الجامعة التكنولوجية

قسم الهندسة الكيمياوية

المرحلة الاولى

الفيزياء

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Physics in general:

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Motion:

In this chapter, you will learn more about motion, a field of study called *kinematics*. You will become familiar with concepts such as velocity, acceleration and displacement. For now, the focus is on how things move, not what causes them to move. Later, you will study *dynamics*, which centers on forces and how they affect motion. Dynamics and kinematics make up *mechanics*, the study of force and motion.

Two key concepts in this chapter are velocity and acceleration. Velocity is how fast something is moving (its speed) **and** in what direction it is moving. Acceleration is the rate of change in velocity. In this chapter, you will have many opportunities to learn about velocity and acceleration and how they relate. To get a feel for these concepts, you can experiment by using the two simulations on the right. These simulations are versions of the tortoise and hare race. In this classic parable, the steady tortoise always wins the race. With your help, though, the hare stands a chance. (After all, this is your physics course, not your literature course.)

In the first simulation, the tortoise has a head start and moves at a constant velocity of three meters per second to the right. The hare is initially stationary; it has zero velocity. You set its acceleration □ in other words, how much its velocity changes each second. The acceleration you set is constant throughout the race. Can you set the acceleration so that the hare crosses the finish line first and wins the race? To try, click on Interactive 1, enter an acceleration value in the entry box in the simulation, and press GO to see what happens. Press RESET if you want to try again. Try acceleration values up to 10 meters per second squared. (At this acceleration, the velocity increases by 10 meters per second every second. Values larger than this will cause the action to occur so rapidly that the hare may quickly disappear off the screen.) It does not really matter if you can cause the hare to beat this rather fast-moving tortoise. However, we do want you to try a few different rates of acceleration and see how they affect the hare’s velocity. Nothing particularly tricky is occurring here; you are simply observing two basic properties of motion:

velocity and acceleration.

In the second simulation, the race is a round trip. To win the race, a contestant needs to go around the post on the right and then return to the starting line. The tortoise has been given a head start in this race. When you start the simulation, the tortoise has already rounded the post and is moving at a constant velocity on the homestretch back to the finish line. In this simulation, when you press GO the hare starts off moving quickly to the right. Again, you supply a value for its acceleration. The challenge is to supply a value for the hare's acceleration so that it turns around at the post and races back to beat the tortoise. We have given you a fair number of concepts in this introduction. These fundamentals are the foundation of the study of motion, and you will learn much more about them shortly. The definition of displacement is precise: the direction and length of the **shortest** path from the **initial** to the **final** position of an object's motion. As you may recall from your mathematics courses, the shortest path between two points is a straight line. Physicists use arrows to indicate the direction of displacement. In the illustrations to the right, the arrow points in the direction of the mouse's displacement. Physicists use the Greek letter Δ (delta) to indicate a change or difference. A change in position is displacement, and since *x* represents position, we write Δx to indicate displacement. You see this notation, and the equation for calculating displacement, to the right. In the equation, x_f represents the final position (the subscript f stands for final) and x_i represents the initial position (the subscript i stands for initial). Displacement is a vector. A vector is a quantity that must be stated in terms of its direction and its magnitude. Magnitude means the size or amount. "Move five meters to the right" is a description of a vector. Scalars, on the other hand, are quantities that are stated solely in terms of magnitude, like "a dozen eggs." There is no direction for a quantity of eggs, just an amount.

In one dimension, a positive or negative sign is enough to specify a direction. As mentioned, numbers to the right of the origin are positive, and those to the left are negative. This means displacement to the right is positive, and to the left it is negative. For instance, you can see in Example 1 that the mouse's car starts at the position +3.0 meters and moves to the left to the position −1.0 meters. (We measure the position at the middle of the car.) Since it moves to the left 4.0 meters, its displacement is −4.0 meters. Displacement measures the distance solely between the beginning and end of motion. We can use dance to illustrate this point. Let's say you are dancing and you take three steps forward and two steps back. Although you moved a total of five steps,

your displacement after this maneuver is one step forward. It would be better to use signs to describe the dance directions, so we could describe forward as "positive" and backwards as "negative." Three steps forward and two steps back yield a displacement of positive one step. Since displacement is in part a measure of distance, it is measured with units of length. Meters are the SI unit for displacement

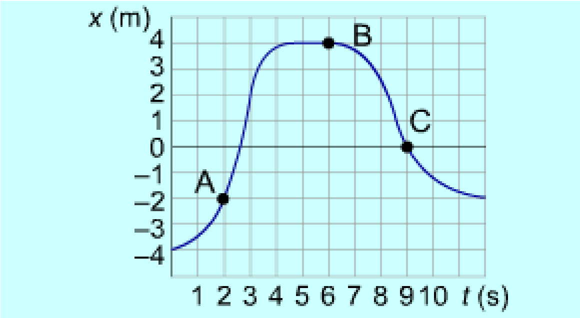
Velocity: Speed and direction.

You are familiar with the concept of speed. It tells you how fast something is going: 55 miles per hour (mi/h) is an example of speed. The speedometer in a car measures speed but does not indicate direction. When you need to know both speed and direction, you use velocity. Velocity is a vector. It is the measure of how fast **and** in which direction the motion is occurring. It is represented by v. In this section, we focus on average velocity, which is represented by v with a bar over it, as shown in Equation 1. A police officer uses the concepts of both speed and velocity in her work. She might issue a ticket to a motorist for driving 36 mi/h (58 km/h) in a school zone; in this case, speed matters but direction is irrelevant. In another situation, she might be told that a suspect is fleeing **north** on I-405 at 90 mi/h (149 km/h); now velocity is important because it tells her both how fast and in what direction.

To calculate an object's average velocity, divide its displacement by the time it takes to move that displacement. This time is called the elapsed time, and is represented by Δt. The direction for velocity is the same as for the displacement. For instance, let's say a car moves positive 50 mi (80 km) between the hours of 1 P.M. and 3 P.M. Its displacement is positive 50 mi, and two hours elapse as it moves that distance. The car's average velocity equals +50 miles divided by two hours, or +25 mi/h (+40 km/h). Note that the direction is positive because the displacement was positive. If the displacement were negative, then the velocity would also be negative. At this point in the discussion, we are intentionally ignoring any variations in the car's velocity. Perhaps the car moves at constant speed, or change direction. In other words, their velocity can change. For example, if you drop an egg off a 40-story building, the egg's velocity will change: It will move faster as it falls. Someone on the building's 39th floor would see it pass by with a different velocity than would someone on the 30th. When we use the word "instantaneous," we describe an object's velocity at a particular instant. In Concept 1, you see a snapshot of a toy mouse car at an instant when it has a velocity of positive six meters per second.

The fable of the tortoise and the hare provides a classic example of instantaneous

versus average velocity. As you may recall, the hare seemed faster because it could achieve a greater instantaneous velocity than could the tortoise. But the hare's long naps meant that its average velocity was less than that of the tortoise, so the tortoise won the race. When the average velocity of an object is measured over a very short elapsed time, the result is close to the instantaneous velocity. The shorter the elapsed time, the closer the average and instantaneous velocities. Imagine the egg falling past the 39th floor window in the example we mentioned earlier, and let's say you wanted to determine its instantaneous velocity at the midpoint of the window. You could use a stopwatch to time how long it takes the egg to travel from the top to the bottom of the window. If you then divided the height of the window by the elapsed time,



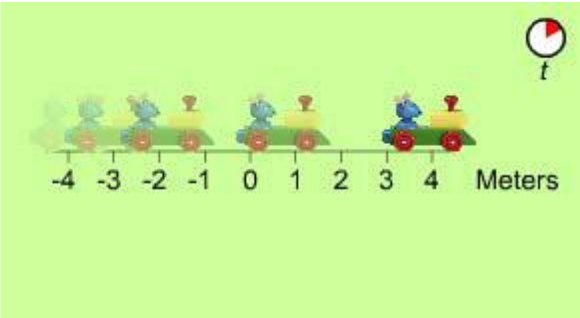
Acceleration: Change in velocity.

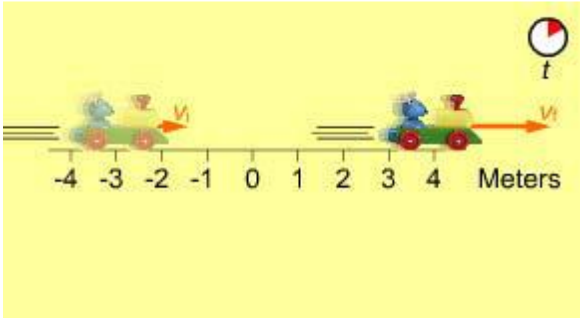
When an object's velocity changes, it accelerates. Acceleration measures the **rate** at which an object speeds up, slows down or changes direction. Any of these variations constitutes a change in velocity. The letter *a* represents acceleration. Acceleration is a popular topic in sports car commercials. In the commercials, acceleration is often expressed as how fast a car can go from zero to 60 miles per hour (97 km/h, or 27 m/s). For example, a current model Corvette® automobile can reach 60 mi/h in 4.9 seconds. There are even hotter cars than this in production.

