

# الجامعة التكنولوجية

## قسم الهندسة الكيميائية

### المرحلة الثالثة

### انتقال كتلة

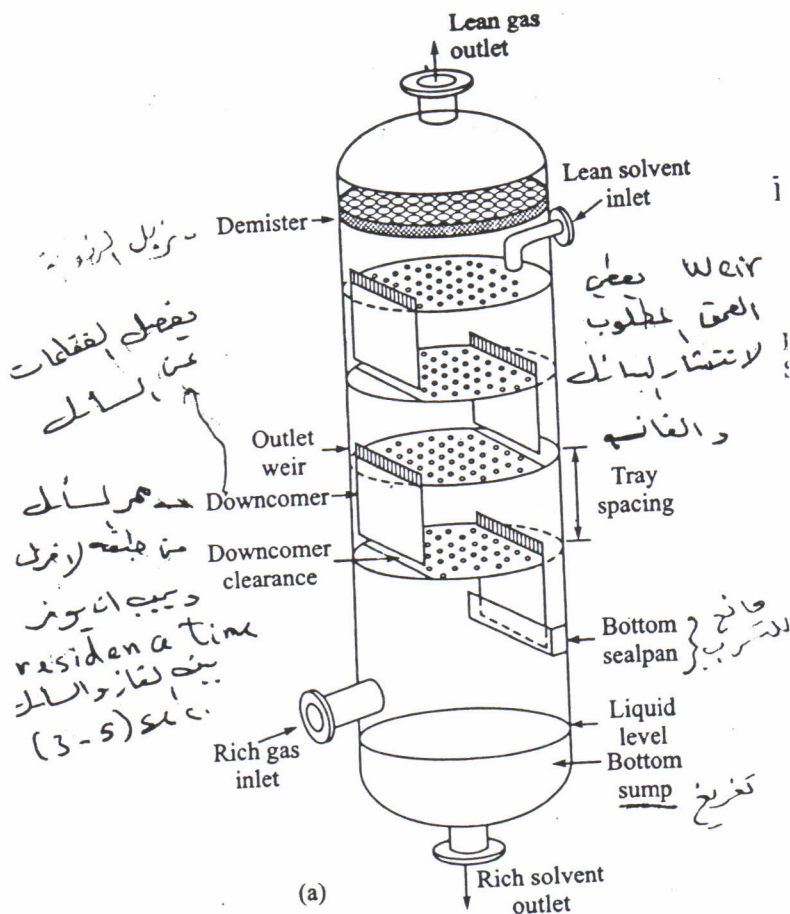
أ.م. د. عامر عزيز

## - Tray or Plate Columns -

A tray column primarily consists of a vertical cylindrical shell and a set of "tower internals" that include:

- 1- Tray or plates on which the gas-liquid contact occurs.
- 2- Arrangements for flow of the liquid from one tray to the lower one through the down-comer.
- 3- Inlet and outlet nozzels for the two phases.

Figure below shows a few essential parts of a (sieve-tray) column.



### The Trays - (Plate)

A tray has two major functions:

① It allows the gas to flow through the holes, the gas bubbles through the liquid to form "gas-liquid dispersion", the tray holds the dispersion.

② The tray separates the column into a number of compartments, each of which constitutes a stage. M.T. between the phases occurs on a tray. Therefore trays as a whole constitute the

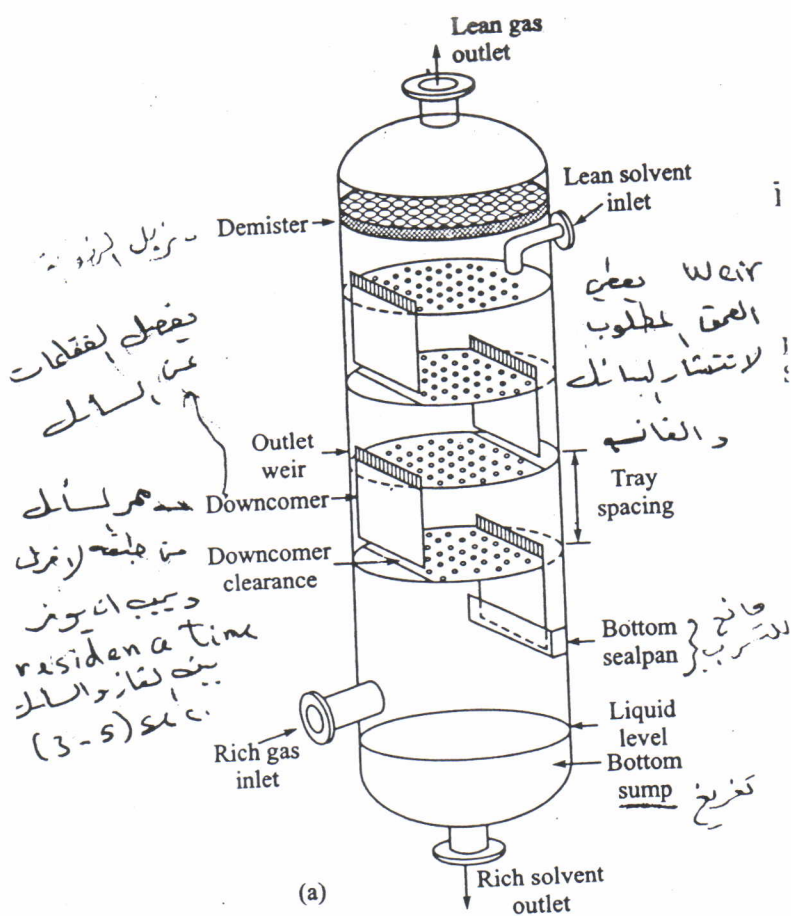
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"heart of a column". The performance of a column depends upon the performance of the trays.



The Types of trays :- ① Bubble - Cap tray.  
② Sieve - Tray. ③ Valve - Tray.

Stage :- Any device or combination of devices in which two immiscible phases are brought into contact in order to achieve mass transfer of one or more solute from one phase to other is called "Stage". An efficient contact of the phases in a stage tends to bring them to equil<sup>m</sup>. If the contacting is so efficient that the phases reach equil<sup>m</sup> when they leave, the stage is called an "ideal stage".

The stage efficiency gives a measure of how close to equil<sup>m</sup> the phases may reach in a stage.

An ideal stage has 100% efficiency.

### Difference between Packed and Plate Columns

- 1 - Plate col. provide more positive contact between the two fluid phases by repeated mixing and separation. whereas packed col. may be subject to by-passing or back-mixing.
- 2 - Plate col. can handle greater liquid loads without flooding.
- 3 - s s are more easily cleaned.
- 4 - Packed col. give lower press. drop for gas flow, which is of particular importance in vacc. operations.
- 5 - liquid hold-up is less in packed col.
- 6 - Packed col. may be more economical in processing.
- 7 - Small diameter packed col. are usually cheaper than plate col. of the same size.



# Types of Dispersion

## ① Spary

سرعة غاز عالية  
وسائل واجهة  
فيصبح عند  
dispersion لـ  
غاز continuous

## ② Froth

سرعة سائل عاليه  
نسبياً فيصبح حدود  
Spary للغاز ولكنها  
ليست كافيه لتصبح  
emulsion  
الفقاعات

## ③ Emulsion

سرعة سائل عاليه  
والغاز سرعته قليله  
وبالتالي فقاعات الغاز  
تبقى عالقه في سائل

