# Lecture No.11: Fluid motion in the presence of solid particles; Fixed & Fluidized Beds

## 1. Relative motion between a fluid and a single particle

The relative motion is considered the following cases are covered:

- a stationary particle in a moving fluid;
- a moving particle in a stationary fluid;
- a particle and a fluid moving in opposite directions;
- a particle and a fluid both moving in the same direction but at different velocities.

Assume flow around a spherical particle of diameter ***dp, as shown in Fig.1*** the appropriate definition of the Reynolds No. is :

$$Re_p = \frac{\rho u_p d_p}{\mu} \qquad \textbf{(1)}$$

where $u_p$ is the speed of the particle relative to the fluid.

P density of fluid, and μ viscosity of fluid

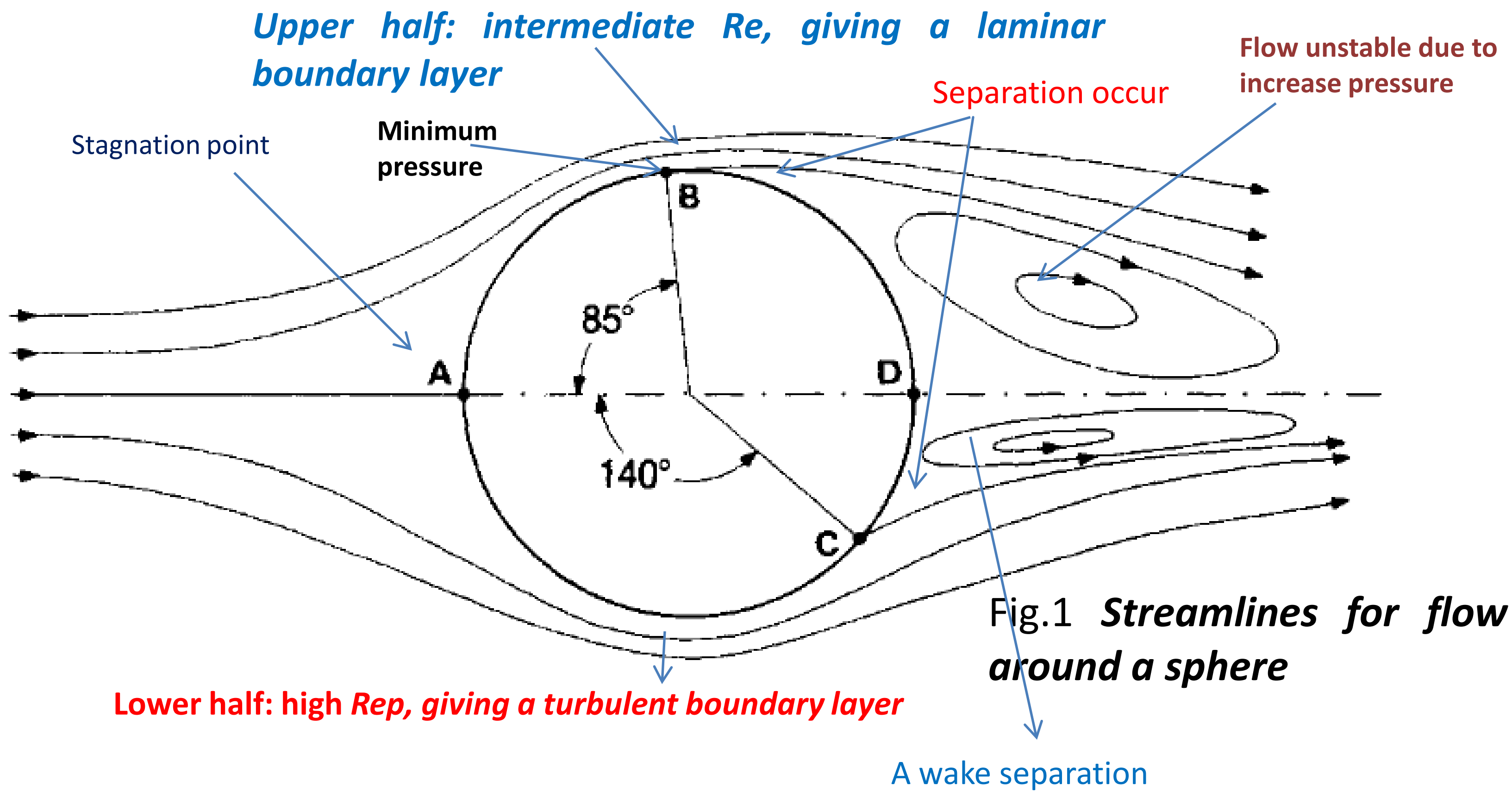


Fig.1 ***Streamlines for flow around a sphere***

Note: The whole of the wake is a region of relatively low pressure, very close to that at the point of separation, and much lower than the pressure near point A. Therefore the force arising from this pressure difference is known as from drag because it is due to the (bluff) shape of the particle. Then the total drag force is a combination of:

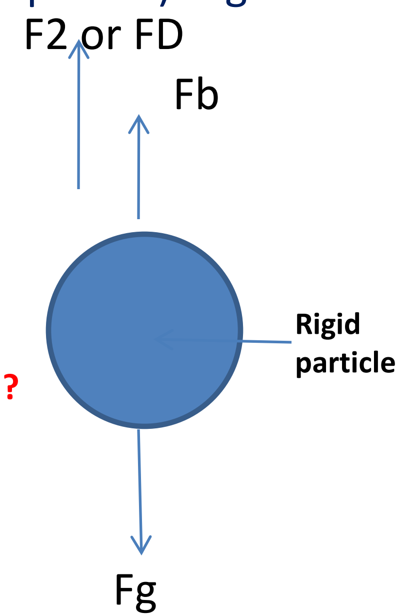
- **Skin friction and**
- **Form drag**

## 2. Terminal settling or Falling velocity of the Particle $u_p$ $u_t$

Consider a spherical particle as shown in Fig.2 of diameter dp and density ρp falling with a velocity up under the influence of gravity in a fluid of density p. The net gravitational force F1 on the particle (gravity force Fg-buoyancy Fb Archimedes' principle=F1) is given by the equation:

$$F_1 = \frac{\pi d_p^3}{6}(\rho_p - \rho)g \qquad \textbf{(2)}$$

volume of spherical particle




The retarding force or drag force *F2 on the particle from the fluid is given by:*

$$F_2 = C_d S_p \frac{\rho u_p^2}{2} \qquad (3)$$