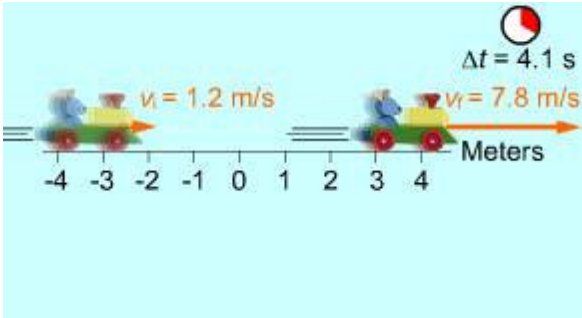
To calculate average acceleration, divide the change in instantaneous velocity by the elapsed time, as shown in Equation 1. To calculate the acceleration of the Corvette, divide its change in velocity, from 0 to 27 m/s, by the elapsed time of 4.9 seconds. The car accelerates at an average rate of 5.5 m/s per second. We typically express this as 5.5 meters per second squared, or 5.5 m/s2. (This equals 18 ft/s2, and with this observation we will cease stating values in both measurement systems, in order to simplify the expression of numbers.) Acceleration is measured in units of length divided by time squared. Meters per second squared (m/s2) express acceleration in SI

units. Let's assume the car accelerates at a constant rate; this means that each second the Corvette moves 5.5 m/s faster. At one second, it is moving at 5.5 m/s; at two seconds, 11 m/s; at three seconds, 16.5 m/s; and so forth. The car's velocity increases by 5.5 m/s every second. Since acceleration measures the change in **velocity**, an object can accelerate even while it is moving at a constant **speed**. For instance, consider a car moving around a curve. Even if the car's speed remains constant, it accelerates because the change in the car's direction means its velocity (speed plus direction) is changing. Acceleration can be positive or negative. If the Corvette uses its brakes to go from +60 to 0 mi/h in 4.9 seconds, its velocity is decreasing just as fast as it was increasing before. This is an example of negative acceleration.

You may want to think of negative acceleration as "slowing down," but be careful! Let's say a train has an initial velocity of **negative** 25 m/s and that changes to **negative** 50 m/s. The train is moving at a faster rate (speeding up) but it has negative acceleration. To be precise, its negative acceleration causes an increasingly negative velocity. Velocity and acceleration are related but distinct values for an object. For example, an object can have **positive** velocity and **negative** acceleration. In this case, it is slowing down. An object can have zero velocity, yet be accelerating. For example, when a ball bounces off the ground, it experiences a moment of zero velocity as its velocity changes



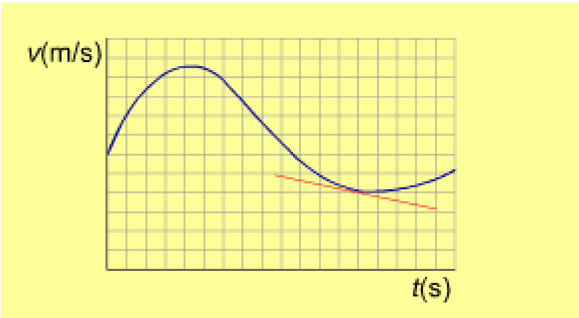


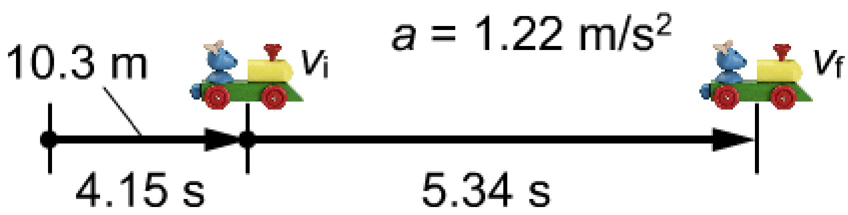


Instantaneous acceleration: Acceleration at a particular moment.

You have learned that velocity can be either average or instantaneous. Similarly, you can determine the average acceleration or the instantaneous acceleration of an object. We use the mouse in Concept 1 on the right to show the distinction between the two. The mouse moves toward the trap and then wisely turns around to retreat in a hurry. The illustration shows the mouse as it moves toward and then hurries away from the trap. It starts from a rest position and moves to the right with increasingly positive velocity, which means it has a positive acceleration for an interval of time. Then it slows to a stop when it sees the trap, and its positive velocity decreases to zero (this is negative acceleration). It then moves back to the left with increasingly negative velocity (negative acceleration again). If you would like to see this action occur again in the Concept 1 graphic, press the refresh button in your browser. We could calculate an average acceleration, but describing the mouse's motion with instantaneous acceleration is a more informative description of that motion. At some instants in time, it has positive acceleration and at other instants, negative acceleration. By knowing its acceleration and its velocity at an instant in time, we can determine whether it is moving toward the trap with increasingly positive velocity, slowing its rate of approach, or moving away with increasingly negative velocity. Instantaneous

acceleration is defined as the change in velocity divided by the elapsed time as the elapsed time approaches zero. This concept is stated mathematically in Equation 1 on the right. Earlier, we discussed how the slope of the tangent at any point on a position-time graph equals the instantaneous velocity at that point. We can apply similar reasoning here to conclude that the instantaneous acceleration at any point on a velocity-time graph equals the slope of the tangent, as shown in Equation 2. Why? Because slope equals the rate of change, and acceleration is the rate of change of velocity. In Example 1, we show a graph of the velocity of the mouse as it approaches the trap and then flees. You are asked to determine the sign of the instantaneous acceleration at four points; you can do so by considering the slope of the tangent to the velocity graph at each point.





Variables

Part 1: Constant velocity

displacement	$\Delta x = 10.3$ m
elapsed time	$\Delta t = 4.15$ s
velocity	v

Part 2: Constant acceleration

initial velocity	$v_i = v$ (calculated above)
acceleration	$a = 1.22$ m/s^2
elapsed time	$\Delta t = 5.34$ s
final velocity	v_f

What is the strategy?

What is the strategy?

1. Use the definition of velocity to find the velocity of the mouse car before it accelerates. The velocity is constant during the first part of the journey.
2. Use the definition of acceleration and solve for the final velocity.

Physics principles and equations

The definitions of velocity and acceleration will prove useful. The velocity and acceleration are constant in this problem. In this and later problems, we use the definitions for average velocity and acceleration without the bars over the variables.

$$v = \Delta x/\Delta t$$

$$a = \Delta v/\Delta t = (v_f - v_i)/\Delta t$$

Step-by-step solution

We start by finding the velocity before the engine fires.

Step	Reason
1. $v = \Delta x/\Delta t$	definition of velocity
2. $v = (10.3\ \text{m})/(4.15\ \text{s})$	enter values
3. $v = 2.48\ \text{m/s}$	divide

Next we find the final velocity using the definition of acceleration. The initial velocity is the same as the velocity we just calculated.

Step	Reason
4. $a = (v_f - v_i)/\Delta t$	definition of acceleration
5. $1.22\ \text{m/s}^2 = \dfrac{v_f - 2.48\ \text{m/s}}{5.34\ \text{s}}$	enter given values, and velocity from step 3
6. $6.51\ \text{m/s} = v_f - 2.48\ \text{m/s}$	multiply by 5.34 s
7. $v_f = 8.99\ \text{m/s}$	solve for v_f

Strategy

First, we will discuss our strategy for this derivation. That is, we will describe our overall plan of attack. These strategy points outline the major steps of the derivation.

1. We start with the definition of acceleration and rearrange it. It includes the terms for initial and final velocity, as well as elapsed time.

2. We derive another equation involving time that can be used to eliminate the time variable from the acceleration equation. The condition of constant acceleration will be crucial here.

3. We eliminate the time variable from the acceleration equation and simplify. This results in an equation that depends on other variables, but not time.

Physics principles and equations

Since the acceleration is constant, the velocity increases at a constant rate. This means the average velocity is the sum of the initial and final

$\overline{v} = (v_i + v_f)/2$

velocities divided by two.

