وزارة التعليم العالي والبحث العلمي

University of Technology

Chemical Engineering Dept.

Electrical Technology

First Class

By

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Electricity is one type of energy, all matter whether solid, liquid or gaseous consists of minute particles known as Atoms. According to the researches, electric current means flow of electrons.

SI units The system of units used in engineering and science (International system of units), usually abbreviated to SI units, and is based on the metric system. This was introduced in 1960 and is now adopted by the majority of countries as the official system of measurement. The basic units in the SI system are listed with their symbols, in Table 1.1.

TABLE 1.1 *Basic SI Units*

Quantity	*Unit*
length	metre, m
mass	kilogram, kg
time	second, s
electric current	ampere, A
thermodynamic temperature	kelvin, K
luminous intensity	candela, cd
amount of substance	mole, mol

SI units may be made larger or smaller by using prefixes which denote multiplication or division by a particular amount. The six most common multiples, with their meaning, are listed in Table 1.2.

TABLE 1.2

Prefix	*Name*	*Meaning*	
M	mega	multiply by 1 000 000	(i.e. $\times 10^6$)
k	kilo	multiply by 1000	(i.e. $\times 10^3$)
m	milli	divide by 1000	(i.e. $\times 10^{-3}$)
μ	micro	divide by 1 000 000	(i.e. $\times 10^{-6}$)
n	nano	divide by 1 000 000 000	(i.e. $\times 10^{-9}$)
p	pico	divide by 1 000 000 000 000	(i.e. $\times 10^{-12}$)

Charge

The **unit of charge** is the coulomb (C) where one coulomb is one ampere second. (1 coulomb $= 6.24 \times 10^{18}$ electrons). The coulomb is defined as the quantity of electricity which flows past a given point in an electric circuit when a current of one ampere is maintained for one second. Thus,

charge, in coulombs $\boxed{Q = It}$

where I is the current in amperes and t is the time in seconds.

Problem 1. If a current of 5 A flows for 2 minutes, find the quantity of electricity transferred.

Quantity of electricity $Q = It$ coulombs

$I = 5$ A, $t = 2 \times 60 = 120$ s

Hence $Q = 5 \times 120 = \mathbf{600\ C}$

Force The **unit of force** is the newton (N) where one newton is one kilogram metre per second squared. The newton is defined as the force which, when applied to a mass of one kilogram, gives it an acceleration of one metre per second squared. Thus,

force, in newtons $\boxed{F = ma}$

where m is the mass in kilograms and a is the acceleration in metres per second squared. Gravitational force, or weight, is mg, where $g = 9.81\ \text{m/s}^2$

> Problem 2. A mass of 5000 g is accelerated at 2 m/s^2 by a force. Determine the force needed.

Force = mass × acceleration

$$= 5\ \text{kg} \times 2\ \text{m/s}^2 = 10\frac{\text{kg m}}{\text{s}^2} = \mathbf{10\ N}$$

> Problem 3. Find the force acting vertically downwards on a mass of 200 g attached to a wire.

Mass = 200 g = 0.2 kg and acceleration due to gravity, $g = 9.81\ \text{m/s}^2$

Force acting downwards = weight = mass × acceleration

$$= 0.2\ \text{kg} \times 9.81\ \text{m/s}^2$$

$$= \mathbf{1.962\ N}$$

Work The **unit of work or energy** is the **joule (J)** where one joule is one newton metre. The joule is defined as the work done or energy transferred when a force of one newton is exerted through a distance of one metre in the direction of the force. Thus

work done on a body, in joules $\boxed{W = Fs}$

where F is the force in newtons and s is the distance in metres moved by the body in the direction of the force. Energy is the capacity for doing work.

Power The **unit of power** is the watt (W) where one watt is one joule per second. Power is defined as the rate of doing work or transferring energy. Thus,

power in watts, $\boxed{P = \dfrac{W}{t}}$

where W is the work done or energy transferred in joules and t is the time in seconds. Thus

energy, in joules, $\boxed{W = Pt}$

Problem 4. A portable machine requires a force of 200 N to move it. How much work is done if the machine is moved 20 m and what average power is utilized if the movement takes 25 s?

Problem 5. A mass of 1000 kg is raised through a height of 10 m in 20 s. What is (a) the work done and (b) the power developed?

Electrical potential and e.m.f.

The **unit of electric potential** is the volt (V) where one volt is one joule per coulomb. One volt is defined as the difference in potential between two points in a conductor which, when carrying a current of one ampere, dissipates a power of one watt, i.e.

$$\text{volts} = \frac{\text{watts}}{\text{amperes}} = \frac{\text{joules/second}}{\text{amperes}} = \frac{\text{joules}}{\text{ampere seconds}} = \frac{\text{joules}}{\text{coulombs}}$$

A change in electric potential between two points in an electric circuit is called a **potential difference**. The **electromotive force (e.m.f.)** provided by a source of energy such as a battery or a generator is measured in volts.

Resistance and conductance

The **unit of electric resistance** is the **ohm (Ω)** where one ohm is one volt per ampere. It is defined as the resistance between two points in a conductor when a constant electric potential of one volt applied at the two points produces a current flow of one ampere in the conductor. Thus,

resistance, in ohms $\boxed{R = \dfrac{V}{I}}$

where V is the potential difference across the two points in volts and I is the current flowing between the two points in amperes.

The reciprocal of resistance is called **conductance** and is measured in siemens (S). Thus,

conductance, in siemens $\boxed{G = \frac{1}{R}}$

where R is the resistance in ohms.

Problem 6. Find the conductance of a conductor of resistance (a) 10 Ω, (b) 5 kΩ and (c) 100 mΩ

Electrical power and energy

When a direct current of I amperes is flowing in an electric circuit and the voltage across the circuit is V volts, then

power, in watts $\boxed{P = VI}$

Electrical energy = Power × time

= VIt **Joules**

Although the unit of energy is the joule, when dealing with large amounts of energy, the unit used is the **kilowatt hour (kWh)** where

1 kWh = 1000 watt hour

= 1000 × 3600 watt seconds or joules

= 3 600 000 J

Problem 7. A source e.m.f. of 5 V supplies a current of 3 A for 10 minutes. How much energy is provided in this time?

Problem 8. An electric heater consumes 1.8 MJ when connected to a 250 V supply for 30 minutes. Find the power rating of the heater and the current taken from the supply.