**Is derived by ?**

**dimensionless drag coefficient is similar to the friction factor in pipe**

projected area $S_p = \pi d_p^2/4$

**For steady flow the F1=F2 *and F2 and opposite and the* particle reaches a constant speed *ut Eqs. 2&3 can be* combined and written as:**

$$\frac{\pi d_p^3}{6}(\rho_p - \rho)g = C_d S_p \frac{\rho u_t^2}{2} \qquad (4)$$

$$u_t = \sqrt{\frac{4 d_p(\rho_p - \rho)g}{3 C_d \rho}} \qquad (5)$$

**<u>terminal velocity or falling velocity</u>**

## 3. Calculation Drag Coefficient Cd:

### A-For Rep < 0.2 (laminar flow or creeping flow, Stokes' region)

the drag coefficient ***Cd is a function of the Reynolds number. For the streamline*** flow range of Reynolds numbers, ***Re,<0.2, the drag force F2 is given by:***

$$F_2 = 3\pi d_p \mu u_t \qquad (6)$$

Therefore Eq.3=Eq.6 to calculate Cd

$$C_d S_p \frac{\rho u_p^2}{2} = 3\pi d_p \mu u_t \quad\Rightarrow\quad C_d = \frac{24}{Re_p} \qquad (7)$$

**Therefore sub. Eq.7 in Eq.5 to calculate ut**

$$\boxed{u_t = \frac{d_p^2(\rho_p - \rho)g}{18\mu}} \qquad (8)$$

**Eq.8 is called Stokes' Equation , apply for Laminar or creeping flow only ,it can be used for Rep< 2**

### B-For 0.2<Rep < 500 or 2<Rep <500 (Intermediate region or Schiller region

$$C_d = \frac{24}{Re_p}(1 + 0.15 Re_p^{0.687}) \qquad (9) \qquad 0.2 < Re_p < 500,$$

$$C_d = \frac{18.5}{Re_p^{0.6}} \qquad (10) \qquad 2 < Re_p < 500$$

### C- For Newton s' region , 500< Rep <200 000, Cd constant=0.44

$$C_d = 0.44 \qquad (11)$$

***Note :***1-When the Reynolds number ***Rep reaches a value of about 300000,*** transition from a laminar to a turbulent boundary layer occurs and the point of separation moves towards the rear of the sphere as discussed above. As a result, the drag coefficient suddenly falls to a value of **0.10 and** remains constant at this value at higher values of ***Rep.***