We will use the definition of acceleration,

a = (vf □ vi)/t

We will also use the definition of average velocity,

$\overline{v} = \Delta x\ /t$

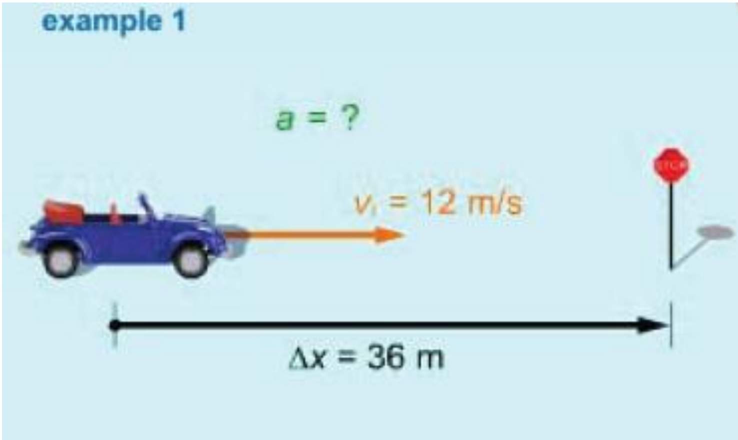
Step-by-step derivation

We start the derivation with the definition of average acceleration, solve it for the final velocity and do some algebra. This creates an equation with the square of the final velocity on the left side, where it appears in the equation we want to derive.

Step	Reason
1. $a = (v_f - v_i)/t$	definition of average acceleration
2. $v_f = v_i + at$	solve for final velocity
3. $v_f^2 = (v_i + at)^2$	square both sides
4. $v_f^2 = v_i^2 + 2v_i at + a^2t^2$	expand right side
5. $v_f^2 = v_i^2 + at(2v_i + at)$	factor out at
6. $v_f^2 = v_i^2 + at(v_i + v_i + at)$	rewrite $2v_i$ as a sum
7. $v_f^2 = v_i^2 + at(v_i + v_f)$	substitution from equation 2

The equation we just found is the basic equation from which we will derive the desired motion equation. But it still involves the time variable t - multiplied by a sum of velocities. In the **next** stage of the derivation, we use two different ways of expressing the average velocity to develop a second equation involving time multiplied by velocities. We will subsequently use that second equation to eliminate time from the equation above.

Step	Reason
8. $\bar{v} = \frac{v_i + v_f}{2}$	average velocity is average of initial and final velocities
9. $\bar{v} = \frac{\Delta x}{t}$	definition of average velocity
10. $\frac{v_i + v_f}{2} = \frac{\Delta x}{t}$	set right sides of 8 and 9 equal
11. $t(v_i + v_f) = 2\Delta x$	rearrange equation



What acceleration will stop the car exactly at the stop sign?

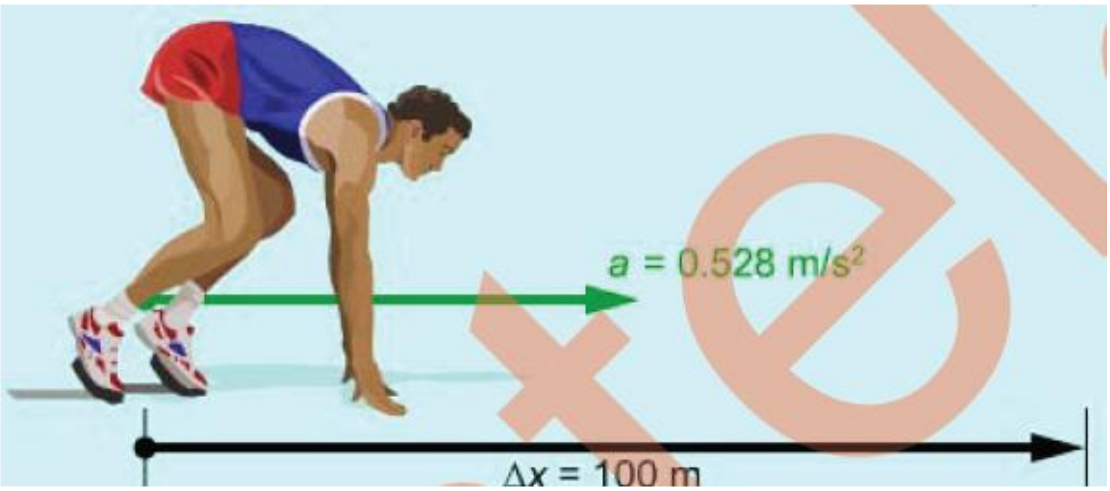
$v_f^2 = v_i^2 + 2a\Delta x$

$a = (v_f^2 - v_i^2)/2\Delta x$

$$a = \frac{(0.0\ \text{m/s})^2 - (12\ \text{m/s})^2}{2\,(36\ \text{m})}$$

$a = -144/72\ \text{m/s}^2$

$a = -2.0\ \text{m/s}^2$

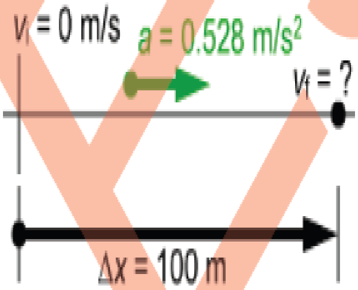


What is the runner's velocity at the end of a 100-meter dash?

You are asked to calculate the final velocity of a sprinter running a 100-meter dash. List the variables that you know and the one you are asked for, and then consider which equation you might use to solve the problem. You want an equation with just one unknown variable, which in this problem is the final velocity.

The sprinter's initial velocity is not explicitly stated, but he starts motionless, so it is zero m/s.

Draw a diagram



Variables

displacement	$\Delta x = 100$ m
acceleration	$a = 0.528$ m/s^2
initial velocity	$v_i = 0.00$ m/s
final velocity	v_f

What is the strategy?

1. Choose an appropriate equation based on the values you know and the one you want to find.
2. Enter the known values and solve for the final velocity.

Physics principles and equations

Based on the known and unknown values, the equation below is appropriate. We know all the variables in the equation except the one we are asked to find, so we can solve for it.

$$v_f^2 = v_i^2 + 2a\Delta x$$

Now we use a second motion equation containing the two velocities, substitute known values, and simplify. This gives us two equations with the two unknowns we want to find.

Step	Reason
4. $\Delta x = \frac{1}{2}(v_i + v_f)t$	second motion equation
5. $11.8\text{ m} = \frac{1}{2}(v_i + v_f)(3.14\text{ s})$	substitute known values
6. $7.52\text{ m/s} = v_i + v_f$	multiply by 2, divide by 3.14 s

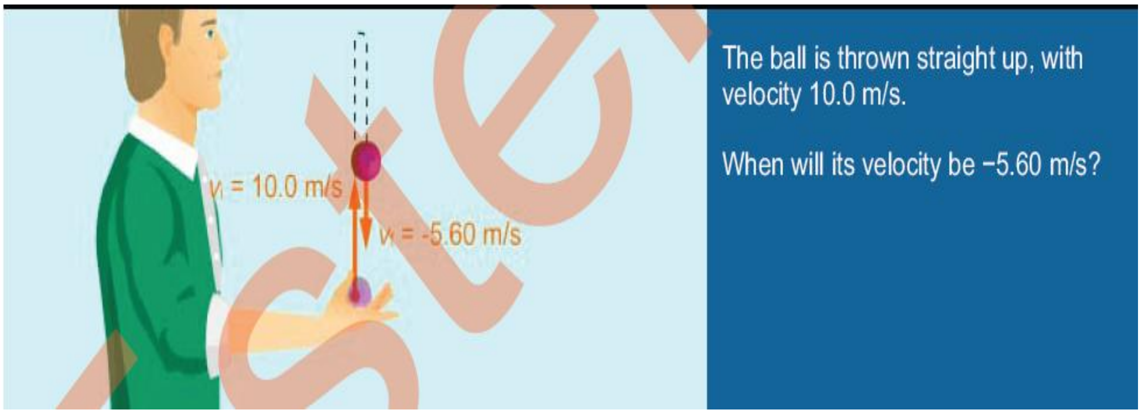
Now we solve the two equations.

Step	Reason
7. $7.52\text{ m/s} = v_i + v_i + 3.80\text{ m/s}$	substitute equation 3 into equation 6
8. $v_i = 1.86\text{ m/s}$	solve for v_i
9. $v_f = v_i + 3.80\text{ m/s} = 5.66\text{ m/s}$	from equation 3

There are other ways to solve this problem. For example, you could use the equation

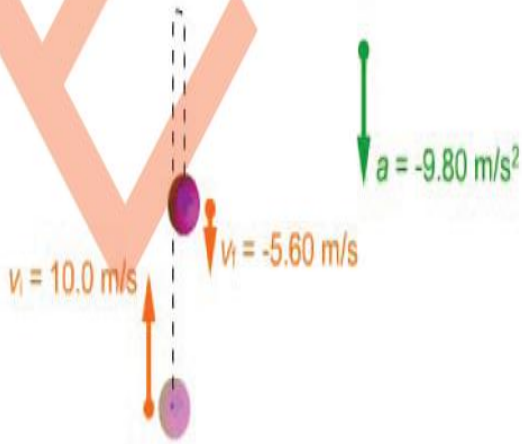
$$\Delta x = v_i t + \frac{1}{2}at^2$$

to find the initial velocity from the displacement, acceleration, and elapsed time. Then you could use the equation $v_f = v_i + at$ to solve for the final velocity.