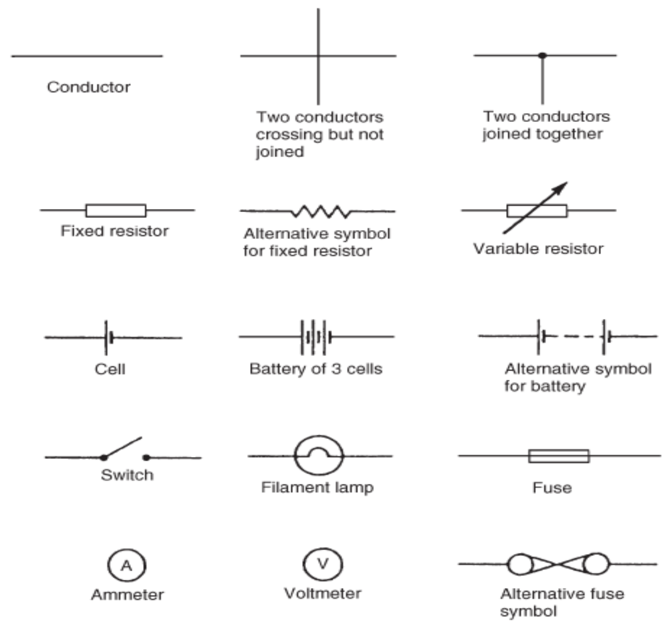
Summary of terms,
:s and their symbols

Quantity	Quantity Symbol	Unit	Unit symbol
Length	l	metre	m
Mass	m	kilogram	kg
Time	t	second	s
Velocity	v	metres per second	m/s or m s^{-1}
Acceleration	a	metres per second squared	m/s^2 or m s^{-2}
Force	F	newton	N
Electrical charge or quantity	Q	coulomb	C
Electric current	I	ampere	A
Resistance	R	ohm	Ω
Conductance	G	siemen	S
Electromotive force	E	volt	V
Potential difference	V	volt	V
Work	W	joule	J
Energy	E (or W)	joule	J
Power	P	watt	W

Electric Circuits

Standard symbols for electrical components

Symbols are used for components in electrical circuit diagrams and some of the more common ones are shown in Figure below:



Basic electrical measuring instruments

An **<u>ammeter</u>** is an instrument used to measure current and must be connected **in series** with the circuit. Figure 2.2 shows an ammeter connected in series with the lamp to measure the current flowing through it. Since all the current in the circuit passes through the ammeter it must have a very **low resistance**.

A **<u>voltmeter</u>** is an instrument used to measure p.d. and must be connected **in parallel** with the part of the circuit whose p.d. is required. In Figure 2.2, a voltmeter is connected in parallel with the lamp to measure the p.d. across it. To avoid a significant current flowing through it a voltmeter must have a very **high resistance**.

An **<u>ohmmeter</u>** is an instrument for measuring resistance.

A **<u>multimeter</u>**, or universal instrument, may be used to measure voltage, current and resistance. An '**<u>Avometer</u>**' is a typical example.

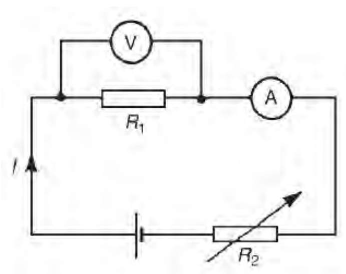
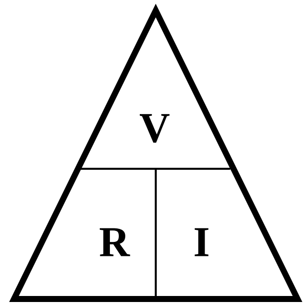


Figure 2.3

Ohm's law

Ohm's law states that the current I flowing in a circuit is directly proportional to the applied voltage V and inversely proportional to the resistance R, provided the temperature remains constant. Thus,



$$I = \frac{V}{R} \text{ or } V = IR \text{ or } R = \frac{V}{I}$$

Problem 3. The current flowing through a resistor is 0.8 A when a p.d. of 20 V is applied. Determine the value of the resistance.

From Ohm's law, resistance $R = \frac{V}{I} = \frac{20}{0.8} = \frac{200}{8} = \mathbf{25\ \Omega}$

Problem 4. Determine the p.d. which must be applied to a 2 kΩ resistor in order that a current of 10 mA may flow.

Resistance $R = 2\ \text{k}\Omega = 2 \times 10^3 = 2000\ \Omega$

Current $I = 10\ \text{mA} = 10 \times 10^{-3}$ A or $\frac{10}{10^3}$ or $\frac{10}{1000}$ A $= 0.01$ A

From Ohm's law, potential difference, $V = IR = (0.01)(2000) = \mathbf{20\ V}$

Problem 5. A coil has a current of 50 mA flowing through it when the applied voltage is 12 V. What is the resistance of the coil?

Resistance, $R = \frac{V}{I} = \frac{12}{50 \times 10^{-3}} = \frac{12 \times 10^3}{50} = \frac{12\,000}{50} = \mathbf{240\ \Omega}$

Problem 6. A 100 V battery is connected across a resistor and causes a current of 5 mA to flow. Determine the resistance of the resistor. If the voltage is now reduced to 25 V, what will be the new value of the current flowing?

Resistance $R = \frac{V}{I} = \frac{100}{5 \times 10^{-3}} = \frac{100 \times 10^3}{5} = 20 \times 10^3 = \mathbf{20\ k\Omega}$

Current when voltage is reduced to 25 V,

$$I = \frac{V}{R} = \frac{25}{20 \times 10^3} = \frac{25}{20} \times 10^{-3} = \mathbf{1.25\ mA}$$

$$\boxed{\boldsymbol{P} = \mathbf{V\,I}}$$

$$P = V\,\frac{V}{R}$$

$$\boxed{P = \frac{V^2}{R}}$$

$$\boldsymbol{P} = \mathbf{V\,I} \;= \mathbf{(IR)\,I}$$

$$\boxed{\boldsymbol{P} = \mathbf{I^2\,R}}$$

Conductors and Insulators

A **conductor** is a material having a low resistance which allows electric current to flow in it. All metals are conductors and some examples include copper, aluminium, brass, platinum, silver, gold and carbon.

An **insulator** is a material having a high resistance which does not allow electric current to flow in it. Some examples of insulators include plastic, rubber, glass, porcelain, air, paper, cork, mica, ceramics and certain oils.

Electrical Power

Power *P* in an electrical circuit is given by the product of potential difference V and current I, The unit of power is the **watt**, **W**. Hence

$$\boxed{P = V \times I \text{ watts}} \qquad \text{……. 2-1}$$

From Ohm's law, V = IR , Substituting for V in equation (2.1) gives:

$$P = (IR) \times I$$

i.e. $$\boxed{P = I^2R \text{ watts}}$$

Also, from Ohm's law, I = V/R, Substituting for I in equation (2.1) gives:

$$P = V \times \frac{V}{R}$$

i.e. $$\boxed{P = \frac{V^2}{R} \text{ watts}}$$

There are thus three possible formulae which may be used for calculating power.

Problem . A 100 W electric light bulb is connected to a 250 V supply. Determine (a) the current flowing in the bulb, and (b) the resistance of the bulb.

Power $P = V \times I$, from which, current $I = \frac{P}{V}$

(a) Current $I = \frac{100}{250} = \frac{10}{25} = \frac{2}{5} = \mathbf{0.4\ A}$

(b) Resistance $R = \frac{V}{I} = \frac{250}{0.4} = \frac{2500}{4} = \mathbf{625\ \Omega}$

(try for other solution for b)

Problem- An electric kettle has a resistance of 30 Ω. What current will flow when it is connected to a 240 V supply? Find also the power rating of the kettle.