***2-***For the most part, solid particles in fluid streams have Reynolds numbers which are much lower than 500.

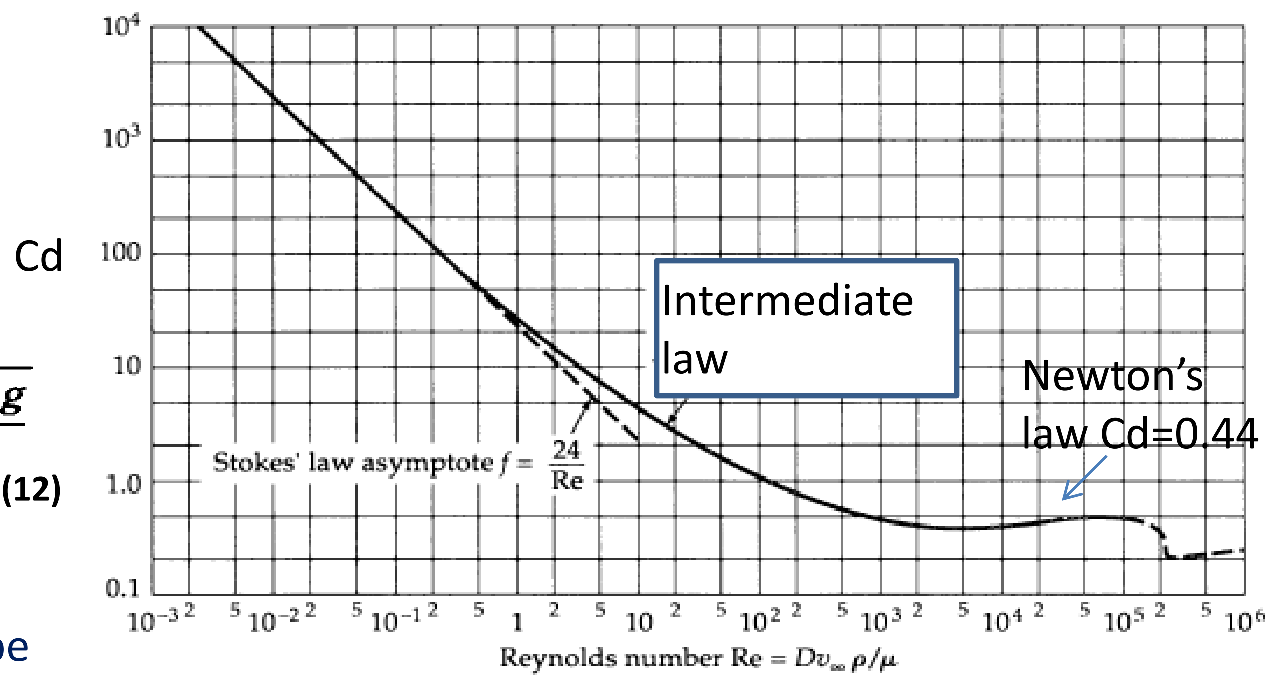
**From above in A, B, & C the relationship between Cd & Rep can be drawn in the following Fig. (note: similar with friction factor with Re in pipe flow:-**

**NOTE** :Eq. 5 is derived for spherical particle but if non spherical shape then Eq5 can be written as following form:

$$u_t = \sqrt{\frac{4 d_p \psi (\rho_p - \rho) g}{3 C_d \rho}} \quad \textbf{(12)}$$

Ψ=shape factor

Ψ=1 for spherical shape



**Example:1** Air at 37.8 C and 101.3 Kpa absolute pressure at a velocity of 23m/s past a sphere having a dp=24mm. What are the drag coefficient and the force on the sphere? μ air=1.9*10-5, ρ=1.137 Kg/m3.