As you see above, a ball is tossed straight up into the air with an initial velocity of positive 10.0 meters per second. You are asked to figure out how long it will take before its velocity is **negative** 5.60 m/s. The ball will have this velocity when it is falling back to the ground.

Draw a diagram



Variables

Be careful with the signs for acceleration and velocity. We use the common convention that upward quantities are positive, and downward negative. The magnitude of the acceleration is the free-fall acceleration constant $g = 9.80\ \text{m/s}^2$.

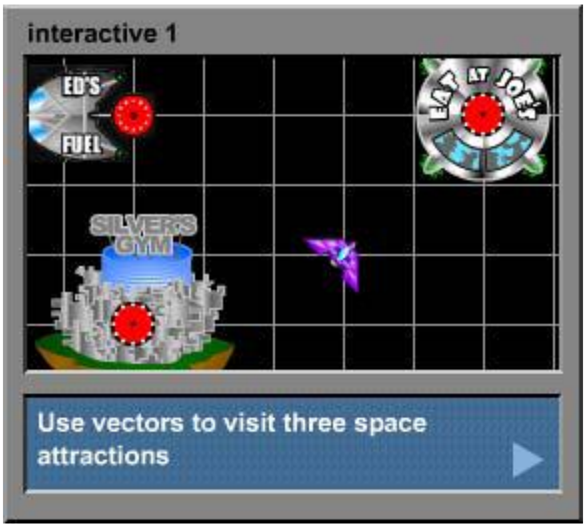
initial velocity	$v_i = 10.0\ \text{m/s}$
final velocity	$v_f = -5.60\ \text{m/s}$
acceleration	$a = -9.80\ \text{m/s}^2$
elapsed time	t

What is the strategy?

1. Choose an appropriate motion equation for the knowns and unknowns.
2. Solve for the elapsed time.

Knowing "how far" or "how fast" can often be useful, but "which way" sometimes proves even more valuable. If you have ever been lost, you understand that direction can be the most important thing to know. Vectors describe "how much" **and** "which way," or, in the terminology of physics, magnitude and direction. You use vectors frequently, even if you are not familiar with the term. "Go three miles northeast" or "walk two blocks north, one block east" are both vector descriptions. Vectors prove crucial in much of physics. For example, if you throw a ball up into the air, you need to understand that the initial velocity of the ball points "up" while the acceleration due to the force of gravity points "down." In this chapter, you will learn the fundamentals of vectors: how to write them and how to combine them using operations such as addition and subtraction. On the right, a simulation lets you explore vectors, in this case displacement vectors. In the simulation, you are the pilot of a small spaceship. There are three locations nearby that you want to visit: a refueling station, a diner, and the local gym. To reach any of these locations, you describe the displacement vector of the spaceship by setting its *x* (horizontal) and *y* (vertical) components. In other words, you set how far horizontally you want to travel, and how far vertically. This is a common way to express a two-dimensional vector.

There is a grid on the drawing to help you determine these values. You, and each of the places you want to visit, are at the intersection of two grid lines. Each square on the grid is one kilometer across in each direction. Enter the values, press GO, and the simulation will show you traveling in a straight line □ along the displacement vector □ according to the values you set. See if you can reach all three places. You can do this by entering displacement values to the nearest kilometer, like (3, 4) km. To start over at any time, press RESET.

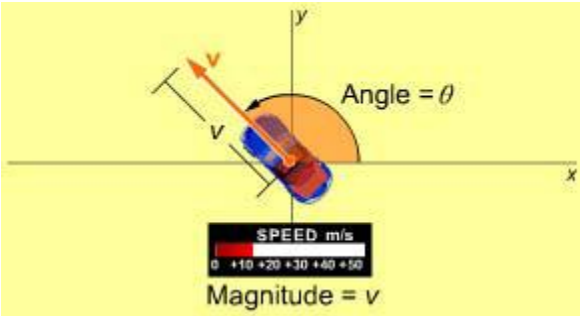


Scalar: A quantity that states only an amount.

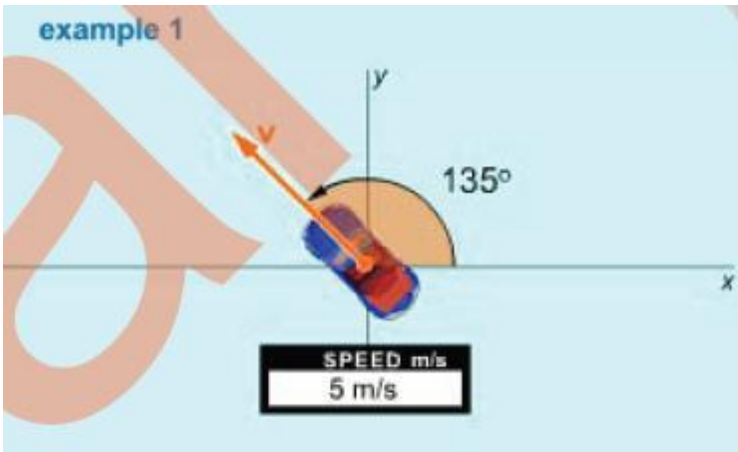
Scalar quantities state an amount: “how much” or “how many.” At the right is a picture of a dozen eggs. The quantity, a dozen, is a scalar. Unlike vectors, there is no direction associated with a scalar □ no up or down, no left or right □ just one quantity, the amount. A scalar is described by a single number, together with the appropriate units. Temperature provides another example of a scalar quantity; it gets warmer and colder, but at any particular time and place there is no “direction” to temperature, only a value. Time is another commonly used scalar.

Speed and distance are yet other scalars. A speed like 60 kilometers per hour says how fast but not which way. Distance is a scalar since it tells you how far away something is, but not the direction.

direction, a negative angle a clockwise direction. For example, 90° represents a quarter turn **counterclockwise** from the positive *x* axis. In other words, a vector with a 90° angle points straight up. We could also specify this angle as □270°. The radian is another unit of measurement for angles that you may have seen before. We will use degrees to specify angles unless we specifically note that we are using radians. (Radians do prove essential at times.)



Polar notation
v is magnitude
θ is angle
Written **v** = (*v*, *θ*)



Write the velocity vector of the car in polar notation.

$\mathbf{v} = (v, \theta)$

$\mathbf{v} = (5\text{ m/s}, 135^{\circ})$

Rectangular notation: Defining a vector by its components.

Often what we know, or want to know, about a particular vector is not its overall magnitude and direction, but how far it extends horizontally and vertically. On a graph, we represent the horizontal direction as x and the vertical direction as y. These are called *Cartesian coordinates*. The x *component* of a vector indicates its extent in the horizontal dimension and the y *component* its extent in the vertical dimension.

Rectangular notation is a way to describe a vector using the components that make up the vector. In rectangular notation, the x and y components of a vector are written inside parentheses. A vector that extends a units along the x axis and b units along the y axis is written as (a, b). For instance (3, 4) is a vector that extends positive three in the x direction and positive four in the y direction from its starting point.

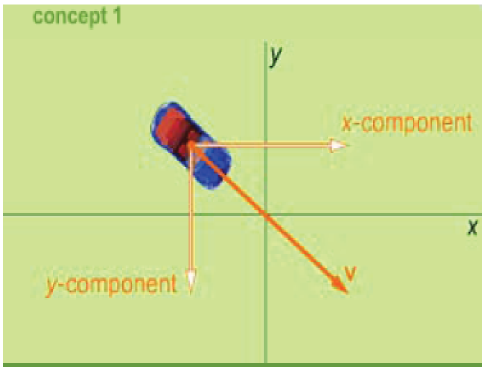
The components of vectors are scalars with the direction indicated by their sign: x components point right (positive) or left (negative), and y components point up (positive) or down (negative). You see the x and y components of a car's velocity vector in Concept 1 at the right, shown as "hollow" vectors. The x and y values define the vector, as they provide direction and magnitude.

For a vector $\mathbf{A}$, the x and y components are sometimes written as A_x and A_y. You see this notation used for a velocity vector $\mathbf{v}$ in Equation 1 and Example 1 on the right.