## Overall M.B :-

$$G_s * Y_{N+1} + L_s X_0 = G_s * Y_1 + L_s X_N$$

$$G_s * Y_{N+1} + L_s * X_0 = G_s Y_T + L_s X_B$$

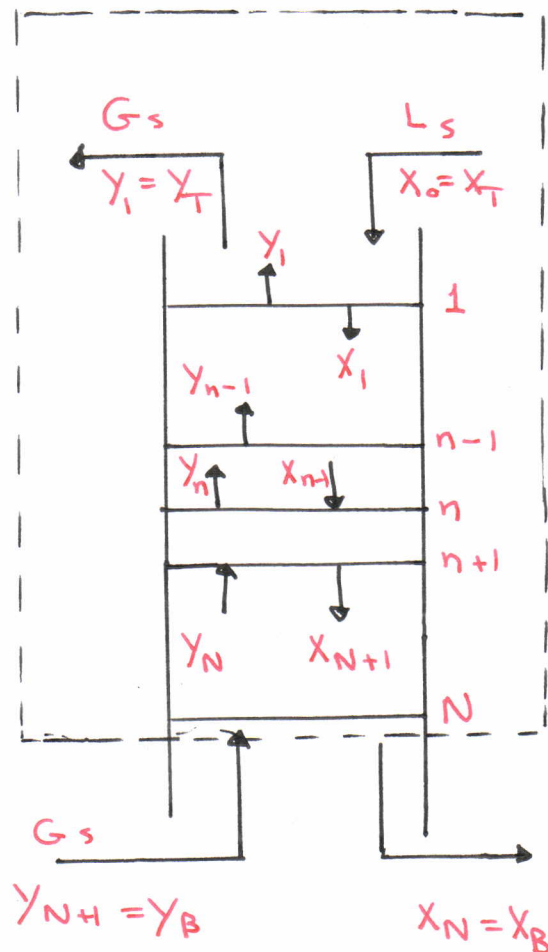
or

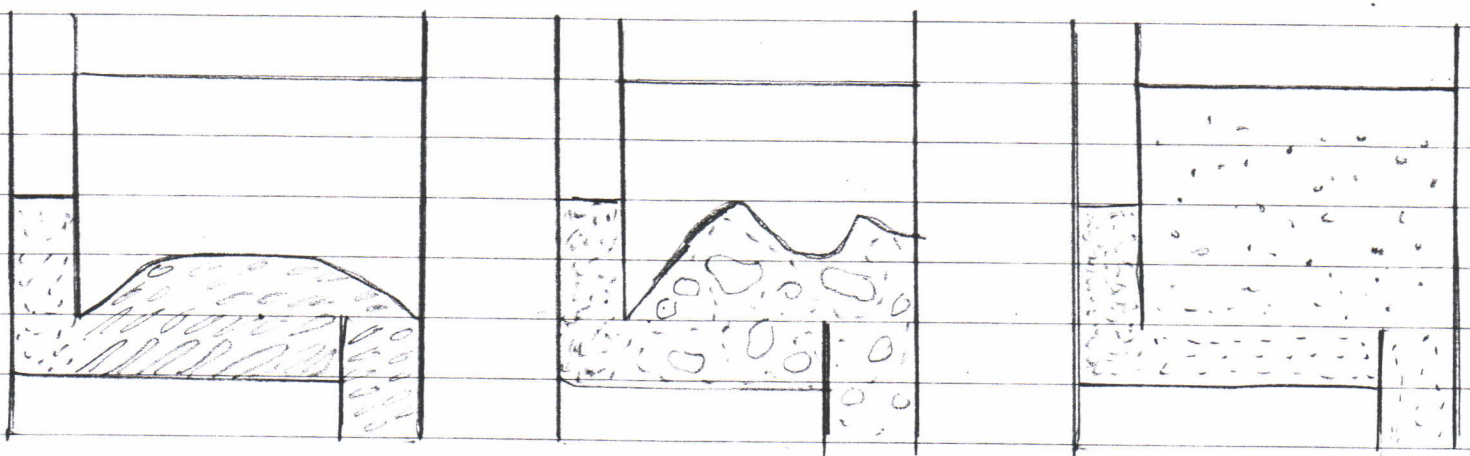
$$G_s * Y_B + L_s * X_T = G_s Y_T + L_s X_B$$

$$G_s (Y_B - Y_T) = L_s (X_B - X_T)$$

$$\frac{L_s}{G_s} = \frac{(Y_B - Y_T)}{(X_B - X_T)} \quad \text{operating}$$

$$Y^* = m X \quad \text{equil}^m. \text{ relation}$$



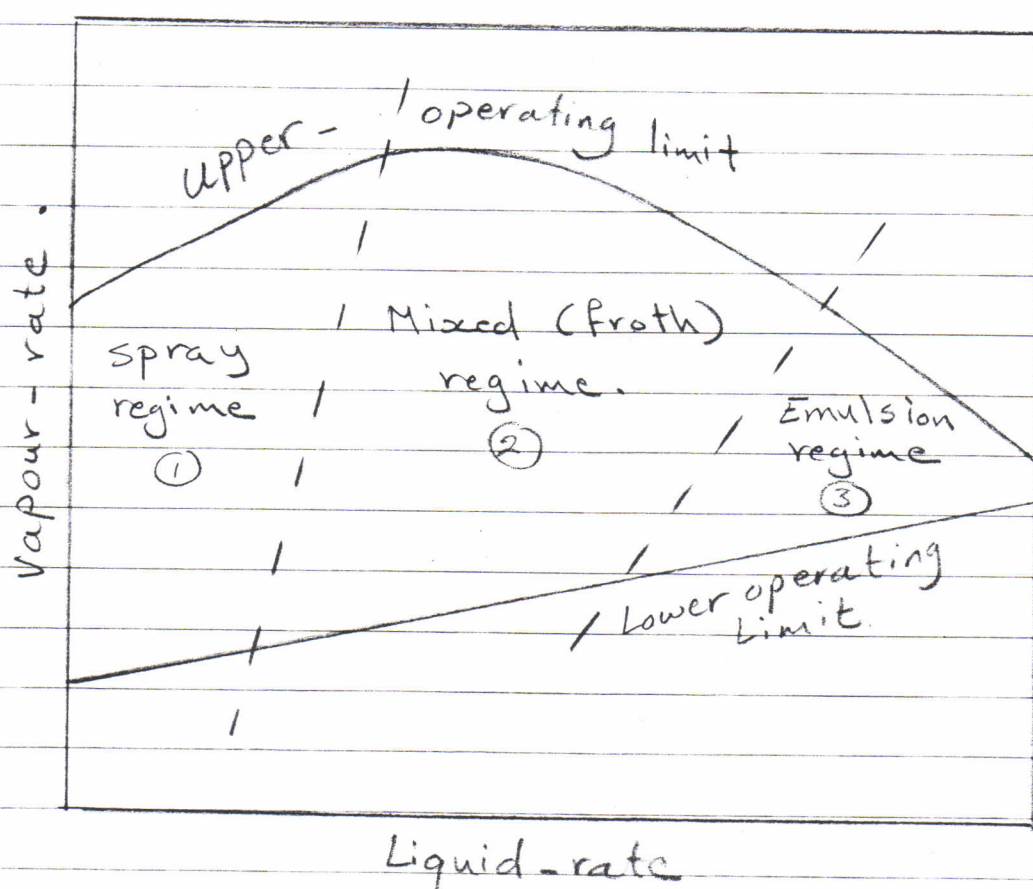


③ Emulsion

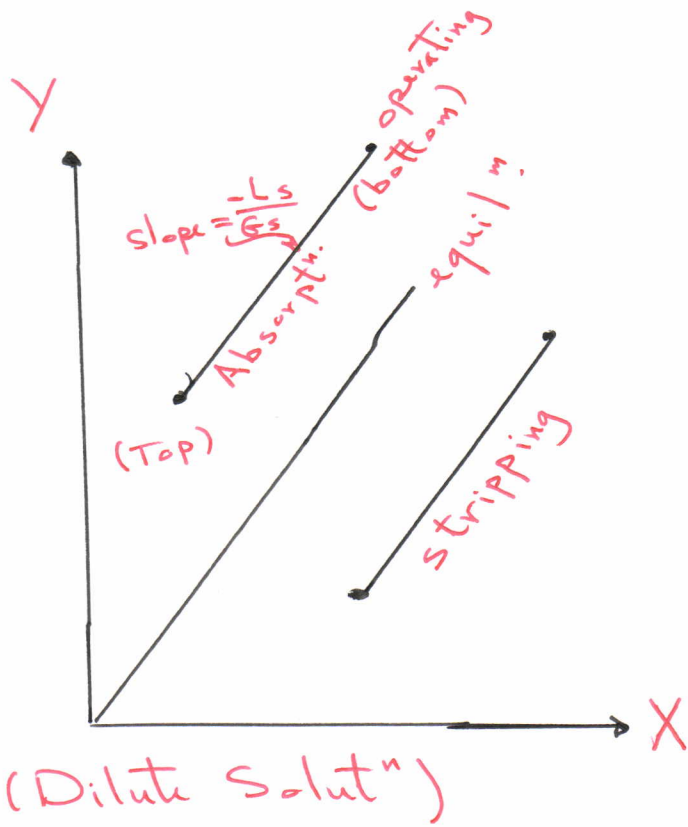
② Froth

① Spray

\* Type of gas-liquid dispersion on a tray.