**Sol:** Calculate Rep=ρ up dp/μ=57810 → Newton's region from Fig. Cd=0.44 ,

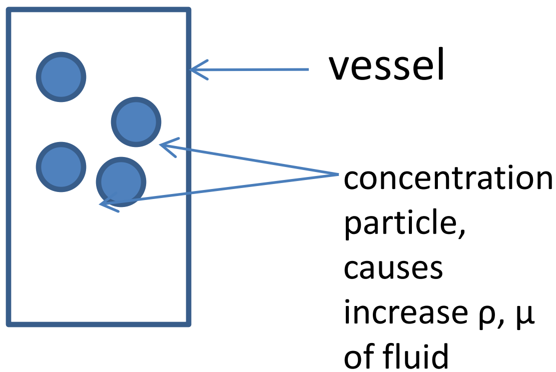
The drag force $$F_2 = C_d S_p \frac{\rho u_p^2}{2}$$

SP=πdp2/4 F2= ?N

## 4. Relative motion between a fluid and a concentration of particles

So far the relative motion between a fluid and a single particle has been considered. This process is called → **free settling.**

When a fluid contains a concentration of particles in a vessel, as shown in Fig the settling of an individual particle may be hindered by the other particles and by the walls. When this is the case, the process is called **hindered settling.**




Richardson and Zaki showed that in the Reynolds number range Rep<0.2, the velocity *uc of a suspension of coarse spherical particles in* water relative to a fixed horizontal plane is given by the equation: →

$$\frac{u_c}{u_t} = \varepsilon^{4.6} \qquad (13)$$

voidage fraction of the suspension

terminal settling velocity of single particle

## Relation Between Two Particles in Fluid:

i. ***If the same diameter dp1=dp2***

*but of different densities settling freely in a fluid of density p in the streamline Reynolds number range Rep<0.2. The ratio of the terminal settling velocities ut1/ut2 is given by Eq. 8 rewritten in the form:*

$$\frac{u_{t1}}{u_{t2}} = \frac{\rho_{p1} - \rho}{\rho_{p2} - \rho} \qquad \textbf{(14)}$$

ii**.** ***If the same density ρp1=ρp2***

But of different diameters settling freely in a fluid of density *p in the streamline* Reynolds number range *Rep<0.2, the ratio of the terminal settling* velocities *ut1/ut2 is given by Eq.8 rewritten in the form:*

$$\frac{u_{t1}}{u_{t2}} = \left(\frac{d_{p1}}{d_{p2}}\right)^2 \qquad \textbf{(15)}$$

iii. ***If ut1=ut2*** in the same fluid in the streamline flow regime if their densities and diameters are related by the following Eq.

$$\frac{d_{p1}}{d_{p2}} = \sqrt{\left(\frac{\rho_{p2} - \rho}{\rho_{p1} - \rho}\right)} \qquad \textbf{(16)}$$

5. **Settling velocity of particle in Centrifugal separator**

A particle of mass =m rotating at a **radius =r** with an angular velocity =ω is subject to a centripetal force $= mr\omega^2$ :h can be made very much > than the vertically directed gravity force Fg= mg.

The terminal settling velocity **ut for a single spherical particle in a** centrifugal separator can be calculated from **Eq.5**

$$u_t = \sqrt{\frac{4d_p(\rho_p - \rho)g}{3C_d\rho}}$$

$$u_t = \sqrt{\frac{4d_p(\rho_p - \rho)r\omega^2}{3C_d\rho}} \qquad (17)$$

***A very small particle may still be in laminar flow in a centrifugal separator, therefore, ut is given by Eq.5:***

$$u_t = \frac{d_p^2(\rho_p - \rho)r\omega^2}{18\mu} \qquad (18)$$

***Effect particles as slurry on viscosity of fluid;*** Einstein showed that the distortion of the streamlines around the particles caused the dynamic viscosity of the slurry to increase according to the following Eq.