Consider the car shown in Example 1 on the right. Its velocity has an x component v_x of 17 m/s and a y component v_y of −13 m/s. We can write the car's velocity vector as (17, −13) m/s.

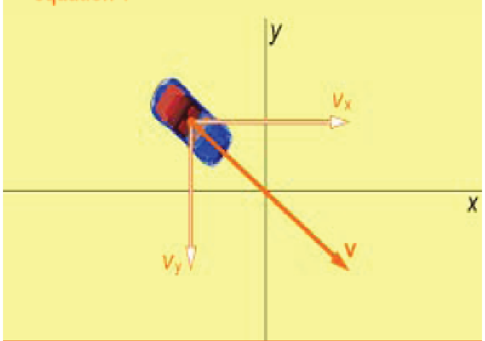
A vector can extend in more than two dimensions: z represents the third dimension. Sometimes z is used to represent distance toward or away from you. For instance, your computer monitor's width is measured in the x dimension, its height with y and your distance from the monitor with z. If you are reading this on a computer monitor and punch your computer screen, your fist would be moving in the z dimension. (We hope

concept 1



Vector components and rectangular notation
x component and *y* component

equation 1

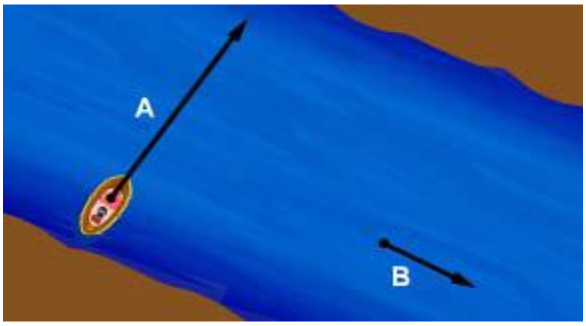


Rectangular notation
v_x is horizontal component

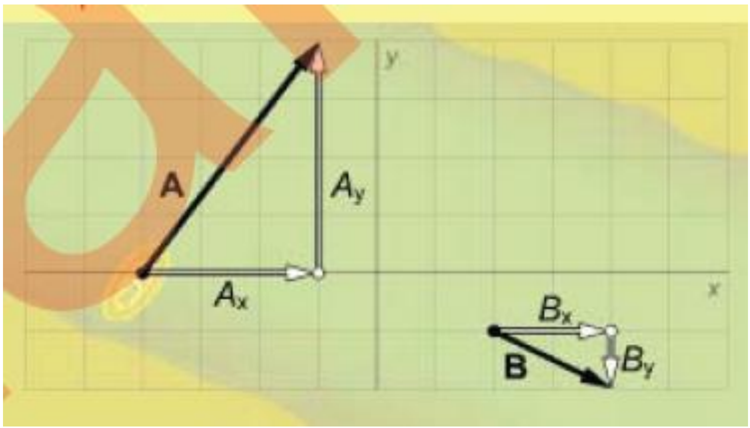
Adding and subtracting vectors by components

You can combine vectors graphically, but it may be more precise to add up their components. You perform this operation intuitively outside physics. If you were a dancer or a cheerleader, you would easily understand the following choreography: "Take two steps forward, four steps to the right and one step back." These are vector instructions. You can add them to determine the overall result. If asked how far **forward** you are after this dance move, you would say "one step," which is two steps

forward plus one step back. You realize that your progress forward or back is unaffected by steps to the left or right. You correctly process left/right and forward/back separately. If a physics-oriented dance instructor asked you to describe the results of your "dancing vector" math, you would say, "One step forward, four steps to the right." You have just learned the basics of vector addition, which is reasonably straightforward: Break the vector into its components and add each component independently. In physics though, you concern yourself with more than dance steps. You might want to add the vector (20, □40, 60) to (10, 50, 10). Let's assume the units for both vectors are meters. As with the dance example, each component is added independently. You add the first number in each set of parentheses: 20 plus 10 equals 30, so the sum along the *x* axis is 30. Then you add □40 and 50 for a total of 10 along the *y* axis. The sum along the *z* axis is 60 plus 10, or 70. The vector sum is (30, 10, 70) meters. If following all this in the text is hard, you can see another problem worked in Example 1 on the right. Although we use displacement vectors in much of this discussion since they may be the most intuitive to understand, it is important to note that all types of vectors can be added or subtracted. You can add two velocity vectors, two acceleration vectors, two force vectors and so on. As illustrated in the example problem, where two velocity vectors are added, the process is identical for any type of vector. Vector subtraction works similarly to addition when you use components. For example, (5, 3) minus (2, 1) equals 5 minus 2, and 3 minus 1; the result is the vector (3, 2).



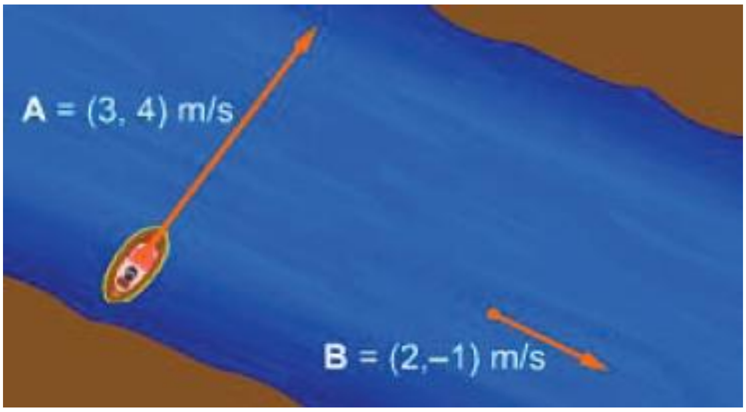
Adding and subtracting vectors by components
Add (or subtract) each component separately



Adding and subtracting vectors by components

$$\mathbf{A} + \mathbf{B} = (A_x + B_x, A_y + B_y)$$
$$\mathbf{A} - \mathbf{B} = (A_x - B_x, A_y - B_y)$$

$\mathbf{A}, \mathbf{B}$ = vectors
A_x, A_y = $\mathbf{A}$ components
B_x, B_y = $\mathbf{B}$ components



The boat has the velocity A in still water. Calculate its velocity as the sum of A and the velocity B of the river's current.

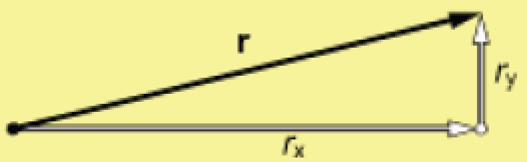
$$\mathbf{v} = \mathbf{A} + \mathbf{B}$$

$$\mathbf{v} = (3, 4)\ \text{m/s} + (2, -1)\ \text{m/s}$$

$$\mathbf{v} = (3 + 2, 4 + (-1))\ \text{m/s}$$

$$\mathbf{v} = (5, 3)\ \text{m/s}$$

equation 1



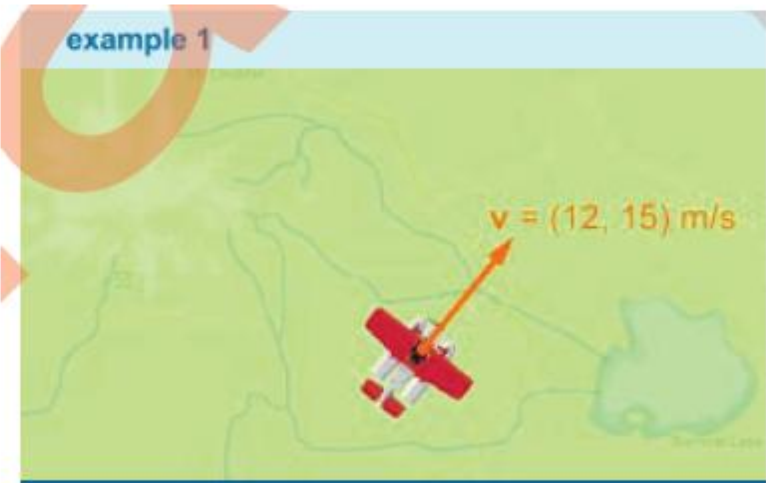
Multiplying a rectangular vector by a scalar

$$\boxed{s\mathbf{r} = (sr_x, sr_y)}$$

s = a scalar

r = a vector

r_x, r_y = **r** components



What is the displacement d of the plane after 5.0 seconds?