\* Flow regimes and operating limits of a sieve tray.



$$G_s(Y - Y_T) = L_s(X - X_T)$$

(G, L) not constant

(G', L') constant

$$G_s = G' = G(1 - y)$$

$$L_s = L' = L(1 - x)$$

$$G_s = \frac{G}{1 + y}, \quad Y = \frac{y}{1 - y} \Rightarrow y = Y(1 - Y)$$

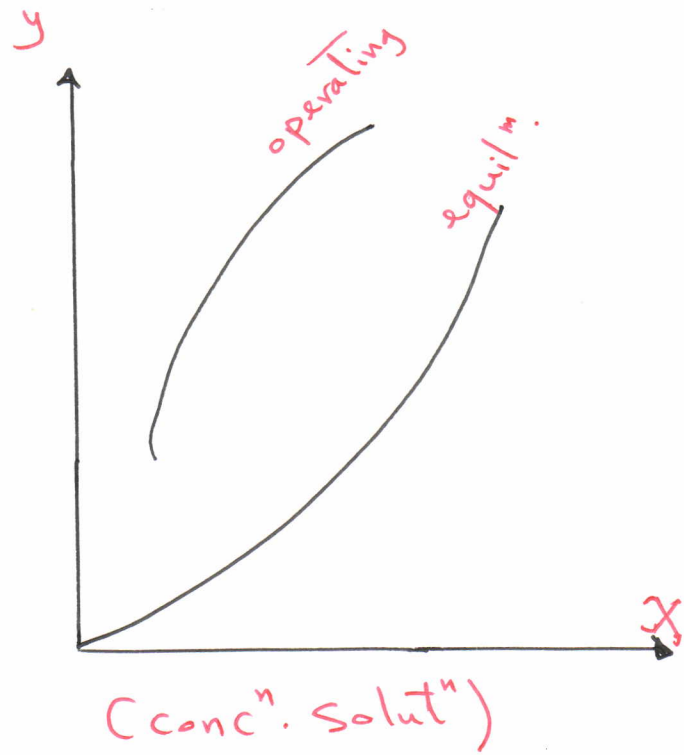
$$L_s = \frac{L}{1 + x}, \quad X = \frac{x}{1 - x} \Rightarrow x = X(1 - X)$$

$G_s$  = rate of flow of phase G on solute-free basis (mole/time)

$L_s =$  s s s s s  $L$  s s s s s

$G =$  s s s s s  $G$  (mol/time)

$L =$  s s s s s  $L$  (



$$G \cdot y - G_T y_T = Lx - L_T x_T$$

(Inlet inert + Carrier)

(Inlet gas (solute only))



# Determination of the No. of Stages in Counter-Current Flow

① Graphical construction. (if both operating and equil<sup>m</sup> are straight line)

- M.B on  $n$ -tray and the top:-

$$G_s(Y_{n+1} - Y_T) = L_s(X_n - X_T)$$

$$Y_{n+1} = \frac{L_s}{G_s} X_n - \frac{L_s}{G_s} X_T + Y_T$$

$$\therefore Y_{n+1} = A X_n + B$$

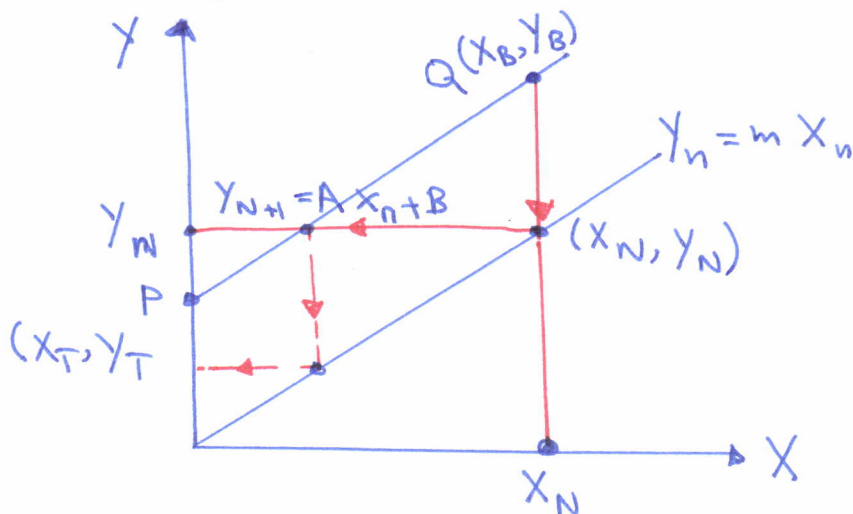
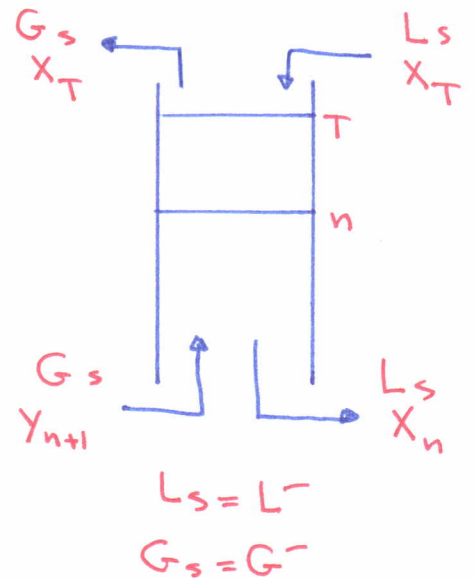
- Total material balance:-

$$G_s(Y_B - Y_T) = L_s(X_B - X_T)$$

$$\frac{L_s}{G_s} = \frac{Y_B - Y_T}{X_B - X_T} \quad \text{operating}$$

- For ideal plate, concentration pair  $(X_n, Y_n)$  should lie on the equil<sup>m</sup> Curve.

$$Y^* = m \cdot X \quad \text{equil}^m \text{ relation.}$$



## ② Algebraic Determination.

- \* Consider the transfer of the solute from gas phase to liquid phase (absorption).
- \* Concentrations are expressed in the mole ratio unit and the phase flow rates are on solute-free basis.
- \* operating line is always linear.

$$y_n = \alpha x_n \text{ at point } (x_n, y_n)$$

Sub. for  $x_n$  in operating equation.

$$y_{n+1} - y_T = \frac{L_s}{G_s} (x_n - x_T)$$

$$y_{n+1} - y_T = \frac{L_s}{G_s} \left( \frac{y_n}{\alpha} - x_T \right)$$

$$y_{n+1} - \bar{A} y_n = y_T - \bar{A} \alpha \cdot x_T \quad \text{--- ①}$$

where  $\bar{A} = \frac{L_s}{\alpha G_s}$

Absorption factor

equation ① is first order - non homogenous linear difference equation.

The final solution for equation ① is :-

$$N = \frac{\log \left[ \left( \frac{y_B - \alpha x_T}{y_T - \alpha x_T} \right) \left( 1 - \frac{1}{\bar{A}} \right) + \frac{1}{\bar{A}} \right]}{\log \bar{A}} \quad \text{--- ②}$$

Kremser-Brown eq. for  $\bar{A} \neq 1$

For  $\bar{A} = 1$ , Solution of equation (1) will be

$$N = \frac{Y_B - Y_T}{Y_T - \alpha \bar{A} X_T} \quad \text{--- (3)}$$

In this case for  $\bar{A} = 1 = \frac{Ls}{\alpha Gs}$ , then slope of operating line will be

$\alpha = \frac{Ls}{Gs}$ , which is the same as the slope of the equilibrium

Curve. So the operating line and equilibrium Curve (line) become parallel, then the concentration change of a phase at each stage is equal. Therefore, the number of plates will be equal to the ratio of the total concentration change over the column to the concentration change over a single stage or plate.

$$N = \Delta C_T / \Delta C_i$$

⊗ In the case of transfer of the solute from liquid phase to gas phase (stripping)

$$N = \frac{\text{Log} \left[ \frac{X_B - \left( \frac{Y_B}{\alpha} \right)}{X_T - \left( \frac{Y_T}{\alpha} \right)} (1 - \bar{A}) + \bar{A} \right]}{\text{Log} \left( \frac{1}{\bar{A}} \right)} \quad \text{--- (4)}$$

where  $\frac{1}{\bar{A}} = \bar{S}$  (stripping factor)



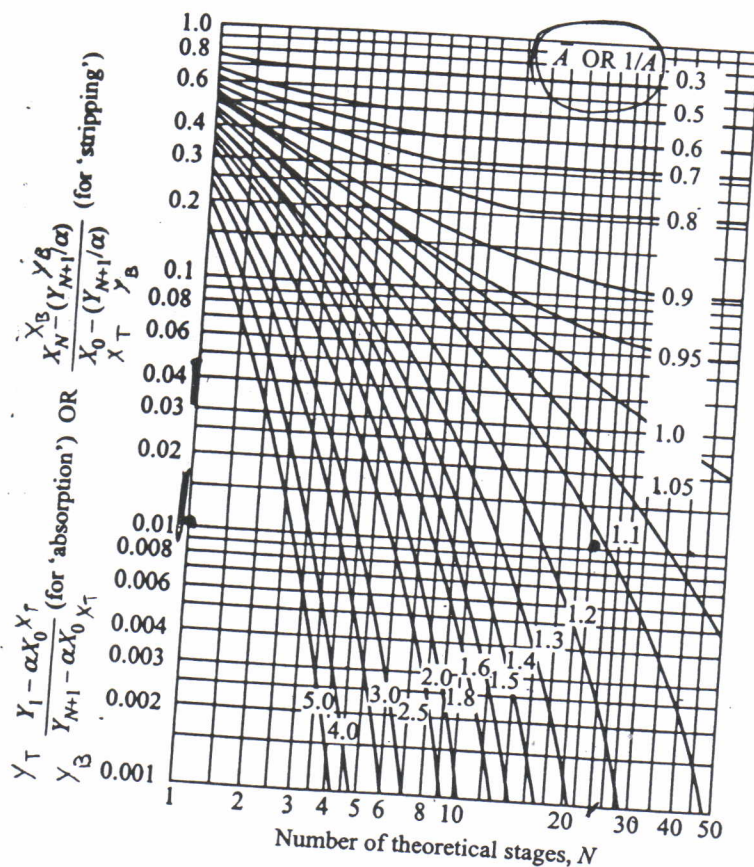
Note 2 if the concentration of solute about (7%)  
equations (3) and (4) can be used noticing:-

$$\bar{A}_T = \frac{L_T}{\alpha G_T} \quad , \quad \bar{A}_B = \frac{L_B}{\alpha G_B}$$

and then  $(\bar{A})$  final will be the geometric mean of  $\bar{A}_T$   
and  $\bar{A}_B$ :-

$$\bar{A} = \sqrt{\bar{A}_T \cdot \bar{A}_B}$$

③ Using Kremser - Chart.