$$\mu = \mu_L(1 + 2.5\alpha) \qquad (19)$$

***viscosity of slurry***

***viscosity of liquid***

***Volume fraction of solid=(1-ε); α=0.02 for low concentration***

## 6. Fluid flow through packed beds, (Fixed Beds)

### *Application in Industry:-*

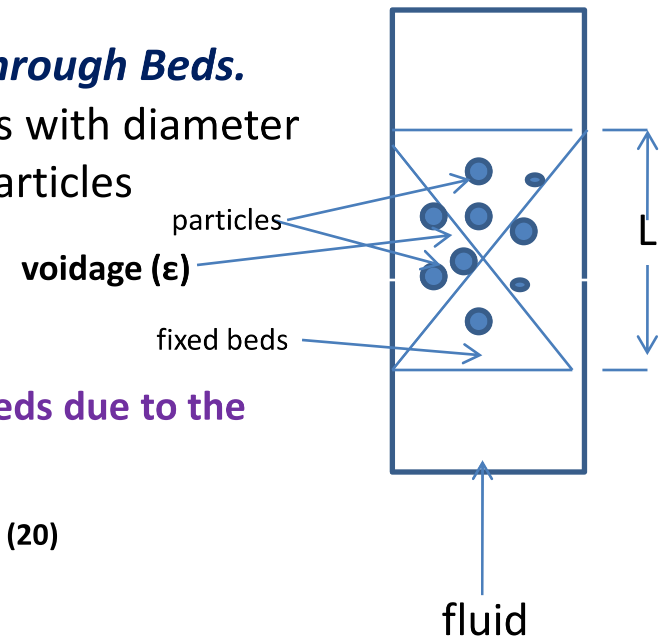
i. Fixed bed catalytic reactors, such as SO2-SO3 converters
ii. Drying columns containing silica gel or molecular sieves, gases are passed through a bed particles
iii. Gas absorption into liquid.
iv. Filtration through a fixed bed.

### *Calculation of Pressure drop (Δp), & Velocity (ub) through Beds.*

Assume in packed bed with length (L), spherical particles with diameter dp, & the surface area (So)=6/dp, ε voidage, 1-ε=solid particles

Thus total surface area $S_o = 6/d_p.$

For non-spherical particles $S_o = 6/(d_p\psi)$




**When fluid flow through packed beds (L), there is Δp across beds due to the resistance caused by the presence of the particles.**

$$d_e = \frac{4\varepsilon}{(1-\varepsilon)S_o} \qquad \textbf{(20)}$$

**equivalent diameter through the bed**

Superficial velocity (u) of fluid through the bed is=Q (volumetric flow rate)/A (whole cross-sectional flow area), therefore the mean velocity of fluid through beds is $u_b = u/\varepsilon$

A Reynolds number for flow through a packed bed

$$Re_b = \frac{\rho u_b d_e}{\mu} \qquad (21)$$

Eq.21 when combined with equation *20 can be written as*

$$Re_b = \frac{4\rho u}{\mu(1-\varepsilon)S_o} \qquad (22)$$

An alternative Reynolds number has been used to correlate data and is defined as

$$Re'_b = \frac{\rho u}{\mu(1-\varepsilon)S_o} \qquad (23)$$

For a packed bed consisting of spherical particles, Eq. *23 can be* written in the form

$$Re'_b = \frac{\rho u d_p}{6\mu(1-\varepsilon)} \qquad (24)$$

The corresponding equation for non-spherical particles is

$$Re'_b = \frac{\rho u d_p \psi}{6\mu(1-\varepsilon)} \qquad (25)$$

A pressure drop Δpf occurs in the bed because of frictional viscous and drag forces, is given by force balance across unit cross-sectional area gives:

$$\Delta P_f \varepsilon = \tau_b L(1-\varepsilon)S_o \qquad (26) \qquad \div\ \rho u_b^2$$

**resistance per unit area of surface be or (R) similar to flow in pipe**

$$\frac{f_b}{2} = \frac{\tau_b}{\rho u_b^2} = \left(\frac{\Delta P_f}{L}\right)\left[\frac{\varepsilon}{(1-\varepsilon)S_o \rho u_b^2}\right] \qquad (27)$$

or since $u_b = u/\varepsilon$ as

$$\frac{f_b}{2} = \frac{\tau_b}{\rho u_b^2} = \left(\frac{\Delta P_f}{L}\right)\left[\frac{\varepsilon^3}{(1-\varepsilon)S_o \rho u^2}\right] \qquad (28)$$

velocity through bed

where $f_b$ is a dimensionless friction factor for flow through a packed bed.