$\mathbf{d} = (5.0 \text{ s})\mathbf{v}$

$\mathbf{d} = (5.0 \text{ s})\ (12 \text{ m/s},\ 15 \text{ m/s})$

$\mathbf{d} = (\ (5.0 \text{ s})(12 \text{ m/s}),\ (5.0 \text{ s})(15 \text{ m/s})\ )$

$\mathbf{d} = (60,\ 75) \text{ m}$

3.10 - Multiplying polar vectors by a scalar

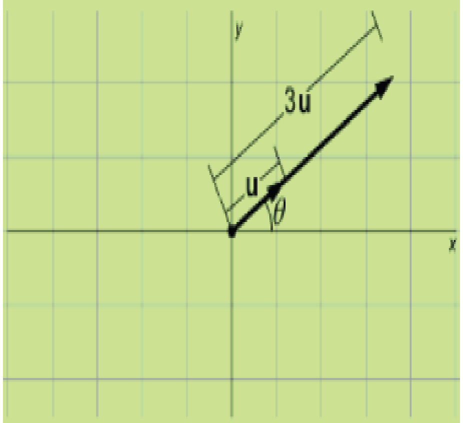
Multiplying a vector represented in polar notation by a positive scalar requires only one multiplication operation: Multiply the magnitude of the vector by the scalar. The angle is unchanged.

Let's say there is a vector of magnitude 50 km with an angle of $30°$. You are asked to multiply it by positive three. This situation is shown in Example 1 to the right. Since you are multiplying by a positive scalar, the angle stays the same at $30°$, and so the answer is 150 km at $30°$.

If you multiply a vector by a negative scalar, multiply its magnitude by the absolute value of the scalar (that is, ignore the negative sign). Then change the direction of the vector by $180°$ so that it points in the opposite direction. In polar notation, since the magnitude is always positive, you add $180°$ to the vector's angle to take its opposite. The result of multiplying (50 km, $30°$) by negative three is (150 km, $\mathbf{210}°$).

If adding $180°$ would result in an angle greater than $360°$, then subtract $180°$ instead. For instance, in reversing an angle of $300°$, subtract $180°$ and express the result as $120°$ rather than $480°$. The two results are identical, but $120°$ is easier to understand.

concept 1



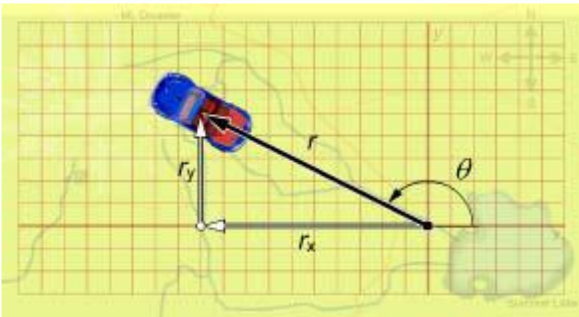
Multiplying polar vector by positive scalar

Multiply vector's magnitude by scalar

Angle unchanged

and the *y* component by multiplying 3.0 km by sin 35°. Here, *x* = (3.0 km)(0.82) and *y* = (3.0 km)(0.57), so the vector in rectangular coordinates is (2.5, 1.7) km. Using the same method with the other vector, 2.0 km at □15° equals (1.9, □0.52) km. The positive *x* component and negative *y* component indicate that this vector points down and to the right, the correct direction for a vector with an angle of □15°. We began this section by asking you how you would add these two vectors. Our work has made this an easier problem: (2.5, 1.7) plus (1.9, □0.52) equals (4.4, 1.2). The units are kilometers.

The *x* and *y* components can be positive or negative. For instance, the *x* component will be negative when the cosine is negative, which it is for angles between 90° and 270°. This corresponds to vectors that have an *x* component which points to the left. The *y* component will be positive when the sine is positive (between 0° and 180°, the vector has an upward *y* component) and negative when the sine is negative (between 180° and 360°, the vector has a downward *y* component). Since it is easy to err, it is a good practice to compare directions and the signs of the components. In Example 1, the negative *x* component is correct, since the car is moving to the left. If we had calculated a negative *y* component, we have erred in our calculations, since the car is clearly moving "up" in the positive *y* direction.



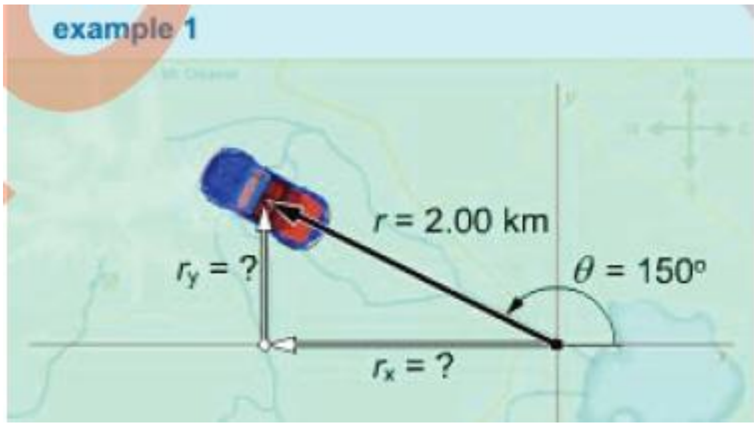
Converting a vector from polar to rectangular notation

To express (r, θ) as (r_x, r_y)

$$r_x = r\cos\theta$$
$$r_y = r\sin\theta$$

r = magnitude, θ = angle

r_x, r_x = components of vector



What is the displacement vector r of the car in rectangular notation?

$r_x = r \cos \theta = (2.00\ \text{km})(\cos 150º)$

$r_x = -1.73\ \text{km}$

$r_y = r \sin \theta = (2.00\ \text{km})(\sin 150º)$

$r_y = 1.00\ \text{km}$

$\mathbf{r} = (x, y) = (-1.73, 1.00)\ \text{km}$

3.12 - Converting vectors from rectangular to polar notation

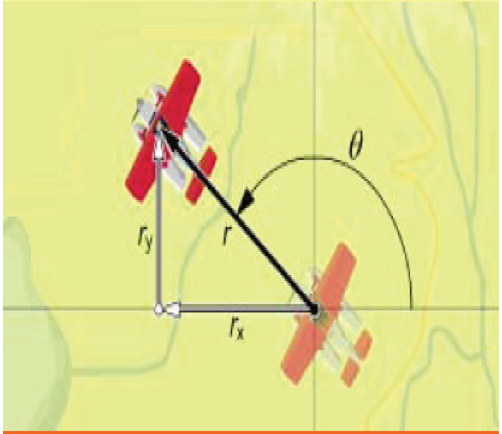
n some counties of the United States, the main roads travel either east-west or north-south. If you wanted to drive from one town to another, the roads might force you to ravel 40 km west and then 30 km north. On the other hand, if you had a plane, you ould fly in a straight line between the two towns, which would be a shorter distance. 'ou would need to know the angle at which to fly and the distance. We work this roblem out in Example 1, but before that, we review the concepts necessary to solve he problem.

o determine the angle and distance, you need to convert from rectangular to polar oordinates. You would use trigonometry to do so. The x and y components represent he legs of a triangle. You need to determine the length of the hypotenuse and the angle he hypotenuse makes with the positive x axis.

n Equation 1 on the right, you see that the Pythagorean theorem is used to calculate he hypotenuse when the two legs are known. The magnitude of the vector (the ypotenuse) is represented with r, and the two legs, called r_x and r_y here, are the omponents of the vector. In the example, the distance in kilometers is the square root f $(-40\text{ km})^2 + (30\text{ km})^2$. That works out to 50 km.

low you can determine the angle, which we represent as θ. As you may recall, the angent function relates the base and height of a right triangle to the angle between the ypotenuse and the base (in this case, the x axis). The angle θ is the arctangent of the

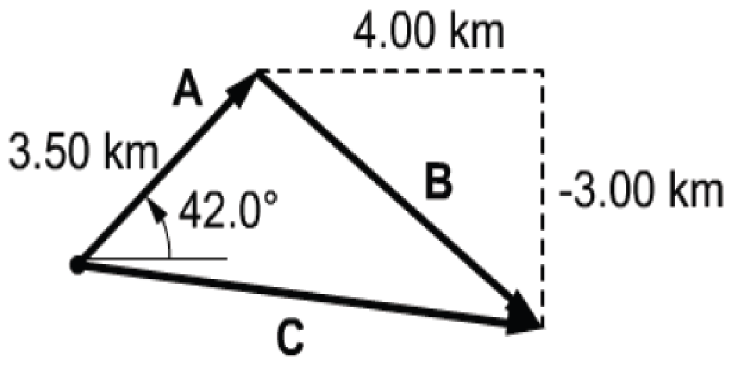
equation 1



Converting rectangular to polar

To express (r_x, r_y) as (r, θ)

Draw a diagram



Variables

We use **A** to indicate the first vector, **B** for the second vector, and **C** for their sum.

	polar notation	rectangular notation
vector **A**	(3.50 km, 42.0°)	(A_x, A_y)
vector **B**	not needed	(4.00, −3.00) km
vector sum **C**	(C, θ)	(C_x, C_y)

What is the strategy?