absorption

$$\frac{Y_T - \alpha X_T}{Y_B - \alpha X_T}$$

$$\frac{X_B - \frac{Y_B}{\alpha}}{X_T - \frac{Y_B}{\alpha}}$$

stripping

## - Tray efficiency -

An ideal stage provides quite an efficient contact between the phases, so that they attain equilb<sup>m</sup>, or the phases leaving an ideal stage are at equilb<sup>m</sup> irrespective of the inlet conc<sup>n</sup>. However, the performance of a real stage will expectedly be different from that of an ideal stage.

The tray efficiency is an indicator of how closely the performance of a real tray approaches that of an ideal tray.

We will consider three kinds of efficiency :-

### ① Point-efficiency (EOG).

The tray efficiency is likely to vary from one location to another on a tray.

$$E_{OG} = 1 - e^{-NTU_{OG}}$$

$$\text{where } NTU_{OG} = \frac{k_y \cdot \bar{a} \cdot Z}{G}$$

كلاهما  $\left\{ \begin{array}{l} k_y = \text{Larg M.T. coeff} \\ \bar{a} = \text{Larg interfacial area} \end{array} \right.$

$Z =$  depth of the froth on the tray.

## ② Murphree-efficiency ( $E_{MG}$ )

$$E_{MG} = \frac{y_{n+1} - y_n}{y_{n+1} - y_n^*}$$

for well mixed condition

There is a relation between ① and ② in two certain cases :-

a - If the liquid well mixed then  $E_{MG} = E_{OG}$

b - If the liquid is in plug flow, then :-

$$E_{MG} = \bar{A} \left[ \exp \left( \frac{E_{OG}}{\bar{A}} \right) - 1 \right] \quad \text{plug flow}$$

## ③ Overall tray efficiency ( $E_o$ ) :-

or (overall column efficiency in section of column) is used to determine No. of real trays.

$$E_o = \frac{\text{No. of ideal trays}}{\text{No. of real trays}}$$

$$\Rightarrow \text{Real No} = \frac{\text{ideal}}{\text{eff.}}$$

$$E_o = \frac{\ln \left[ 1 + E_{MG} \left( \frac{1}{\bar{A}} - 1 \right) \right]}{\ln \left( \frac{1}{\bar{A}} \right)}$$



Ex.(1) :- Absorption column receiving about  $130 \text{ kmol/hr}$  of feed gas containing  $9 \text{ mol}\%$  of solute.

It is required to remove  $93\%$  of solute using  $150 \text{ kmol/hr}$  liquid solvent.

The feed solvent has  $0.4\%$  of solute in it.

The Murphree tray efficiency is to be  $45\%$ .

Equilibrium data are:-

x	0.013	0.033	0.049	0.064	0.074	0.093	0.106
y	0.01	0.026	0.043	0.06	0.073	0.1	0.126

Determine the No. of trays required?

Sol:-

$$x_T = 0.004$$

$$y_T = ?$$

$$y_B = 0.09$$

$$x_B = ?$$

To remove  $93\%$  from  $9\%$  initial

$$y_T = 0.09(1 - 0.93)$$

$$y_T = 0.0063$$

To find  $x_B$  make overall M.B

$$G(y_B - y_T) = L(x_B - x_T)$$

$$130(0.09 - 0.0063) = 150(x_B - 0.004)$$

$$10.881 = 150(x_B - 0.004)$$

$$x_B = 0.07654$$

To find operating line equation  
Make M.B between top of column and any section.

$$G(y - y_T) = L(x - x_T)$$
$$130(y - 0.0063) = 150(x - 0.004)$$

$$y = 1.15x - 0.0017$$
 operating line equation.

Plot equilib<sup>m</sup> line :-

between :-

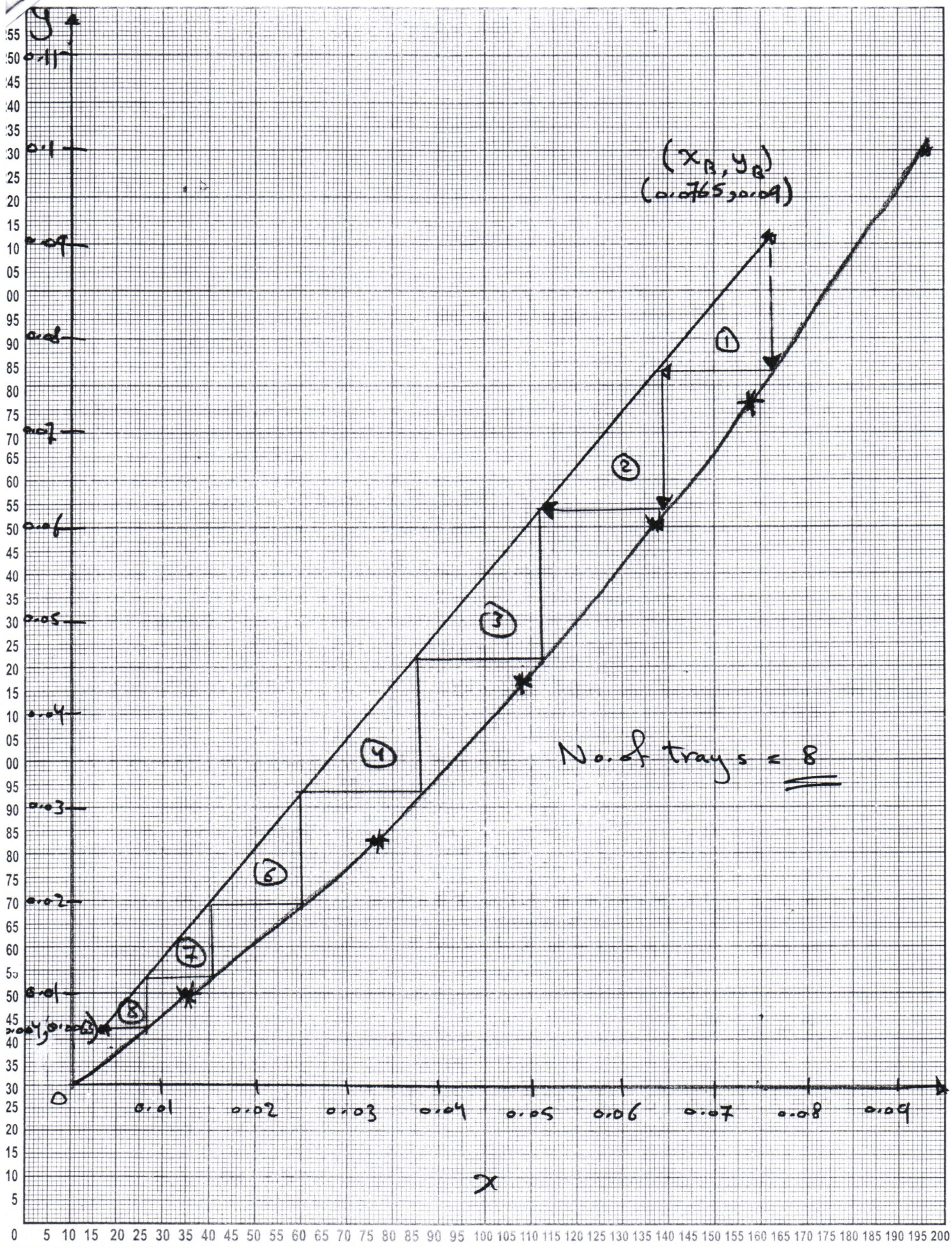
$$(0.004, 0.0063), (0.0765, 0.09)$$

From the Fig. :-

$$\text{No. of theo. trays} = 8$$

$$\text{No. of Actual trays} = \frac{8}{0.45} = 18$$





No. of trays = 8



⑥ to solve analytically :-

$$N = \frac{\log \left\{ \left[ \frac{y_B - \alpha x_T}{y_T - \alpha x_T} \right] \left[ 1 - \frac{1}{\bar{A}} \right] + \frac{1}{\bar{A}} \right\}}{\log \bar{A}}$$