Current, $I = \frac{V}{R} = \frac{240}{30} = \mathbf{8\ A}$

Power, $P = VI = 240 \times 8 = 1920\ \text{W} = \mathbf{1.92\ kW}$

Problem. A current of 5 A flows in the winding of an electric motor, the resistance of the winding being 100 Ω. Determine (a) the p.d. across the winding, and (b) the power dissipated by the coil.

(a) Potential difference across winding, $V = IR = 5 \times 100 = \mathbf{500\ V}$

(b) Power dissipated by coil, $P = I^2R = 5^2 \times 100$

$= \mathbf{2500\ W\ or\ 2.5\ kW}$

(Alternatively, $P = V \times I = 500 \times 5 = \mathbf{2500\ W\ or\ 2.5\ kW}$)

Series and parallel networks

Series circuits

Figure 5.1 shows three resistors R_1, R_2 and R_3 connected end to end, i.e., in series, with a battery source of V volts. Since the circuit is closed a current I will flow and the p.d. across each resistor may be determined from the voltmeter readings V_1, V_2 and V_3

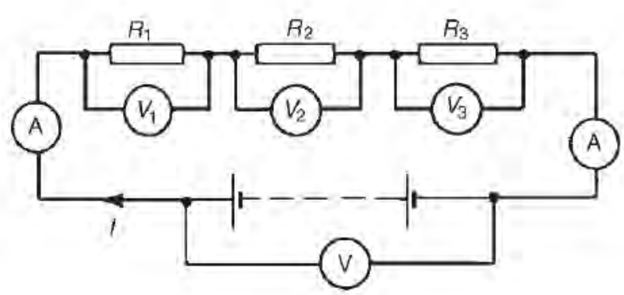


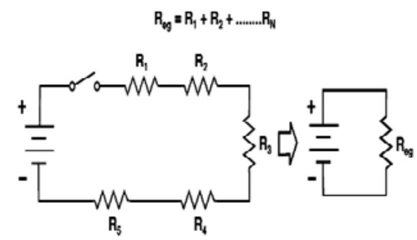
Figure 5.1

In a series circuit:

(a) The current I is the same in all parts of the circuit and hence the same reading is found on each of the two ammeters shown, and

(b) The sum of the voltages V_1, V_2 and V_3 is equal to the total applied voltage, V, i.e.

$$\boxed{V = V_1 + V_2 + V_3}$$



From Ohm's law:

$$V_1 = IR_1,\ V_2 = IR_2,\ V_3 = IR_3 \text{ and } V = IR$$

where R is the total circuit resistance.
Since $V = V_1 + V_2 + V_3$
then $IR = IR_1 + IR_2 + IR_3$
Dividing throughout by I gives

$$\boxed{R = R_1 + R_2 + R_3}$$

Thus for a series circuit, the total resistance is obtained by adding together the values of the separate resistances.

Problem. For the circuit shown in Figure 5.2, determine (a) the battery voltage V, (b) the total resistance of the circuit, and (c) the values of resistance of resistors R1, R2 and R3, given that the p.d.'s across R1, R2 and R3 are 5 V, 2 V and 6 V respectively

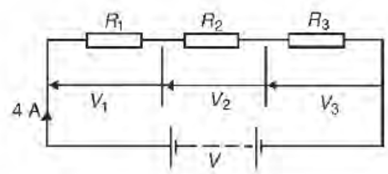


Figure 5.2

(a) Battery voltage $V = V_1 + V_2 + V_3$

$$= 5 + 2 + 6 = \mathbf{13\ V}$$

(b) Total circuit resistance $R = \dfrac{V}{I} = \dfrac{13}{4} = \mathbf{3.25\ \Omega}$

(c) Resistance $R_1 = \dfrac{V_1}{I} = \dfrac{5}{4} = \mathbf{1.25\ \Omega}$

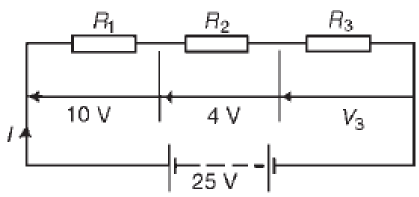
Resistance $R_2 = \dfrac{V_2}{I} = \dfrac{2}{4} = \mathbf{0.5\ \Omega}$

Resistance $R_3 = \dfrac{V_3}{I} = \dfrac{6}{4} = \mathbf{1.5\ \Omega}$

(Check: $R_1 + R_2 + R_3 = 1.25 + 0.5 + 1.5 = 3.25\ \Omega = R$)

Problem. For the circuit shown in Figure 5.3, determine the p.d. across resistor R3. If the total resistance of the circuit is 100 Ω, determine the current flowing through resistor R1. Find also the value of resistor R2

Figure 5.3



P.d. across R_3, $V_3 = 25 - 10 - 4 = \mathbf{11\ V}$

Current $I = \frac{V}{R} = \frac{25}{100} = \mathbf{0.25\ A}$, which is the current flowing in each resistor

Resistance $R_2 = \frac{V_2}{I} = \frac{4}{0.25} = \mathbf{16\ \Omega}$

Parallel networks

In the figure below the three resistors, R1, R2 and R3 connected across each other, i.e., in parallel, across a battery source of V volts.

In a parallel circuit:

(a) The sum of the currents I_1, I_2 and I_3 is equal to the total circuit current, I, i.e. $I = I_1 + I_2 + I_3$, and
(b) The source p.d., V volts, is the same across each of the resistors.

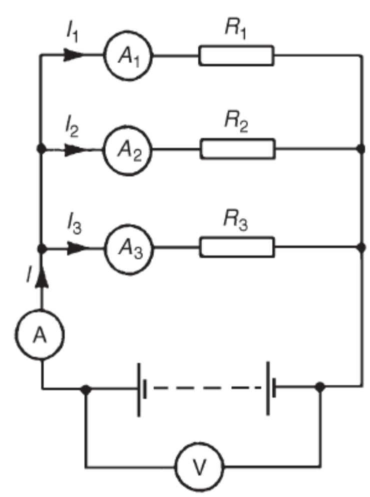
From Ohm's law:

$$I_1 = \frac{V}{R_1},\ I_2 = \frac{V}{R_2},\ I_3 = \frac{V}{R_3} \text{ and } I = \frac{V}{R}$$

where R is the total circuit resistance.

