dots

Separation of a mixture by distillation is based on equilibrium distribution of the components between the liquid and vapor phases. Therefore vap.-liq. equilibria is essential in distillation.

Distillation always used when there is difference in boiling points and volatility.

$$\text{Volatility } (\alpha_A) = \frac{P_A}{x_A} = \frac{y_A \cdot P_T}{x_A}$$

For two components (A, B) to be separated
 $\alpha_A \neq \alpha_B$.

$$\text{Relative Volatility } (\alpha_{AB}) = \alpha = \frac{\alpha_A}{\alpha_B} = \frac{y_A \cdot x_B}{y_B \cdot x_A}$$

For high (α_{AB}), distillation become easier.

When $\alpha_A = \alpha_B$, Special treatment is used, that is called "extractive-distillation".

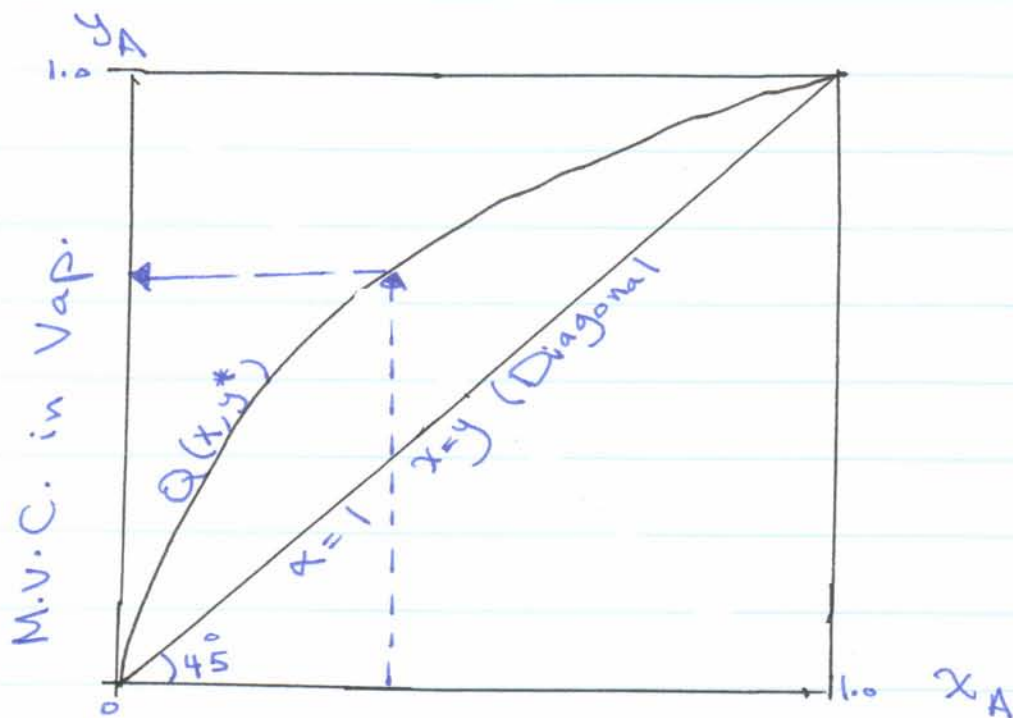
* if $\alpha_{AB} > 1.0$ then (A) is the M.V.C

if $\alpha_{AB} < 1.0$ then (A) is the L.V.C

② Vapour - liquid Equilibria (VLE) :-

هذه دالة تعطينا تركيز البخار الذي هو في حالة توازن مع تلك تركيز
من تراكيز السائل المتكونة من المادتين (A, B).
دائماً نؤخذ عند ضغط ثابت. وهي التي تسمى بعملية التقطير
الذي سيتم استخدامها بالاضمان في عامل (α_{AB}) .
الرسم البياني اعلاه هو (Vap. Liq. Equil^m) مشتق من
رسم بياني تاني به تحت (Boiling point diagram)

M.V.C : more volatile Comp. (Less boiling point)
L.V.C : Less volatile comp. (higher boiling point)



M.V.C in liquid

③ Boiling Point Diagram

① عند ثبوت الضغط :-

المختلبي اسفل يوضع لنا درجات الغليان لتراكيز مختلفة من
الماء . زيه انقلوب في لتر كبير سيغير درجة الغليان .

(T_A) درجة غليان الماء (A) ، لنقي .

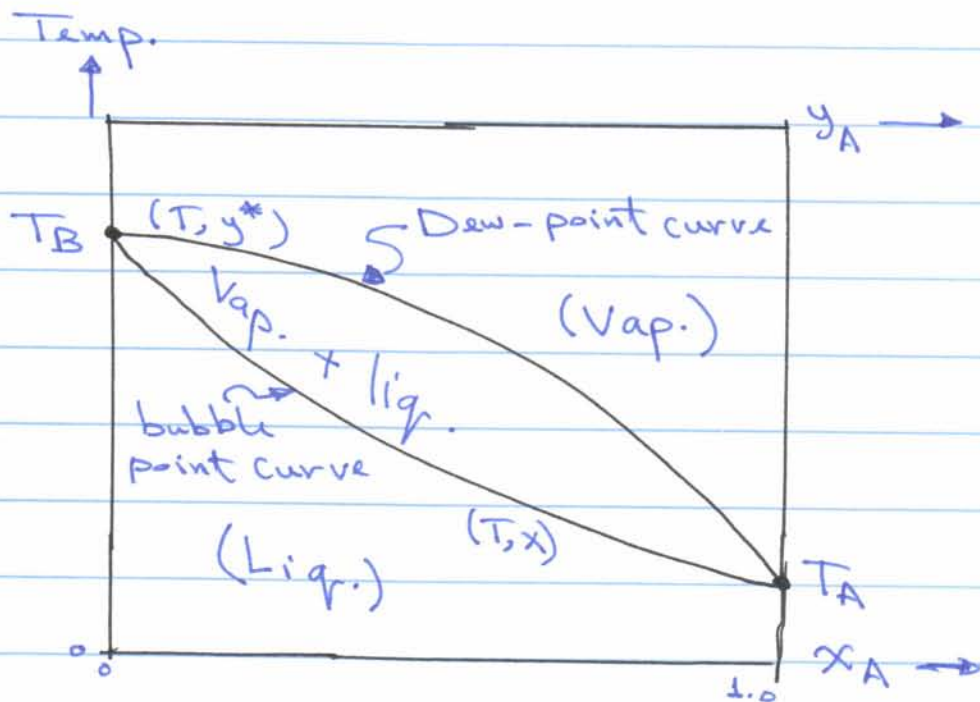
(T_B) درجة غليان المادة (B) ، لنقي .

المختلبي العلوي يعطي درجة الحرارة التي يبدأ بها الغليان بالتكثف .
كل هذا المختلبيات يلتقيان في (T_A) ، (T_B) لأن المادة لنقية
لدرجة غليان ثابتة ، لتكثف .

كل نقطة اسفل المختلبي اسفل تكون في طور سائل .

كل نقطة هناك المختلبي الاعلى تكون في طور بخار .

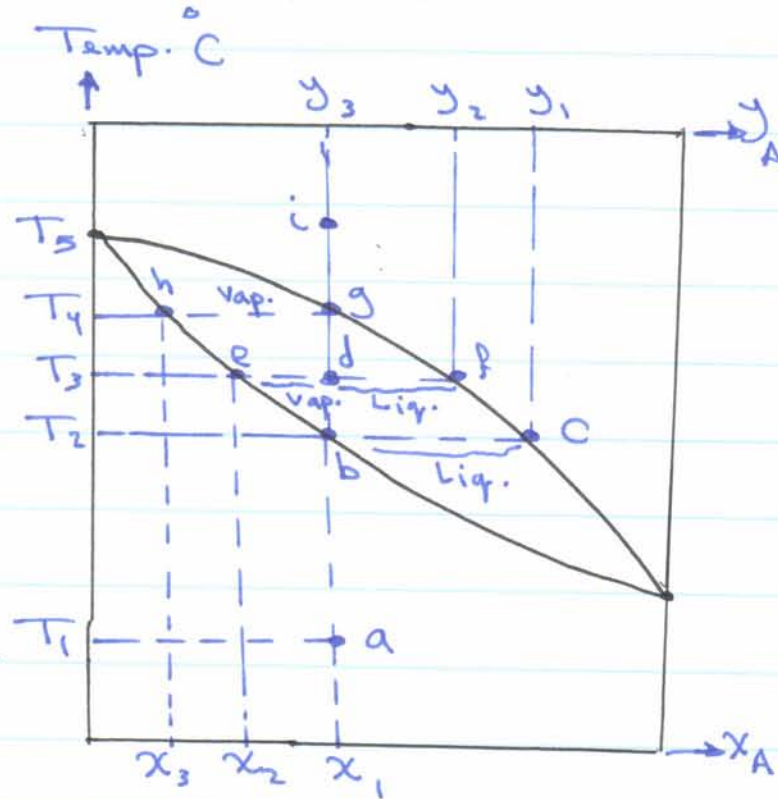
بين المختلبيين هناك مزيج من البخار والسائل .



لو عندنا سائل معين بدرجة حرارة معينة ونبدأ بتغيير درجة الحرارة تدريجياً.

- الآن لو نظرنا عندنا سائل بدرجة حرارة (T_1) وتركيز (X_1) النقطة (a).

- الآن لو رفعنا درجة الحرارة من (T_1) إلى (T_2) فهو يبقى في الحالة السائلة حتى نصل إلى نقطة (b).
هنا تظهر أول فقاعة بالظهور على سطح السائل وتركيزها في طور البخار (y_1) . النقطة (c) تركيز أو قاعه من البخار.



- الآن ترفع الحرارة حتى نقطة (d) بدرجة حرارة (T_3) ، هنا تزداد كمية البخار ويصبح لدينا مزيج من السائل والبخار، تركيز السائل محدد في النقطة (e) وتركيز البخار بالنقطة (f) ويكون (y_2) .

أول فقاعة ظهرت في (b, c) وتقل كلما ارتفعنا إلى الأعلى.

$$\frac{Liq.}{vap.} = \frac{dF}{de} \quad \text{where } F = \text{vapor (sat.)}, e = \text{liq. (sat.)}$$

- The line (ef) is called "Tie-line" (Enthalpy of liq. & vap. at equilibrium)

- section length / section length ed or

$\frac{df}{de}$ is called "Lever-arm rule" (Flash vaporization)

- رفع درجة الحرارة إلى النقطة (g) ، هنا السائل يتبخر كله ويحدث عندنا تبخر تام ، والنقطة (h) تمثل تركيز البخار عند نقطة السائل يتبخر.

- رفع درجة الحرارة عن نقطة لومبول إلى النقطة (i) يصبح لدينا حالة (Super - heated - vapour) مع ملاحظة أنه تركيز البخار يظل يساوي تركيزه في (g) .

* الآن لو عندنا بخار ونريد أن نكثفه تدريجياً :-

- عندنا بخار في (T₅) وتركيزه (x₁) . لو قللنا الحرارة إلى (T₄) عنده نقطة (g) حيث تمثل أدنى درجة تكثف إلى سائل تركيزها عند (h) .

- نقلل الحرارة إلى (T₃) عنده النقطة (d) ، يستقر البخار بالتكثف ونغير لدينا فزييم من البخار والسائل لأن البخار غير كافٍ لتكثف كل البخار .

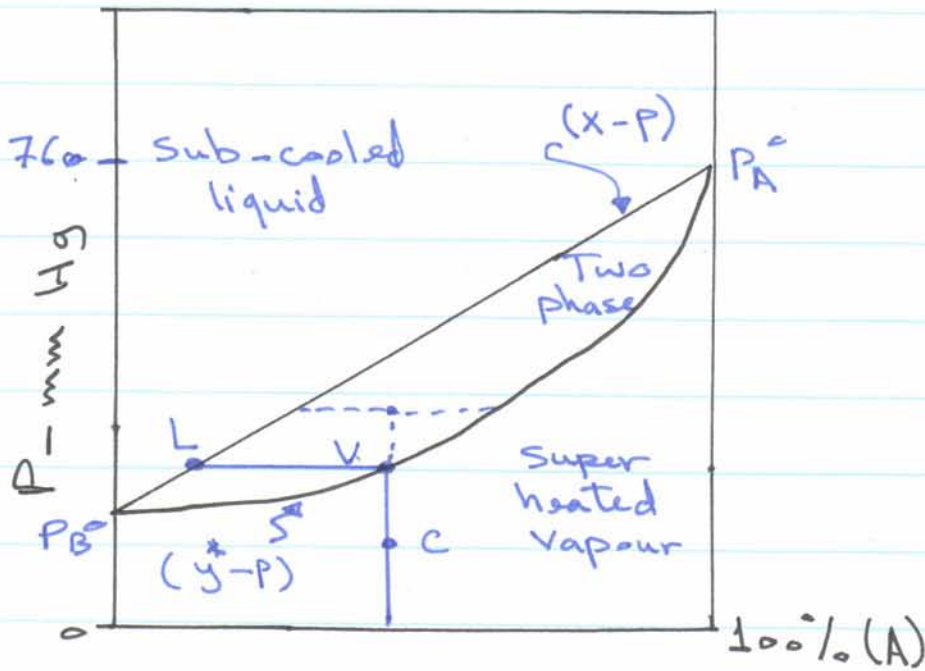
- عند (b) نحول كل البخار إلى سائل ويحدث عندنا فقط سائل :-
(Total condensation)

- الآن لو استمرينا لفاتح (T₁) نصل إلى تركيز ثابت والعمليه هي (sub-cooled liq.)

- من المحاليف المذكورين السابقين هناك خمس حالات لل Feed ظهرت عند استخدام :-

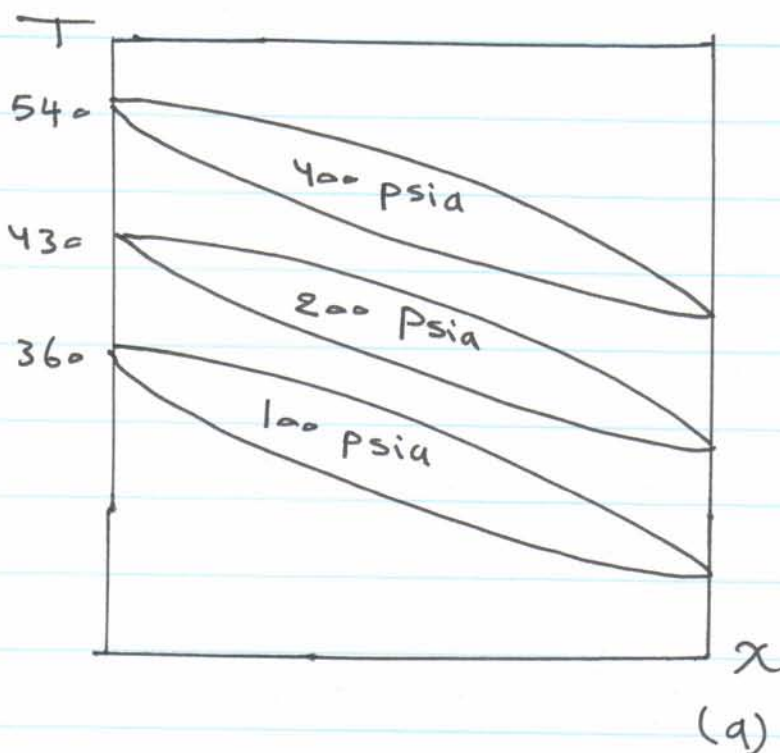
- 1- sub-cooled , 2- at bubble point
- 3- partial vaporized , 4- dew point ,
- 5- Super heated.

③ عند ثبوت درجة الحرارة :-



L: First drop of liquid appear after compression

Effect of pressure on boiling point diagram.



Increasing the pressure will cause the two lines to become closer and closer

How to Find (Vap. - Liq. Equilb^m. data) analytically.

① Raoult's Law (for liq. mixture)

- For component (A) :-

$$P_A = P_A^\circ \cdot x_A$$

- For component (B) :-

$$P_B = P_B^\circ \cdot x_B$$

- For a binary mixture :-

where :- P_A° : vap. press

P_A = parti. press

x_A = mole fraction
in liq. phase

$$x_A = \frac{P_T - P_B^\circ}{P_A^\circ - P_B^\circ} \quad \text{--- (1)}$$

H.W :- Try to find the above relation.

② Dalton's Law (ideal gas)

$$P_T = \sum P_i$$

for binary mixture :-

$$P_T = P_A + P_B$$

$$\therefore P_A = y_A \cdot P_T$$

$$P_A = P_A^\circ \cdot x_A$$

$$\therefore y_A = \frac{P_A^\circ \cdot x_A}{P_T} \quad \text{--- (2)}$$

③ Using relative volatility (α_{AB}) or (α)

$$\alpha_{AB} = \frac{y_A \cdot x_B}{y_B \cdot x_A} = \frac{y_A (1 - x_A)}{(1 - y_A) \cdot x_A}$$

$$y_A = \frac{\alpha \cdot x_A}{1 + x_A (\alpha - 1)} \quad \text{--- ③}$$

$$x_A = \frac{y_A}{\alpha - (\alpha - 1) y_A} \quad \text{--- ④}$$

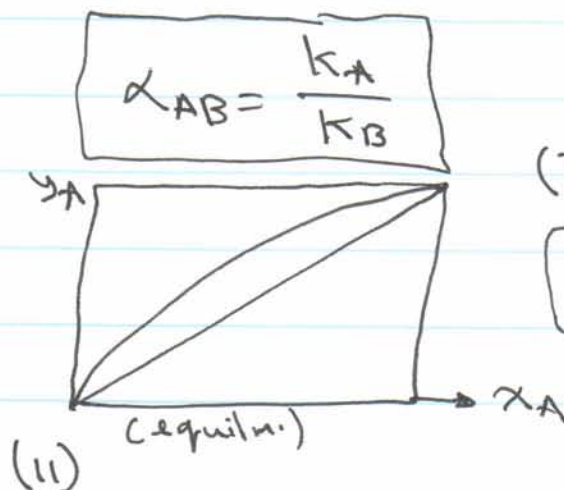
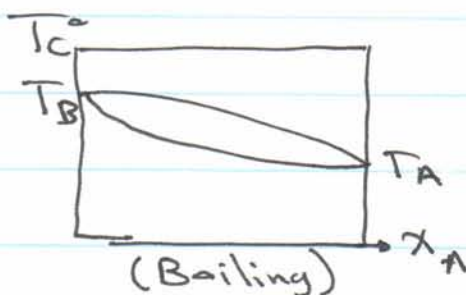
④ Using - equilib^m constant (k_i) (Henry's Law)
or (equilib^m vaporizⁿ ratio) or (distillⁿ coeff.)

$$P_A = H \cdot x_A \Rightarrow y_A = \frac{H}{P_T} x_A \quad (\text{for hydrocarbon mixture})$$

$$y_A = k_A \cdot x_A \quad \text{where then} \quad k_A = \frac{y_A}{x_A} \quad (\text{for multi-component mixture}).$$

k = called (k) value, it is function of (T) and (P).

For ideal system



$$\alpha_{AB} = \frac{k_A}{k_B}$$

(Typical system)

$$x_A = \frac{1 - k_B}{k_A - k_B}$$

$$y_A = \frac{k_A - k_A k_B}{k_A - k_B}$$

Notes

For multicomponent system =

- ① if $\sum P_i < P_T$ liq. is sub cooled, below B.Pt
if $\sum P_i > P_T$ s s super heated, above B.Pt

② $y_i = k_i x_i$

- ③ if $\sum y_i = 1.0$ liq. at bubble Pt.

- ④ if $\sum y_i < 1.0$ liq. is sub cooled.

- ⑤ if $\sum y_i > 1.0$ liq. is super heated

- ⑥ if $\sum y_i / k_i = 1.0$ at dew-point-vap.

- ⑦ if $\sum y_i / k_i < 1.0$ vap. is super heated

- ⑧ if $\sum y_i / k_i > 1.0$ sub cooled

- ⑨ Some times, it is required to calculate (P_i°) if it is not given.

$$\ln P_A^\circ = A - \frac{B}{C+T} \quad \text{Antoine-equation}$$

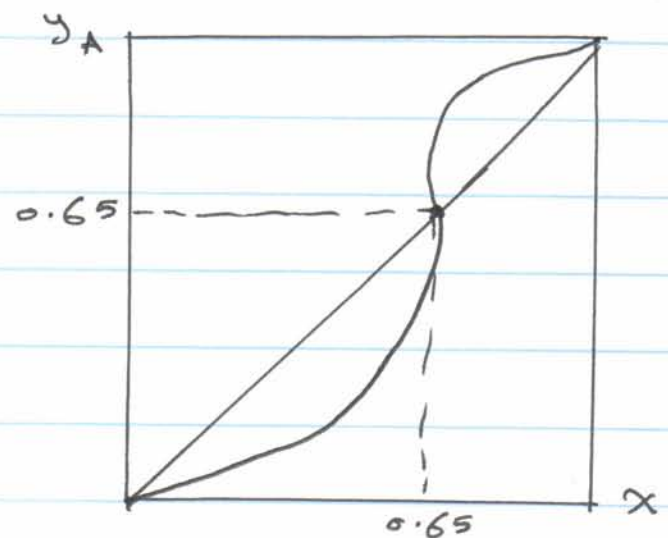
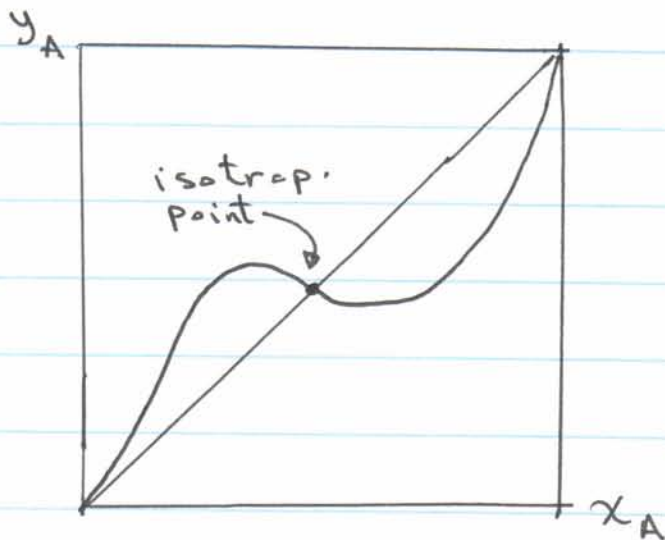
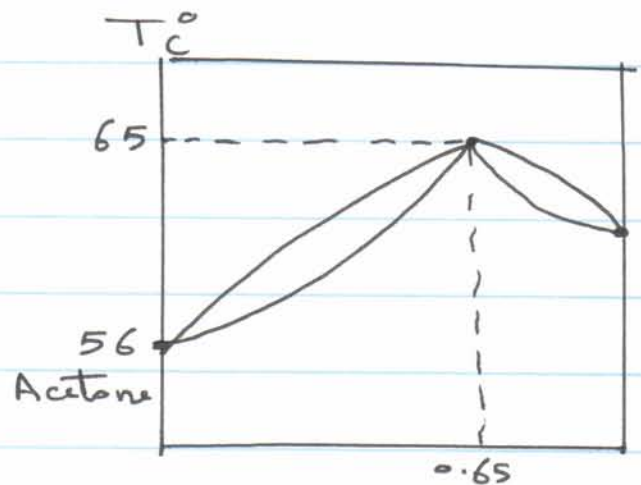
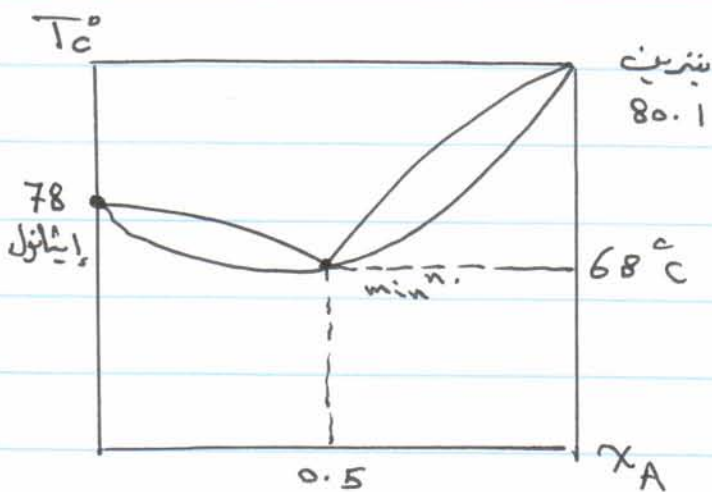
where :- P_A° in mmHg.
 T in $^\circ\text{C}$.