$$\bar{A} = \frac{L}{\alpha G} \quad , \quad \alpha = \text{slope} \approx 1.1$$

$$\bar{A} = \frac{150}{1.1 \times 130} = 1.049$$

$$N = \frac{\log \left\{ \left[ \frac{0.09 - 1.1 \times 0.004}{0.0063 - 1.1 \times 0.004} \right] \left[ 1 - \frac{1}{1.049} \right] + \frac{1}{1.049} \right\}}{\log 1.049}$$

$$N = \frac{\log 3.2}{\log 1.049} \rightarrow N \approx 25$$

$\therefore \text{real No. of trays} = \frac{25}{0.45} \approx 56$

⑦ Using Kremser - Chart

$$\bar{A} \approx 1.05$$

$$\frac{y_T - \alpha x_T}{y_B - \alpha x_T} = \frac{0.0063 - 1.1 \times 0.004}{0.09 - 1.1 \times 0.004} = \frac{1.9 \times 10^{-3}}{8.5 \times 10^{-2}}$$

$$= 0.02$$

From chart find (N) from (1.05) and (0.02)

$$N \approx 25$$

$$\text{Real No. of trays} = \frac{25}{0.45} \approx 56$$

Ex. (2) :- It is required to remove 99% of the solute (C) from a solution of (C) in gas (G) by using a pure solvent liquid (L) in a counter-Current Cascade. The feed containing 8% C in the mixture enters the column at the bottom at a rate of 5500 kg/hr. The solvent enters at the top at a rate of 7685 kg/hr.

- Write down the equation of the operating line.
- Determine the number of trays required to perform the separation if the overall tray efficiency 40%.
- The equil<sup>m</sup>. relation  $y = 1.32 X$ .

Sol. Feed rate (mixture) = 5500 kg/hr  
99% to be removed  $\therefore$  remaining (0.01)

$$y_B = 0.08$$

$$y_T = 0.08 \times 0.01 = 0.0008$$

$$x_T = 0 \text{ (Pure Solvent)}$$

Now making overall M.B to find ( $x_B$ )

$$G(y_B - y_T) = L(x_B - x_T)$$

$$5500(0.08 - 0.0008) = 7685(x_B - 0)$$

$$x_B = 0.056$$

To find operating line

$$G(y - y_T) = L(x - x_T)$$

$$5500(y - 0.0008) = 7685(x - 0)$$

$$y = 1.39x + 0.0008 \quad \text{operating line}$$

We have two points

$(0, 0.0008)$  and  $(0.056, 0.08)$

\* To find No. of plates using all previous methods

① Graphical method :-

a - Draw equil<sup>m</sup> curve using  $y = 1.32x$

(y) varies from  $(0.0008 \text{ to } 0.08)$

(x) varies from  $(0, \text{ to } 0.056)$

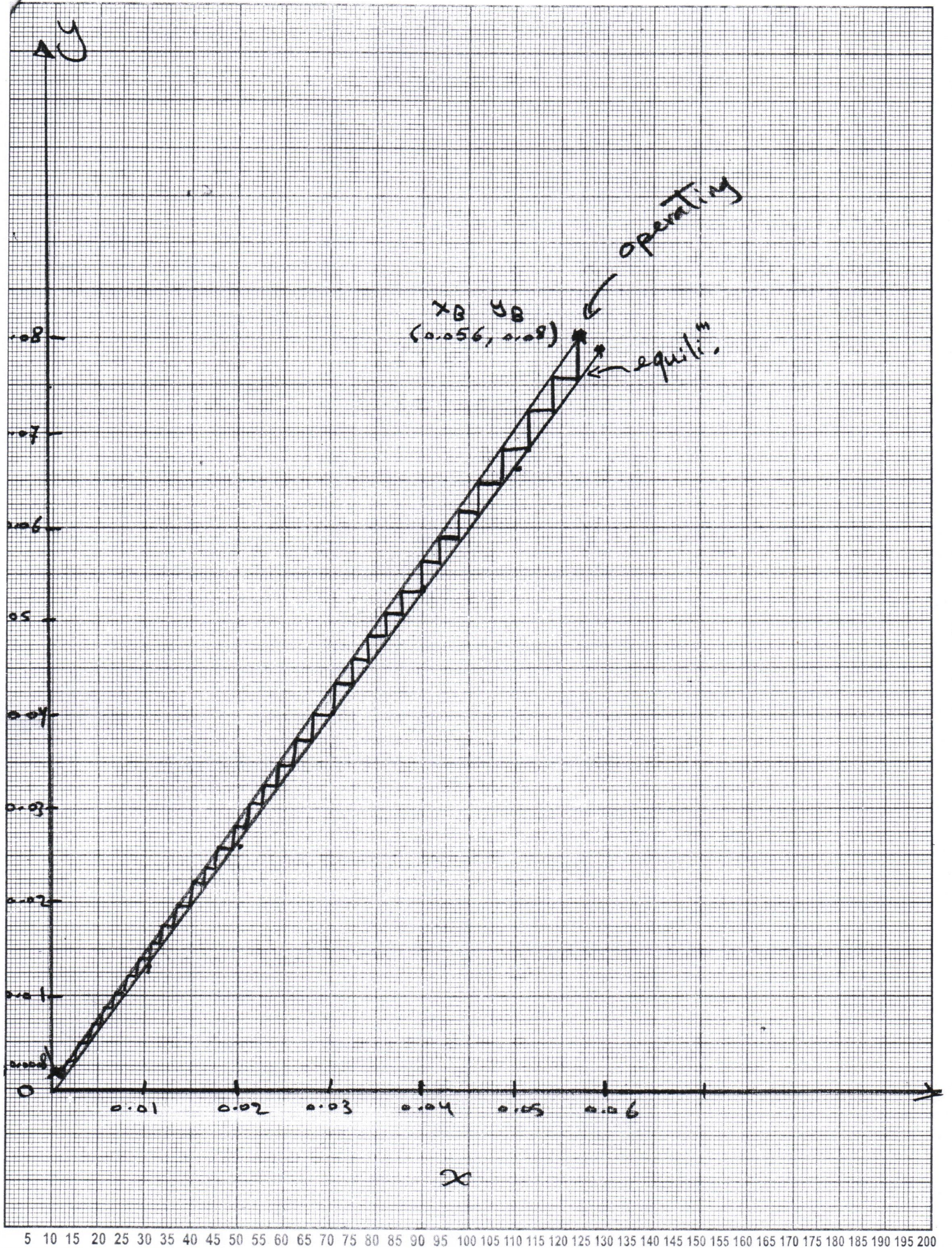
b - Draw operating line according to relation found

$$\text{Total No. of trays} = 33$$

$$\text{Real No. of trays} = \frac{33}{0.4}$$

$$\approx 83$$







② Analytical method :-

$$N = \frac{\text{Log} \left[ \frac{y_B - \alpha x_T}{y_T - \alpha x_T} \left( 1 - \frac{1}{A^-} \right) + \left( \frac{1}{A^-} \right) \right]}{\text{Log } A^-}$$

$$y_B = 0.08, \quad y_T = 0.0008, \quad x_T = 0, \quad \alpha = 1.32$$

$$A^- = \frac{L}{\alpha \cdot G} = \frac{7685}{(1.32) \times 5500} \Rightarrow \bar{A} = 1.058$$

using Kremser eq. for  $\bar{A} \neq 1$

$$N = \frac{\text{Log} \left[ \left( \frac{0.08 - 0}{0.0008 - 0} \right) \left( 1 - \frac{1}{1.058} \right) + \frac{1}{1.058} \right]}{\text{Log } 1.058}$$

$$N \approx 34$$

$$\text{No. of real plates} = \frac{34}{0.4} = 85$$

③ Using Kremser Chart

Calculate the term for y-axis for absorption

$$\frac{y_T - \alpha x_T}{y_B - \alpha x_T} = \frac{0.0008 - 0}{0.08 - 0} = 0.01$$

$$\text{For } \bar{A} \approx 1.06$$

$$N \approx 35, \quad \text{No. of real plates} = \frac{35}{0.4} = 88$$

Q<sub>3</sub>:- A sieve tray tower is being designed for a gas absorption process. The entering gas contains 1.8% (molar) of (A) which must be absorbed. The gas should leave the tower containing no more than 0.1% (molar) of (A). The liquid to be used as absorbent initially contains 0.01% of (A). The system obeys Henry's law with  $m = 1.41$ . At the bottom of the tower, the molar liquid-to-gas ratio = 2.115, and at the other extreme is (2.326). For these conditions, it has been seen that  $EMGE = 0.65$ . Find No. of trays?