**It is calculate as following:**

For laminar flow where $Re'_b \leq 2$

$$\frac{f_b}{2} = \frac{5}{Re'_b} \qquad (29)$$

The transition to turbulent flow is gradual. Turbulence commences initially in the largest channels and eventually extends to the smaller channels. For the complete range of Reynolds number Carman **gave the Eq.**

$$\frac{f_b}{2} = \frac{5}{Re'_b} + \frac{0.4}{(Re'_b)^{0.1}} \qquad (30)$$

**<u>Note:</u> OR f can be calculate by drawn f vs. Re'b as shown in following Fig. (Ref. VOL.2 p.197 5ed.)**

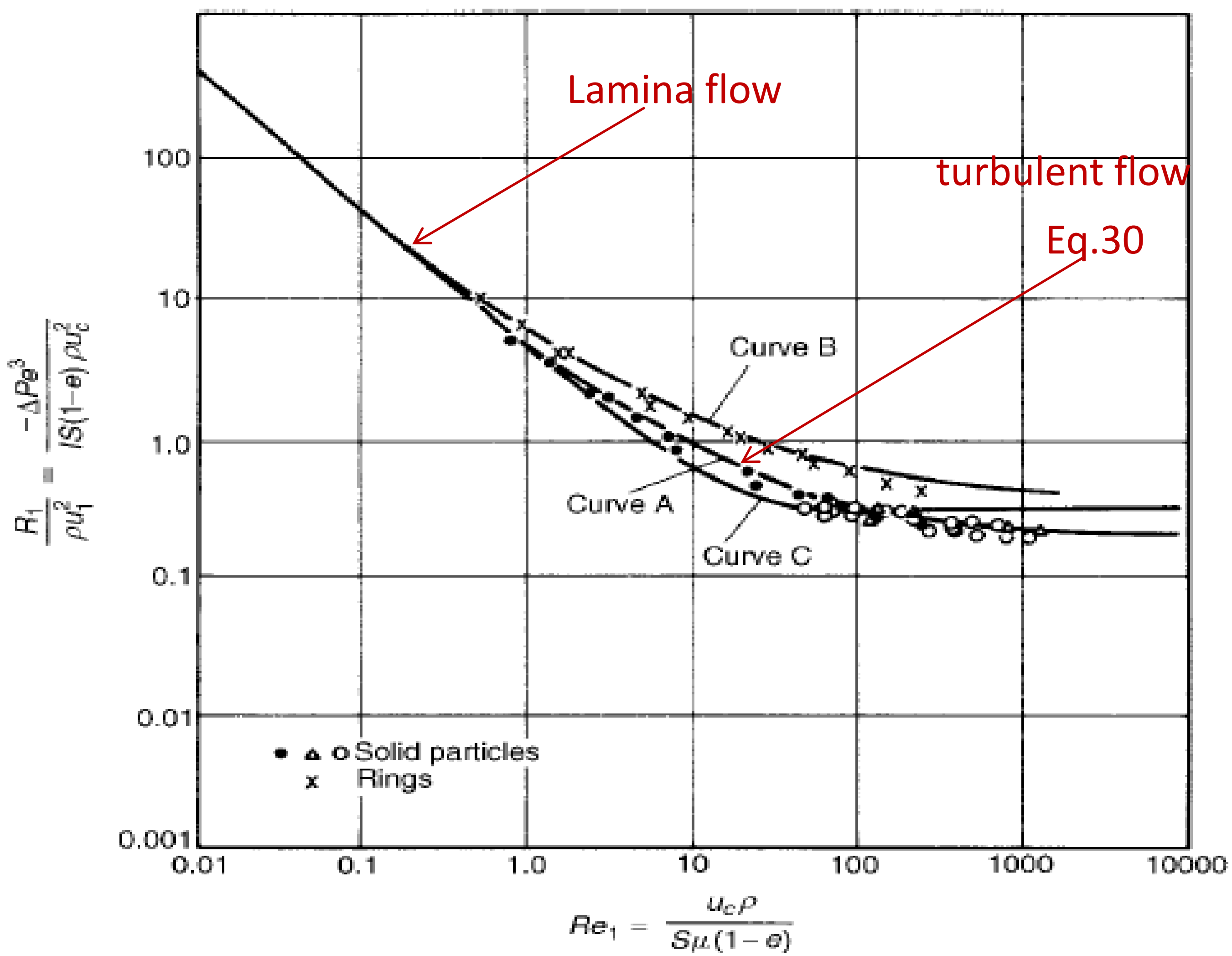



Figure 4.1. Carman's graph of $R_1/\rho u_1^2$ against $Re_1$

## *OR Δp & u calculated by Carman's-Kozeny Eqn. as following:*

The Hagen–Poiseuille equation for steady laminar flow of Newtonian fluids in pipes and tubes can be written as