(A, B, c) are constants.

Deviation from ideality and formation of Azeotropes

Deviation from ideality may be of two types:-

- Positive - deviation, causing min^m. boiling azeotropes
- Negative - deviation, causing max^m. boiling azeotropes



Case ①:

Benzene - Ethanol
system

Case ②:

Acetone - chloroform
system

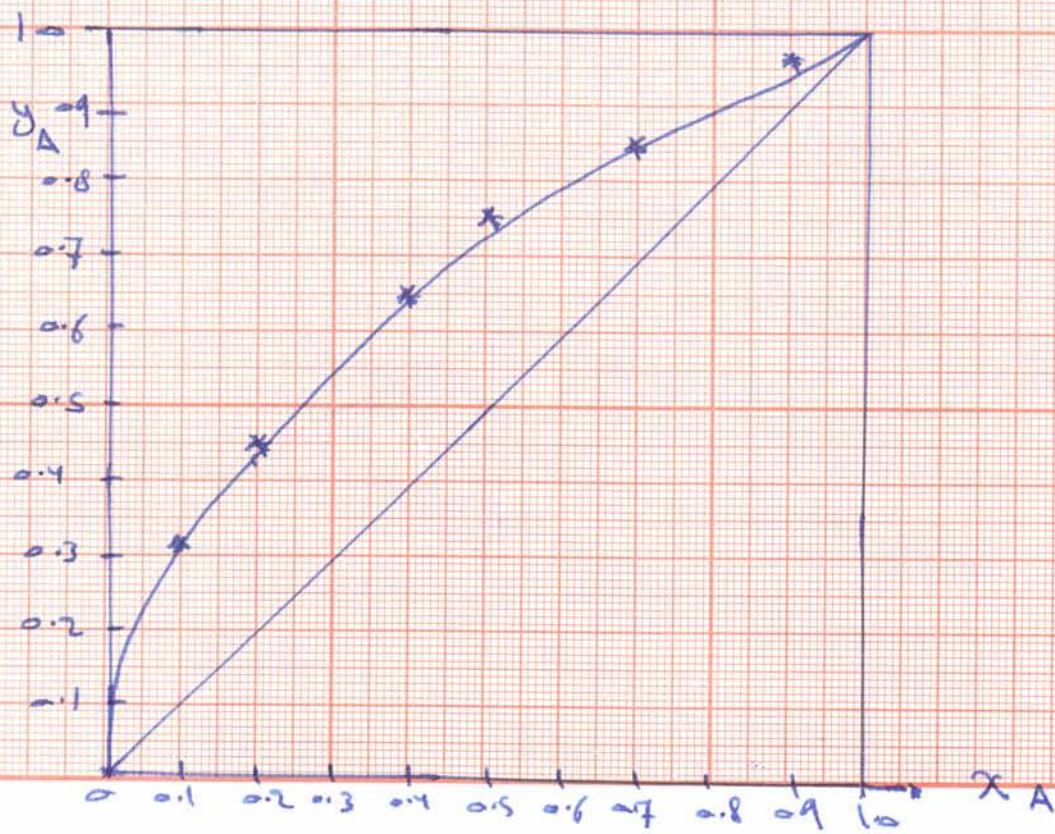
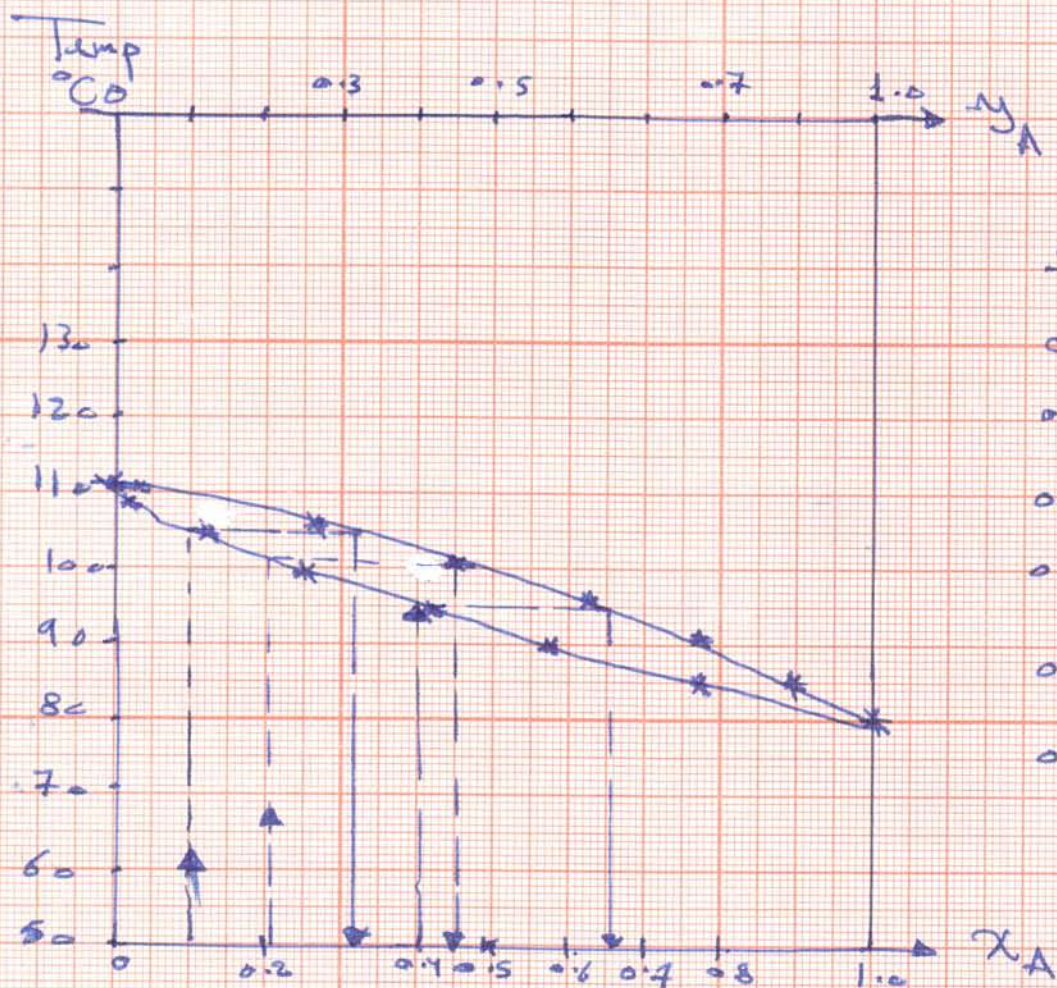
Ex. 2 - Calculate and plot the boiling point and equilibrium diagrams of the system of benzene-toluene at total press. of 1 atm. Assuming all mixtures are ideal mixtures. The vap. press. data of the system as a

<u>Temp. °C</u>	<u>P_A° (Benzene)</u>	<u>P_B° (Toluene)</u>
80.1	760	300
85.0	877	354
90.0	1016	405
95.0	1168	475
100.0	1344	557
105.0	1532	645
110.0	1748	743
110.6	1800	760

Sol. - $P_T = 1 \text{ atm} = 760 \text{ mm Hg.}$

$$x_A = \frac{P_T - P_B^\circ}{P_A^\circ - P_B^\circ}, \quad y_A = \frac{P_A^\circ \cdot x_A}{P_T}$$

<u>Temp.</u>	<u>x_A</u>	<u>y_A</u>	<u>Temp.</u>	<u>x_A</u>	<u>y_A</u>
80.1	1	1	105.0	0.12	0.24
85.0	0.782	0.90	110.0	0.016	0.038
90.0	0.581	0.77	110.6	0.0	0.0
95.0	0.411	0.63			
100.0	0.25	0.45			



Distillation Processes

Distillation processes can be classified according to:

A - Type of separation method

- 1 - Differential distillation.
- 2 - Flash or equilibrium distillation.
- 3 - Rectification distillation.

B - Number and type of component.

- 1 - Binary distillation
- 2 - Multicomponent distillation.
- 3 - Complex distillation.
- 4 - Azeotropic and extractive distillation.

C - Operating design and conditions.

- 1 - Batch distillation.
- 2 - Continuous distillation.
- 3 - Vacuum distillation.
- 4 - Steam distillation.

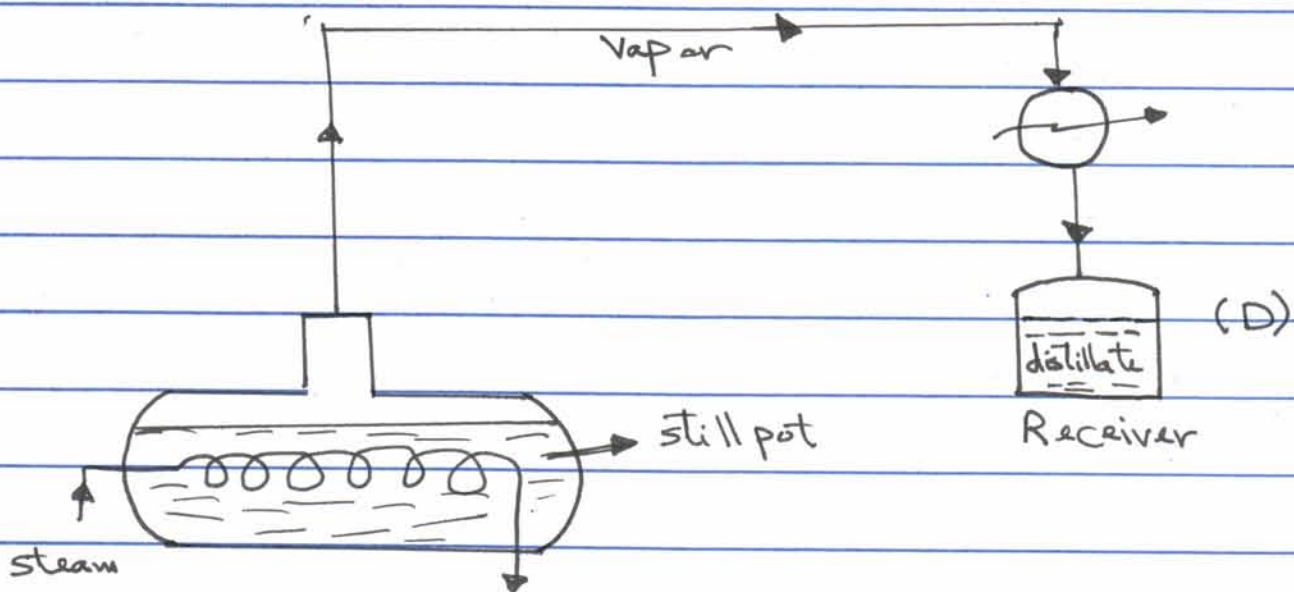
1- Differential Distillation (Batch).

Feed is charged to the still pot to which heat is supplied continuously through a steam jacket or a steam coil. As the mixture boils, it generates a vapor richer in the M.V.C. But as boiling continues, concentrations of M.V.C in the liquid decrease. It is generally assumed that equilibrium vaporization occurs in the still. The vapor is led to a condenser and the condensate or top product is collected in a receiver.

At the beginning, the condensate will be rich in the M.V., but the concentrations of the M.V. in it will decrease as the condensate is usually withdrawn intermittently having products or cuts of different concentrations.

Batch distillation is used when the feed rate is not large enough to justify installⁿ of a continuous distillation unit.

The schematic of a batch distillation setup is shown below



Batch or Differential or Rayleigh distillⁿ.

Overall M.B.:-

$$F = D + W \quad \text{--- (1)}$$

No. of moles present in the still = No. of moles in Vapor + No. of moles in residue

$$F * x = dF * y + (F - dF)(x - dx) \quad \text{--- (2)}$$

where :- F = No. of moles of a mixture in the still.

x = mole fraction in liq. phase.

y = s s s Vapor phase.

dF = amount of liquid vaporize at any time.

$(F - dF)$ = amount of residue.

$(x - dx)$ = mole fraction of residue.

$$F/x = y dF + \cancel{x dF} - x dF - F dx + \cancel{dx dF} \quad \text{Small}$$

$$F dx = (y - x) dF$$

$$\frac{dF}{F} = \frac{dx}{y-x} \Rightarrow \int_F^W \frac{dF}{F} = \int_{x_f}^{x_w} \frac{dx}{y-x}$$

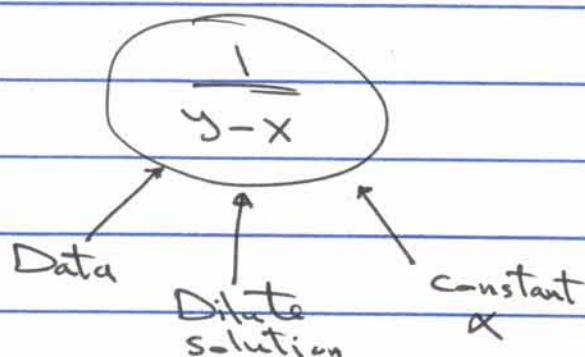
$$\ln \frac{W}{F} = \int_{x_f}^{x_w} \frac{dx}{y-x}$$

————— (3) (Rayleigh Eq.)

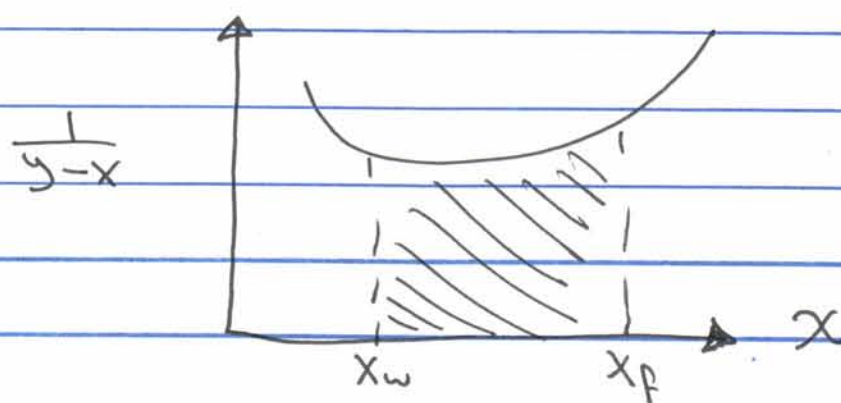
For the R.H.S term:-

the relation between (y) and (x) is an equilibrium relation, So:-

- * $y = m \cdot x$
- * $y = k \cdot x$
- * data (x, y)



(i) if the equilibrium data are known, then by plotting $\frac{1}{y-x}$ vs. (x) and getting area under curve.



(19)

② For dilute solution when

$$y = m x$$

$$\left(\frac{W}{F} \right)^{m-1} = \frac{x_w}{x_f} \quad (4)$$

③ For mixture of constant (α)

$$y = \frac{\alpha \cdot x}{1 + (\alpha - 1)x}$$

$$\therefore \int \frac{dx}{y-x} = \int \frac{dx}{\frac{\alpha \cdot x}{1 + (\alpha - 1)x} - x}$$

$$= \int \frac{1 + (\alpha - 1)x}{(\alpha - 1)(x - x^2)} dx$$

$$= \underbrace{\int \frac{dx}{x(\alpha - 1)(1 - x)}}_{\text{Solved by Partial Fractions}} + \int \frac{(\cancel{\alpha - 1})x}{x(\cancel{\alpha - 1})(1 - x)} dx$$

$\nwarrow \ln \frac{1}{1-x}$

$$\therefore \ln \left(\frac{W}{F} \right) = \frac{1}{\alpha - 1} \ln \left(\frac{x_w(1 - x_f)}{x_f(1 - x_w)} \right) + \ln \left(\frac{1 - x_f}{1 - x_w} \right)$$

\nwarrow for constant (α) $\nearrow (s)$

②

Ex. (1) :- 1000 kg of mixture containing 60% wt of ethanol and 40% wt of water, is subjected to batch distillⁿ. (1 atm) to produce a residue concentration of 5% wt. ethanol. Calculate

- 1- Amount of distilled and residue (D, W).
 - 2- Composition of total distilled obtained (x_D or y_D)
- V.L.E. for (ethanol-H₂O) system at 1 atm is :-

\bar{x} :	0.025	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7
\bar{y} :	0.225	0.36	0.516	0.655	0.71	0.74	0.76	0.78	0.81

Sol. :- $F = 1000 \text{ kg}$, $x_f = 0.6$, $x_w = 0.05$

$$\ln \frac{W}{F} = \int_{x_f}^{x_w} \frac{dx}{y-x}$$

From equilibrium data : $(1 / (\bar{y} - \bar{x}))$ was calculated

$(1 / (\bar{y} - \bar{x}))$: 5.0 3.22 2.4 2.2 2.4 2.9 3.7 5.8 9.1

To plot, we will use simple rule :-

Let $n=6$, $h = (x_f - x_w) / n \Rightarrow h = \frac{0.6 - 0.05}{6} = 0.09$

Area under-curve from (0.6) to (0.05)

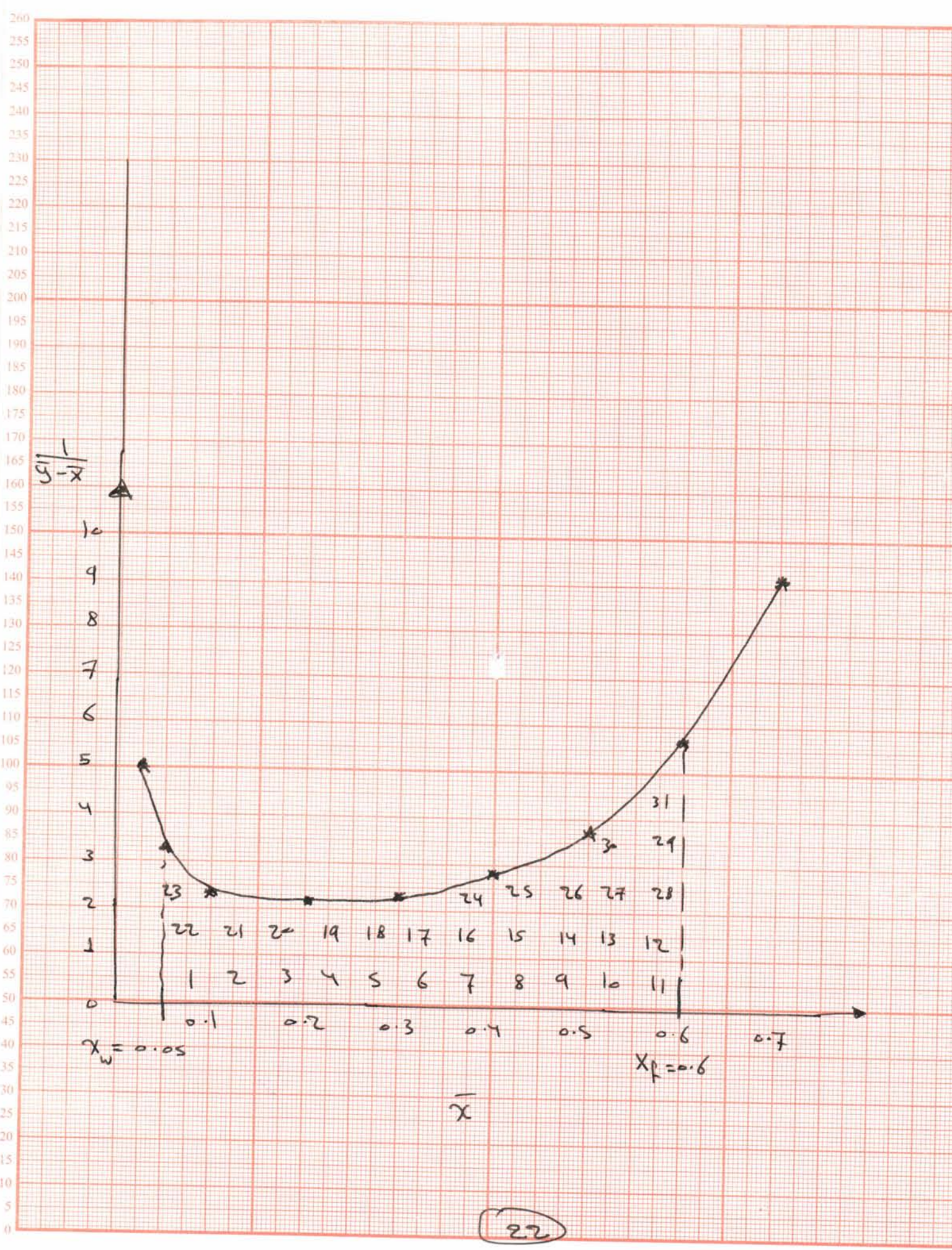
$$\text{Area} = (h/3) * \sum y$$

$$A = \frac{0.09}{3} * 31 \Rightarrow$$

$$A = 0.93 = \int \frac{dx}{\bar{y} - \bar{x}}$$

$$\ln \frac{W}{F} = -0.93 \Rightarrow \frac{W}{1000} = 0.394$$

(21)



$$W = 394 \text{ kg}$$

$$\text{Distillate (D)} = 1000 - 394 = 606 \text{ kg}$$

2- To find distillate composition:-

$$1000 \times 0.6 = 606 \times X_D + 394 \times 0.05$$

$$600 = 606 \times X_D + 19.7$$

$$606 X_D = 580.3$$

$$X_D = 0.957$$

Ex. (2) - A charge of 50 kmol of a mixture of benzene and chlorobenzene, having 55% of the L.V. is to be batch distillation.

- If 25 moles of the solution is vaporized and condensed as the distillate, Calculate the concⁿ. of accumulated distillate.