Since $I = I_1 + I_2 + I_3$

then, $\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$



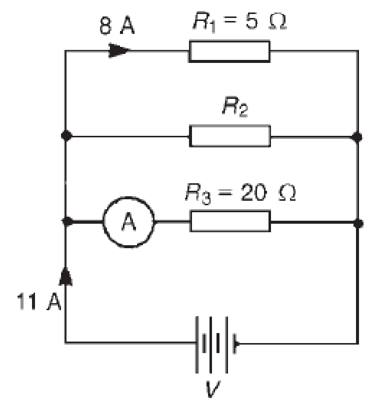
Dividing throughout by V gives:

$$\boxed{\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

This equation must be used when finding the total resistance R of a parallel circuit. For the special case of **two resistors in parallel:**

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_2 + R_1}{R_1 R_2} \qquad \text{Hence} \quad \boxed{R = \frac{R_1 R_2}{R_1 + R_2}}$$

Problem. For the circuit shown in Figure below, determine (a) the reading on the ammeter, and (b) the value of resistor R2



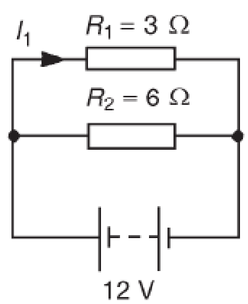
P.d. across R1 is the same as the supply voltage V.
Hence supply voltage, V = 8 * 5 = 40 V

(a) Reading on ammeter, $I = \frac{V}{R_3} = \frac{40}{20} = \mathbf{2\ A}$

(b) Current flowing through $R_2 = 11 - 8 - 2 = 1$ A

Hence, $R_2 = \frac{V}{I_2} = \frac{40}{1} = \mathbf{40\ \Omega}$

Problem. Two resistors, of resistance 3 Ω and 6 Ω, are connected in parallel across a battery having a voltage of 12 V. Determine (a) the total circuit resistance and (b) the current flowing in the 3 Ω resistor.



(a) The total circuit resistance R is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{3} + \frac{1}{6}$$

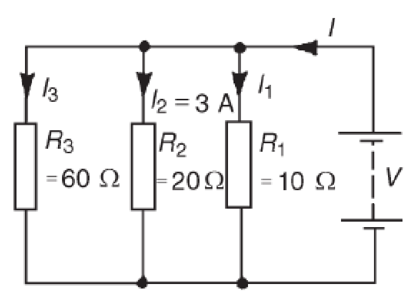
$$\frac{1}{R} = \frac{2+1}{6} = \frac{3}{6}$$

Hence, $R = \frac{6}{3} = \mathbf{2\ \Omega}$

$$\left(\text{Alternatively, } R = \frac{R_1 R_2}{R_1 + R_2} = \frac{3 \times 6}{3 + 6} = \frac{18}{9} = \mathbf{2\ \Omega}\right)$$

(b) Current in the 3 Ω resistance, $I_1 = \frac{V}{R_1} = \frac{12}{3} = \mathbf{4\ A}$

Problem. For the circuit shown in the figure below , find (a) the value of the supply voltage V and (b) the value of current I.



(a) P.d. across 20 Ω resistor = $I_2 * R_2 = 3 * 20 = 60$ V, hence supply voltage V = 60 V since the circuit is connected in parallel.

(b) Current $I_1 = \frac{V}{R_1} = \frac{60}{10} = 6$ A; $I_2 = 3$ A

$$I_3 = \frac{V}{R_3} = \frac{60}{60} = 1 \text{ A}$$

Current $I = I_1 + I_2 + I_3$ and hence $I = 6 + 3 + 1 = \mathbf{10\ A}$

Alternatively, $\frac{1}{R} = \frac{1}{60} + \frac{1}{20} + \frac{1}{10} = \frac{1+3+6}{60} = \frac{10}{60}$

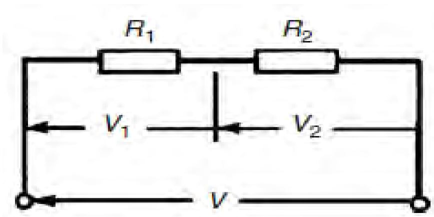
Hence total resistance $R = \frac{60}{10} = 6\ \Omega$

Current $I = \frac{V}{R} = \frac{60}{6} = \mathbf{10\ A}$

Potential divider

The voltage distribution for the circuit shown in Fig. (a) is given by:

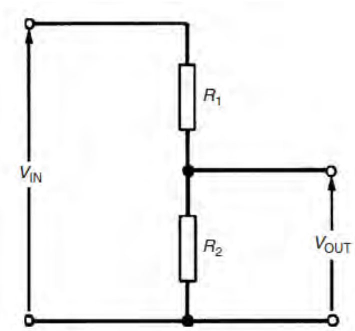
$$V_1 = \left(\frac{R_1}{R_1 + R_2}\right)V \quad \text{and} \quad V_2 = \left(\frac{R_2}{R_1 + R_2}\right)V$$



(a)

The circuit shown in Fig.(b) is often referred to as a **potential divider** circuit. Such a circuit can consist of a number of similar elements in series connected across a voltage source, voltages being taken from connections between the elements. Frequently the divider consists of two resistors as shown in Fig.(b), where

$$V_{OUT} = \left(\frac{R_2}{R_1 + R_2}\right)V_{IN}$$



(b)

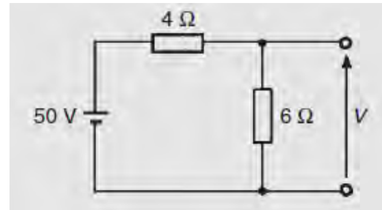


Fig 1

Problem: Determine the value of voltage *V* shown in Fig 1. below

The figure 1 may be redrawn as shown in Fig 2., and

$$\text{voltage } V = \left(\frac{6}{6+4}\right)(50) = \mathbf{30\ V}$$

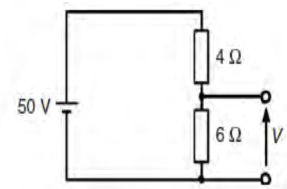
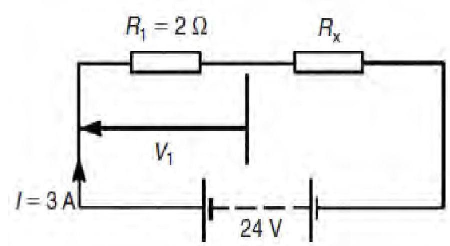


Fig 2

Problem 5. Two resistors are connected in series across a 24V supply and a current of 3A flows in the circuit. If one of the resistors has a resistance of 2Ω determine (a) the value of the other resistor, and (b) the p.d. across the 2Ω resistor. If the circuit is connected for 50 hours, how much energy is used?