$$u = \left(\frac{\Delta P_f}{L}\right)\frac{d_i^2}{32\mu} \qquad *$$

**Eqn. * is rewritten for packed bed as following**

$$u_b = \left(\frac{\Delta P_f}{L}\right)\left(\frac{1}{32\mu}\right)\left[\frac{16\varepsilon^2}{(1-\varepsilon)^2 S_o^2}\right] \qquad (31)$$

or, since $u_b = u/\varepsilon$

$$u = \left(\frac{\Delta P_f}{L}\right)\left(\frac{1}{2\mu}\right)\left[\frac{\varepsilon^3}{(1-\varepsilon)^2 S_o^2}\right] \qquad (32)$$

Eq.32 does not hold for flow through packed beds and should be replaced by Eqn:

$$\boxed{u = \left(\frac{\Delta P_f}{L}\right)\left(\frac{1}{K_c\mu}\right)\left[\frac{\varepsilon^3}{(1-\varepsilon)^2 S_o^2}\right]} \qquad (33)$$

Carman–Kozeny equation

Eq.33 can also be written in the form Δp

$$\Delta P_f = (K_c\mu L)\left[\frac{(1-\varepsilon)^2 S_o^2}{\varepsilon^3}\right]u \qquad (34)$$

Kc parameter 3.5-5.5 but most **value 5**

**If Kc=5, S=6/dp Eq. 34 becomes**

$$\boxed{\Delta P_f = (180\,\mu L)\left[\frac{(1-\varepsilon)^2}{\varepsilon^3 d_p^2}\right]u} \qquad (35)$$

## Example 2

A gas of density $\rho = 1.25$ kg/m$^3$ and dynamic viscosity $\mu = 1.5 \times 10^{-5}$ Pa s flows steadily through a bed of spherical particles of diameter $d_p = 0.005$ m. The bed has a height of 3.00 m and a voidage of $\frac{1}{3}$. The superficial velocity $u = 0.03$ m/s. Calculate the Reynolds number and the frictional pressure drop over the bed.

*Calculations*

$$\text{Reynolds number } Re'_b = \frac{\rho u d_p}{6\mu(1-\varepsilon)}$$

Substituting the given values

$$Re'_b = \frac{(1.25 \text{ kg/m}^3)(0.03 \text{ m/s})(0.005 \text{ m})(3)}{(6)(1.50 \times 10^{-5} \text{ Pa s})(2)}$$

$$= 3.125$$

The frictional pressure drop is given by

$$\Delta P_f = (180\mu L)\left[\frac{(1-\varepsilon)^2}{\varepsilon^3 d_p^2}\right] u$$

Given that

$$(1-\varepsilon)^2 = \tfrac{4}{9}$$

$$\frac{(1-\varepsilon)^2}{\varepsilon^3} = 12$$

$$d_p^2 = 2.5 \times 10^{-5} \text{ m}^2$$

$$u = 0.03 \text{ m/s}$$

$$\mu = 1.5 \times 10^{-5} \text{ Pa s}$$

$$L = 3.0 \text{ m}$$

$$\Delta P_f = (180)(1.5 \times 10^{-5} \text{ Pa s})\frac{(3.0 \text{ m})(12)(0.03 \text{ m/s})}{(2.5 \times 10^{-5} \text{ m}^2)}$$

$$= \underline{116.6 \text{ Pa}}$$

# Example 3

## 7. Fluidization

If a fluid is passed upwards in laminar flow through a packed bed of solid particles the superficial velocity u is related to the pressure drop Δp by Eq.33

$$\boxed{u = \left(\frac{\Delta P_f}{L}\right)\left(\frac{1}{K_c \mu}\right)\left[\frac{\varepsilon^3}{(1-\varepsilon)^2 S_o^2}\right]}$$