- If the concentration of the accumulated product is found to be 72 mol% benzene, calculate its amount.

$$\text{Take } (X_{\text{Benzene}}) = 4.15$$

Sol. :- $F = 50 \text{ kmol}$

$$X_f = 1 - 0.55 \Rightarrow x_f = 0.45$$

① 25 Mole to be vaporized $\therefore D = 25 \text{ kmol}$

$$F = D + W \Rightarrow W = 25 \text{ kmol}$$

(23)

To find x_D ($y_{av.}$)
using eqn. (5) to find x_w

$$\ln\left(\frac{W}{F}\right) = \frac{1}{\alpha-1} \ln\left(\frac{x_w(1-x_F)}{x_F(1-x_w)}\right) + \ln\left(\frac{1-x_F}{1-x_w}\right)$$

$$\ln\left(\frac{25}{50}\right) = \frac{1}{4.15-1} \ln\left(\frac{x_w(1-0.45)}{0.45(1-x_w)}\right) + \ln\left(\frac{1-0.45}{1-x_w}\right)$$

$$-0.7 = 0.32 \ln\left(\frac{0.55 x_w}{0.45(1-x_w)}\right) + \ln\left(\frac{0.55}{1-x_w}\right)$$

$$\therefore x_w = 0.218$$

From M.B., x_D can be found.

$$F \cdot x_F = D \cdot x_D + W \cdot x_w$$

$$50 \times 0.45 = 25 \times x_D + 25 \times 0.218$$

$$\therefore x_D = y_{av.} = 0.682$$

⑥ if $y_{av.} = x_D = 0.72$, find (D)

$$F = D + W \Rightarrow 50 = D + W \Rightarrow \boxed{D = 50 - W}$$

From M.B.:-

$$F \cdot x_F = D \cdot x_D + W \cdot x_w$$

$$50 \times 0.45 = D \times 0.72 + W \cdot x_w$$

sub. for
(D)

$$22.5 = 0.72(50 - W) + W \cdot x_w$$

$$\therefore W = \frac{13.5}{0.72 - x_w}$$

sub. for (W) in eq. (5) :-

$$\ln \frac{(13.5 / (0.72 - x_w))}{50} = \frac{1}{4.15 - 1} \ln \left(\frac{x_w (1 - 0.45)}{0.45 (1 - x_w)} \right) + \ln \left(\frac{1 - 0.45}{1 - x_w} \right)$$

$$\therefore x_w = 0.309$$

$$\therefore W = 32.85 \text{ kmol}$$

$$\therefore D = F - W$$

$$= 50 - 32.85$$

$$D = 17.15 \text{ kmol}$$

Ex. (3) :- A mixture of 100 mole containing 5% mole of n-pentane and rest is n-heptane, is distilled under differential conditions at 1 atm, until (40 mole) is distilled. What are the compositions of liquid left and total vapour distilled?

X:	1.0	0.867	0.594	0.398	0.25	0.145	0.05	0.0
y:	1.0	0.987	0.925	0.836	0.7	0.521	0.27	0.0

Sol. :- Overall M.B

$$F = D + W \Rightarrow 100 = 40 + W$$

$$W = 60 \text{ mole.}$$

$$\ln \frac{F}{W} = \int_{x_w}^{x_f} \frac{dx}{y^* - x} \Rightarrow \ln \frac{100}{60} = \int_{x_w}^{0.5} \frac{dx}{y^* - x}$$

$$0.51 = \int_{x_w}^{0.5} \frac{dx}{y^* - x}$$

assume $x_w = 0.3$, y^* from the Fig.

assume

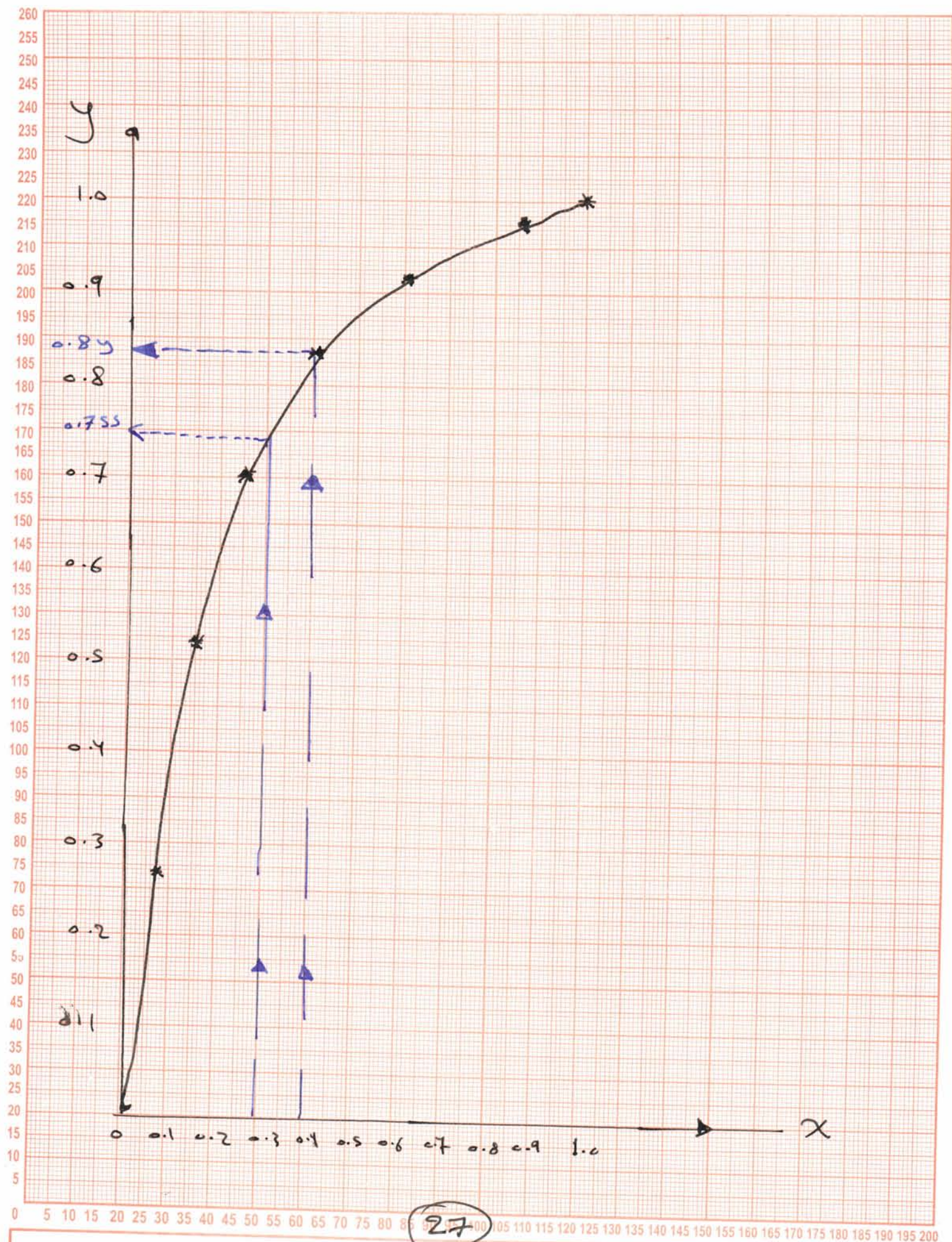
X:	0.3	0.35	0.4	0.45	0.5
----	-----	------	-----	------	-----

From Fig.

y:	0.755	0.8	0.84	0.86	0.89
----	-------	-----	------	------	------

$y^* - x$:	0.455	0.45	0.44	0.41	0.39
-------------	-------	------	------	------	------

$\frac{1}{y^* - x}$:	2.197	2.22	2.27	2.44	2.56
	(F ₀)	(F ₁)	(F ₂)	(F ₃)	(F _n)



27

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الموضوع:

المادة:

$$A = \frac{h}{3} [F_0 + 4 \sum_{\text{odd}} + 2 \sum_{\text{even}} + F_n]$$

$$A = \frac{0.05}{3} [2.197 + 4 \times (2.22 + 2.44) + 2 \times 2.27 + 2.56]$$

$$A = 0.47 \neq 0.51$$

Assume another $x_w = 0.277$

$x:$	0.277	0.373	0.389	0.444	0.5
$y:$	0.73	0.78	0.815	0.855	0.889
$y-x:$	0.453	0.407	0.426	0.411	0.389

$$\frac{1}{y-x} : 2.207 \quad 2.23 \quad 2.3474 \quad 2.433 \quad 2.57$$

$$h = \frac{0.5 - 0.277}{5} = 0.045$$

$$A = \frac{0.045}{3} [2.207 + 4 \times (2.23 + 2.433) + 2 \times 2.347 + 2.57]$$

$$A = 0.42 \neq 0.51$$

$\therefore x_w = 0.3$ we take this value

$$F \times x_p = W \times x_w + D \times y_D$$

$$100 \times 0.5 = 60 \times 0.3 + y_D \times 40$$

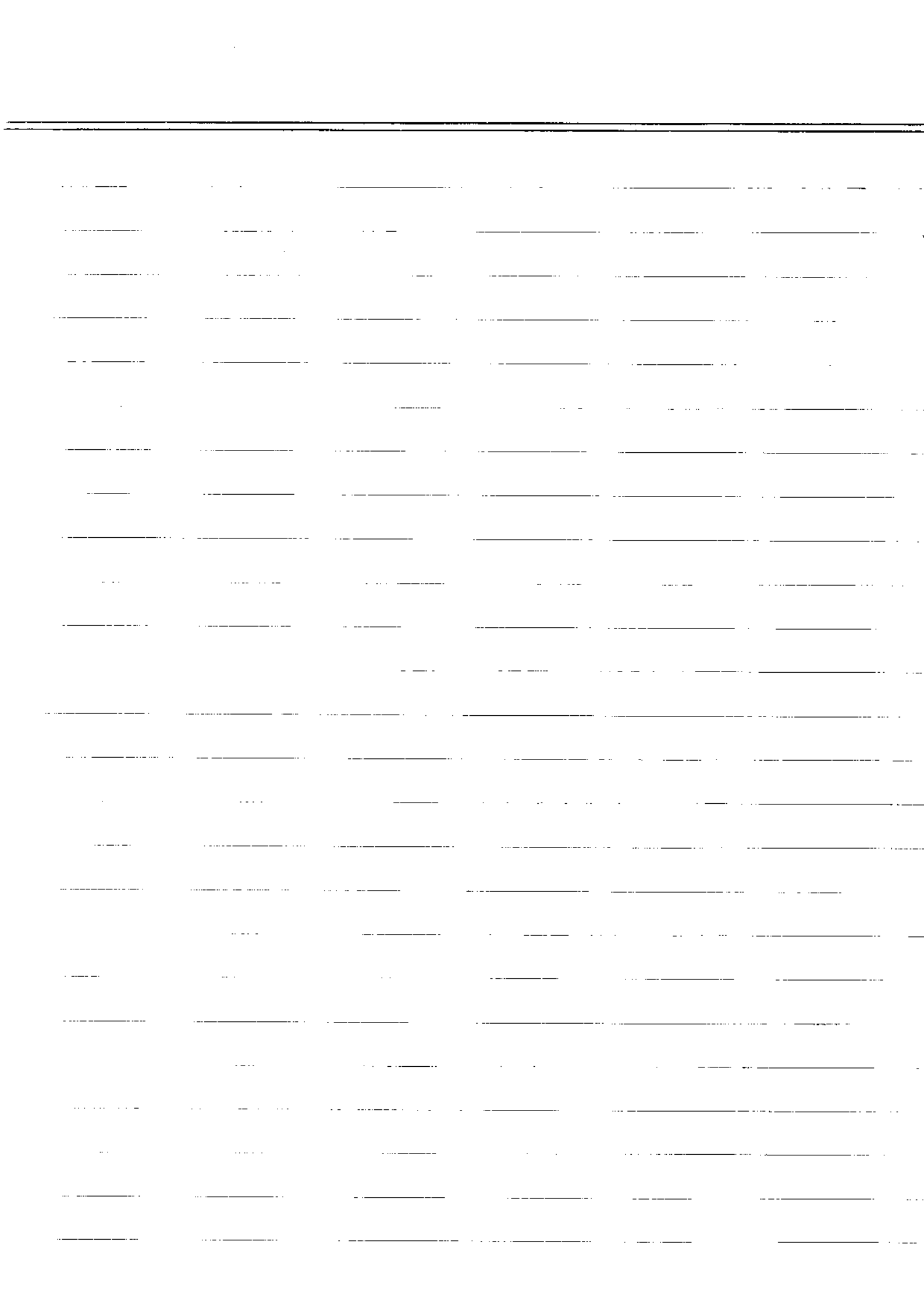
$$y_D = 0.8$$

2- Flash (equilibrium) or Integral Distillation

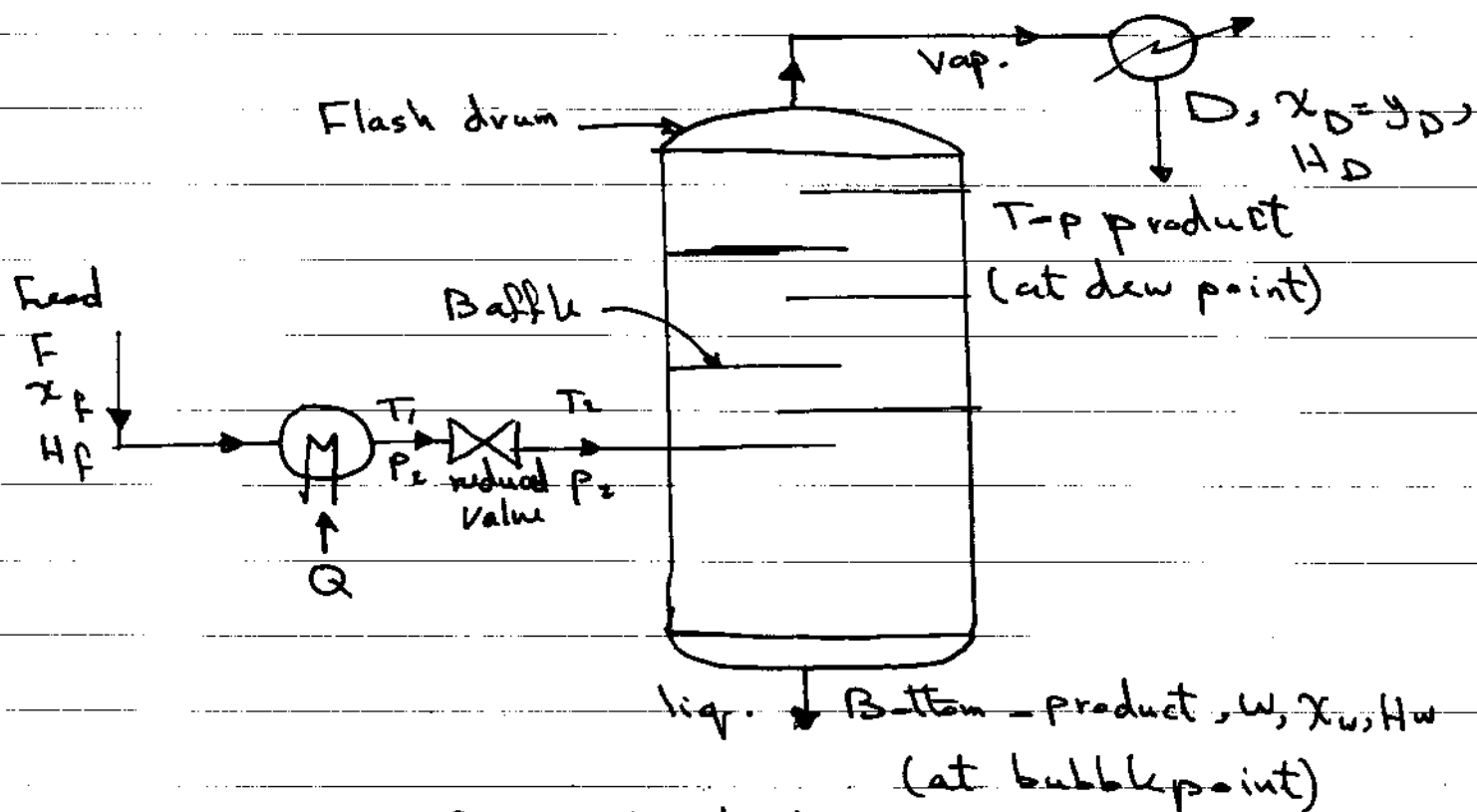
If a sufficiently hot liquid mixture is throttled into a vessel, a part of it will vaporize. The vapour produced will be richer in the M.V.C, and thus partial separation of the desired components will be achieved. This is called "Flash vaporization or equilb^m vaporization". The feed is first heated in a heat exchanger under pressure and then led to a flash drum by throttling where partial vaporization of the feed occurs under reduced pressure. The vapour leaves the drum at the top and is condensed to get the top product. The fraction having a higher concentration of the L.V. leaves the drum as a liquid bottom product. It is often assumed that the liquid and vapour streams leaving the flash drum are in equilibrium, therefore the process is also called "equilibrium - vaporization".

Assumption:

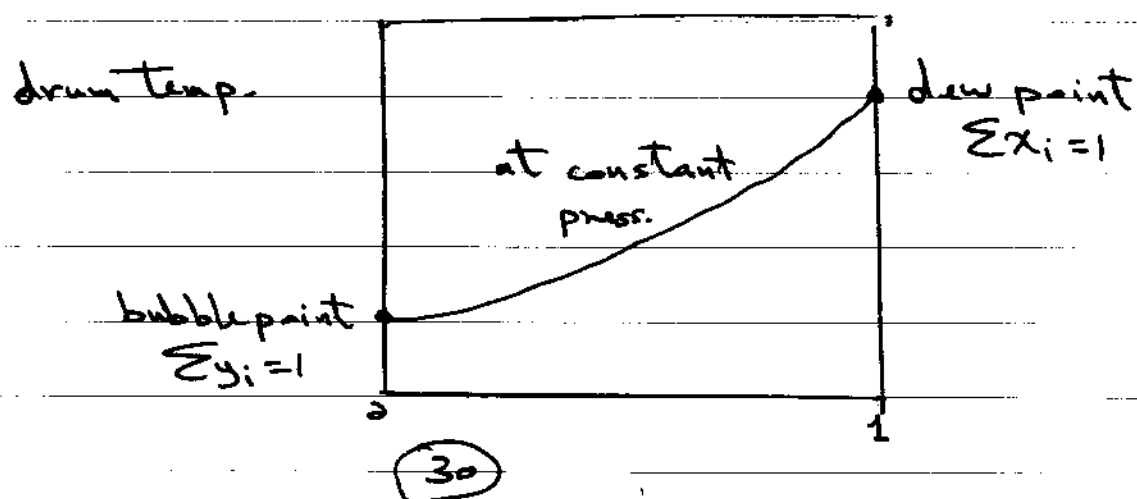
- 1- large contact surface between liq. and vapor, and vapor after forming should contact liquid till equilibrium occurs



- 2- Temp. and press. of vapour out = Temp. and press. of liquid out (at equilibrium).
- 3- All components exist in liquid should be exist in vapour, for vapour $\sum y_i = 1$ and liquid $\sum x_i = 1$.



"Schematic of Flash distillation Unit"





Overall M.B =

$$F = D + W$$

$$F \cdot x_F = D \cdot y_D + W \cdot x_W$$

$$F \cdot x_F = D \cdot y_D + (F - D) x_W$$

$$x_F = \frac{D}{F} y_D + \frac{(F - D)}{F} x_W$$

$$x_F = f_v \cdot y_D + (1 - f_v) x_W$$

$$y_D = \frac{x_F}{f_v} - \frac{1 - f_v}{f_v} x_W \quad \text{operating line equation for binary system.}$$

$$\text{Slope} = - \frac{1 - f_v}{f_v}, \quad \text{intercept with (y) axis} = \frac{x_F}{f_v}$$

(*) For multicomponent system, equilib^m relation will be:-

$$y_i = k_i x_i \Rightarrow x_i = y_i / k_i \Rightarrow (\text{sub. for } x_W) :=$$

$$y_i = \frac{x_{Fi}}{f_v} - \frac{1 - f_v}{f_v} \frac{y_i}{k_i}$$

$$\therefore y_i = \frac{k_i x_{Fi}}{f_v (k_i - 1) + 1} \quad \text{for multi component.}$$

Assume a value for $(f_v) < 1.0$, and calculate (y_i) for each component.



if $\sum y_i = 1$ then value assumed for (P_v) is correct.

* For ideal solutions where:

$$P_A = P_A^\circ \cdot x_A \quad \text{and} \quad K_A = \frac{y_A}{x_A}$$

then:-

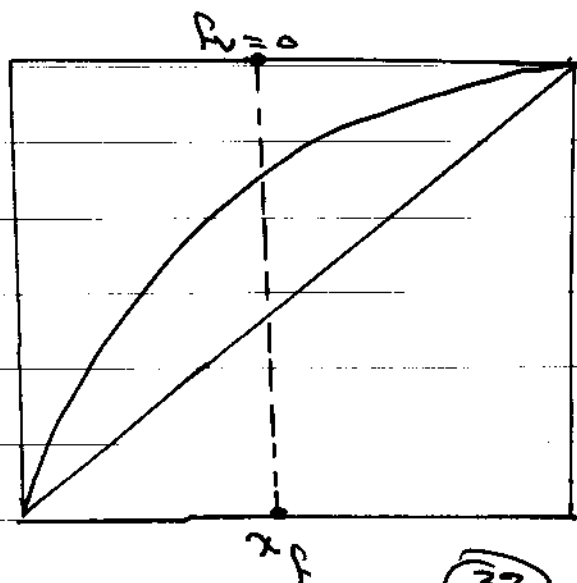
$$y_i = \frac{x_{Fi}}{P_v + \frac{P_T}{P_i^\circ} (1 - P_v)}$$

For ideal solutions

* if there is no-vaporization, then $D=0$

$$P_v = \frac{D}{F} = 0, \text{ then slope} = \infty$$

\therefore Vertical line



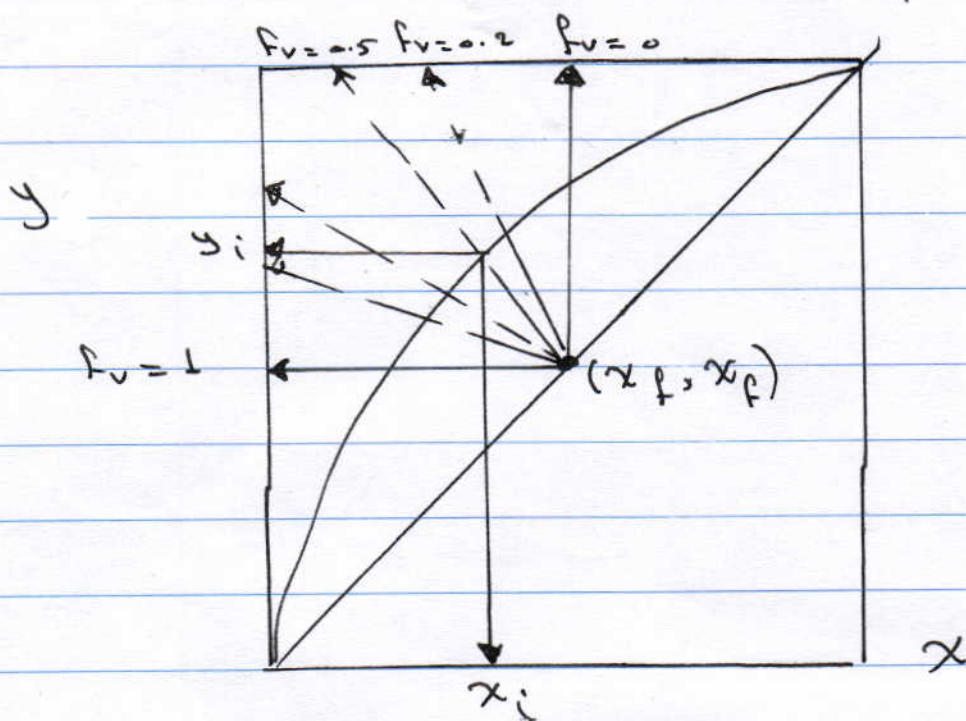
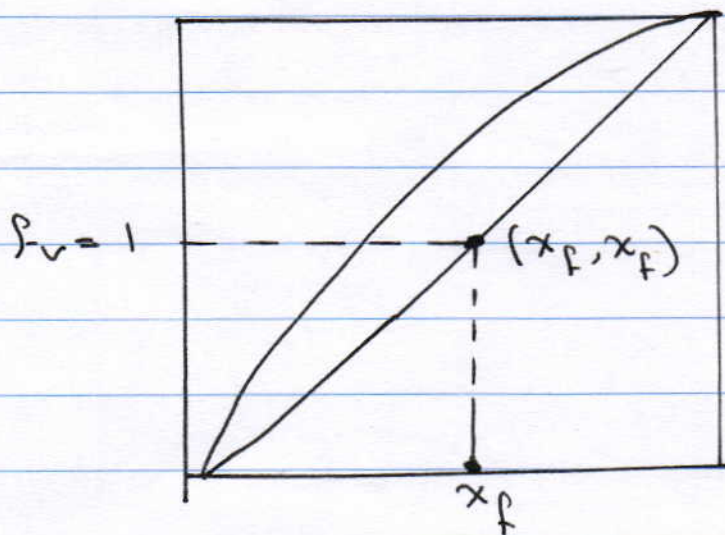
\therefore composition of product = composition of feed.