The circuit diagram is shown in Fig. below



(a) Total circuit resistance:

$$R = \frac{V}{I} = \frac{24}{3} = 8\,\Omega$$

Value of unknown resistance,

$$R_x = 8 - 2 = \mathbf{6\ \Omega}$$

(b) P.d. across 2 Ω resistor,

$$V_1 = IR_1 = 3 \times 2 = \mathbf{6\ V}$$

Alternatively, from above,

$$V_1 = \left(\frac{R_1}{R_1 + R_x}\right) V$$

$$= \left(\frac{2}{2+6}\right)(24) = 6\ \text{V}$$

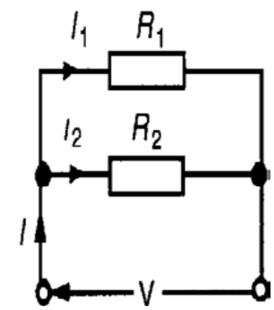
$$\text{Energy used} = \text{power} \times \text{time}$$

$$= (V \times I) \times t$$

$$= (24 \times 3\ \text{W})(50\ \text{h})$$

$$= 3600\ \text{Wh} = \mathbf{3.6\ kWh}$$

Current division



For the circuit shown in Fig. below, the total circuit resistance, *R*T is given by:

$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

and $$V = IR_T = I\left(\frac{R_1 R_2}{R_1 + R_2}\right)$$

Current $$I_1 = \frac{V}{R_1} = \frac{I}{R_1}\left(\frac{R_1 R_2}{R_1 + R_2}\right)$$

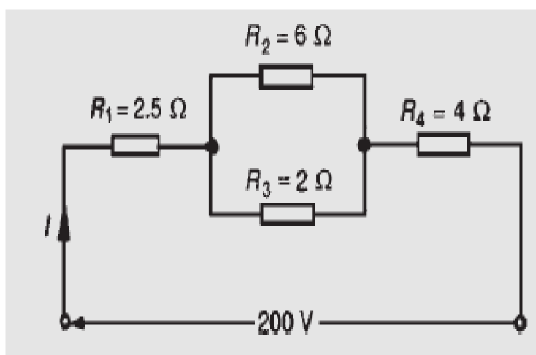
$$= \left(\frac{\mathbf{R_2}}{\mathbf{R_1 + R_2}}\right)(\mathbf{I})$$

Similarly,

current $$I_2 = \frac{V}{R_2} = \frac{I}{R_2}\left(\frac{R_1 R_2}{R_1 + R_2}\right)$$

$$= \left(\frac{\mathbf{R_1}}{\mathbf{R_1 + R_2}}\right)(\mathbf{I})$$

Problem 12. For the series-parallel arrangement shown in Fig. 5.24, find (a) the supply current, (b) the current flowing through each resistor and (c) the p.d. across each resistor.



(a) The equivalent resistance R_x of R_2 and R_3 in parallel is:

$$R_x = \frac{6 \times 2}{6 + 2} = 1.5\,\Omega$$

The equivalent resistance R_T of R_1, R_x and R_4 in series is:

$$R_T = 2.5 + 1.5 + 4 = 8\,\Omega$$

Supply current

$$I = \frac{V}{R_T} = \frac{200}{8} = \mathbf{25\,A}$$

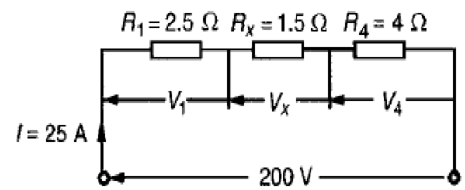
(b) The current flowing through R_1 and R_4 is 25 A. The current flowing through R_2

$$= \left(\frac{R_3}{R_2 + R_3}\right) I = \left(\frac{2}{6+2}\right) 25$$
$$= \mathbf{6.25\,A}$$

The current flowing through R_3

$$= \left(\frac{R_2}{R_2 + R_3}\right) I$$
$$= \left(\frac{6}{6+2}\right) 25 = \mathbf{18.75\,A}$$

(c) The equivalent circuit of Fig. above is shown in Fig. below:



p.d. across R_1, i.e.

$$V_1 = IR_1 = (25)(2.5) = \mathbf{62.5\,V}$$

p.d. across R_x, i.e.

$$V_x = IR_x = (25)(1.5) = \mathbf{37.5\,V}$$

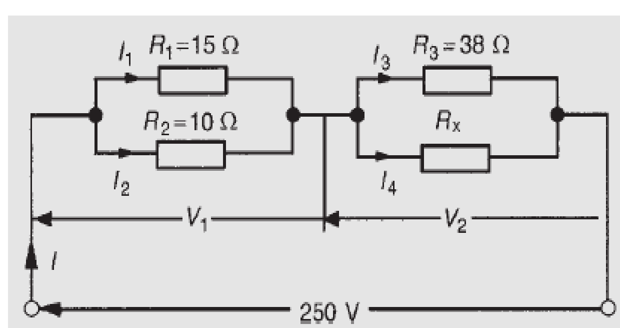
p.d. across R_4, i.e.

$$V_4 = IR_4 = (25)(4) = \mathbf{100\,V}$$

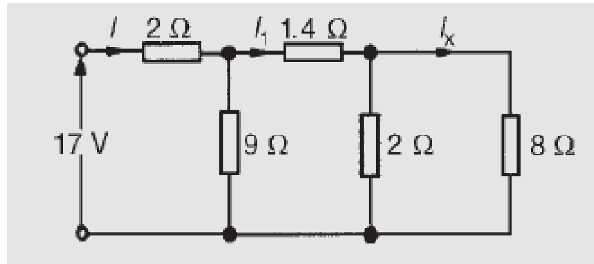
Hence the p.d. across R_2

$$= \text{p.d. across } R_3 = \mathbf{37.5\,V}$$

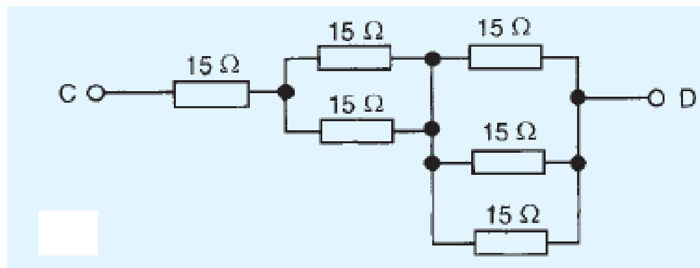
H.W1 . For the circuit shown in Fig. below calculate (a) the value of resistor *R*x such that the total power dissipated in the circuit is 2.5kW, (b) the current flowing in each of the four resistors.



H.W. 2 For the arrangement shown in Fig. below , find the current *I*x.



Find the equivalent resistance between terminals C and D of the circuit shown in Fig. below:



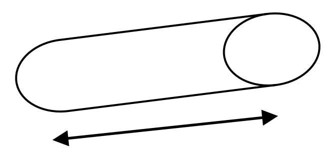
Resistance and resistivity

The resistance of an electrical conductor depends on 4 factors, these being: (a) the length of the conductor, (b) the cross-sectional area of the conductor, (c) the type of material and (d) the temperature of the material.
Resistance, R, is directly proportional to length, *l*, of a conductor, i.e. R α *l*. Thus, for example, if the length of a piece of wire is doubled, then the resistance is doubled. Resistance, R, is inversely proportional to cross-sectional area, a, of a conductor, i.e. R α 1/a. Thus, for example, if the cross-sectional area of a piece of wire is doubled then the resistance is halved.