As the fluid velocity is increased the drag on the particles increases and a point is reached when the viscous frictional and drag forces on the particles become= to the weight of the particles in the fluid stream. This start of fluidization, the bed become fluidized is known ***incipient fluidization*** or ***minimum fluidization***, and the velocity in this case is called ***minimum fluidization velocity* umf. Then the force balance umf is given by the following Eqns.:**

$$(\Delta P)_{mf} = (1-\varepsilon_{mf})(\rho_p - \rho)L_{mf}g \qquad (36)$$

void fraction at minimum fluidization

$$u_{mf} = \left[\frac{(\rho_p - \rho)g}{K_c \mu}\right]\left[\frac{\varepsilon_{mf}^3}{(1-\varepsilon_{mf})S_o^2}\right] \qquad (37)$$

Kc=5 & S=6/dp

***minimum fluidization velocity* umf**

If the velocity is further increased, the bed expands , then the following cases occur:

*i.* ***Particulate fluidization***, (at all velocities of liquid & low gas velocity).

*ii.* ***Bubbling fluidization***, (at higher gas velocity), similar to gas bubbles in a boiling liquid. This case also called aggregative fluidization

**Note**: In this case velocity of fluid is given by Eq.35 but use uf & ε instead of umf & εmf

The relationship between Δp vs. u is shown in Fig. (Ref.Vol2 p.232

As the fluid velocity is increased, the bed expands and solid particles become entrained. Initially the smaller particles only are carried away. If the fluid is sufficiently increased, all particles will become entrained, then velocity particles is given by Eq.4.

$$u_t = \sqrt{\frac{4d_p(\rho_p - \rho)g}{3C_d\rho}}$$

***Therefore in practice must be mean velocity of fluid is umf < u < up***

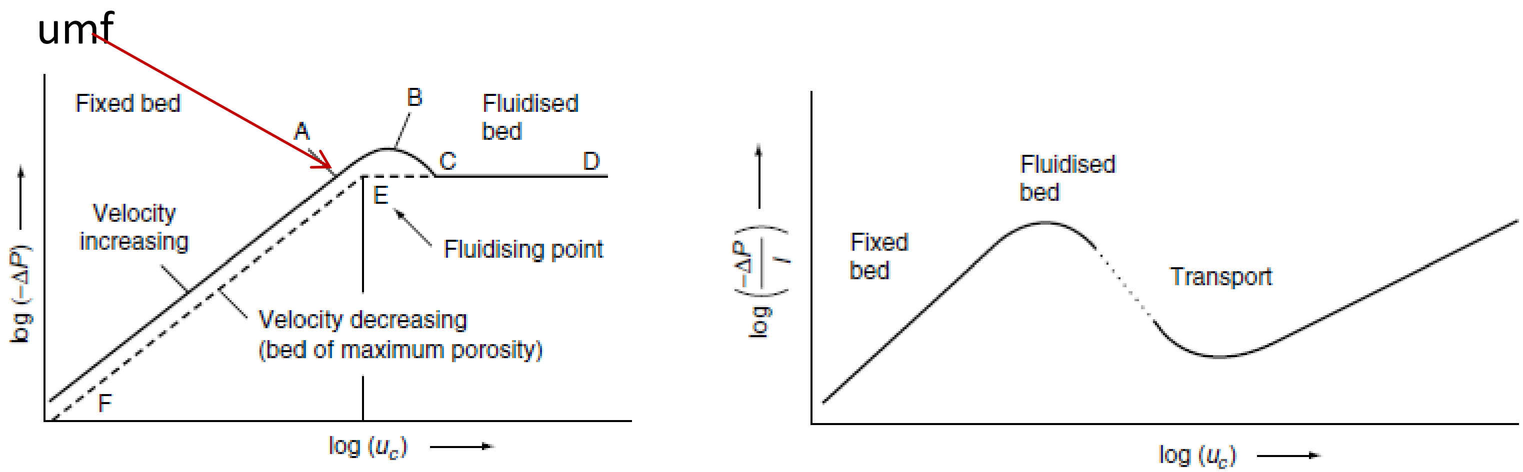



Fig. Pressure drop over fixed & fluidized beds