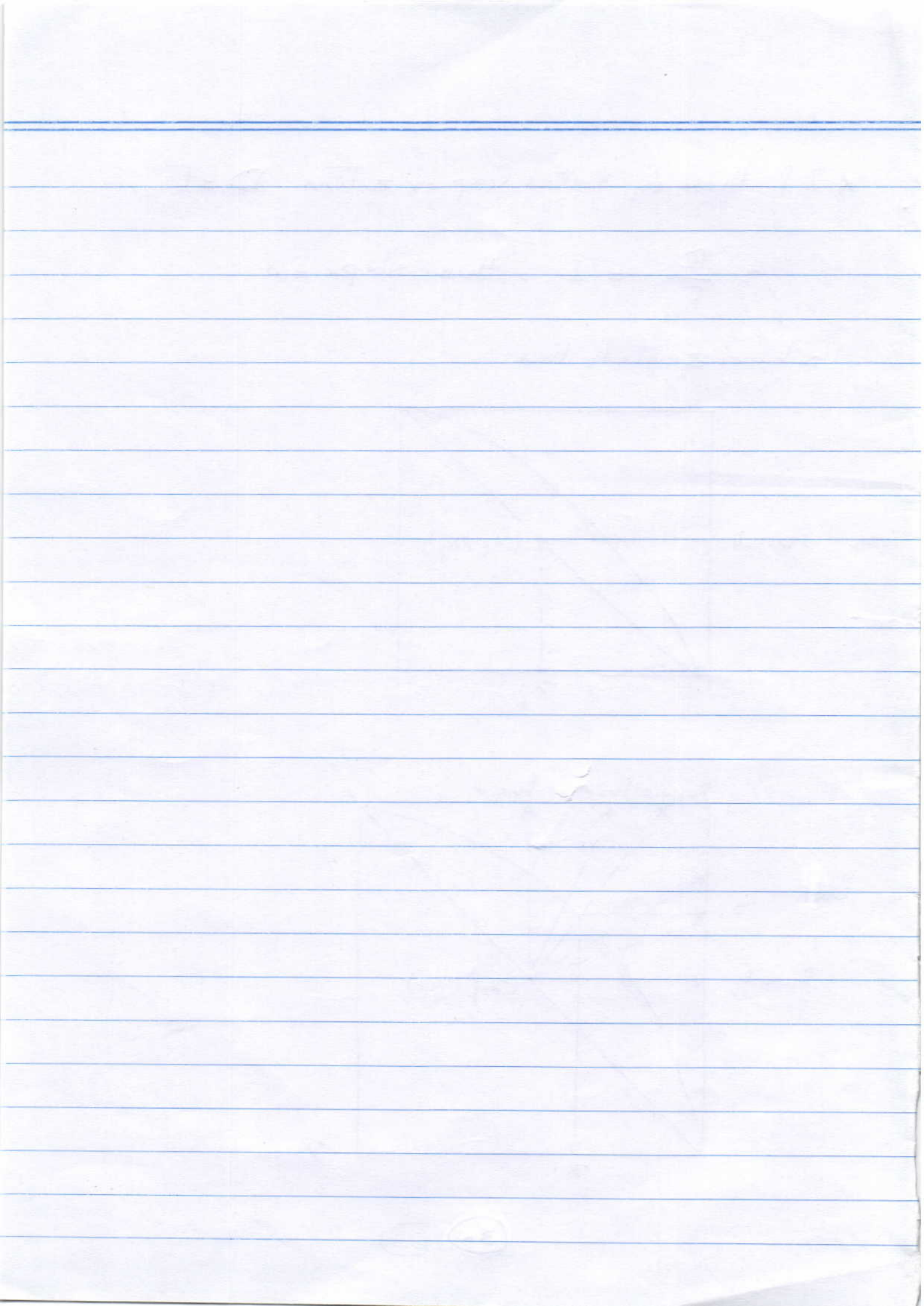


* If there is total vaporization $D \leq F$

$$\therefore \frac{D}{F} = 1, \text{ then slope} = 0$$

= horizontal line





Ex. (1) = What fraction of a liquid mixture containing 10% mole propane, 65% mole n-butane, 25% mole n-pentane, would be vaporized in a flash vaporizⁿ process at 40°F and press. of 600 mm Hg. Assume liquid solution an ideal and vapour as an ideal.

Vapour pressure at 40°F is:-

$$P_{\text{propane}}^{\circ} = 3800 \text{ mm Hg}, \quad P_{\text{n-but.}}^{\circ} = 820 \text{ mm Hg}$$

$$P_{\text{n-pent.}}^{\circ} = 140 \text{ mm Hg}$$

Sol. = For ideal solution $\Rightarrow y_i = \frac{x P_i}{f_v + \frac{P_i}{P_i^{\circ}} (1 - f_v)}$

Let $f_v = 0.3$

$$y_1 = \frac{0.1}{0.3 + \frac{600}{3800} (1 - 0.3)} = 0.244$$

$$y_2 = \frac{0.65}{0.3 + \frac{600}{820} (1 - 0.3)} = 0.8$$

$$y_3 = \frac{0.25}{0.3 + \frac{600}{140} (1 - 0.3)} = 0.076$$

$$\therefore \sum y_i = 1.119$$

Assume $f_v = 0.49$

$$y_1 = 0.17, \quad y_2 = 0.75, \quad y_3 = 0.08$$

$$\therefore \sum y_i = \underline{\underline{1.0}}$$

In the first part of the paper, the author discusses the
 the role of the author in the writing process.
 The author is not a passive observer of the writing process,
 but an active participant who is constantly making choices
 and decisions about what to write and how to write it.
 The author's role is to create a narrative that is both
 informative and engaging for the reader.

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 informative and engaging for the reader.

Ex. (2) :- It is desired to separate a binary mixture by distillation, the composition of the mixture 0.5 mol%. Calculate the fraction vaporized (f_v) required to obtain a top product composition 0.75 mol. fraction when using flash distillation. Equilibrium curve $y = 1.2x + 0.3$ with the range of liquid composition (0.3 to 0.8)

Sol. :- $y = 1.2x + 0.3$

∴ vapour in contact with liquid then :-

$$y_D = 1.2 x_w + 0.3$$

For $y_D = 0.75$

$$\therefore 0.75 = 1.2 x_w + 0.3 \Rightarrow x_w = 0.375$$

∴ We have now two points (0.75, 0.375) and (0.5, 0.5)

∴ Slope of (f_v) line can be found.

$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{0.75 - 0.5}{0.375 - 0.5} = -2$$

$$\therefore \text{slope} = -\left[\frac{1 - f_v}{f_v}\right] = -2$$

$$\therefore \underline{\underline{f_v = 0.33}}$$

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Ex.(3) :- It is required to vaporize 50% of the following liquid mixture :-

60% Benzene, 30% Toluene, 10% Xylene.

What would be the composition of vapour if this mixture is subjected to flash distillation process at 1 atm, assuming an ideal liquid solution and gas.

Vapour - pressure data as follows :-

<u>Temp. °C</u>	<u>P_{Benz.}</u>	<u>P_{Tol.}</u>	<u>P_{Xyl.}</u>
60		139	51
70	540	206	78
80	756	287	116
90	1008	404	168
100	1338	557	238
110	1740	741	330
120	2215	990	449

Sol. ; Hints :-

- 1 - Take any temp. given, then multiply mole fraction (X_i) by each (P_i°) for each component and find (P_i) to each.
- 2 - make summation (Σ) for (P_i) to find total press.
- 3 - If this total press. = total press. given = 760 mm Hg or (1 atm), then (T) chosen is correct.

gives a lot of information at a glance (2) x 3

... and it is very easy to read and understand
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1929	1930	1931	1932
12	121		12
25	25	25	25
35	35	35	35
45	45	45	45
55	55	55	55
65	65	65	65
75	75	75	75
85	85	85	85
95	95	95	95
105	105	105	105
115	115	115	115
125	125	125	125

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4- From this (T) found, ($P_A^\circ, P_B^\circ, P_C^\circ$) are known.

5- Now, assume many values for (P_v) and find y_i , according to :-

$$y_i = \frac{x_i}{P_v + \frac{P_T}{P_i^\circ} (1 - P_v)}$$

6- Plot (y_i) Vs (P_v), and find (P_v) at ($y_i = 1$)

if we take (assume) $T = 90$

1- find $P_1^\circ, P_2^\circ, P_3^\circ$ from fig.

2- then calculate P_i from

$$P_1 = P_1^\circ \cdot x_1 \leftarrow 0.6$$

$$P_2 = P_2^\circ \cdot x_2 \leftarrow 0.3$$

$$P_3 = P_3^\circ \cdot x_3 \leftarrow 0.1$$

3- then if $\sum P_i \approx 1 \text{ atm}$, then $\therefore T_{\text{ass.}} = \text{correct}$

4- then Take, $P_1^\circ, P_2^\circ, P_3^\circ$, \Rightarrow Assume $P_v = 0.4$

5- find y_1, y_2, y_3 from $y_i = \frac{x_i}{P_v + \frac{P_T}{P_i^\circ} (1 - P_v)}$

6- if $\sum y = 1$ $\Rightarrow P_{v, \text{ass.}} = \text{correct}$

... and ... (T) ...

$$(v-1) \frac{g}{f} = v$$

(12.1) to (12) ...

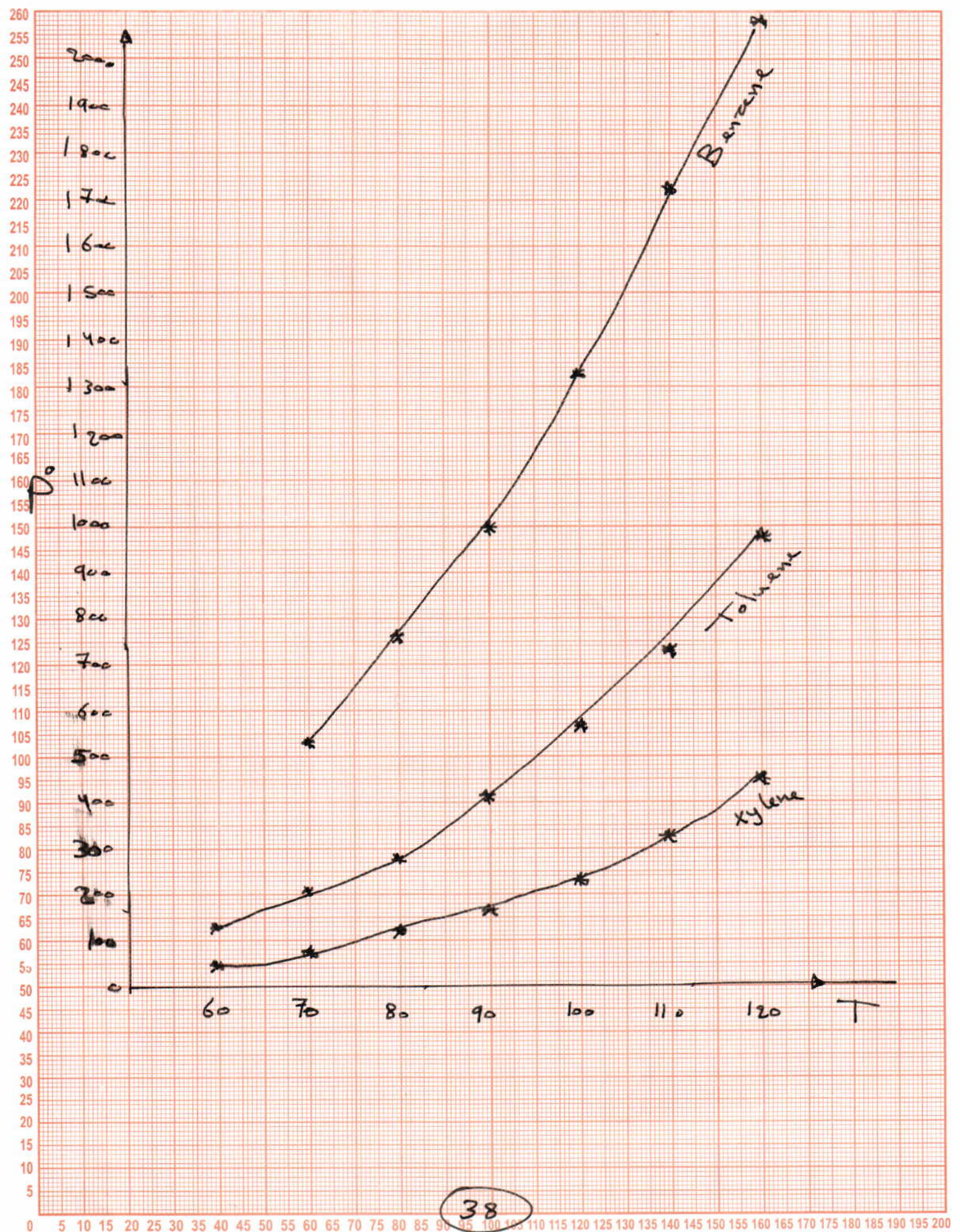
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... ..

... ..

$$(v-1) \frac{g}{f} = v$$

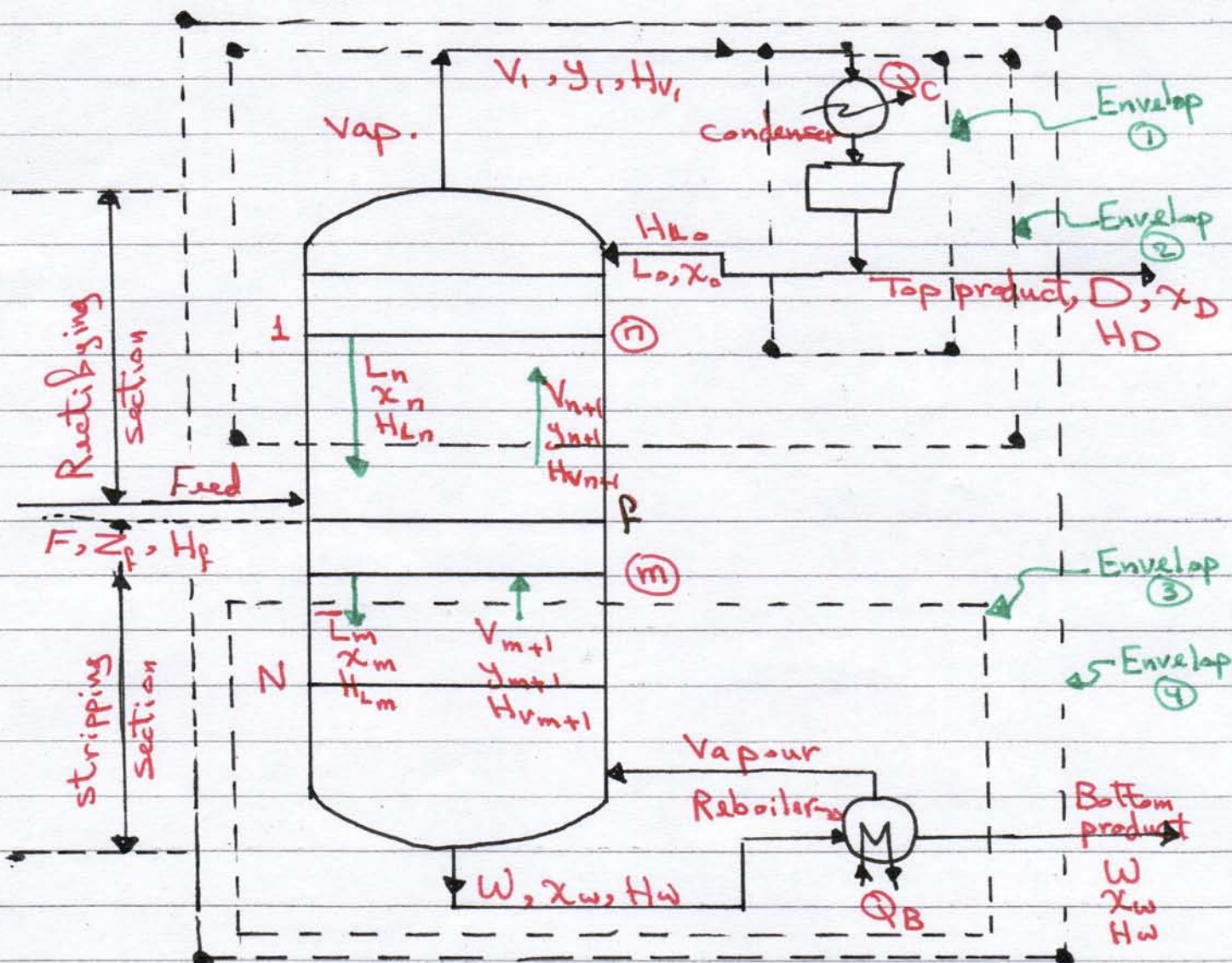
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③ Continuous - multistage - Fractionation of binary mixture

Separation of a volatile liquid mixture to relatively pure products is very often done in a continuous fractionating column. A continuous column is much more effective than multistage flash or batch distillation. A tray column and flow rates and the concⁿ. of the vapour and the liquid phases at different trays are shown in Fig. :-



- * For a binary liquid mixture, the feed (liquid, vapour or two-phase mixture) containing component A and B. (A) is the M.V.C enters the column at a suitable location. The liquid stream flows down the column from one tray to the next lower tray, the vapour stream flows up bubbling through the liquid on the trays.
- * the vapour from the top tray is condensed and the condensate is collected in a reflux drum. A part of this liquid is drawn as the "top-product" and the other part is fed back to the top tray (no. 1) as reflux. The top product contains the M.V.C (A) and a little of the L.V.C. (B).
- * The liquid from the bottom tray goes to a reboiler where it is partly vaporized, the vapour is fed back to the tower and the liquid part is continuously withdrawn as the bottom product. The bottom product is rich in the L.V.C. (B) and has a small amount of (A) in it.
- * Transport of the M.V.C (A) occurs from the liquid to the vapour, while transport of the L.V.C (B) occurs from the vapour to the liquid phase. Thus, a distillⁿ. column involves counter-diffusion of the components (not necessarily E.M.D). As the vapour flows up, it becomes richer in (A), similarly, the liquid becomes richer in the L.V.C (B) as it flows down the column.

* In the section of the column above the feed point, the conc. of the M.V.C. is larger than that in the feed. This means that the vap. is enriched or purified by discarding the L.V.C. (B) into the down flowing liquid. So the section above the feed tray is called the "rectifying or enriching" section.

For the section below, the feed tray, the M.V.C. is removed or stripped out of the liquid, so this is called the "stripping section".

- * Variables, parameters and factors involved in the design of a trayed distill. tower.
- The flow rate, composition and state of the feed.
- The required degree of separation.
- The reflux ratio and the condition of the reflux.
- The operating pressure and the allowable press. drop across the column.
- Tray type and column internals.

Material and Energy Balance Equations

Reflux ratio can be defined as:-

$$R = \frac{L_o}{D} \Rightarrow \boxed{L_o = D \cdot R} \quad \text{--- (1)}$$

* Now consider envelope (1) in fig. (1), enclosing the condenser and reflux:-

- Total material balance:-

$$V_1 = L_o + D = D \cdot R + D \Rightarrow \boxed{V_1 = D(R+1)} \quad \text{--- (2)}$$

- Component (A) balance:-

$$V_1 y_1 = L_o x_o + D \cdot x_D \quad \text{--- (3)}$$

- Energy balance:-

$$V_1 \cdot H_{V_1} = L_o H_{L_o} + D \cdot H_D + Q_c \quad \text{--- (4)}$$

From eqn^s (2) and (4):-

$$D(R+1) H_{V_1} = L_o H_{L_o} + D \cdot H_D + Q_c$$

$$\therefore \boxed{Q_c = D[(R+1) H_{V_1} - R \cdot H_{L_o} - H_D]} \quad \text{--- (5)}$$

↳ Condenser heat load eqnⁿ.

* For envelope (2), enclosing a part of the rectifying and the condenser.

- Total material balance

$$V_{n+1} = L_n + D \quad \text{--- (6)}$$

- Component (A) balance:-

$$V_{n+1} \cdot y_{n+1} = L_n \cdot x_n + D \cdot x_D \quad \text{--- (7)}$$

- Energy-balance:-

$$V_{n+1} \cdot H_{V_{n+1}} = L_n \cdot H_{L_n} + D \cdot H_D + Q_c \quad \text{--- (8)}$$

* For envelope (3), enclosing a part of stripping section and the reboiler.