Since R α *l* and R α 1/a then R α *l*/a. By inserting a constant of proportionality into this relationship the type of material used may be taken into account. The constant of proportionality is known as the **resistivity** of the material and is given the symbol ρ (Ω m). Thus,

$$\text{resistance} \quad \boxed{R = \frac{\rho l}{a}\ \textbf{ohms}}$$

Resistivity varies with temperature and some typical values of resistivities measured at about room temperature are given below:
Copper $1.7 * 10^{-8}$ Ω m (or 0.017 μ Ω m)
Aluminium $2.6 * 10^{-8}$ Ω m (or 0.026 μ Ω m)
Carbon (graphite) $10 * 10^{-8}$ Ω m (or 0.10 μ Ω m)
Glass $1 * 10^{10}$ Ω m (or 104 μ Ω m
Mica $1 * 10^{13}$ Ω m (or 107 μ Ω m)



Note that good conductors of electricity have a low value of resistivity and good insulators have a high value of resistivity.

Problem. The resistance of a 5 m length of wire is 600Ω . Determine (a) the resistance of an 8 m length of the same wire, and (b) the length of the same wire when the resistance is 420Ω .

(a) Resistance, R, is directly proportional to length, *l*, i.e. R α *l* Hence, 600 Ω α 5 m or 600 = k * 5 , where k is the coefficient of proportionality. Hence,

$$k = \frac{600}{5} = 120$$

When the length l is 8 m, then resistance

$$R = kl = (120)(8) = \mathbf{960\ \Omega}$$

(b) When the resistance is 420 Ω, $420 = kl$, from which,

$$\text{length } l = \frac{420}{k} = \frac{420}{120} = \mathbf{3.5\ m}$$

Problem . A piece of wire of cross-sectional area 2 mm^2 has a resistance of 300Ω . Find (a) the resistance of a wire of the same length and material if the cross-sectional area is 5 mm^2, (b) the cross-sectional area of a wire of the same length and material of resistance 750Ω

Resistance R is inversely proportional to cross-sectional area, a, i.e. $R \propto \frac{1}{a}$

Hence $300\ \Omega \propto \frac{1}{2\ \text{mm}^2}$ or $300 = (k)\left(\frac{1}{2}\right)$,

from which, the coefficient of proportionality, $k = 300 \times 2 = 600$

(a) When the cross-sectional area $a = 5\ \text{mm}^2$ then $R = (k)\left(\frac{1}{5}\right)$

$$= (600)\left(\tfrac{1}{5}\right) = \mathbf{120\ \Omega}$$

(Note that resistance has decreased as the cross-sectional is increased.)

(b) When the resistance is 750 Ω then $750 = (k)(1/a)$, from which

cross-sectional area, $a = \frac{k}{750} = \frac{600}{750} = \mathbf{0.8\ mm^2}$

Temperature coefficient of resistance

Effect of temperature on resistance

The effect of rise in temperature is

1. The resistance increase when temperature increases in metal like copper and iron, from this we can understand that pure metals have positive temperature co-efficient.
2. In alloys like magnesium and Eureka resistance increase is relatively small with increase in temperature.
3. In Electrolyte, Insulators, mica, glass and rubber resistance decreases with increase is temperature. Hence they have negative temperature - coefficient of resistance.

The symbol used for the temperature coefficient of resistance is α ,
Some typical values of temperature coefficient of resistance measured at 0°C are given below:

Copper 0.0043/°C	Aluminium 0.0038/°C
Nickel 0.0062/°C	Carbon - 0.000 48/°C

If the resistance of a material at 0°C is known the resistance at any other temperature can be determined from:

$$\boxed{R_\theta = R_0(1 + \alpha_0\theta)}$$

where R_0 = resistance at 0°C

R_θ = resistance at temperature θ°C

α_0 = temperature coefficient of resistance at 0°C

Problem. A coil of copper wire has a resistance of 100 Ω when its temperature is 0°C. Determine its resistance at 70°C if the temperature coefficient of resistance of copper at 0°C is 0.0043/°C

Problem. An aluminium cable has a resistance of 27Ω at a temperature of 35°C. Determine its resistance at 0°C. Take the temperature coefficient of resistance at 0°C to be 0.0038/°C

Problem. A carbon resistor has a resistance of 1 kΩ at 0°C. Determine its resistance at 80°C. Assume that the temperature coefficient of resistance for carbon at 0°C is -0.0005/°C

If the resistance at 0°C is not known, but is known at some other temperature θ_1, then the resistance at any temperature can be found as follows:

$$R_1 = R_0(1 + \alpha_0\theta_1) \text{ and } R_2 = R_0(1 + \alpha_0\theta_2)$$

Dividing one equation by the other gives:

$$\boxed{\frac{R_1}{R_2} = \frac{1 + \alpha_0\theta_1}{1 + \alpha_0\theta_2}}$$

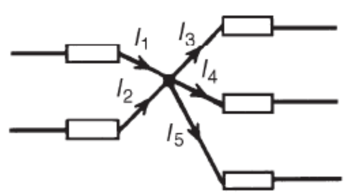
where R_2 = resistance at temperature θ_2

Problem Some copper wire has a resistance of 200Ω at 20°C. A current is passed through the wire and the temperature rises to 90°C. Determine the resistance of the wire at 90°C, assuming that the temperature coefficient of resistance is 0.004/°C at 0°C

Kirchhoff's laws

(a) ***Current Law.*** At any junction in an electric circuit the total current flowing towards that junction is equal to the total current flowing away from the junction, i.e. Σ I= 0.
Thus, referring to Figure

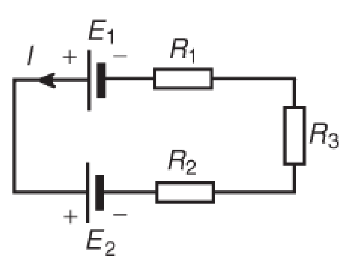
$$I_1 + I_2 = I_3 + I_4 + I_5 \quad \text{or} \quad I_1 + I_2 - I_3 - I_4 - I_5 = 0$$



Σ I =0

(b)***Voltage Law***. In any closed loop in a network, the algebraic sum of the voltage drops (i.e. products of current and resistance) taken around the loop is equal to the resultant e.m.f. acting in that loop. Thus, referring to Figure below, :

$$E_1 - E_2 = IR_1 + IR_2 + IR_3$$

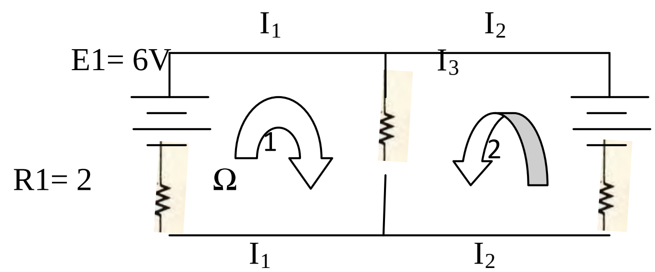


$\Sigma IR = \Sigma E$

Applying first law:
$\Sigma I = 0$

$I_1 + I_2 - I_3 = 0$

$I_3 = I_1 + I_2$ ……………….1



Applying second law:

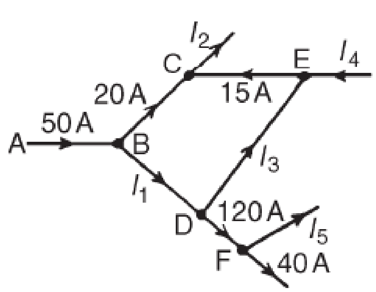
Loop 1
$I_3 * R_3 + I_1 * R_1 = E_1$
$(I_1 + I_2)\,10 + I_1 * 2 = 6$
$12\ \ I_1 + 10\,I_2$

Loop 2
$-R_2I_2 - I_3R_3 = -E_2$
$R_2I_2 + I_3R_3 = E_2$
$3I_2 + (I_1 + I_2)\,10 = 4$ ………..3

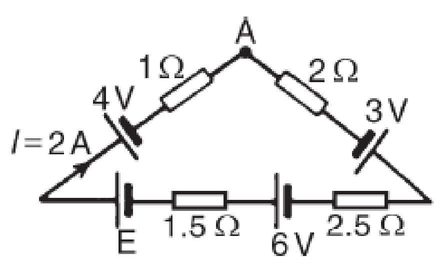
ABCDEFA,
$-R_2I_2 - R_1I_1 = E_1 - E_2$ (ملاحظة: $I_1R_1 - R_2I_2 = E_1 - E_2$)
$-3\,I_2 + 2\,I_1 = 6-4$
$2\,I_1 - 3\,I_2 = 2$ ………………..4

Sub. Any two equations 2 or 3 or 4:
$I_1 = 0.6786$ A
$I_2 = 0.2193$ A

Problem . (a) Find the unknown currents marked in Figure (a). (b) Determine the value of e.m.f. *E* in Figure (b).



(a)



(b)

(a) Applying Kirchhoff's current law:

For junction B: $50 = 20 + I_1$. Hence $I_1 = \mathbf{30\ A}$

For junction C: $20 + 15 = I_2$. Hence $I_2 = \mathbf{35\ A}$

For junction D: $I_1 = I_3 + 120$

i.e. $30 = I_3 + 120$. Hence $I_3 = \mathbf{-90\ A}$

(i.e. in the opposite direction to that shown in Figure 13.3(a))

For junction E: $I_4 + I_3 = 15$

i.e. $I_4 = 15 - (-90)$. Hence $I_4 = \mathbf{105\ A}$

For junction F: $120 = I_5 + 40$. Hence $I_5 = \mathbf{80\ A}$

(b) Applying Kirchhoff's voltage law and moving clockwise around the loop of Figure (b) starting at point A:

$$3 + 6 + E - 4 = (I)(2) + (I)(2.5) + (I)(1.5) + (I)(1)$$
$$= I(2 + 2.5 + 1.5 + 1)$$

i.e. $5 + E = 2(7)$, since $I = 2$ A

Hence $E = 14 - 5 = \mathbf{9\ V}$

Mesh

A mesh is a loop that does not contain other loops. Meshes are an important aid to certain analysis methods. In **Figure A**, the circuit with loops 1, 2, and 3 consists of two meshes: loops 1 and 2 are meshes, but loop 3 is not a mesh, because it encircles both loops 1 and 2. The one-loop circuit of **Figure A** is also a one-mesh circuit.

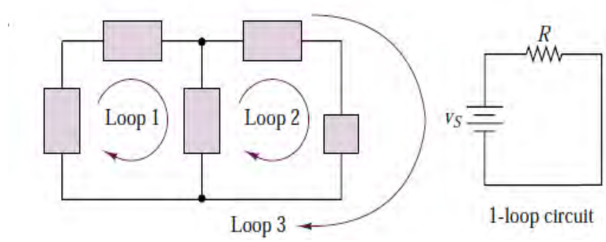


Figure A

THE MESH CURRENT METHOD

The idea is to write the appropriate number of independent equations, using mesh currents as the independent variables. Analysis by mesh currents consists of defining the currents around the individual meshes as the independent variables. Subsequent application of Kirchhoff's voltage law around each mesh provides the desired system of equations.

In the mesh current method, we observe that a current flowing through a resistor in a specified direction defines the polarity of the voltage across the resistor, as illustrated in

Figure 1, and that the sum of the voltages around a closed circuit must equal zero, by KVL. Once a convention is established regarding the direction of current flow around a mesh, simple application of KVL provides the desired equation. Figure 2, illustrates this point.

The number of equations one obtains by this technique is equal to the number of meshes in the circuit. All branch currents and voltages may subsequently be obtained from the mesh currents, as will presently be shown. Since meshes are easily identified in a circuit, this method provides a very efficient and systematic procedure for the analysis of electrical circuits.

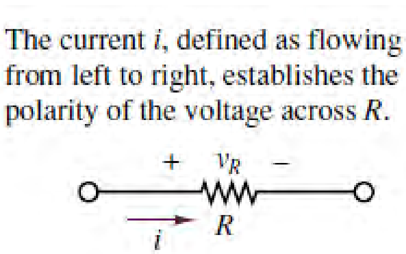


Fig.1

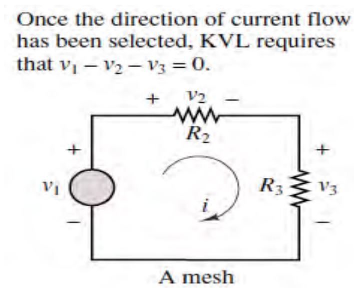


Fig.2

Mesh Current Analysis Method

1. Define each mesh current consistently. We shall always define mesh currents clockwise, for convenience.
2. Apply KVL around each mesh, expressing each voltage in terms of one or more mesh currents.
3. Solve the resulting linear system of equations with mesh currents as the independent variables.

To illustrate the mesh current method, consider the simple two-mesh circuit shown in Figure 3. This circuit will be used to generate two equations in the two unknowns, the mesh currents i_1 and i_2. It is instructive to first consider each mesh by itself. Beginning with mesh 1, note that the voltages around the mesh have been assigned in Figure 4, according to the direction of the mesh current, i_1.

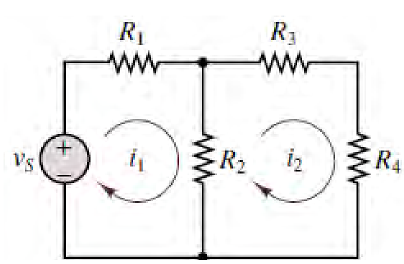


Fig.3

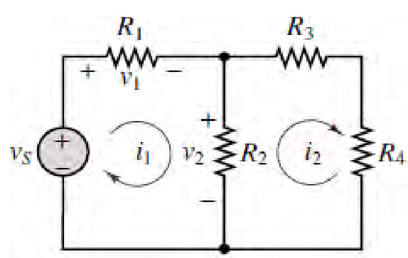


Fig.4

Mesh 1: KVL requires that $v_S - v_1 - v_2 = 0$, where $v_1 = i_1R_1$, $v_2 = (i_1 - i_2)R_1$.