Sol.:-

$$A_1 = \frac{2.115}{1.41} = 1.5, \quad A_2 = \frac{2.32}{1.41} = 1.65$$

$$\text{then } A = \sqrt{1.5 \times 1.65} = 1.57$$

$$y_B = 0.018, \quad y_T = 0.001, \quad x_T = 0.0001$$

analytically:-

$$N = \frac{\log \left[ \frac{y_B - m x_T}{y_T - m x_T} \left( 1 - \frac{1}{A} \right) + \frac{1}{A} \right]}{\log A}$$

$$\therefore N_{\text{theo.}} = 4.64$$

$$E_o = \frac{\ln \left[ 1 + EMGE \left( \frac{1}{A} - 1 \right) \right]}{\ln (1/A)} = 0.596$$

$$\text{and } N_{\text{act.}} = \frac{4.64}{0.59} = 7.79 \approx 8 \text{ tray.}$$

## **Packed column**

Absorption is frequently conducted in packed column, particularly when:

- (1) The required column diameter is less than 2 ft.
- (2) The pressure drop must be low, as for a vacuum service.
- (3) Corrosion consideration favors the use of ceramic or polymeric material.
- (4) Low liquid holdup is desirable.

The countercurrent packed towers operates in a different manner from plated towers. In packed towers the fluids are in contact continuously in their path through the tower, while in plated towers the fluids are contacted occasionally. So, packed columns are continuous differential contacting devices that do not have the physically distinguishable stages found in tray towers. Thus, packed columns are best analyzed by mass transfer considerations rather than by the equilibrium – stage concept. Nevertheless, in practice packed – towers performance is often analyzed on the basis of equivalent equilibrium stages using a packed height equivalent to a theoretical (equilibrium) plates (stage), called the HETP.

The most important reason to use packed towers in absorption is to provide a large contact area as possible as can. There are many types of packing such as, Rasching rings, Lessing rings, Bert Saddles, Pall ring, and many others, most of these types of packing are made of cheap inert materials such as glass, ceramic, graphite or, plastic, but sometimes it may be made of stainless –steel.



## Calculations of the height of packing (Z)

### For dilute mixtures :

Consider mass transfer occur in the column of cross sectional area equal to ( S ) (  $\text{m}^2$  ), containing packing of specific surface area equal to ( a ), (  $\text{m}^2 / \text{m}^3$  ).

Specific surface area = total surface area / volume of the column

Total surface area = surface area of one pack \* number of packing

Volume of column = S \* dZ

Total mass transfer area in height dZ (Interfacial area for transfer) = a . S . dZ

The total no. of moles of (A) transfer per unit area per unit time =  $N_A$

Total moles of (A) transfer / time =  $N_A * a * S * dZ$

According to Whitman two-film theory:-

$$N_A = K_{OG} (P_A - P_A^*) \quad \dots\dots\dots(1)$$

$$N_A * a * S * dZ = K_{OG} (P_A - P_A^*) * a * S * dZ \quad \dots\dots\dots(2)$$

The equation is based on gas phase and is used in calculating the height of the packing (Z) are applied only for dilute or weak solution which leads to the assumption that  $L_m$  &  $G_m$  are constant through out the solution. Don't forget that the solutions can Be considered dilute if the mole fraction of the solute in the inlet streams (gas & liquid) are less than 0.1 (i.e.  $\leq 10\%$ ). The change in gas composition of (A) through the height (dZ) equal to ( $dy_A$ ).

c



Total no. of moles of (A) transfer / time =  $N_A * a * S * dZ$

$$N_A * a * S * dZ = G_m * dy_A \quad \dots\dots\dots(3)$$

$$G_m * dy_A = K_{OG} (P_A - P_A^*) * a * S * dZ \quad \dots\dots\dots(4)$$

Divide the right hand side of the above equation by  $(P_T / P_T)$ :-

$$G_m * dy_A = K_{OG} (y_A - y_A^*) P_T * a * S * dZ \quad \dots\dots\dots(5)$$

$$\int_0^Z dZ = \left( \frac{G_m}{K_{OG} * P_T * a * S} \right) \int_{y_{AT}}^{y_{AB}} \frac{dy_A}{(y_A - y_A^*)}$$

Integration eq.:-

$$Z = \left( \frac{G_m}{K_{OG} * P_T * a * S} \right) * \int_{y_{AT}}^{y_{AB}} \frac{dy_A}{(y_A - y_A^*)} \quad \dots\dots\dots(6)$$

$Z = (HTU)_{OG} * (NTU)_{OG}$  [Height of packing for dilute solution for gas phase]

Where:-  $(HTU)$  = height of transfer unit.

$(NTU)$  = number of transfer unit.

$K_{OG}$  = is constant depends on the physical properties of gas and hydrodynamic properties of fluid.

$$G_m = \text{constant for dilute solution} = \left( \frac{G_{top} + G_{Bott.}}{2} \right).$$

$P_T$  = constant, no friction losses.

$K_{OG}, K_{OL}$  = volumetric overall mass transfer coefficient ( Kmole/ sec. mole fraction.  $m^3$  packing).

H.W :- Derive the  $(Z)$  equation for liquid phases:-

$Z = (HTU)_{OL} * (NTU)_{OL}$  [Height of packing for dilute solution for liquid phase].

## Summary:-

### For dilute solutions:-

### For gases ( Overall Mass Transfer):-

$$Z = (HTU)_{OG} * (NTU)_{OG} \quad [\text{Height of packing for dilute solution for gas phase}]$$

$$(HTU)_{OG} = \left( \frac{G_m}{K_{OG} * P_T * a * S} \right)$$

$$(NTU)_{OG} = \int_{y_{AT}}^{y_{AB}} \frac{dy_A}{(y_A - y_A^*)}$$

### For liquids (Overall Mass Transfer):-

$$Z = (HTU)_{OL} * (NTU)_{OL} \quad [\text{Height of packing for dilute solution for liquid phase}]$$

$$(HTU)_{OL} = \left( \frac{L_m}{K_{OL} * C_T * a * S} \right)$$

$$(NTU)_{OL} = \int_{X_{AT}}^{X_{AB}} \frac{dX_A}{(X_A^* - X_A)}$$

Where the integration of  $(NTU)_{OG} = \int_{y_{AT}}^{y_{AB}} \frac{dy_A}{(y_A - y_A^*)}$  is :-

$$(NTU)_{OG} = \left( \frac{y_B - y_T}{(y - y^*)_{Lm}} \right)$$

$$(y - y^*)_{Lm} = \frac{(y_B - y_{B^*}) - (y_T - y_{T^*})}{\ln \frac{(y_B - y_{B^*})}{(y_T - y_{T^*})}}$$

**For gases (Individual Mass Transfer):-**

$Z = (HTU)_G * (NTU)_G$  [Height of packing for dilute solution for gas phase]

$$(HTU)_G = \left( \frac{G_m}{K_G * P_T * a * S} \right)$$

$$(NTU)_G = \int_{y_{AT}}^{y_{AB}} \frac{dy_A}{(y_A - y_{Ai})}$$

**For liquids (Individual Mass Transfer):-**

$Z = (HTU)_L * (NTU)_L$  [Height of packing for dilute solution for liquid phase]

$$(HTU)_L = \left( \frac{L_m}{K_L * C_T * a * S} \right)$$

$$(NTU)_L = \int_{X_{AT}}^{X_{AB}} \frac{dX_A}{(X_{Ai} - X_A)}$$

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The integration of  $(NTU)_G = \int_{y_{AT}}^{y_{AB}} \frac{dy_A}{(y_A - y_{Ai})}$  is :-

$$(NTU)_G = \left( \frac{y_B - y_T}{(y - y_i)_{Lm}} \right) \text{ where } (y - y_i)_{Lm} = \frac{(y_B - y_{Bi}) - (y_T - y_{Ti})}{\ln \frac{(y_B - y_{Bi})}{(y_T - y_{Ti})}}$$

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**Notice:** 1)  $\frac{L_m}{G_m}$  slope of operating line =  $\frac{(y_B - y_T)}{(x_B - x_T)}$  from ( M. B. ).

2)  $m$  = slope of equilibrium line =  $\frac{(y_B^* - y_T^*)}{(x_B - x_T)}$  from ( equilibrium data ).