- Total material balance:-

$$\bar{L}_m = \bar{V}_{m+1} + W \quad \text{--- (9)}$$

- Component (A) balance:-

$$\bar{L}_m \cdot x_m = \bar{V}_{m+1} \cdot y_{m+1} + W \cdot x_w \quad \text{--- (10)}$$

- Energy balance:-

$$\bar{L}_m \cdot H_{L_m} + Q_B = \bar{V}_{m+1} \cdot H_{V_{m+1}} + W \cdot H_w \quad \text{--- (11)}$$

⊛ Envelope (4) the entire column:-

- Total material balance:-

$$F = D + W \quad \text{--- (12)}$$

- Component (A) balance:-

$$F \cdot z_F = D \cdot x_D + W \cdot x_w \quad \text{--- (13)}$$

- Energy balance:-

$$F \cdot H_F + Q_B = D \cdot H_D + W \cdot H_w + Q_c \quad \text{--- (14)}$$

* All the above equations can be solved algebraically to determine the number of ideal trays required.

Determination of the number of trays using McCabe-Thiele method "Graphical-method"

The McCabe-Thiele method involves a graphical solution of the material balance equations, together with the equilibrium relation or equilibrium data.

Assumptions :-

1 - The most important assumption, is that the molar rate of overflow of the liquid from one tray to another is constant over any section.

$L_0 = L_1 = L_2 \dots L_m = L = \text{constant}$ in rectifying section.

$\bar{L}_m = \bar{L}_{m+1} = \dots \bar{L}_N = \bar{L} = \dots$ in stripping section

2 - Heat loss from the column is negligible.

It is to be noted that if there is heat loss or gain, there will be a accompanying condensation or vaporization within the column, and the flow rates will vary along the column as a result.

The major steps of the graphical construction in this method are :-

- (*) The equil^m. curve using the available data.
- (*) The operating lines for rectifying and stripping sections.
- (*) The steps between the equil^m. and operating lines to find out the number of ideal plates and trays.

* The Rectifying section:-

From eq. (7) :-

$$V \cdot y_{n+1} = L \cdot x_n + D \cdot x_D \quad \text{--- (15)}$$

From eq. (1) :-

$$R = \frac{L}{D} = \frac{L}{D}$$

$$\therefore (R+1) = \frac{V}{D} \quad \text{from eqn. (2)}$$

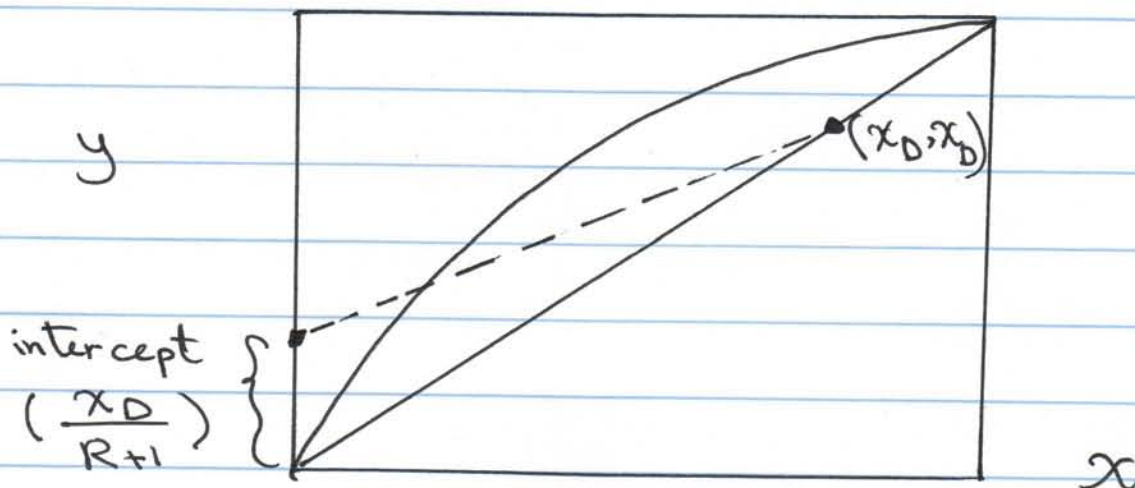
$$y_{n+1} = \frac{L}{V} x_n + \frac{D}{V} x_D$$

$$= \frac{L/D}{L/D} x_n + \frac{x_D}{V/D}$$

$$\therefore \boxed{y_{n+1} = \frac{R}{R+1} x_n + \frac{x_D}{R+1}} \quad \text{--- (16)}$$

Eq. (16) is straight line eq. with slope = $\frac{R}{R+1}$ and intercept $\left(\frac{x_D}{R+1}\right)$ on the y-axis.

To plot the line use point (x_D, x_D) on the diagonal and the intercept term, rather than using the slope.



* The stripping section :-

From eq. (16) :-

$$\bar{L} \cdot x_m = \bar{V} \cdot y_{m+1} + W \cdot x_w \quad \text{--- (17)}$$

Putting $\bar{V} = \bar{L} - W \rightarrow \text{T.M.B}$ $\bar{L} = \bar{V} + W$

$$\therefore y_{m+1} = \frac{\bar{L}}{\bar{V}} x_m - \frac{W}{\bar{V}} x_w$$

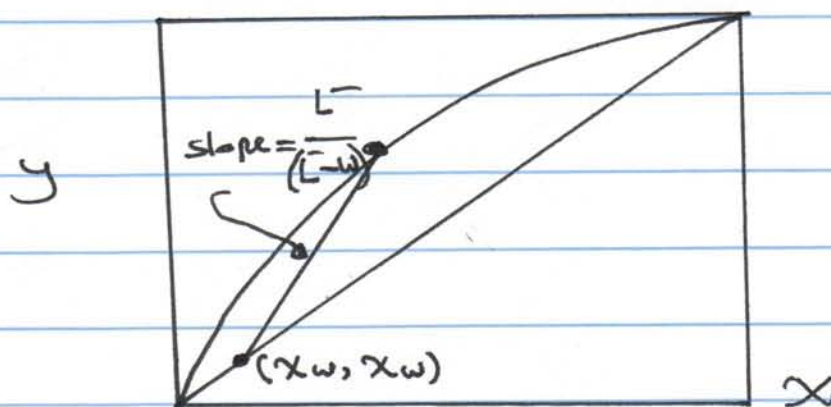
$$y_{m+1} = \frac{\bar{L}}{\bar{L} - W} x_m - \frac{W}{\bar{L} - W} x_w \quad \text{--- (18)}$$

This is the eq. for the stripping section, passes through point (x_w, x_w) , with slope equal to $\bar{L}/(\bar{L} - W)$.

Similar to reflux ratio defined in a rectified section, we may define a quantity called "Boil-up-ratio" (R_v), for this section.

$$R_v = \frac{\text{moles of vap. leaving the reboiler per hour.}}{\text{moles of liq. drawn as the bott. product per hour.}}$$

$$R_v = \frac{\bar{V}}{W} \quad \text{--- (19)}$$



(46)

Ex. (1) :- (How to draw operating lines for both sections)

A mixture of benzene and toluene containing 40 mol% benzene is to be separated continuously in a tray tower at a rate of 200 kmol/hr. The top product should have 95 mol% of benzene and the bottom must not contain more than 4 mol% of it. The reflux is a saturated liquid, and the reflux ratio of 2.0 is maintained.

The feed is a saturated liquid (at its bubble point). Obtain and plot the operating lines for both sections. What is the boil-up ratio?

The (V.L.E) data at 101.3 kPa are :-

x:	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
y:	0	0.21	0.38	0.51	0.63	0.72	0.79	0.85	0.91	0.96	1.0

Sol. $F = 200 \text{ kmol/hr}$, $z_F = 0.4$, $x_D = 0.95$, $x_W = 0.04$, $R = 2$

Total M.B.:-

$$F = D + W \Rightarrow 200 = D + W \Rightarrow W = 200 - D$$

Benz. M.B.:-

$$F \cdot z_F = D \cdot x_D + W \cdot x_W$$

$$200 \times 0.4 = D \times 0.95 + W \times 0.04$$

$$80 = 0.95D + 0.04(200 - D)$$

$$\therefore \underline{D = 79.12 \text{ kmol/hr}}, \quad \underline{W = 120.9 \text{ kmol/h}}$$

$$R = \frac{L_o}{D} = 2 \Rightarrow L_o = 158.24 \text{ kmol/hr}$$

$$V_1 = D(R+1) \Rightarrow V_1 = 237.3 \text{ kmol/hr}$$

$V_1 = V$ Feed is liquid, and vap. rate remain constant in Rect. section.

In stripp. section :-

$$\bar{L} = L_0 + 200$$

$$\bar{L} = 158.2 + 200 = 358.2 \frac{\text{kmol}}{\text{hr}} \text{ and } V = \bar{V} = 237.3 \frac{\text{kmol}}{\text{hr}}$$

operating lines :-

(a) rectifying section

$$y_{n+1} = \frac{R}{R+1} x_n + \frac{x_D}{R+1}$$

$$y_{n+1} = \frac{2}{3} x_n + \frac{0.95}{3}$$

$$y_{n+1} = 0.667 x_n + 0.317 \quad (*)$$

(b) Stripping section :-

$$y_{m+1} = \frac{\bar{L}}{\bar{L}-W} x_m - \frac{W}{\bar{L}-W} x_w$$

$$y_{m+1} = \frac{358.2}{358.2 - 120.9} x_m - \frac{120.9}{358.2 - 120.9} \times 0.04$$

$$y_{m+1} = 1.509 x_m - 0.0204 \quad (**)$$

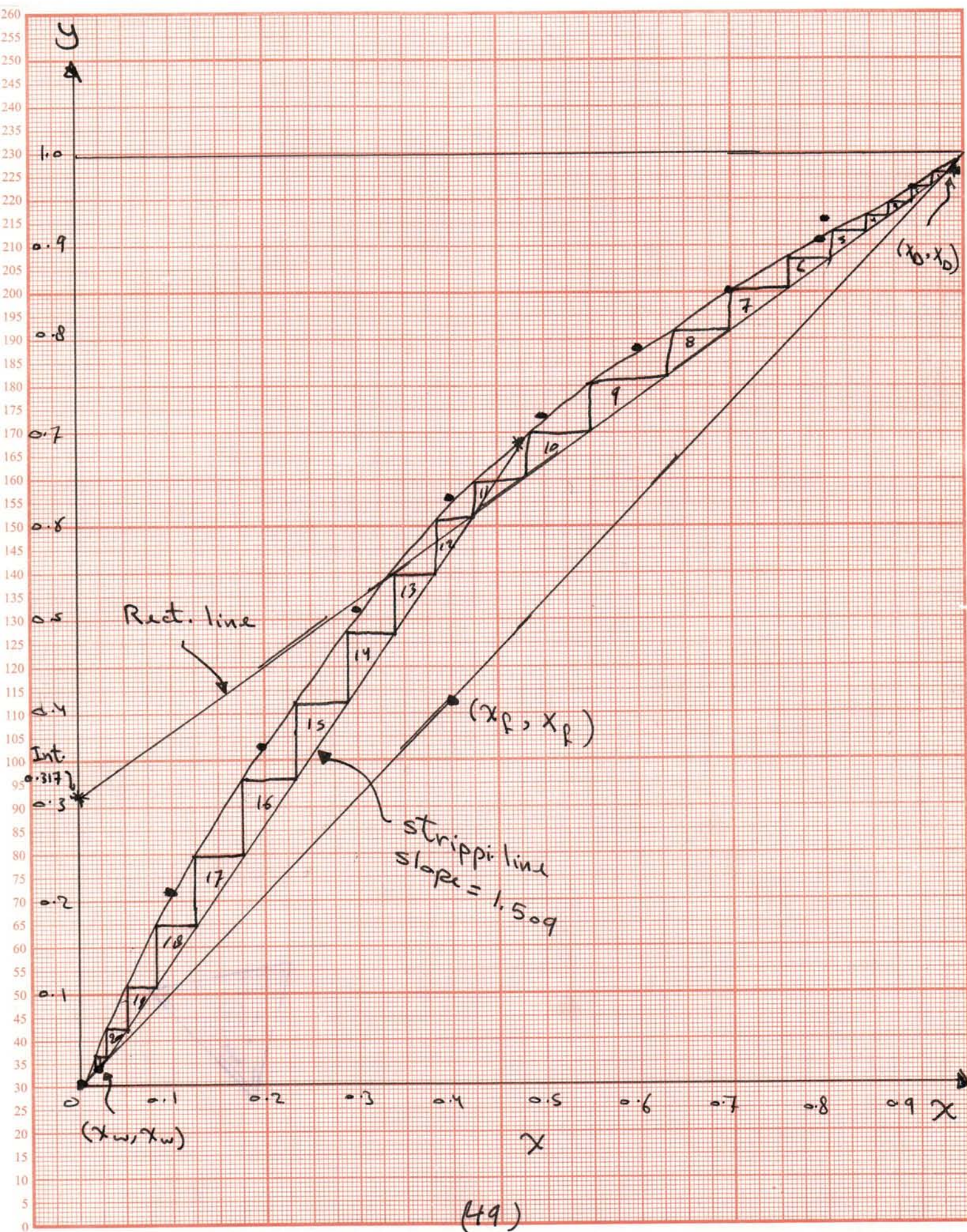
Now plot equilibrium data and both operating lines.

$$\text{The boil-up ratio } R_V = \frac{\bar{V}}{W} = \frac{237.3}{120.9}$$

$$R_V = 1.963$$

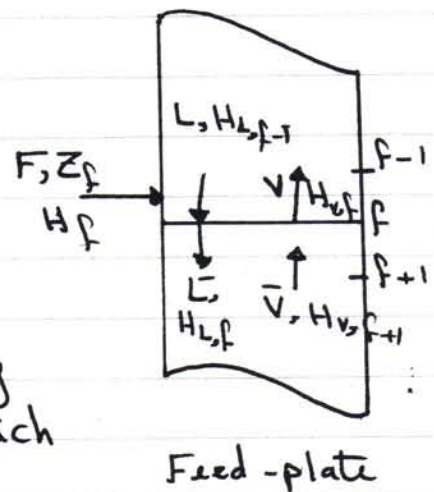
∴ From Fig. The number of stages :- (21)

(48)



The Feed Line :-

As the feed enters the column, the liquid and vapour flow rates undergo step changes depending up on the state of the feed (how much liq. and vap.).



We shall write the material and energy balance equations over the plate to which the feed is introduced, this plate will be written by suffix "f".

T.M.B :-

$$F + L + \bar{V} = \bar{L} + V \quad \text{--- (20)}$$

Energy - Balance :-

$$F \cdot H_F + L \cdot H_{L,f-1} + \bar{V} \cdot H_{V,f+1} = \bar{L} \cdot H_{L,f} + V \cdot H_{V,f} \quad \text{--- (21)}$$

$$\left. \begin{aligned} \text{Assume :- } H_{L,f-1} &= H_{L,f} = H_L \\ H_{V,f+1} &= H_{V,f} = H_V \end{aligned} \right\} \quad \text{--- (22)}$$

Now substitute (22) in (21) :-

$$F \cdot H_F + L \cdot H_L + \bar{V} \cdot H_V = \bar{L} \cdot H_L + V \cdot H_V$$

$$(\bar{L} - L) H_L = (\bar{V} - V) H_V + F \cdot H_F \quad \text{--- (23)}$$

From eq. (20) and (23) :-

$$\frac{\bar{L} - L}{F} = \frac{H_F - H_V}{H_L - H_V} = \frac{H_V - H_F}{\underbrace{H_V - H_L}_{-x}} = q \quad \text{--- (24)}$$

Super heated or sub cooled

on the basis of eq. (24), we may define :-

$$q = \frac{\text{Heat required to convert 1 mole feed to sat. vap.}}{\text{molar heat of vaporiz}^n \text{ of the sat. liquid.}}$$

Now if :-

- a) $q = 1$ liquid at bubble pt. (sat. liq.)
 b) $q = 0$ vapour at dew. pt (sat. vap.)

c) if the feed is a two-phase mixture (liq. + vap.), then (q) represent the fraction of liquid in it.

$z_F(1-q)$ gives a measure of the "quality" of feed.

Simplifying and recombination equations :-

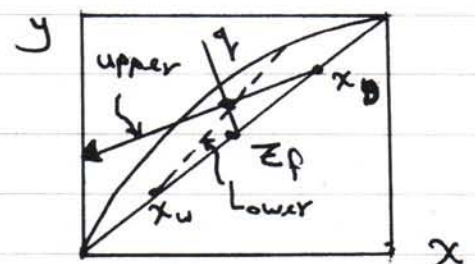
$$\boxed{y = \frac{q}{q-1} x - \frac{z_F}{q-1}} \quad \dots (25) \text{ Feed line equation sub cooled or super heated}$$

$\frac{q}{q-1}$ = slope of the line, governed by the nature of the feed

$-\frac{z_F}{q-1}$ = intersection of the operating lines, with y-axis

$\frac{z_F}{q}$ = = = = = x-axis

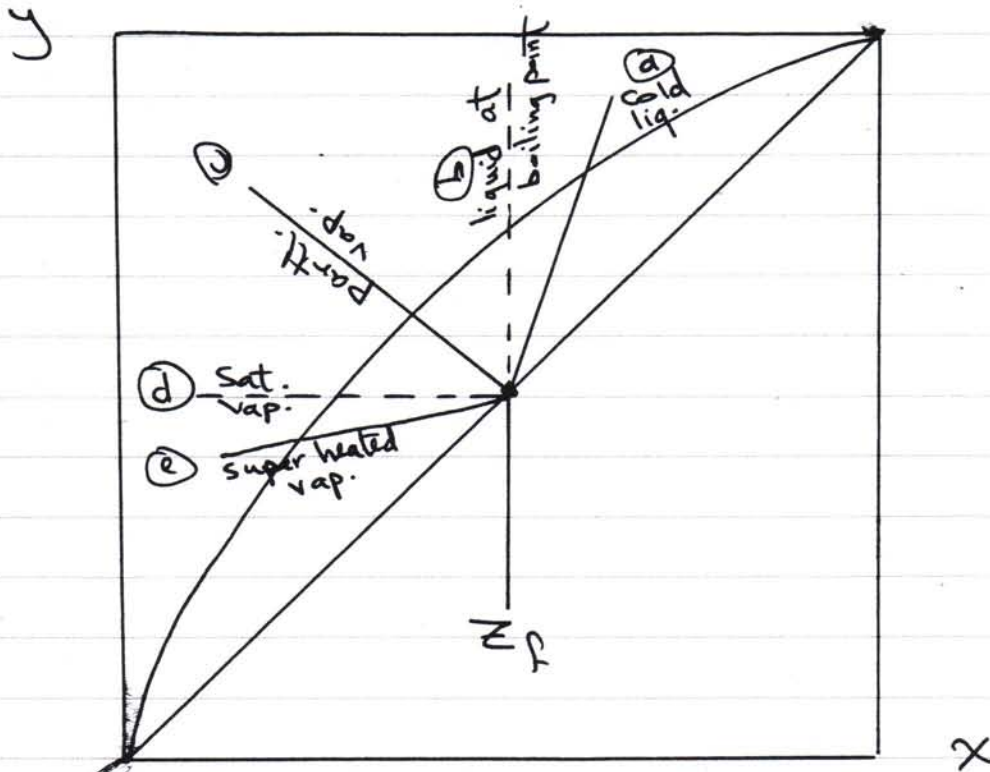
Notes :- The point of intersection of upper operating line (Rect. line) and feed line, can be joined with (x_w, x_w) to draw stripping section line.



Feed Conditions :-

- Cold Feed (liquor)
- Feed at boiling point
- Feed partly vapour
- Feed saturated vapour
- Feed superheated Vapour

(q)	shape	slope
$q > 1$	/	(+ve)
$q = 1$		(∞)
$0 < q < 1$	\	(-ve)
$q = 0$	—	(0)
$q < 0$	/	(+ve)



Effect of the feed condition at fixed reflux ratio.

Eqⁿ of q-line

point of q line

(a) $y = \frac{q}{q-1} x - \frac{z_f}{q-1}$

$(z_f, z_f), (\frac{z_f}{q}, 0)$

(b) $x = z_f$

vertical through (z_f) .

(c) $y = -(\frac{1-f_v}{f_v}) x + \frac{z_f}{f_v}$

$(z_f, z_f), (0, \frac{z_f}{f_v})$.

(d) $y = z_f$

horiz. through (z_f) .

(e) $y = \frac{q}{q-1} x - \frac{z_f}{q-1}$

$(z_f, z_f), (0, \frac{z_f}{1-q})$

(H.W) Try to obtain the above relations.

① If the feed is saturated liquid, $q=1$, the slope of the feed line is (∞) , and feed line will be vertical line through (Z_f, Z_f) .

② If the feed is a saturated vapour, $q=0$, the slope of the feed line is zero, and feed line will be horizontal line through (Z_f, Z_f) .

③ If the feed is a mixture of liquid and vapor or superheated vap. or subcooled liquid, then the slope of the feed line can be calculated using equation (24), where :-

$$H_L = C_{p_s} \cdot M_{av} \cdot (T - T_o) + \Delta H_s \quad \text{----- (26)}$$

Enthalpy of liquid.

H_L = molar enthalpy of solution at temp. T , kJ/kmol .

C_{p_s} = Specific heat of solution, $\text{kJ/kg} \cdot \text{K}$.

M_{av} = average molecular weight of solution.

T_o = ref. Temp.

ΔH_s = heat of solution at ref. Temp. T_o , kJ/kmol .

$$H_v = \sum y_i^* \cdot M_i [C_{p_i} (T - T_o) + \lambda_i] \quad \text{----- (27)}$$

Enthalpy of vapour.

H_v = Enthalpy of the vapour, kJ/kmol

λ = heat of vaporization, kJ/kg .

$H_F = \sum \text{Latent heat} + \text{Sensible heat}$

$$H_F = \sum (x_i \cdot M_i \cdot \lambda_i) + (x_i \cdot C_{p_i} \cdot (T - T_o)) \quad \text{----- (28)}$$

H_F = Enthalpy of the feed.

Ex. (2) :-(How to draw feed line for different feed conditions)

A mixture of benzene and toluene containing 58% mole benzene is to be separated in a continuous column operating at 1 atm total press. Draw the feed line for the following feed condition:-

- (a) Saturated vapour. (at dew point).
- (b) Saturated liquid. (at bubble point).
- (c) 65 mass % vapour. (partial vaporiza)
- (d) vapour at 120°C . (super-heated)
- (e) liquid at 50°C . (subcooled)

Given that:-

$$C_{P(B)L} = 146.5 \text{ kJ/kmol.K}$$

$$C_{P(B)V} = 97.6 \text{ kJ/kmol.K}$$

$$M_B \lambda_{(B)} = 30,770 \text{ kJ/kmol.}$$

$$C_{P(T)L} = 170 \text{ kJ/kmol.K}$$

$$C_{P(T)V} = 124.3 \text{ kJ/kmol.K}$$

$$M_T \lambda_{(T)} = 32,120 \text{ kJ/kmol.}$$

$$\text{ref. Temp.} = 90^{\circ}\text{C}$$

Sol. :-

Using eq. (25) to draw feed line.

$$y = \frac{q}{q-1} x - \frac{Z_F}{q-1}$$

① For saturated vapour. $\therefore q = 0$

Slope = $\frac{0}{0-1} \Rightarrow$ horizontal line through point F
(Z_F, Z_F).