According to the sign convention, then, the voltages v_1 and v_2 are defined as shown in Figure 4. Now, it is important to observe that while mesh current i_1 is equal to the current flowing through resistor R_1 (and is therefore also the branch current through R_1), it is not equal to the current through R_2. The branch current through R_2 is the difference between the two mesh

currents, $i_1 - i_2$. Thus, since the polarity of the voltage v_2 has already been assigned, according to the convention discussed in the previous paragraph, it follows that the voltage v_2 is given by:

$$v_2 = (i_1 - i_2)R_2 \quad \text{.........1}$$

Finally, the complete expression for mesh 1 is

$$v_S - i_1R_1 - (i_1 - i_2)R_2 = 0 \quad \text{.........2}$$

The same line of reasoning applies to the second mesh. Figure 5 depicts the voltage assignment around the second mesh, following the clockwise direction of mesh current i_2. The mesh current i_2 is also the branch current through resistors R_3 and R_4; however, the current through the resistor that is shared by the two meshes, R_2, is now equal to $(i_2 - i_1)$, and the voltage across this resistor is:

$$v_2 = (i_2 - i_1)R_2 \quad \text{..........3}$$

and the complete expression for mesh 2 is:

$$(i_2 - i_1)R_2 + i_2R_3 + i_2R_4 = 0 \quad \text{.........4}$$

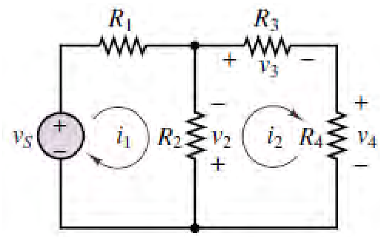


Fig.5

Why is the expression for v_2 obtained in equation 3 different from equation 2? The reason for this apparent discrepancy is that the voltage assignment for each mesh was dictated by the (clockwise) mesh current. Thus, since the mesh currents flow through R_2 in opposing directions, the voltage assignments for v_2 in the two meshes will also be opposite. This is perhaps a potential source of confusion in applying the mesh current method; you should be very careful to carry out the assignment of the voltages around each mesh separately.

Combining the equations for the two meshes, we obtain the following system of equations:

$$(R_1 + R_2)i_1 - R_2i_2 = v_S$$
$$-R_2i_1 + (R_2 + R_3 + R_4)i_2 = 0$$

These equations may be solved simultaneously to obtain the desired solution, namely, the mesh currents, i_1 and i_2.

Problem

Find the mesh currents in the circuit of Figure below:

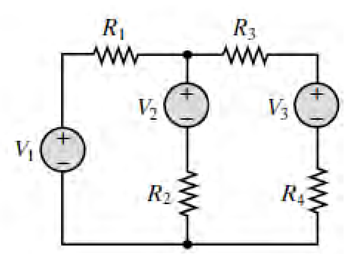


Fig.6

Solution

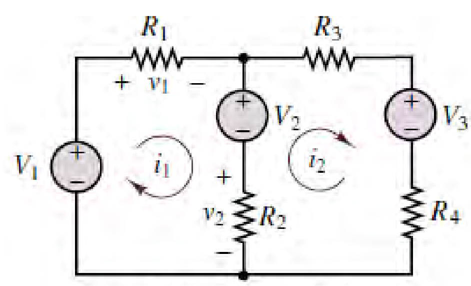
Known Quantities: Source voltages; resistor values.

Find: Mesh currents.

Schematics, Diagrams, Circuits, and Given Data: $V_1 = 10$ V; $V_2 = 9$ V; $V_3 = 1$ V; $R_1 = 5\ \Omega$; $R_2 = 10\ \Omega$; $R_3 = 5\ \Omega$; $R_4 = 5\ \Omega$.

Assumptions: Assume clockwise mesh currents i_1 and i_2.

Analysis: The circuit of Figure 6 will yield two equations in two unknowns, i_1 and i_2. It is instructive to consider each mesh separately in writing the mesh equations; to this end, Figure 7 depicts the appropriate voltage assignments around the two meshes, based on the assumed directions of the mesh currents. From Figure7, we write the mesh equations:



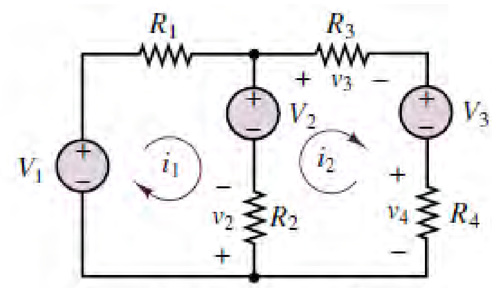


Fig.7

$$V_1 - R_1 i_1 - V_2 - R_2(i_1 - i_2) = 0$$
$$R_2(i_2 - i_1) + V_2 - R_3 i_2 - V_3 - R_4 i_2 = 0$$

Rearranging the linear system of the equation, we obtain

$$15i_1 - 10i_2 = 1$$
$$-10i_1 + 20i_2 = 8$$

which can be solved to obtain i_1 and i_2:

$$i_1 = 0.5 \text{ A} \quad \text{and} \quad i_2 = 0.65 \text{ A}$$

Comments: Note how the voltage v_2 across resistor R_2 has different polarity in Figure 7, depending on whether we are working in mesh 1 or mesh 2.

Mesh Analysis with Current Sources

Mesh analysis is particularly effective when applied to circuits containing voltage sources exclusively; however, it may be applied to mixed circuits, containing both voltage and current sources, the method is illustrated by solving the circuit shown in Figure 1. The first observation in analyzing this circuit is that the presence of the current source requires that the following relationship hold true: $i_1 - i_2 = 2 \text{ A}$ …..1

If the unknown voltage across the current source is labeled vx , application of KVL around mesh 1 yields:

$$10 - 5i_1 - v_x = 0 \quad \text{.....2}$$

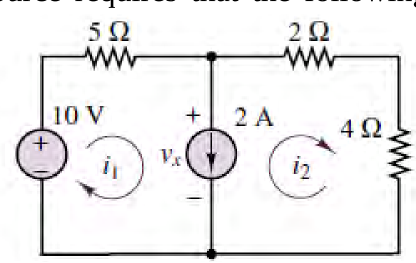


Fig.1

While KVL around mesh 2 dictates that

$$v_x - 2i_2 - 4i_2 = 0 \quad \text{........ 3}$$

Substituting equation 3 in equation 2, and using equation 1, we can then obtain the system of equations:

$$\begin{aligned} 5i_1 + 6i_2 &= 10 \\ -i_1 + i_2 &= -2 \end{aligned} \qquad \text{...... 4}$$

which we can solve to obtain:

$$\begin{aligned} i_1 &= 2 \text{ A} \\ i_2 &= 0 \text{ A} \end{aligned} \qquad \text{..........5}$$

Note also that the voltage across the current source may be found by using either equation 2 or equation 3; for example, using equation 3,

$$v_x = 6i_2 = 0 \text{ V} \qquad \text{...........6}$$