### Relation between overall and individual mass transfer coefficient

$$\frac{1}{K_{OG}} = \frac{1}{K_G} + \frac{H}{K_L} \quad \dots\dots\dots(1) \quad (\text{in gas phase})$$

$$\frac{1}{K_{OL}} = \frac{1}{K_G} + \frac{1}{H.K_L} \quad \dots\dots\dots(2) \quad (\text{in liquid phase})$$

Multiply equation (1) by  $(G_m / a \cdot s \cdot P_T)$ :-

$$\frac{G_m}{K_{OG} \cdot a \cdot s \cdot P_T} = \frac{G_m}{K_G \cdot a \cdot s \cdot P_T} + \frac{H \cdot G_m}{K_L \cdot a \cdot s \cdot P_T} \quad \dots\dots\dots(3)$$

$$(\text{HTU})_{OG} = (\text{HTU})_G + \frac{H \cdot G_m}{K_L \cdot a \cdot s \cdot P_T} \cdot \frac{L_m}{G_m} \cdot \frac{C_T}{C_T}$$

$$(\text{HTU})_{OG} = (\text{HTU})_G + (\text{HTU})_L \cdot \frac{G_m}{L_m} \cdot \frac{H \cdot C_T}{P_T} \quad \dots\dots\dots(4)$$

$$\therefore P_A = H \cdot C_A \rightarrow \frac{P_A}{P_T} = \frac{H \cdot C_A}{P_T} \rightarrow \therefore y_A = \frac{H \cdot C_T}{P_T} \cdot x_A$$

$$\therefore y_A = m \cdot x_A \quad \dots\dots\dots \text{straight line equation}$$

$$(\text{HTU})_{OG} = (\text{HTU})_G + (\text{HTU})_L \cdot m \cdot \frac{G_m}{L_m} \quad \dots\dots\dots(5) \quad (\text{For gas phase})$$

Same steps can be applied to equation (2) for liquid phase to get :

$$(\text{HTU})_{OL} = (\text{HTU})_L + (\text{HTU})_G \cdot \frac{L_m}{m \cdot G_m} \quad \dots\dots\dots(6) \quad (\text{For liquid phase})$$

Where  $\left(\frac{L_m}{m \cdot G_m}\right)$  is the absorption factor (A).



## How to find the intermediate concentration

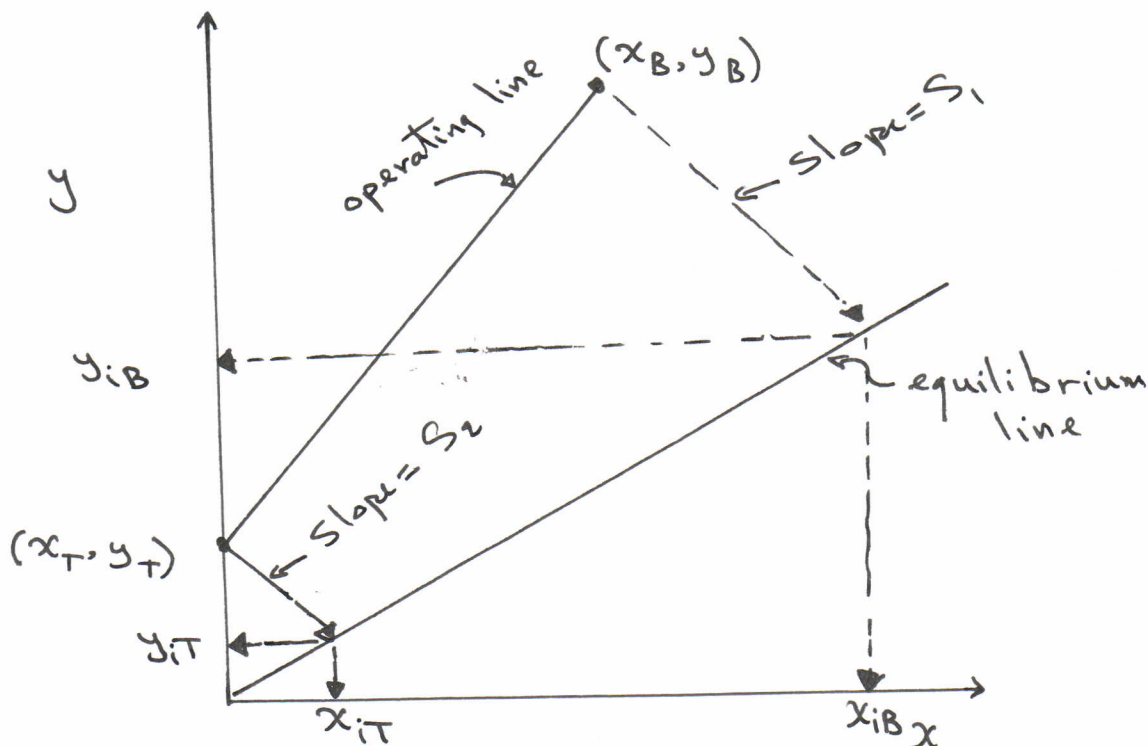
To find  $(x_{iT}, y_{iT})$  and  $(x_{iB}, y_{iB})$ , we must follow these steps:-

- 1- Plot the equilibrium data.
- 2- Plot  $p(x_T, y_T)$  and  $q(x_B, y_B)$ .
- 3- Plot the line  $(pq)$ , which represents the operating line.
- 4- Calculate the slopes which are:

$$S_1 = \frac{-K_x \cdot a / (1 - x_B)}{K_y \cdot a / (1 - y_B)}$$

$$S_2 = \frac{-K_x \cdot a / (1 - x_T)}{K_y \cdot a / (1 - y_T)}$$

- 5- From point  $p(x_T, y_T)$  and slope  $S_2$ , intersect the equilibrium line which gives  $(x_{iT}, y_{iT})$  and from point  $q(x_B, y_B)$  and slope  $S_1$ , intersect the equilibrium line which gives  $(x_{iB}, y_{iB})$ .



## Height Equivalent To A Theoretical Plate ( H. E. T. P )

( H. E. T. P ) : packed height required to achieve the same separation as on the ( n th ) tray.

( H. E. T. P ) is often used to characterize the performance of a packing.

A good packing has a small ( H. E. T. P ).

( H. E. T. P ) range from ( 1 – 3 ).

$$( H. E. T. P ) = ( HTU )_{OG} * \left[ \frac{Ln ( m. \frac{Gm}{Lm} )}{( m. \frac{Gm}{Lm} ) - 1} \right]$$

$$( H. E. T. P ) * N = Z \quad \rightarrow \quad N = Z / ( H. E. T. P )$$

( H. E. T. P ) depends on :-

- 1- Type and size of packing.
- 2- Gas and liquid flow rates.
- 3- Transport properties.
- 4- Equilibrium relation.
- 5-Uniformity of liquid and gas distribution.

## Minimum Liquid Flow Rate In Adsorption Column

Reducing liquid flow rate  $\rightarrow$  reducing slope  $(L/G)$  of operating line.

This mean  $(x_B)$  increase  $\rightarrow$  tall column  $\rightarrow (\infty)$ .

( less driving force , absorption more difficult ).

If liquid flow reduced until it reach equilibrium line at  $(R)$ , this give minimum liquid flow and driving force = zero.

So  $(P_T R)$  give minimum slope or  $(L/G)_{\min}$ .

$$(L/G)_{\text{op.}} = (1.1 - 1.5) * (L/G)_{\min}.$$

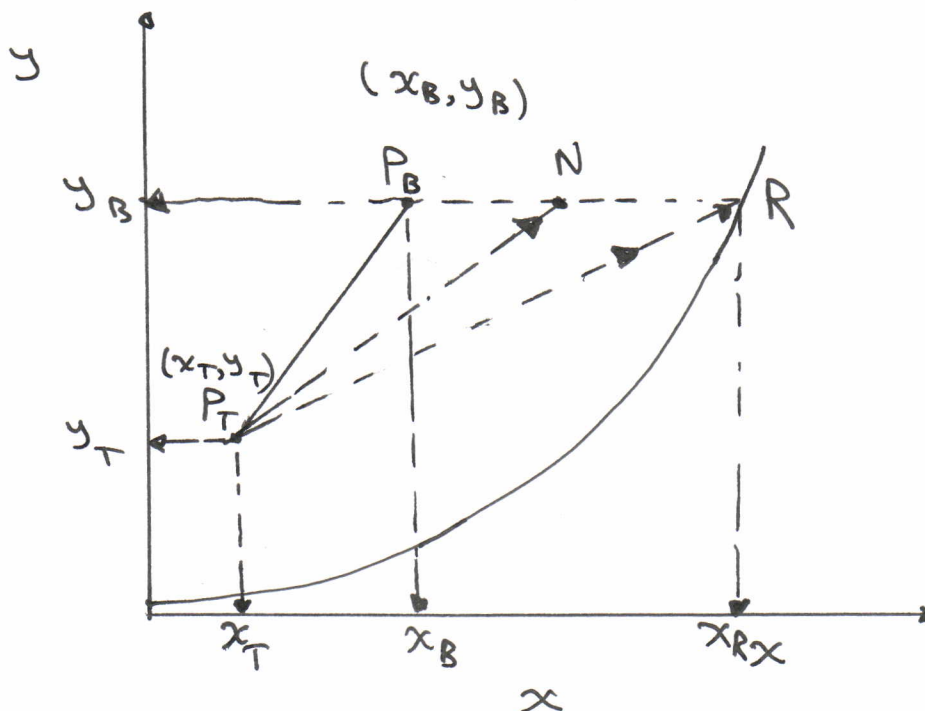
### Steps of calculations:-

1- Find  $(x_B^*)$  from  $y_B = m \cdot x_B^*$ .