② For saturated liquid. $\therefore q = 1$

slope = $\frac{q}{q-1} = \infty \Rightarrow$ vertical line through point F.

③ For 65% vap. $\Rightarrow \therefore$ feed contain 35% liquid.

$$\therefore q = 0.35 \Rightarrow \text{slope} = \frac{0.35}{0.35 - 1} = -0.538$$

$$\text{intercept} = \frac{-Z_F}{q-1} \Rightarrow \text{inter.} = \frac{-0.58}{0.35 - 1} = 0.892$$

We have (F) point and intercept.

④ For vapour at $120^\circ\text{C} \Rightarrow$ super heated vapour.

\therefore Eq. (24) is to be used.

$$q = (H_v - H_F) / (H_v - H_L)$$

$$\Delta H_s = 0 \quad (\text{pure component})$$

$$H_L = C_{p_{sal}} \cdot M_{av} (T - T_0) + \cancel{\Delta H_s}^0$$

For the solution at its bubble pt ($T = 90^\circ\text{C}$), take $T_0 = 90$; then :-

$$\boxed{H_L = 0}$$

$$\text{for } \Delta T = 0$$

⊛ Enthalpy of vapour :- For $x = 0.58$, $y^* = 0.78$ (from y^*)

$$H_v = y_B^* \cdot M_B [C_{p_B} (T - T_0) + \lambda_B] + y_T^* M_T [C_{p_T} (T - T_0) + \lambda_T]$$

$$H_v = 0.78 * 30770 + (1 - 0.78) * 32120$$

$$H_v = 31,067 \text{ kJ/kmol.}$$

⊛ Feed enthalpy :-

We assume that 0.58 kmol Benzene and 0.42 kmol of toluene are vaporized separately at 90°C , the vapours are heated to 120°C and then mixed

$$H_F = 0.58 * 30770 + 0.58 * 97.6 (120 - 90) + [0.42 * 32120 + 0.42 (124.3) (120 - 90)]$$

$$H_F = 34601$$

$$\text{Now :- } q = \frac{H_v - H_F}{H_v - H_L}$$

(57)

$$q = \frac{31067 - 34601}{31067 - 0} = -0.114$$

$$\text{intercept} = \frac{-Z_F}{q-1} = \frac{-0.58}{-0.114-1} = 0.521$$

② The Feed is subcooled at 50°C

We assume that 0.58 kmol of (B) and $(0.42) \text{ kmol of (T)}$ are cooled from ref. Temp.

$(T_0 = 90^\circ\text{C})$ to $(T = 50^\circ\text{C})$ and then mixed to get 1 kmol of feed .

$$\therefore H_F = 0.58 \times 146.5 \times (50 - 90) + 0.42 \times 170 (50 - 90)$$

$$H_F = -6240 \text{ kJ/kmol.}$$

$$\therefore q = \frac{31067 - (-6240)}{31067 - 0} = 1.2$$

$$\text{intercept with x-axis} = \frac{Z_F}{q} = \frac{0.58}{1.2} = 0.484$$

Note:- ① For aliq. feed, it is introduced just above the feed tray. But a vapour feed is introduced just below it

② If the feed is a mixture of liq. and vap., it is desirable that it is separated into the vap., and the liq. phases first. The liq. part should enter the column just above feed tray, and the vap. part just below it. However, this is not always done in practice, and a mixed feed is often introduced as a whole over the feed tray.

Ex. (3) :- Determination of No. of ideal trays.

A stream of aqueous methanol having 45 mol% CH_3OH is to be separated into a top product having 96 mol% methanol and a bottom liquid with 4% methanol.

The feed is at its bubble point and the operating pressure is 101.3 kPa. A reflux ratio of 1.5 is used.

(a) Determine the No. of ideal trays.

(b) Find the No. of real trays if overall tray eff. is 40%. On which real tray should the feed introduced.

The equilib^m. Data for the system.

X :	0	0.02	0.04	0.06	0.08	0.1	0.2	0.3	...	0.7	0.8	0.91
y :	0	0.134	0.23	0.304	0.365	0.41	0.58	0.66	...	0.87	0.92	0.961
T :	100	96.4	93.5	91.2	89.3	87.7	84.4	78.0	...	69.3	67.6	66 64

Solution :- $x_D = 0.96$, $x_W = 0.04$, $z_F = 0.45$

Plot equil^m. data, the points D(0.96, 0.96), W(0.04, 0.04) and F(0.45, 0.45) are located on the x-y plane.

(a) The upper operating line is drawn through the point (D) with the intercept with the y-axis

$$\text{Intercept} = \frac{x_D}{R+1} = \frac{0.96}{1.5+1} = 0.384$$

\therefore Feed is a saturated liquid, then feed line is vertical line through point (F).

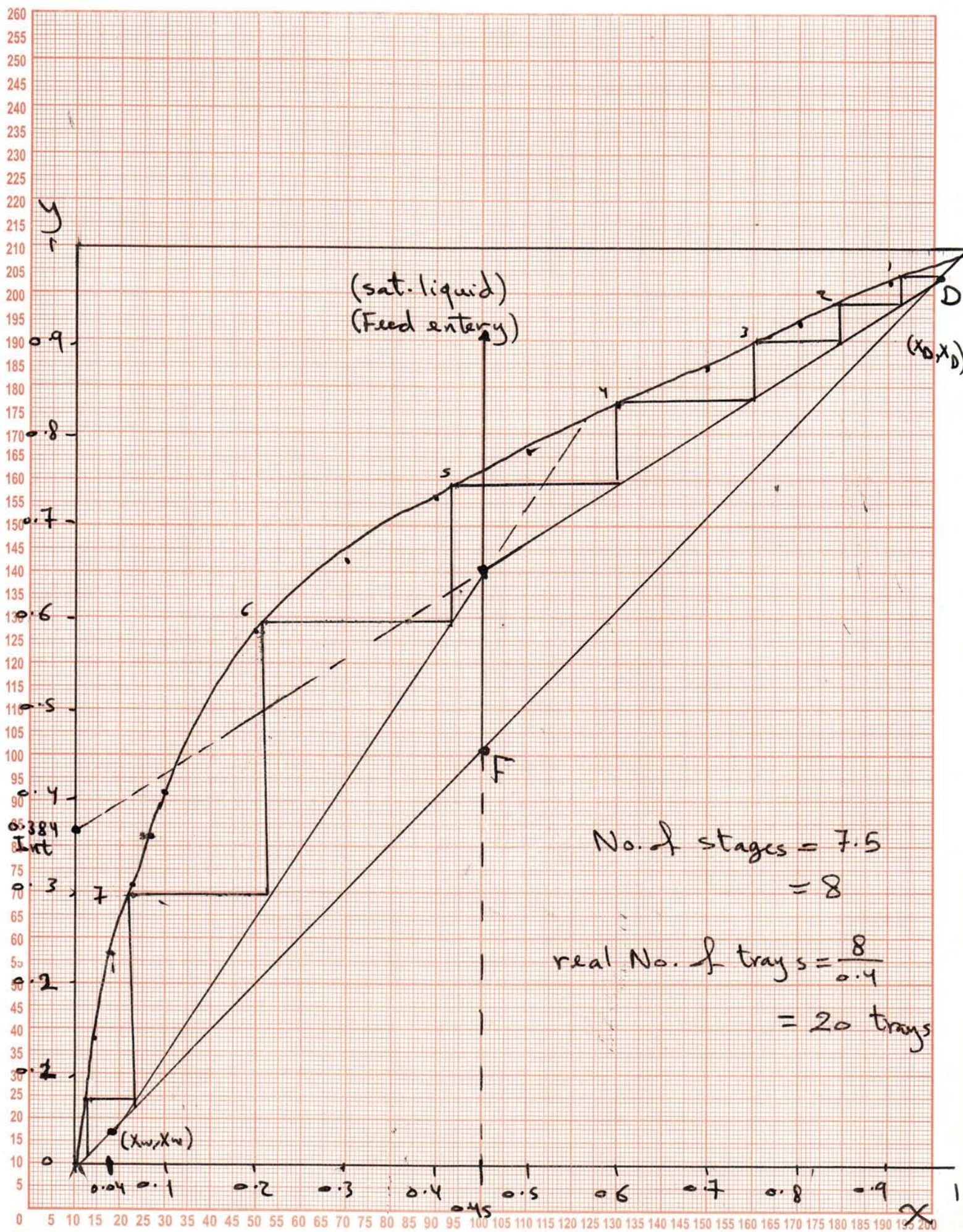
* The upper operating line meets the feed line at point (M).

* The point (M) and (W) are joined to get the lower or stripping section operating line.

⑥ No. of ideal plates = 7.5 From the figure
 ≈ 8

No. of real plates = $8 / 0.4$
 $= 20$ trays plus the reboiler
which is assumed to act like an ideal stage.

* Feed enters in a place between (4-5)



(62)

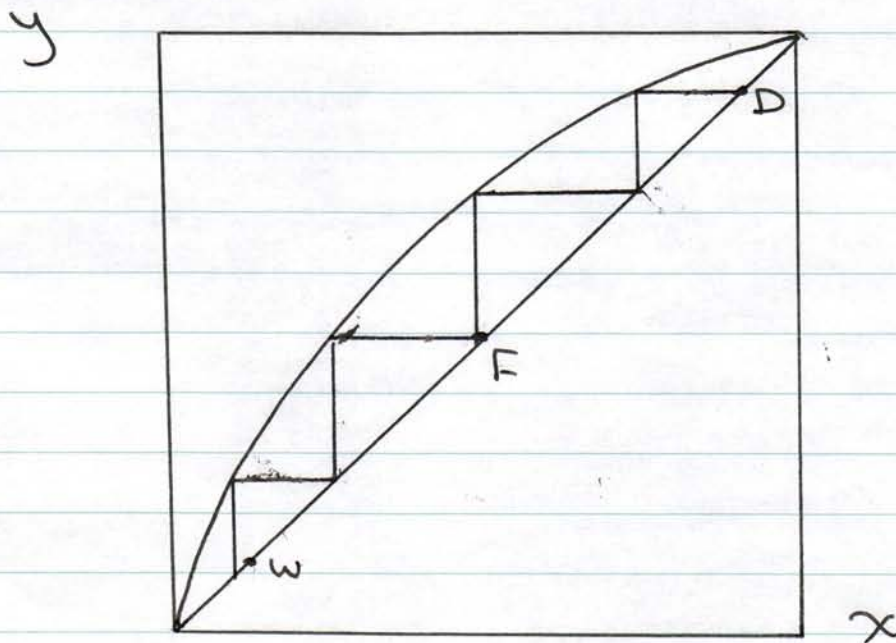
Analytical Determination of The No. of Ideal Stages.

(a) Total Reflux \Rightarrow Give (N_{\min})

$\therefore R = \frac{L_0}{D}$ and at total (R) where no distillate is accumulated then $D = 0$

$\therefore R = \infty$ and No. product is drawn from reboiler

At total reflux, the slope of the rectifying section operating line is $\left(\frac{R}{R+1}\right)$ for $R \rightarrow \infty$. The slope of the line is unity and pass through the point (x_D, x_D) on the diagonal. Therefore, the operating line coincides with the diagonal. So does the stripping section line. The No. of ideal stages is obtained by stair case construction between equilibrium line and the diagonal. Total reflux is very often used during the startup of the column till steady state condition reached.



No. of plates at total R.
(min^m. No. of plates)

Fenske's equation 2-

This equation can be used to calculate theoretically the "Minimum. No. of Trays" if the relative volatility remains constant ($\alpha_{avg.}$)

$$y_n = \frac{R}{R+1} x_{n+1} + \frac{x_D}{R+1} \quad \text{at } R=\infty$$

$\therefore y_n = x_{n+1}$ for rectifying section
Same $y_m = x_{m+1}$ \rightarrow stripping section

$$\alpha = \frac{y_A / x_A}{y_B / x_B} \Rightarrow \boxed{\frac{y_{AW}}{y_{BW}} = \alpha_o \left(\frac{x_{AW}}{x_{BW}} \right)}$$

vap. rise from the reboiler (y_{AW}) have a relation with liquid fall in stage one, joining in operating line.

$$x_{A1} = y_{AW} \quad , \quad x_{B1} = y_{BW}$$

$$\frac{x_{A1}}{x_{B1}} = \frac{y_{AW}}{y_{BW}}$$

$$\boxed{\frac{x_{A1}}{x_{B1}} = \alpha_o \left(\frac{x_{AW}}{x_{BW}} \right)}$$

Same for stage one:-

$$\alpha_1 = \frac{y_{A1} / x_{A1}}{y_{B1} / x_{B1}}$$

$$\frac{x_{A2}}{x_{B2}} = \alpha_0 \alpha_1 \left(\frac{x_{Aw}}{x_{Bw}} \right)$$

continue to stage (n)

$$\left(\frac{x_A}{x_B} \right)_D = \underbrace{\alpha_0 \alpha_1 \alpha_2 \dots \alpha_n}_{\alpha_{avg.}} \left(\frac{x_A}{x_B} \right)_w$$

$\hookrightarrow \alpha_{avg.} = \text{avg. volatility of the M.V.C}$

$$N_{min.} = \frac{\log \left[\frac{x_D(1-x_w)}{x_w(1-x_D)} \right]}{\log \alpha_{avg.}} - 1$$

Fenske's equation (at total Reflux)

⑥ Minimum Reflux Ratio (R_m) for (N_{∞})

① Graphically :-

Can be defined as that ratio at which an infinite No. of stages are needed to obtain the desired overhead and bottom products.

The determination of (R_m) is based on identifying the "Pinch point".

For a particular reflux ratio (R_1), DE_1 is the enriching section operating line with slope $= (R/(R+1))$. It intersects the feed line at point (M_1), WM_1 is the stripping section operating line.

As the reflux ratio (R_1) decreases to (R_2) the slope of U.D.L decreases, but intercept increases, the point (E_1) moves to (E_2). DM_2 is the U.D.L and WM_2 is the S.D.L, and they intersect the feed line at M_2 . Then the driving force is less and the No. of theo. trays will be more.

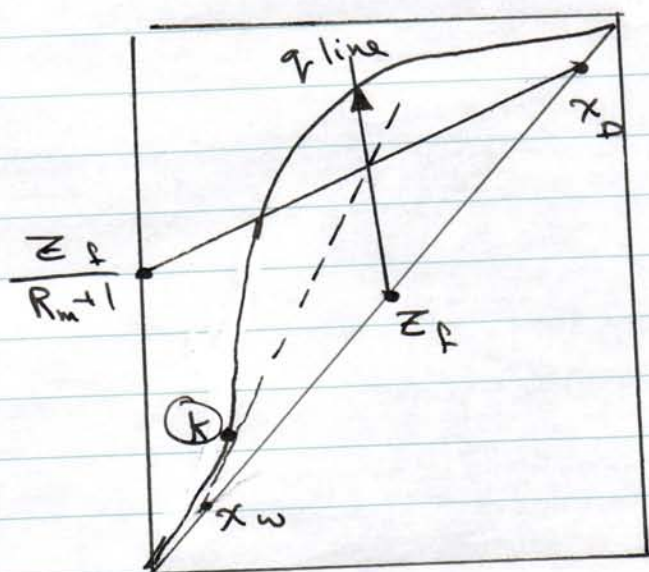
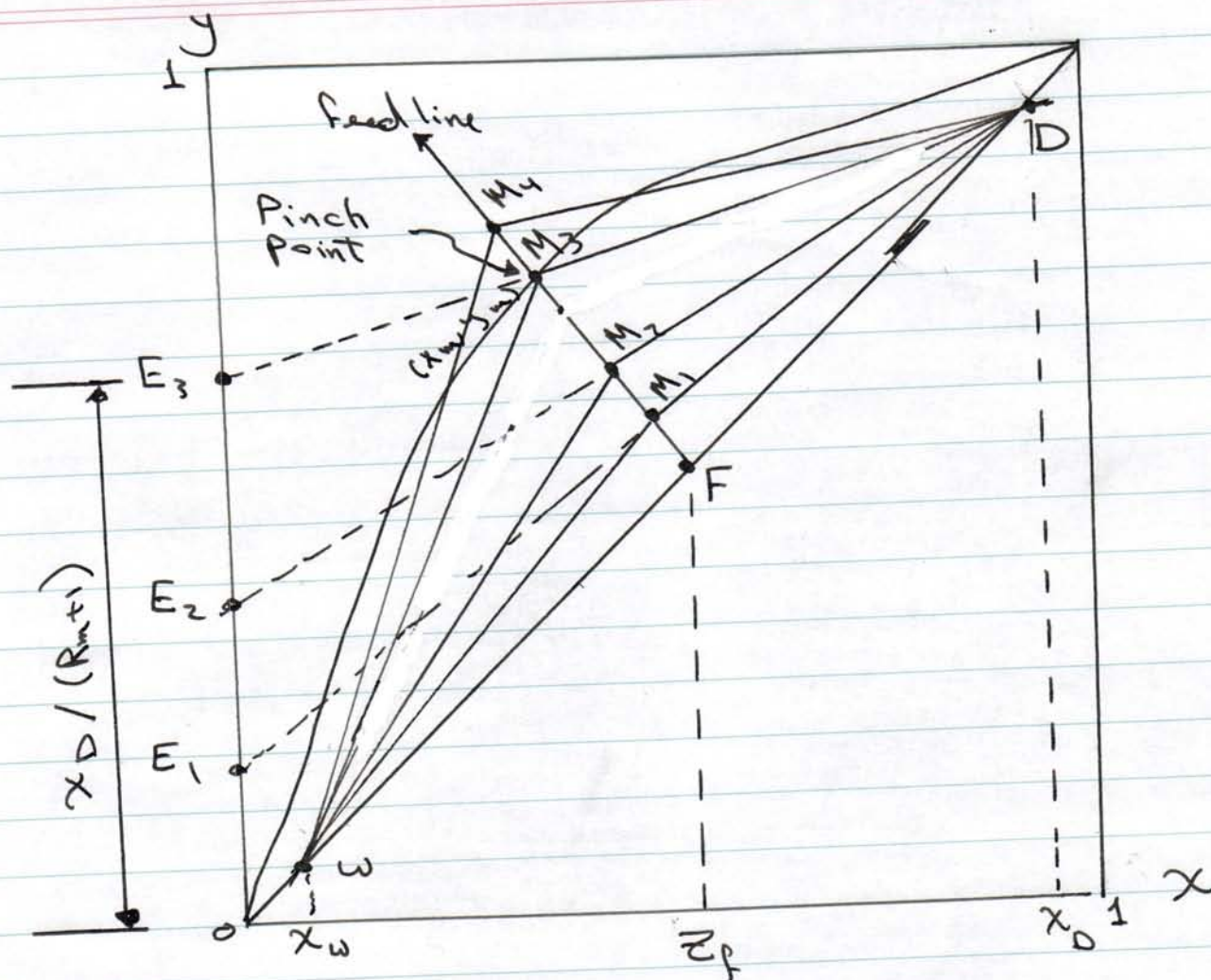
If the reflux ratio is gradually reduced, to (E_3), then line DE_3 intersect the feed line at (M_3) which lies on the equilibrium curve.

Then driving force will be zero at M_3 , it is the "Pinch point".

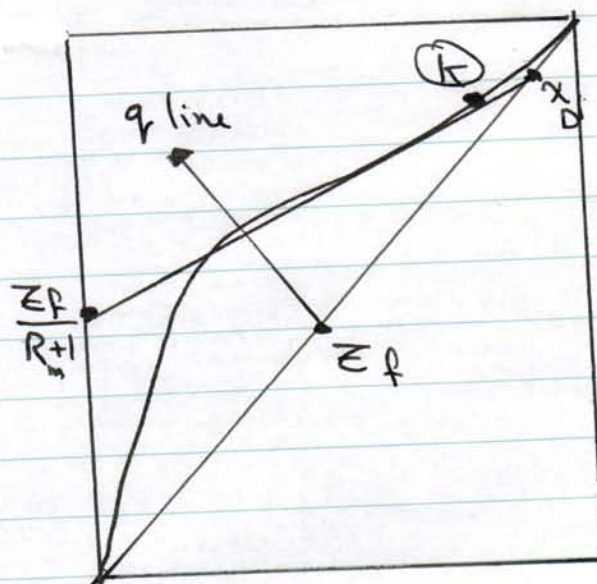
The No. of Theo. plates will be infinite.

The operating line (DM_3) corresponds to the min. reflux ratio.

For further reduced in (R) the operating line will intersect the feed line at a point above the equilibrium curve, this is impossible.



Pinch Point for stripping section



Pinch Point for Rectifying section

② Under Wood Method For Calculating R_{min}

A mathematical method to determine (R_{min}) for ideal solutions at constant α .

$$y_m = \frac{R_m}{R_m + 1} x_m + \frac{x_D}{R_m + 1} \quad \text{--- (1) U.D.L}$$

$$y_m = \frac{q}{q-1} x_m - \frac{z_F}{q-1} \quad \text{--- (2) Feed.L}$$

$$y_m = \frac{\alpha \cdot x_m}{1 + (\alpha - 1) x_m} \quad \text{--- (3) Equil.^m L}$$

From eq. (1) and (2) :-

$$\frac{R_m}{R_m + 1} x_m + \frac{x_D}{R_m + 1} = \frac{q}{q-1} x_m - \frac{z_F}{q-1}$$

$$x_m = \frac{x_D (q-1) + z_F (R_m + 1)}{R_m + q}$$

$$y_m = \frac{R_m z_F + q x_D}{R_m + q}$$

Sub. in eq. (3) :-

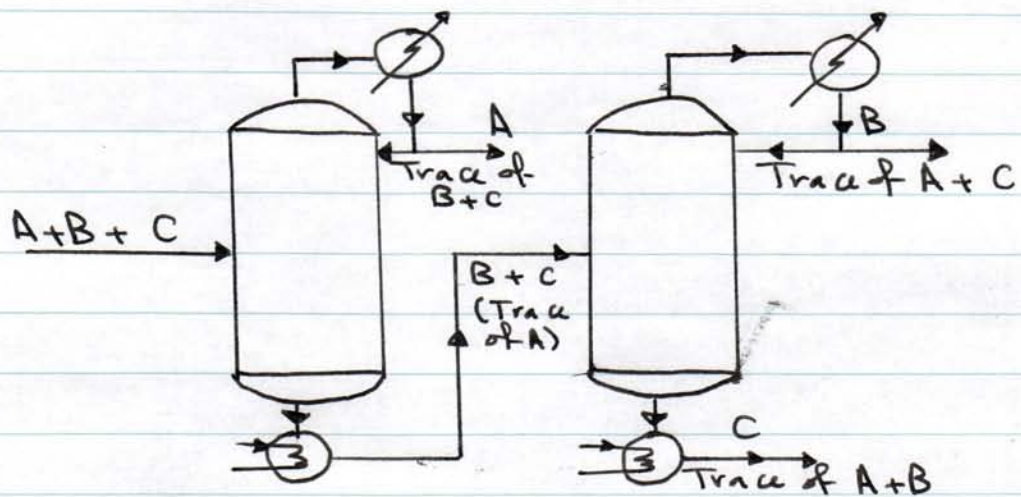
$$\frac{R_m Z_f + q x_D}{R_m (1 - Z_f) + q (1 - x_D)} = \alpha \frac{x_D (q - 1) + Z_f (R_m + 1)}{(R_m + 1) (1 - Z_f) + (q - 1) (1 - x_D)}$$

Underwood equation

$$R_m = \frac{1}{\alpha - 1} \left[\frac{x_D}{Z_f} - \frac{\alpha (1 - x_D)}{(1 - Z_f)} \right] \quad \text{For Sat. liquid}$$

$$R_m = \frac{1}{\alpha - 1} \left[\frac{\alpha \cdot x_D}{Z_f} - \frac{1 - x_D}{1 - Z_f} \right] - 1 \quad \text{For Sat. vapour.}$$

- Multi component Distillation -



Key-Component :-

The two components of the feed whose concentrations are specified in the distillate and in the bottom product are called the key-component.