1. Convert the first vector **A** to rectangular notation.
2. Add vectors **A** and **B** by adding their components. This will give you the resulting displacement **C** in rectangular notati
3. Convert **C** to polar notation. Check to make sure the angle is in the right quadrant.

Mathematics principles

Polar to rectangular conversion

$$r_x = r\cos\theta$$

$$r_y = r\sin\theta$$

Rectangular to polar conversion

$$r = \sqrt{r_x^2 + r_y^2}$$

$$\theta = \arctan(r_y / r_x)$$

Adding vectors

$$\mathbf{A} + \mathbf{B} = (A_x + B_x, A_y + B_y)$$

Step-by-step solution

We start by converting vector **A** to rectangular notation.

Step	Reason
1. $A_x = A \cos\theta$	x component of vector
2. $A_x = (3.50\text{ km})(\cos 42.0°)$	enter values
3. $A_x = 2.60\text{ km}$	cosine, multiplication
4. $A_y = A \sin\theta$	y component of vector
5. $A_y = (3.50\text{ km})(\sin 42.0°)$	enter values
6. $A_y = 2.34\text{ km}$	sine, multiplication
7. $\mathbf{A} = (2.60, 2.34)\text{ km}$	combine components

Mathematics principles

Polar to rectangular conversion

$$r_x = r\cos\theta$$

$$r_y = r\sin\theta$$

Rectangular to polar conversion

$$r = \sqrt{r_x^2 + r_y^2}$$

$$\theta = \arctan(r_y/r_x)$$

Adding three vectors

$$\mathbf{A} + \mathbf{B} + \mathbf{C} = (A_x + B_x + C_x, A_y + B_y + C_y)$$

Step-by-step solution

First, we convert vector $\mathbf{A}$ to rectangular notation.

Step	Reason
1. $A_x = A \cos \theta$	*x* component of vector
2. $A_x = (6.10\text{ N})(\cos 55.0°)$	enter values
3. $A_x = 3.50\text{ N}$	evaluate
4. $A_y = A \sin \theta$	*y* component of vector
5. $A_y = (6.10\text{ N})(\sin 55.0°)$	enter values
6. $A_y = 5.00\text{ N}$	evaluate
7. $\mathbf{A} = (3.50, 5.00)\text{ N}$	combine components

We use steps similar to those above to convert **B** to rectangular notation.

Step	Reason
8. $B_x = (9.00\text{ N})(\cos 274°)$	enter values
9. $B_x = 0.628\text{ N}$	evaluate
10. $B_y = (9.00\text{ N})(\sin 274°)$	enter values
11. $B_y = -8.98\text{ N}$	evaluate
12. $\mathbf{B} = (0.628, -8.98)\text{ N}$	combine components

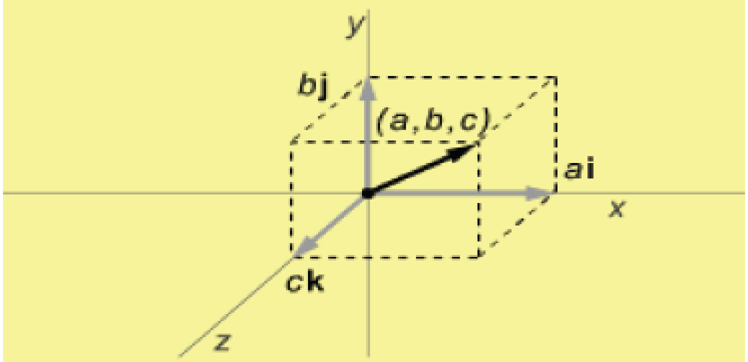
Now we add the *x* components of all three vectors and set the sum equal to zero. This lets us solve for the *x* component of vector **C**.

Step	Reason
13. $\mathbf{A}+\mathbf{B}+\mathbf{C}=\mathbf{0}$	vector sum is zero
14. $A_x+B_x+C_x=0$	sum of the x components is zero
15. $3.50+0.628+C_x=0$	enter values
16. $C_x=-4.13$ N	solve for C_x

Similarly, we find the y component of $\mathbf{C}$.

Step	Reason
17. $A_y+B_y+C_y=0$	sum of the y components is zero
18. $5.00+-8.98+C_y=0$	enter values
19. $C_y=3.98$ N	solve for C_y

equation 1



Unit vectors

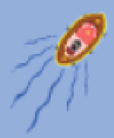
$$(a, b, c)=a\mathbf{i}+b\mathbf{j}+c\mathbf{k}$$

a, b, c = vector components
$\mathbf{i}, \mathbf{j}, \mathbf{k}$ = unit vectors

example 1

$x(t) = 3t + 4$ meters

$y(t) = 4t^2 - 2t + 1$ meters



The *x* and *y* components of the boat's displacement are defined by the two equations shown. Express the displacement at 3.0 seconds as a vector r in unit vector notation.

$x(3.0) = 3(3.0) + 4 = 13$ m

$y(3.0) = 4(3.0)^2 - 2(3.0) + 1 = 31$ m

$\mathbf{r} = (13\mathbf{i} + 31\mathbf{j})$ m

Equations

$$\Delta \mathbf{r} = \mathbf{r}_f - \mathbf{r}_i$$

$$\bar{\mathbf{v}} = \frac{\Delta \mathbf{r}}{\Delta t}$$

$$\bar{\mathbf{a}} = \frac{\Delta \mathbf{v}}{\Delta t}$$

Range equation

$$\Delta x = \frac{-v^2 \sin 2\theta}{a_y}$$

Shooting angle equation

$$\theta = \tfrac{1}{2} \arcsin\left(\frac{-a_y \Delta x}{v^2}\right)$$

Relative velocity equation

Introduction

Objects can speed up, slow down, and change direction while they move. In short, they accelerate. A famous scientist, Sir Isaac Newton, wondered how and why this occurs. Theories about acceleration existed, but Newton did not find them very convincing. His skepticism led him to some of the most important discoveries in physics. Before Newton, people who studied motion noted that the objects they observed on Earth always slowed down. According to their theories, objects possessed an internal property that caused this acceleration. This belief led them to theorize that a force was required to keep things moving.

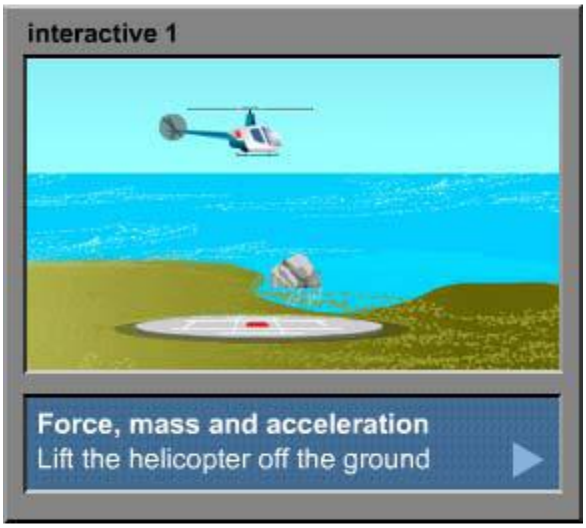
This idea seems like common sense. Moving objects do seem to slow down on their own: a car coasts to a stop, a yo-yo stops spinning, a soccer ball rolls to a halt. Newton, however, rejected this belief, instead suggesting the opposite: The nature of

objects is to continue moving unless some force acts on them. For instance, Newton would say that a soccer ball stops rolling because of forces like friction and air resistance, not because of some property of the soccer ball. He would say that if these forces were **not** present, the ball would roll and roll and roll. A force (a kick) is required to start the ball's motion, and a force such as the frictional force of the grass is required to stop its motion. Newton proposed several fundamental principles that govern forces and motion. Nearly 300 years later, his insights remain the foundation for the study of forces and much of motion. This chapter stands as a testament to a brilliant scientist. At the right, you can use a simulation to experience one of Newton's fundamental principles: his law relating a net force, mass and acceleration. In the simulation, you can attempt some of the basic tasks required of a helicopter pilot. To do so, you control the **net** force upward on the helicopter. When the helicopter is in the air, the net force equals the lift force minus its weight. (The lift force is caused by the interaction of the spinning blades with the air, and is used to propel the helicopter upward.) The net force, like all forces, is measured in newtons (N). When the helicopter is in the air, you can set the net force to positive, negative, or zero values. The net force is negative when the helicopter's lift force is less than its weight. When the helicopter is on the ground, there cannot be a negative net force because the ground opposes the downward force of the helicopter's weight and does not allow the helicopter to sink below the Earth's surface. The simulation starts with the helicopter on the ground and a net force of 0 N. To increase the net force on the helicopter, press the up arrow key (□) on your keyboard; to decrease it, press the down arrow key (□). This net force will continue to be applied until you change it. To start, apply a positive net force to cause the helicopter to rise off the ground. Next, attempt to have the helicopter reach a constant vertical velocity. For an optional challenge, have it hover at a constant height of 15 meters, and finally, attempt to land (not crash) the helicopter. Once in the air, you may find that controlling the craft is a little trickier than you anticipated □ it may act a little skittish. Welcome to (a) the challenge of flying a helicopter and (b) Newton's world. Here are a few hints: Start slowly! Initially, just use small net forces. You can look at the acceleration gauge to see in which direction you are accelerating. Try to keep your acceleration initially between plus or minus 0.25 m/s2. This simulation is designed to help you experiment with the relationship between force and acceleration. If you find that achieving a

constant velocity or otherwise controlling the helicopter is challenging read on! You will gain insights as you do.