2- Find  $(L/G)_{\min} = \frac{(y_B - y_T)}{(x_B^* - x_T)}$ .

3- Find  $(L/G)_{\text{op.}} = (1.1 - 1.5) * (L/G)_{\min}$ .

4- Finally find  $(x_B)$  from  $(L/G)_{\text{op.}} = \frac{(y_B - y_T)}{(x_B - x_T)}$ .





Ex.(1) :- Acetone is being absorbed by water in a packed tower having cross-sectional area ( $0.18 \text{ m}^2$ ) at (S.C). The inlet air contains  $2.6 \text{ mol}\%$  acetone and outlet  $0.5 \text{ mol}\%$ . The gas flowrate  $13.65 \text{ kmol inert air/hr}$ . The pure water inlet flow is  $45.36 \text{ kmol/hr}$ . Film coefficient for the given flow in the tower are:

$$K_y \cdot a = 3.78 \times 10^{-2} \text{ kmol/m}^2 \cdot \text{s} \cdot \text{mol fraction and}$$

$$K_x \cdot a = 6.16 \times 10^{-2} \text{ s s s s}$$

The equilibrium line equation  $y = 1.186 X$

1 - Calculate the tower height using  $K_y \cdot a$  &  $K_x \cdot a$

2 - s s , s s  $K_{OG} \cdot a$

Sol. Overall M.B. to find ( $X_B$ ) :-

$$L \cdot X_T + G \cdot y_B = L \cdot X_B + G \cdot y_T$$

$$45.65 \times 0 + 13.65 \times 0.026 = 45.65 \times X_B + 13.65 \times 0.005$$

$$0.355 = 45.65 X_B + 0.0683$$

$$X_B = 0.0063$$

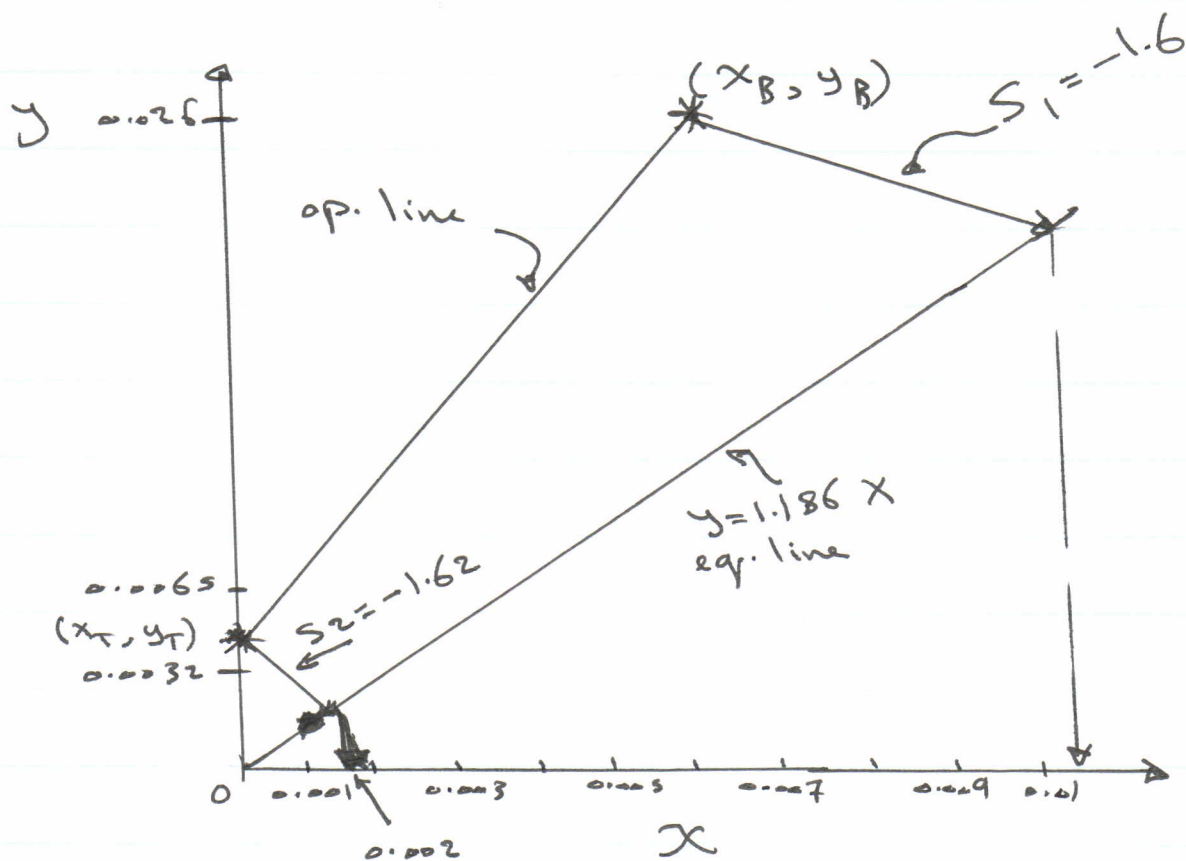
$$\textcircled{1} Z = (HTU)_G \times (NTU)_G$$

$$(HTU)_G = \frac{G_m}{K_y \cdot a \cdot s} = \frac{0.0038}{3.78 \times 10^{-2} \times 0.186} = 0.54 \text{ m}$$

$$(NTU)_G = \frac{y_B - y_T}{(y - y_i)_{Lm}}$$

$$(y - y_i)_{lm} = \frac{\Delta y_B - \Delta y_T}{\ln \frac{\Delta y_B}{\Delta y_T}} = \frac{(y_B - y_{iB}) - (y_T - y_{iT})}{\ln \frac{y_B - y_{iB}}{y_T - y_{iT}}}$$

To find  $(y_{iB} \text{ \& } y_{iT})$



To find  $S_1$  from  $(x_B, y_B)$  &  $S_2$  from  $(x_T, y_T)$

$$S_1 = \frac{-K_x \cdot a / (1 - x_B)}{K_y \cdot a / (1 - y_B)} = -1.6$$

$$S_2 = \frac{-K_x \cdot a / (1 - x_T)}{K_y \cdot a / (1 - y_T)} = -1.62$$

- Intercept of  $(S_1)$  with E.L. gives :-

$$x_{iB} = 0.013, y_{iB} = 0.0154$$

- Intercept of  $(S_2)$  with E.L. gives :-

$$x_{iT} = 0.0018, y_{iT} = 0.002$$

$$\therefore (y - y_i)_{Lm} = 0.006$$

$$NTU)_G = \frac{0.026 - 0.005}{0.006} = 3.5$$

$$\therefore Z = 0.54 * 3.5 = \underline{\underline{1.91 \text{ m}}}$$

$$\textcircled{2} Z = HTU)_{OG} \cdot NTU)_{OG}$$

$$HTU)_{OG} = \frac{G_m}{K_{OG} \cdot a \cdot S \cdot P_T}$$

$$NTU)_{OG} = \frac{y_B - y_T}{(y - y^*)_{Lm}}$$

$$(y - y^*)_{Lm} = \frac{(y_B - y_B^*) - (y_T - y_T^*)}{\ln \frac{(y_B - y_B^*)}{(y_T - y_T^*)}}$$



$$y_B^* = 1.186 x_B, \quad y_T^* = 1.186 x_T^{\rightarrow 0}$$

To find  $(K_{OG})$ :-

$$\frac{1}{K_{OG} \cdot a} = \frac{1}{K_y \cdot a} + \frac{m}{K_x \cdot a} \quad (\text{To find } K_y \cdot a \text{ and } K_x \cdot a)$$

$$K_y \cdot a = \frac{\bar{K}_y \cdot a}{y_{im}} \quad \text{and} \quad K_x \cdot a = \frac{\bar{K}_x \cdot a}{x_{im}}$$

$$y_{im} = \frac{(1-y_{iB})(1-y_B)}{\ln \frac{(1-y_{iB})}{(1-y_B)}}, \quad x_{im} = \frac{(1-x_B) - (1-x_{iB})}{\ln \frac{1-x_B}{1-x_{iB}}}$$

$$K_{OG} \cdot a = 2.188 \times 10^{-2} \text{ kmol/s.m}^3 \cdot \text{mol.fract}^n$$

$$\boxed{Z = 1.94 \text{ m}}$$