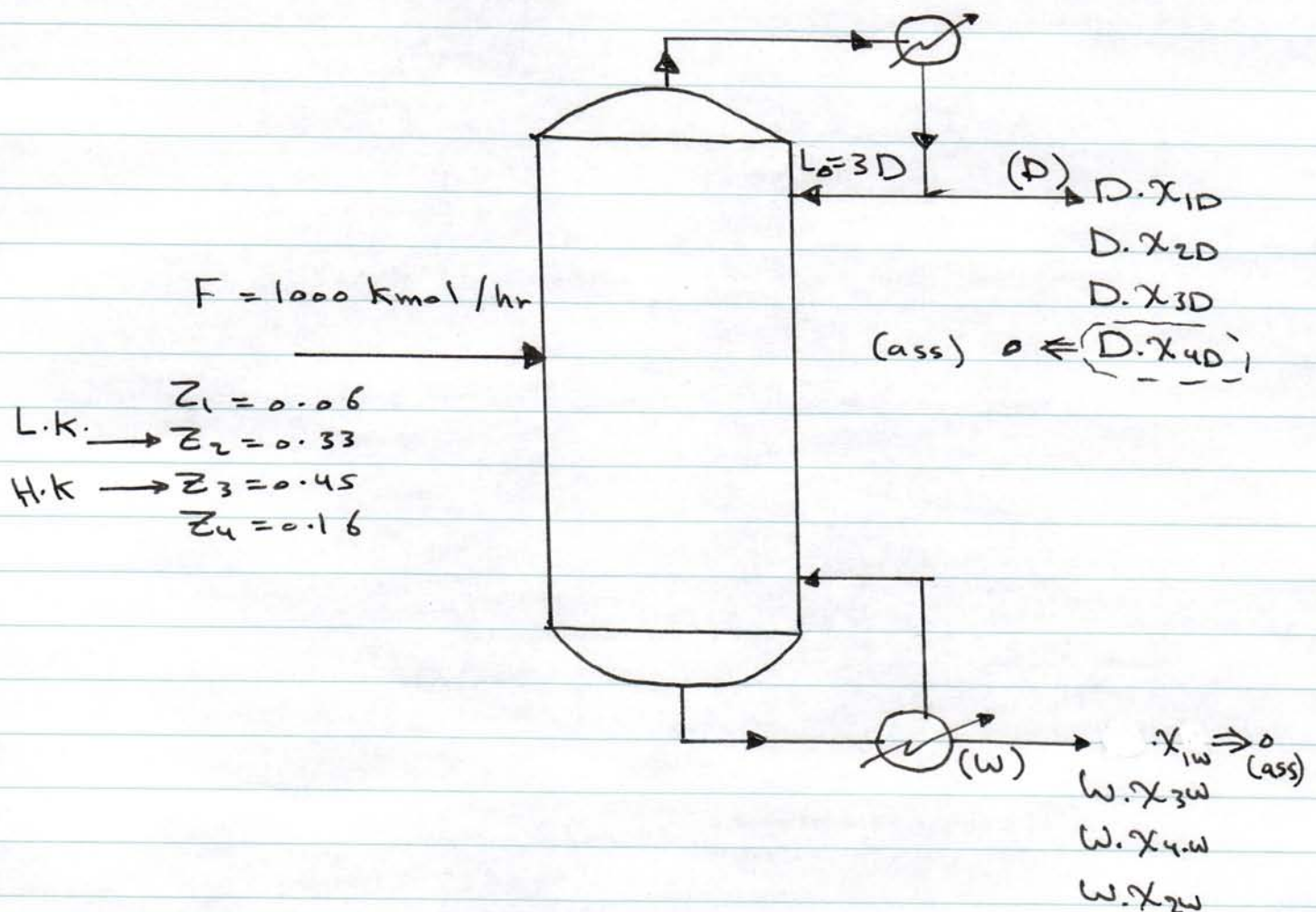
The more volatile of the two keys that concentrates in the distillate is the light key (LK), the less volatile one is the heavy key (HK). All other components which get distributed between the distillate and the bottoms are (non-keys). A non-key more volatile than the (LK) is called a light nonkey (LNK), a non key less volatile than the heavy key is a heavy non-key (HNK).

In some separation problems, there may be one or more components having volatilities intermediate between the (LK) and (HK), these are called (intermediate-keys) or (distributed keys).

Ex. (1) :- (Distribution of the keys and non-keys)

It is required to separate a saturated quaternary mixture containing propane ($z_1 = 0.06$), n-butane ($z_2 = 0.33$), n-pentane ($z_3 = 0.45$) and n-hexane ($z_4 = 0.16$), at a rate of 1000 kmol/hr . The total press. = 1 atm . The reflux ratio is $R = 3.0$. It is desired to recover 99% of the butane in the distillate and 99.5% of the pentane in the bottoms. Calculate:-

- The flow rates and composition of the distillate and the bottom product.
- the condensation rate and the boil-up rate, if (R) assumed to be at bubble point.



Sol.:- There are eight unknowns, $[D, W, x_{1D}, x_{2D}, x_{3D}, x_{1W}, x_{2W}, x_{3W}]$.

(LK) is the butane

(HK) is the pentane

(LNK) is the propane

(HNK) is the hexane

For (L.K):-

We want 99% of comp. (2) to be recovered in (D),

$$0.99 \times 1000 \times 0.33 = D \cdot x_{2D}$$

$$D \cdot x_{2D} = 326.7 \text{ kmol/hr}$$

$$\therefore W \cdot x_{2W} = F \cdot z_2 - D \cdot x_{2D}$$

$$= (1000 \times 0.33) - 326.7$$

$$W \cdot x_{2W} = 3.3 \text{ kmol/hr}$$

L.K.C

For (H.V)

99.5% to be recovered of (3).

$$0.995 \times 1000 \times 0.45 = W \cdot x_{3W}$$

$$W \cdot x_{3W} = 447.75 \text{ kmol/hr}$$

$$D \cdot x_{3D} = (1000 \times 0.45) - 447.75$$

$$D \cdot x_{3D} = 2.25 \text{ kmol/hr}$$

H.K.C

For (L.N.K)

$$F \cdot z_1 = D \cdot x_{1D} + W \cdot x_{1W}$$

$$1000 \times 0.06 = D \cdot x_{1D} + \text{Zero}$$

$$D \cdot x_{1D} = 60 \text{ kmol/hr}$$

$$\text{Total (D)} = D \cdot x_{1D} + D \cdot x_{2D} + D \cdot x_{3D} + D \cdot x_{4D}$$

$$D = 60 + 326.7 + 2.25 \Rightarrow D = 388.95 \text{ kmol/hr}$$

(72)

Now (x_{1D}, x_{2D}, x_{3D}) can be found $(x_{1D} = \frac{D \cdot x_{1D}}{D}, \dots)$

$$F = D + W$$

$$1000 = 388.95 + W \Rightarrow W = 611.05 \text{ kmol/hr}$$

$$\sum x_{iw} = 1$$

$$x_{1w} + x_{2w} + x_{3w} + x_{4w} = 1$$

$$0 + \frac{3.3}{611.05} + \frac{447.75}{611.05} + x_{4w} = 1$$

$$x_{4w} = 0.2618$$

Component	(Z)	x_{iD}	x_{iw}
Propane(1)	0.06	0.1543	0.0000
butane(2)	0.33	0.84	0.0055
pentane(3)	0.45	0.0057	0.7327
hexane(4)	0.16	0.0000	0.2618

⑥ rate of condensation = $R \cdot D$

$$= 3 \times 388.95 = 1166.85 \text{ kmol/hr}$$

⑦ at bubble point, then the vaporization rate in the reboiler = rate of condensation

$$\therefore \text{boil-up} = 1166.85 \text{ kmol/hr}$$

Approximate Methods Calculation - The (FUG) Technique.

The most important approximate methods of calculation collectively called the Fenske-Underwood-Gilliland (FUG) method.

* Fenske Equation:-

For A(LK) and B(HK) in multicomponent mixtures:-

$$N_{\min} = \frac{\log \left[\frac{(x_{AD} \cdot x_{BW})}{x_{BD} \cdot x_{AW}} \right]}{\log (\alpha_{AB})_{\text{avg}}} - 1 \quad \text{--- (*)}$$

N_{\min} = min. number of trays including the reboiler

$$\alpha_{AB}|_{\text{avg}} = \frac{\alpha_{A|LK}}{\alpha_{B|HK}} \quad \text{and} \quad \alpha|_{\text{avg}} = (\alpha_{A \text{ Top}} \cdot \alpha_{A \text{ Bott.}})^{0.5}$$

- For fractional recovery of the key component instead of concentration then:-

$$N_{\min} = \frac{\log \left[\frac{P_{A,D} \cdot P_{B,W}}{(1-P_{A,D})(1-P_{B,W})} \right]}{\log (\alpha_{AB})_{\text{avg}}} \quad \text{--- (**)}$$

Once the (N_{\min}) is determined, it is easy to calculate the distribution of a non-key or (intermediate), by substituting ($P_{B,W}$) by any other component, or using eq. (*).

Ex. (2) :- [the min. Number of trays and distribution of the non-key]

A feed mixture containing six component, $z_1 = 0.032$, $z_2 = 0.068$, $z_3 = 0.17$, $z_4 = 0.3$, $z_5 = 0.32$ and $z_6 = 0.11$, is to be separated by distillation so that 98.5% of comp. (3) goes to distillate and 98% of comp. (5) goes to bottom product.

- 1- Determine the min. number of trays required.
- 2- Composition of the top and bottom products.

Given that: $\alpha_{15} = 3.15$, $\alpha_{25} = 2.75$, $\alpha_{35} = 2.35$
 $\alpha_{45} = 1.4$, $\alpha_{55} = 1.0$, $\alpha_{65} = 0.75$

Feed flow rate 1000 kmol/hr.

Sol. 2 - Comp. (3) is the L.k
 comp. (5) is H.k

- For the (L.k) :- 98.5% to be recov.

$$0.985 \times 1000 \times 0.17 = D \cdot X_{3D} = 167.45$$

$$W \cdot X_{3W} = F \cdot z_3 - D \cdot X_{3D}$$

$$= 1000 \times 0.17 - 167.45$$

$$W \cdot X_{3W} = 2.55 \text{ kmol/hr}$$

- For the (H.k): 98% to be recov.

$$0.98 \times 1000 \times 0.32 = W \cdot X_{5W} = 313.6$$

$$D \cdot X_{5D} = 1000 \times 0.98 - 313.6 = 6.4$$

$$N_{\min} = \frac{\log \left[\frac{X_{3D} \cdot X_{5W}}{X_{5D} \cdot X_{3W}} \right]}{\log \alpha_{35}} = \frac{\log \left[\frac{D \cdot X_{3D} \cdot W \cdot X_{5W}}{D \cdot X_{5D} \cdot W \cdot X_{3W}} \right]}{\log (2.35)}$$

$$N_{\min} = 9.45 \quad (75)$$

or ($N_{min.}$) can be found by using eq. (**):-

$$f_{3D} = 0.985, f_{5W} = 0.98$$

$$N_{min} = \frac{\log [(0.985 * 0.98) / (1 - 0.985)(1 - 0.98)]}{\log 2.35}$$

$$N_{min.} = 9.45$$

② To find the distribution of comp. (1) consider pair (1-3) and using Fenske - eq.

$$N_{min} = 9.45 = \frac{\log \left[\frac{x_{1D} \cdot x_{3W}}{x_{3D} \cdot x_{1W}} \right]}{\log \alpha_{13}} = \frac{\log \left[\frac{D \cdot x_{1D} \cdot W \cdot x_{3W}}{D \cdot x_{3D} \cdot W \cdot x_{1W}} \right]}{\log \alpha_{13} \leftarrow \frac{\alpha_1}{\alpha_3}}$$

$$9.45 = \frac{\log \left[\frac{D \cdot x_{1D} * 2.55}{W \cdot x_{1W} * 167.45} \right]}{\log (3.15 / 2.35)}$$

$$\frac{D \cdot x_{1D}}{W \cdot x_{1W}} = 1046.6 \Rightarrow D \cdot x_{1D} = 1046.6 W \cdot x_{1W}$$

From mat. balance:-

$$F \cdot Z_1 = D \cdot x_{1D} + W \cdot x_{1W}$$

$$1000 * 0.032 = 1046.6 \cdot W \cdot x_{1W} + W \cdot x_{1W}$$

$$W \cdot x_{1W} = 0.0305 \quad \text{and} \quad D \cdot x_{1D} = 31.969$$

Similarly, for pairs (2-5), (4-5) and (6-3) and using Fenske's equation:-

$$\left. \begin{array}{l} D \cdot x_{2D} = 67.766 \\ W \cdot x_{2W} = 0.234 \end{array} \right\} \left. \begin{array}{l} D \cdot x_{4D} = 98.74 \\ W \cdot x_{4W} = 201.26 \end{array} \right\} \left. \begin{array}{l} D \cdot x_{6D} = 3.2 \times 10^{-5} \\ W \cdot x_{6W} = 110 \end{array} \right\}$$

$$\text{Now } D = \sum D x_{iD} \Rightarrow \boxed{D = 372.325 \text{ kmol/hr}}$$

$$W = 1000 - 372.325$$

$$\boxed{W = 627.67 \text{ kmol/hr}}$$

<u>Component</u>	<u>α_{is}</u>	<u>Z_i</u>	<u>$D \cdot x_{iD}$</u>	<u>$W \cdot x_{iW}$</u>	<u>x_{iD}</u>	<u>x_{iW}</u>
1	3.15	0.032	31.969	0.0305	0.086	4.86×10^{-5}
2	2.75	0.068	67.767	0.234	0.182	3.73×10^{-4}
3	2.35	0.17	167.45	2.55	0.449	4.06×10^{-3}
4	1.4	0.3	98.74	201.26	0.266	0.3206
5	1.0	0.32	6.4	313.6	0.0172	0.4996
6	0.75	0.11	3.44×10^{-4}	110.0	9.2×10^{-8}	0.175
		1.0	372.325	627.67	0.999	0.999

$$\sum D \cdot x_{iD} + W \cdot x_{iW} = F$$

$$999.995 = 1000 \text{ kmol/hr}$$

The Underwood - Equation for Min. Reflux

In a column for distillⁿ. of a binary mixture with min^m. reflux, the pinch point occurs at the feed tray if the solution is ideal or nearly ideal.

For multicomponent system, the pinch point appears at the feed tray if all the components are distributed between the top and the bottom products.

However, if one or more of the non-keys appear in only one of the products, separate pinch points in the rectifying and stripping sections may occur

- For rectifying section :-

$$V_{\min} = \sum \frac{\alpha_i \cdot D \cdot x_{iD}}{\alpha_i - \phi_{\text{underwood constant}}}, \quad \phi = \frac{L_{\min}}{V_{\min} \cdot K_{HK}}$$

- For stripping section :-

$$-\bar{V}_{\min} = \sum \frac{\alpha_i \cdot W \cdot x_{iW}}{\alpha_i - \bar{\phi}}, \quad \bar{\phi} = \frac{\bar{L}_{\min}}{\bar{V}_{\min} \cdot K_{HK}}$$

To find (ϕ) in order to find V_{\min} , then we can use Underwood eq. for multicomponent

$$1 - q = \sum \frac{\alpha_i z_i}{\alpha_i - \phi} \quad \text{--- (*) Underwood - eq.}$$

where (q) fraction liquid in feed, depends on the inlet feed condition.

The min^m. liquid flow rate can be found where

$$L_{\min} = V_{\min} - D$$

Then (R_{\min}) can be calculated as :-

$$R_{\min} = \frac{L_{\min}}{D} \quad \text{or} \quad R_{\min} = \frac{V_{\min}}{D} - 1$$

In solving (underwood) equation to find values for (ϕ), the following cases deserve attention :-

Case ① :-

The non-keys are either too light or too heavy to distribute, all (LNK) go to the top, all the (HNK) go to the bottom, (No intermediate key), then :-

$\alpha_{HK} < \phi < \alpha_{LK}$, then find V_{\min} and

finally find total (D) = $\sum D \cdot x_{iD}$

Case ② :-

Distribution of the intermediate key between (LK) and (H.k).

This case can be solved by taking the two roots of eq. (*) and find (V_{\min}) for each root, and by equating (V_{\min}) for each root, the value of ($D \cdot x_{iD}$) can be found.

Ex. (3) :- (The min. reflux ratio)

A saturated liquid feed at 100 kmol/hr containing 35% benzene (1), 35 mol% toluene (2) and 30% ethyl benzene (3), is to be fractionated to recover 97% of benzene at the top and 95% of the toluene in the bottom product. The total pressure atmospheric, and reflux returned is at bubble point. Determine :-

- 1- Min. No. of trays using Fenske equation.
- 2- Min. Reflux ratio using Underwood equation.

The average relative volatilities with respect to comp. (2) are :-

$$\alpha_{12} = 2.4, \text{ and } \alpha_{32} = 0.48$$

Sol. :- $P_{1D} = 0.97$, $P_{2W} = 0.95$ then

$$1- N_{\min} = \frac{\log \left[\frac{P_{1D} \times P_{2W}}{(1-P_{1D})(1-P_{2W})} \right]}{\log \alpha_{AB} \leftarrow \alpha_{12}}$$

$$N_{\min} = \frac{\log [(0.97 \times 0.95) / (1-0.97)(1-0.95)]}{\log 2.4} = \boxed{7.33}$$

To calculate the fraction of comp (3) removed at the bottom, then :-

$$N_{\min} = \frac{\log [P_{1D} \times P_{3W} / (1-P_{1D})(1-P_{3W})]}{\log \alpha_{13}} = 7.33$$

$$7.33 = \frac{\log [0.97 \times P_{3W} / (1-0.97)(1-P_{3W})]}{\log (\alpha_{12} / \alpha_{32})}$$

$$\therefore P_{3W} = 0.9997$$

(80)

2- To calculate (R_{min})

Since most of the comp. (3) goes to the bottoms ($P_{3w} = 0.999$), then we assume that it does not distribute. then case (1) will be considered.

$$1 - q = \frac{\alpha_1 Z_1}{\alpha_1 - \phi} + \frac{\alpha_2 Z_2}{\alpha_2 - \phi} + \frac{\alpha_3 Z_3}{\alpha_3 - \phi}$$

\therefore Saturated liquid, then $q = 1$

$$0 = \frac{(2.4)(0.35)}{2.4 - \phi} + \frac{(1)(0.35)}{1 - \phi} + \frac{(0.48)(0.3)}{0.48 - \phi}$$

$$\boxed{\phi_1 = 1.465}, \quad \phi_2 = 0.589$$

\therefore comp. (3) does not go to distillate $\Rightarrow \alpha_{LK} > \phi > \phi_{HK}$

$$\therefore \boxed{\phi = 1.465}$$

$$\text{Now: } D \cdot X_{1D} = 0.97 * F * Z_1 \\ = 0.97 * 100 * 0.35$$

$$\boxed{D \cdot X_{1D} = 33.95}$$

$$W \cdot X_{2W} = 0.95 * F * Z_2 \\ = 0.95 * 100 * 0.35$$

$$\boxed{W \cdot X_{2W} = 33.25}$$

$$D \cdot X_{2D} = F \cdot Z_2 - W \cdot X_{2W} \\ = 100 * 0.35 - 33.25$$

$$\boxed{D \cdot X_{2D} = 1.75}$$

$$\boxed{D \cdot X_{3D} = 0}$$

(81)

$$D_{\text{total}} = 33.95 + 1.75 + \text{Zero}$$

$$D = 35.7 \text{ kmol/hr}$$

$$V_{\min} = \frac{\alpha_1 D x_{1D}}{\alpha_1 - \phi} + \frac{\alpha_2 D x_{2D}}{\alpha_2 - \phi} + \frac{\alpha_3 D x_{3D}}{\alpha_3 - \phi}$$

$$V_{\min} = \frac{(2.4)(33.95)}{(2.4) - (1.465)} + \frac{(1)(1.75)}{(1) - (1.465)} + 0 = \boxed{83.4}$$

$$V_{\min} = D(R_{\min} + 1)$$

$$83.4 = 35.7(R_{\min} + 1)$$

$$\boxed{R_{\min} = 1.336}$$

or

$$R_{\min} = \frac{L_{\min}}{D}$$

$$= \frac{V_{\min} - D}{D}$$

$$= \frac{83.4 - 35.7}{35.7}$$

$$\boxed{R_{\min} = 1.336}$$

Now if we try to solve the example using case (2)
 $D \cdot x_{1D} = 33.95$, $D \cdot x_{2D} = 1.75$ but $D \cdot x_{3D} \neq 0$

For $\phi = 1.465$

$$V_{\min} = \frac{(2.4)(33.95)}{2.4 - 1.465} + \frac{(1)(1.75)}{1 - 1.465} + \frac{(0.48)(D \cdot x_{3D})}{0.48 - 1.465} \quad \dots (1)$$

For $\phi = 0.589$

$$V_{\min} = \frac{(2.4)(33.95)}{2.4 - 0.589} + \frac{(1)(1.75)}{1 - 0.589} + \frac{(0.48)(D \cdot x_{3D})}{0.48 - 0.589} \quad \dots (2)$$

Solving (1) and (2) yield a negative value for $D \cdot x_{3D}$ which insures that case (1) is more suitable, for calculating R_{\min} .

Ex. (4) :- (Distribution of the intermediate Key)

A saturated liquid feed flowing at 100 kmol/hr containing 38 mol% benzene (1), 17 mol% toluene (2) and 45 mol% Cumene (3) is to be separated to recover 99.7% benzene in the distillate and 99.9% of the cumene in the bottom. Calculate the minimum reflux ratio and the distribution of the intermediate key.

$$\alpha_{12} = 2.28, \quad \alpha_{22} = 1, \quad \alpha_{32} = 0.22$$

Sol.

$F = 100$ kmol/hr at bubble point

$$Z_1 = 0.38, \quad Z_2 = 0.17, \quad Z_3 = 0.55$$

99.7% of comp. (1) as distillate \Rightarrow

$$\begin{aligned} D X_{1D} &= 0.997 \times F \times Z_1 \\ &= 0.997 \times 100 \times 0.38 \end{aligned}$$

$$\therefore D \cdot X_{1D} = 37.886$$

99.9% of comp. (3) as waste \Rightarrow

$$\begin{aligned} D X_{3D} &= (1 - 0.999) \times F \times Z_3 \\ &= 1 \times 10^{-3} \times 100 \times 0.55 \end{aligned}$$

$$\therefore D \cdot X_{3D} = 0.055$$

$$1 - q = \sum \frac{\alpha_i Z_i}{\alpha_i - \phi} \quad \therefore q = 1$$

$$0 = \frac{(2.28)(0.38)}{2.28 - \phi} + \frac{(1)(0.17)}{1 - \phi} + \frac{(0.22)(0.45)}{0.22 - \phi}$$

(83)

$$\phi_1 = 1.233, \phi_2 = 0.358$$

\therefore To luene will distribute between Top and bottom, then case (2) must be applied:

V_{\min} for ϕ_1 :

$$V_{\min} = \frac{(2.28)(37.88)}{2.28 - 1.233} + \frac{(1)(D \cdot X_{2D})}{1 - 1.233} + \frac{(0.22)(0.055)}{0.22 - 1.233}$$

V_{\min} for ϕ_2 :

$$V_{\min} = \frac{(2.28)(37.88)}{(2.28) - (0.358)} + \frac{(1)(D \cdot X_{2D})}{1 - 0.358} + \frac{(0.22)(0.055)}{0.22 - 0.358}$$

$$\therefore D \cdot X_{2D} = 6.4337 \Rightarrow V_{\min} = 54.877$$

$$\therefore D_T = 44.375 \text{ kmol/hr}$$

$$\therefore R_{\min} = 0.237$$

Gilliland - Correlation For the Number of Trays

Gilliland made a tray-by-tray calculation to determine the number of ideal stages (N) (included Reboiler) required to separate a multicomponent mixture by correlating (N) total by N_{min} and R_{min} , R .

$$X = \frac{R - R_{min}}{R + 1}$$

$$Y = 1 - \exp \left[\left(\frac{1 + 54.4 X}{11 + 117.2 X} \right) \left(- \frac{X - 1}{X^{0.5}} \right) \right]$$

$$Y = \frac{N - N_{min}}{N + 1}$$

Also feed stage location could be expressed as :-

$$\frac{N_R}{N_S} = \left[\left(\frac{Z_{HK}}{Z_{LK}} \right) \left(\frac{x_{LK.W}}{x_{HK.D}} \right)^2 \frac{W}{D} \right]^{0.206}$$

$$N_{Total} = N_R + N_S$$

N_R = no. of trays in the Rectifying section.

N_S = s s s s s stripping section

Ex. (5) :-

A Feed mixture containing 35% benzene, 35% toluene and 30% Cumene (all mol %), is to be fractionated at a rate of 100 kmol/hr to recover 98% of benzene in the distillate and 98.5% of the toluene in the bottom product. The feed is saturated vapour and the reflux at bubble point. Calculate:-

1- N_{min} .

2- R_{min} .

3- The number of ideal stages if a reflux ratio 1.3 times the R_{min} is used. Specify feed stage.

Given:-

$$P_{total} = 1 \text{ atm.}$$

$$\begin{array}{l} \text{top:} \quad 2.55 \\ \text{bott:} \quad 2.25 \end{array} \left. \vphantom{\begin{array}{l} \text{top:} \\ \text{bott:} \end{array}} \right\} \alpha_{AB}$$

$$\begin{array}{l} 0.254 \\ 0.311 \end{array} \left. \vphantom{\begin{array}{l} 0.254 \\ 0.311 \end{array}} \right\} \alpha_{CB}$$

Sol. :-

$$f_{AD} = 0.98, \quad f_{BW} = 0.985$$

$$F = 100, \quad z_A = 0.35, \quad z_B = 0.35, \quad z_C = 0.3$$

$$q = 0 \text{ (Sat. vap.)}$$

$$D \cdot x_{AD} = 0.98 * 100 * 0.35$$

$$\boxed{D \cdot x_{AD} = 34.3} \Rightarrow \boxed{W \cdot x_{AW} = 0.7 \text{ kmol}}$$

$$W \cdot x_{BW} = 0.985 * 100 * 0.35$$

$$\boxed{W \cdot x_{BW} = 34.47} \Rightarrow \boxed{D \cdot x_{BD} = 0.53 \text{ kmol}}$$

$$D_{\text{total}} = 34.3 + 0.53 + 0$$

$$\boxed{D_T = 34.83 \text{ kmol}} \Rightarrow \boxed{W = 65.17 \text{ kmol}}$$

$$\therefore x_{A,D} = 0.985$$

$$x_{A,W} = 0.0107$$

$$\textcircled{a} N_{\min} = \frac{\text{Log} \left[\frac{f_{AD} \times f_{BW}}{(1-f_{AD})(1-f_{BW})} \right]}{\text{Log } \alpha_{AB} |_{\text{avg.}}}$$

(α_{AB}) at top differ than (α_{AB}) at bottom, then

$$\alpha_{AB} |_{\text{avg.}} = \left(\alpha_{AB} |_{\text{top}} \cdot \alpha_{AB} |_{\text{Bot}} \right)^{0.5}$$

$$= (2.55 \times 2.25)^{0.5} \Rightarrow \alpha_{AB} |_{\text{avg.}} = 2.4$$

$$\alpha_{CB} |_{\text{avg.}} = (0.254 \times 0.311)^{0.5} \Rightarrow \alpha_{CB} |_{\text{avg.}} = 0.281$$

$$N_{\min} = \frac{\text{Log} \left[\frac{0.98 \times 0.985}{(1-0.98)(1-0.985)} \right]}{\text{Log } (2.4)} = \underline{\underline{9.2}}$$

$$\text{Ideal trays + the reboiler} = \boxed{9.2}$$

(Now we can check for comp. (c) by calculating its fraction in the distillate by using N_{\min}).

⑥ R_{min} .

To calculate (R_{min}) we must first find (ϕ)

$$1 - q = 1 - 0 = \frac{(0.35)(2.4)}{2.4 - \phi} + \frac{(0.35)(1.0)}{(1.0) - \phi} + \frac{(0.3)(0.28)}{(0.28 - \phi)}$$

$$\boxed{\phi = 1.44} \quad (\text{because (HNK) is non-distributing})$$

$$V_{min} = \frac{(2.4)(34.3)}{2.4 - 1.44} + \frac{(1.0)(0.53)}{1.0 - 1.44} + 0$$

$$\boxed{V_{min} = 84.54 \text{ kmol}}$$

$$L_{min} = V_{min} - D \Rightarrow 84.54 - 34.83$$

$$\boxed{L_{min} = 49.71 \text{ kmol}}$$

$$R_{min} = \frac{L_{min}}{D} = \frac{49.71}{34.83} \Rightarrow \boxed{R_{min} = 1.43}$$

⑦ To find No. of trays.

$$R_{act.} = 1.3 R_{min}$$

$$R = 1.3 \times 1.43 \Rightarrow \boxed{R = 1.86}$$

$$X = \frac{R - R_{min}}{R + 1} = \frac{1.86 - 1.43}{1.86 + 1}$$

$$\boxed{X = 0.1503}$$

(88)

$$Y = 1 - \exp \left[\frac{1 + (54.4)(0.1503)}{11 + (117.2)(0.1503)} \left(\frac{0.1503 - 1}{(0.1503)^{0.5}} \right) \right]$$

$$Y = 0.505$$

$$Y = \frac{N - N_{\min}}{N + 1} = 0.505 = \frac{N - 9.2}{N + 1}$$

$$N = 19.6$$

To find feed plate location:

$$\frac{N_R}{N_S} = \left[\left(\frac{Z_{HK}}{Z_{LK}} \right) \left(\frac{x_{LK.W}}{x_{H.K.D}} \right)^2 \left(\frac{W}{D} \right) \right]^{0.206}$$

$$= \left[\left(\frac{0.35}{0.35} \right) \left(\frac{0.0107}{0.015} \right)^2 \left(\frac{65.17}{34.83} \right) \right]^{0.206}$$

$$\frac{N_R}{N_S} = 0.99 \Rightarrow N_R = 0.99 \times N_S$$

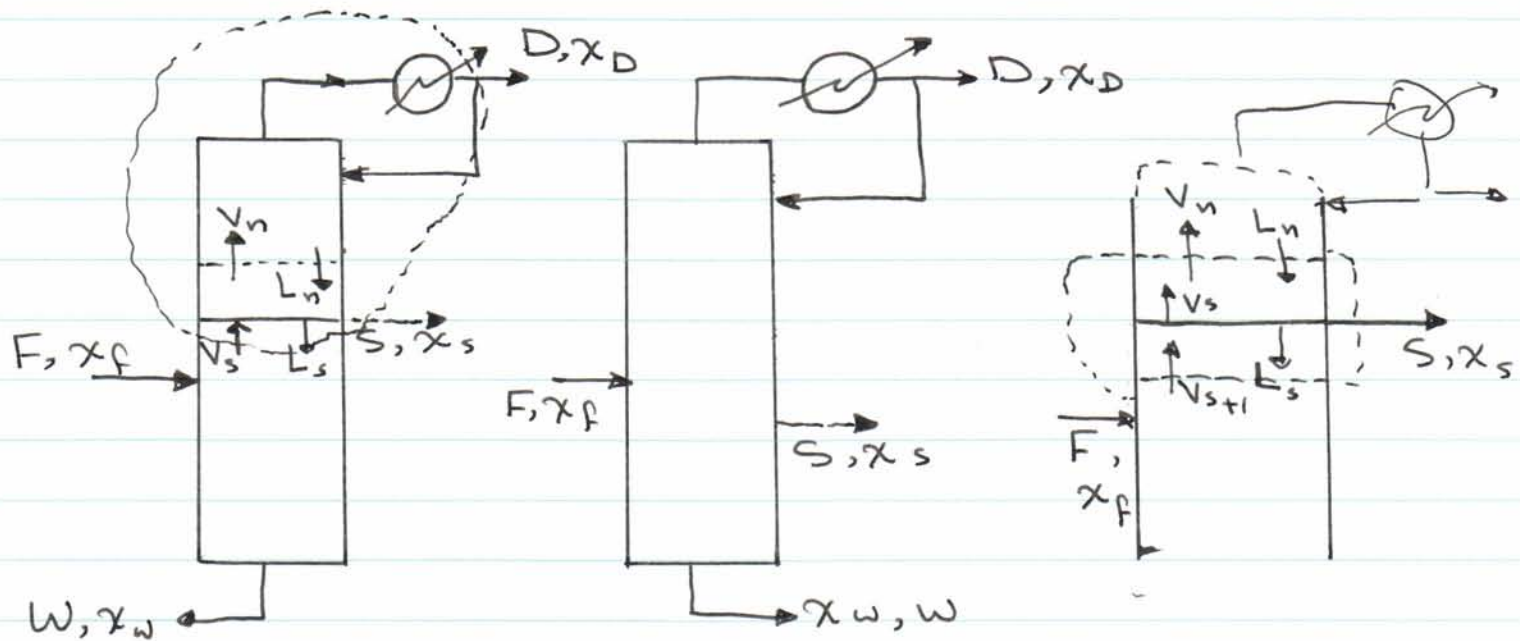
$$N_R + N_S = N_T = 19.6 \Rightarrow N_S = 19.6 - N_R$$

$$\therefore N_R = 9.85, N_R = 9.5$$

\therefore Feed tray is (10th) tray from the top.

Multiple Feed and Sidestreams

Sidestream is defined as any product stream other than the overhead product and the residue.



① Above feed tray

② below feed tray

The operating line for the upper section..

$$y_n = \frac{L_n}{V_n} x_{n+1} + \frac{D \cdot x_D}{V_n}$$

M.B. between Top and side stream.

$$V_n = V_{n+1} = V_{s+1} = V_s$$

$$\therefore V_s = L_s + S + D$$

$$V_s \cdot y_n = L_s \cdot x_{n+1} + S \cdot x_s + D \cdot x_D$$

$$\therefore y_n = \frac{L_s}{V_s} x_{n+1} + \frac{S \cdot x_s + D \cdot x_D}{V_s} \quad \text{Side stream equation}$$

Since (s) always withdrawn or removed as liq. then:-
 $L_s = L_n - S$ and $V_s = V_n$

Ex. (1) "Side stream"

A mixture of water and ethyl alcohol containing 0.16 mole fraction alcohol, is continuously distilled in fractionator column to give product containing 0.77 mol. fractⁿ. alcohol and a waste of 0.02 mol. fractⁿ. alcohol. It is proposed to withdraw 25% of the alcohol in the entering stream as side product with 0.5 mol. fractⁿ. alcohol. Determine the No. of theoretical plates required and side stream location if the feed is liquor at the boiling point, given that $R=2$.

Sol. Basis = 100 kmol feed.

As the (S.S) composition is to be (0.5) then there are (8 kmol) in that stream

$$\rightarrow 100 * 0.16 * 0.25 = 0.5 S$$

O.M.B :-

$$F = D + W + S \Rightarrow 100 = D + W + 8$$
$$\therefore D + W = 92$$

C.M.B :-

$$100 * 0.16 = 0.77 D + 0.02 W + 8 * 0.5$$
$$12 = 0.77 D + 0.02 W$$

$$\therefore D = 13.5 \text{ kmol.}, W = 78.45 \text{ kmol.}$$

* In the section between the (s.s) and top of column

$$R = L_n / D \Rightarrow L_n = 27.0$$

$$V_n = L_n + D \Rightarrow V_n = 40.50$$

* For section between (S.S) and Feed

$$V_s = V_n = 40.50$$

$$L_n = L_s + S \Rightarrow L_s = 19.0$$

* In the bottom

$$L_m = L_s + F \Rightarrow L_m = 119.0$$

∴ Feed is sat. liq. then ∴

$$V_m = L_m - W \Rightarrow V_m = 40.55$$

∴ slope always (L/V), the slope for each part can be found.

$$U.O.L. \quad \text{slope} = \frac{L_n}{V_n} = \frac{27.0}{40.50} = 0.67$$

نرسم خط لقطه (x_d, x_d) وبالخط 0.67 حتى نقطه q-line عند S.S.L. بعد ذلك نمتجه الى

$$\text{slope} = \frac{19.0}{40.50} = 0.47$$

حتى نقطه تقاطع q-line مع U.O.L. وبالخط 0.47 نبدأ بتوجيه خط مستقيم الى قيم $x_f = 0.16$. وبعد ذلك نمتجه الى L.O.L.

$$L.O.L. \Rightarrow \text{slope} = \frac{119.0}{40.55} = 2.92$$

ومن نقطه $x_f = 0.16$ وبالخط 2.92 أو

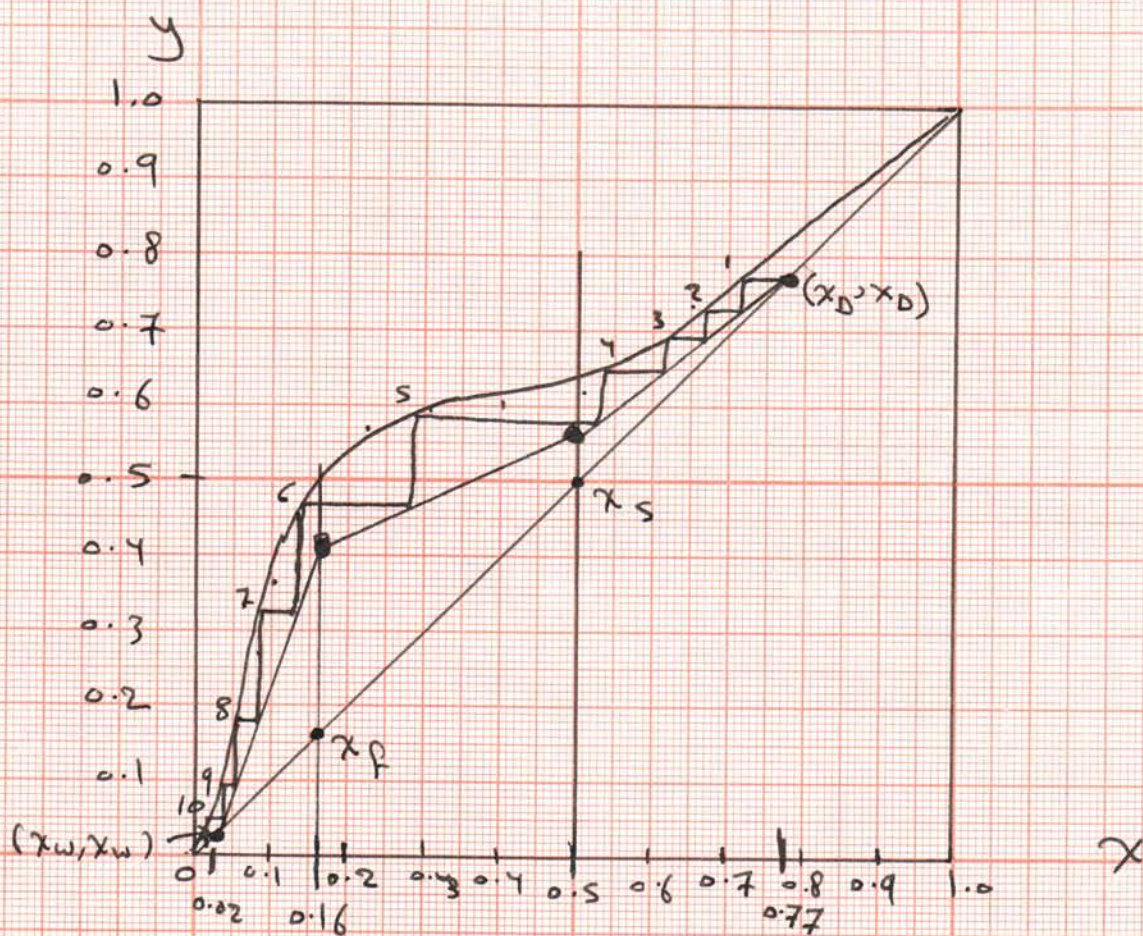
من نقطه $x_f = 0.16$ الى قيم $x_w = 0.02$ نصل خطاً مستقيماً.

وبعد ذلك نبدأ عند x_D بخط افقي ويليهِ عمودي الى ان نصل x_w

نجد قيم N

From Fig. ∴

$$N = 10$$



Ex. (2) :- "Two Feed"

For Ethanol & N-Propanol system, Find the amounts of D & W, the actual No. of plates and the heat duties for the partial condenser & reboiler for the following data:-

$F_1 = 750 \text{ kmol/hr}$ sat. liquid $x_{F_1} = 0.65$ mol. fract.

$F_2 = 27900 \text{ kg/hr}$ Sat. vap. $x_{F_2} = 0.24$ wt. fract.

$x_D = 0.96$, $x_W = 0.04$

$R = \text{twice the minimum reflux ratio}$

$\lambda_{\text{ethanol}} = 38770 \text{ J/mol.}$

$\lambda_{\text{propanol}} = 41784 \text{ J/mol.}$

Efficiency = 70%

$x : 0 \quad 0.1 \quad 0.2 \quad 0.3 \quad 0.5 \quad 0.7 \quad 0.8 \quad 0.9 \quad 1.0$

$y : 0 \quad 0.19 \quad 0.34 \quad 0.47 \quad 0.67 \quad 0.83 \quad 0.89 \quad 0.95 \quad 1.0$

Sol. :-

$$x_{F_2} = \frac{0.24/46}{\frac{0.24}{46} + \frac{0.76}{60}} = 0.29$$

From Fig. :-

$$\frac{x_D}{R_{\min} + 1} = 0.4 \Rightarrow \text{Therefore } R_{\min} = 1.4$$

$$R = 2 R_{\min} \Rightarrow R = 2 * 1.4 \Rightarrow R = 2.8$$

$$\frac{x_D}{R+1} = \frac{0.96}{2.8+1} = 0.25$$

$$F_2 = \frac{27900}{0.29 \times 46 + 0.71 \times 60} = 498.15 \text{ kmol/hr}$$

$$F_1 + F_2 = D + W \Rightarrow 750 + 498.15 = 1248.15 = D + W$$

$$F_1 \cdot x_{F_1} + F_2 \cdot x_{F_2} = D \cdot x_D + W \cdot x_D$$

$$750 \times 0.65 + 498.15 \times 0.29 = D \times 0.96 + (1248.15 - D) \times 0.01$$

$$\therefore D = 632.65 \text{ kmol/hr}$$

$$W = 615.50 \text{ kmol/hr}$$

$$\therefore R = \frac{L_n}{D} \Rightarrow 2.8 = \frac{L_n}{632.65} \Rightarrow L_n = 1771.42 \frac{\text{kmol}}{\text{hr}}$$

$$V_n = L_n + D = 1771.42 + 632.65 \Rightarrow V_n = 2404.07 \frac{\text{kmol}}{\text{hr}}$$

$\hookrightarrow V_n = V_p$

$$\therefore V_w = V_m = V_p - F_2 = 2404.07 - 498.15 = 1906 \frac{\text{kmol}}{\text{hr}}$$

For first feed

$$F_1 + V_F = L_F + D$$

$$F_1 \cdot x_{F_1} + V_F y_F = L_F \cdot x_F + D \cdot x_D$$

$$y_F = \frac{L_F}{V_F} x_F + \frac{D \cdot x_D - F_1 \cdot x_{F_1}}{V_F}$$

$$V_F = V_n = 2404.07 \frac{\text{kmol}}{\text{hr}}$$

$$\text{intercept} = \frac{632.65 \times 0.96 - 750 \times 0.65}{2404.07} = \frac{607.344 - 487.5}{2404.07}$$

$$\text{intercept} = 0.05$$

slope for the second feed :-

$$\text{slope} = \frac{L_m}{V_m} = \frac{L_F + F_2 \leftarrow \text{sat. vap.} = 0}{V_m} \Rightarrow \frac{L_F}{V_m} \Rightarrow$$

$$\text{slope} = \frac{F_1 + L_n}{V_m} = \frac{750 + 1771.42}{1906}$$

$$\text{slope} = 1.32$$

From Fig., the No. of plates = 14

$$\text{The actual No. of plates} = \frac{14}{0.7} = 20 \text{ trays}$$

$$q_r = V_w \cdot \lambda_w$$

$$\lambda_w = \lambda_e \cdot y_{we} + \lambda_p y_p = 38770 * 0.07 + 41784 * 0.93$$

$$\lambda_w = 41573.02 \text{ kJ/kmol}$$

$$q_r = V_w \cdot \lambda_w$$

$$= 1906 * 41573.02$$

$$q_r = 7.92 * 10^7 \text{ kJ/hr} = 22000 \frac{\text{kJ}}{\text{s}} = 22000 \text{ kW}$$

$$q_D = L_n \lambda_R$$

$$\lambda_R = \lambda_R \cdot \lambda_e + (1 - \lambda_R) \lambda_p$$

$$= 0.92 * 38770 + 0.08 * 41784$$

$$\lambda_R = 39011.12 \text{ kJ/kmol}$$

$$q_D = 1771.42 * 39011.12$$

$$= 6.91 * 10^7 \frac{\text{kJ}}{\text{hr}} = 19195.8 \frac{\text{kJ}}{\text{s}} = 19195.8 \text{ kW}$$

$$\approx 19 \text{ MW}$$