1 – Force *Force:* Loosely defined as "pushing" or "pulling."

Your everyday conception of force as pushing or pulling provides a good starting point for explaining what a force is. There are many types of forces. Your initial thoughts may be of forces that require direct contact: pushing a box, hitting a ball, pulling a wagon, and so on. Some forces, however, can act without direct contact. For example, the gravitational force of the Earth pulls on the Moon even though hundreds of thousands of kilometers separate the two bodies. The gravitational force of the Moon, in turn, pulls on the Earth.



observe no net force acting on the cup. However, the nature of observations made in an accelerating reference frame is a topic far removed from this chapter's focus, and this marks the end of our discussion of reference frames in this chapter.

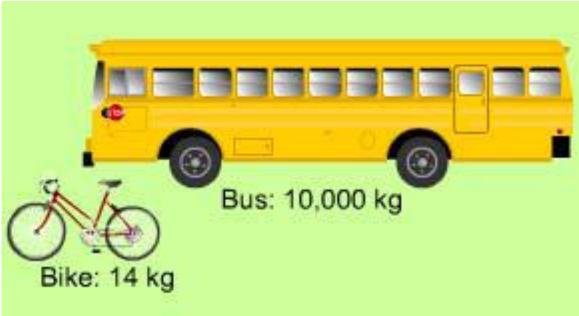
3 – Mass *Mass:* A property of an object that determines how much it will resist a change in velocity.

Newton's second law summarizes the relationship of force, mass and acceleration.

Mass is crucial to understanding the second law because an object's mass determines how much it resists a change in velocity. More massive objects require more net force to accelerate than less massive objects. An object's resistance to a change in velocity is called its *inertial mass*. It requires more force to accelerate the bus on the right at, say, five m/s2 than the much less massive bicycle.

A common error is to confuse mass and weight. Weight is a force caused by gravity and is measured in newtons. Mass is an object's resistance to change in velocity and is

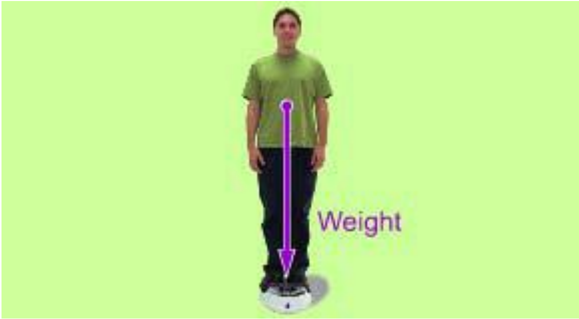
measured in kilograms. An object's weight can vary: Its weight is greater on Jupiter's surface than on Earth's, since Jupiter's surface gravity is stronger than Earth's. In contrast, the object's mass does not change as it moves from planet to planet. The kilogram (kg) is the SI unit of mass.



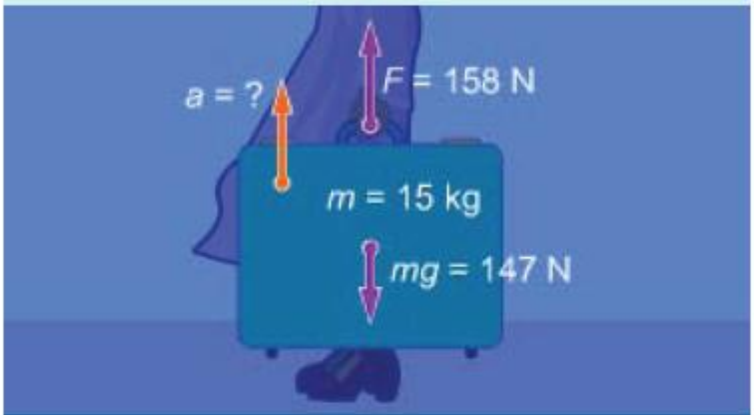
Gravitational force: weight *Weight:* The force of gravity on an object.

We all experience weight, the force of gravity. On Earth, by far the largest component of the gravitational force we experience comes from our own planet. To give you a sense of proportion, the Earth exerts 1600 times more gravitational force on you than does the Sun. As a practical matter, an object's weight on Earth is defined as the gravitational force the Earth exerts on it. Weight is a force; it has both magnitude and direction. At the Earth' surface, the direction of the force is toward the center of the Earth. The magnitude of weight equals the product of an object's mass and the rate of free fall acceleration due to gravity. On Earth, the rate of acceleration g due to gravity is 9.80 m/s2. The rate of free fall acceleration depends on a planet's mass and radius, so it varies from planet to planet. On Jupiter, for instance, gravity exerts more force than on Earth, which makes for a greater value for free fall acceleration. This means you would weigh more on Jupiter's surface than on Earth's.

Scales, such as the one shown in Concept 1, are used to measure the magnitude of weight. The force of Earth's gravity pulls Kevin down and compresses a spring. This scale is calibrated to display the amount of weight in both newtons and pounds, as shown in Equation 1. Forces like weight are measured in pounds in the British system. One newton equals about 0.225 pounds. A quick word of caution: In everyday conversation, people speak of someone who "weighs 100 kilograms," but kilograms are units for mass, not weight. Weight, like any force, is measured in newtons. A person with a mass of 100 kg weighs 980 newtons.



example 1



What is the suitcase's acceleration?

$\Sigma \mathbf{F} = m\mathbf{a}$

$F + (-mg) = ma$

$a = (F - mg)/m$

$a = (158\ \text{N} - 147\ \text{N})/(15\ \text{kg})$

$a = 0.73\ \text{m/s}^2$ (upward)

Above you see "Rocket Guy," a superhero who wears a jet pack. The jet pack provides an upward force on him, while Rocket Guy's weight points downward.

Variables

All the forces on Rocket Guy are directed along the y axis.

thrust	$F_T = 1250$ N
weight	$-mg = -905$ N
mass	m
acceleration	a

What is the strategy?

1. Determine the net force on Rocket Guy.
2. Determine Rocket Guy's mass.
3. Use Newton's second law to find his acceleration.

Physics principles and equations

Newton's second law

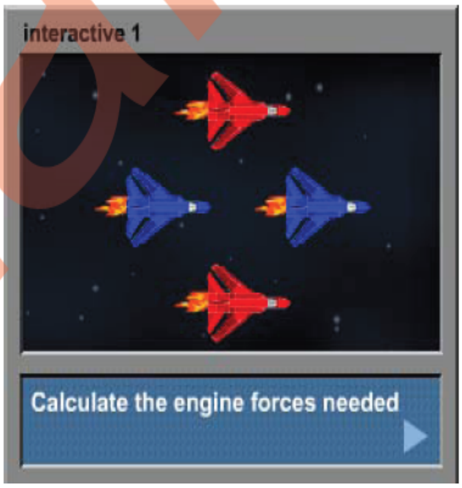
$$\Sigma \mathbf{F} = m\mathbf{a}$$

5.9 - Interactive problem: flying in formation

The simulation on the right will give you some practice with Newton's second law. Initially, all the space ships have the same velocity. Their pilots want all the ships to accelerate at 5.15 m/s^2. The red ships have a mass of 1.27×10^4 kg, and the blue ships, a mass of 1.47×10^4 kg. You need to set the amount of force supplied by the ships' engines so that they accelerate equally. The masses of the ships do not change significantly as they burn fuel.

Apply Newton's second law to calculate the engine forces needed. The simulation uses scientific notation; you need to enter three-digit leading values. Enter your values and press GO to start the simulation. If all the ships accelerate at 5.15 m/s^2, you have succeeded. Press RESET to try again.

If you have difficulty solving this problem, review Newton's second law.



concept 1

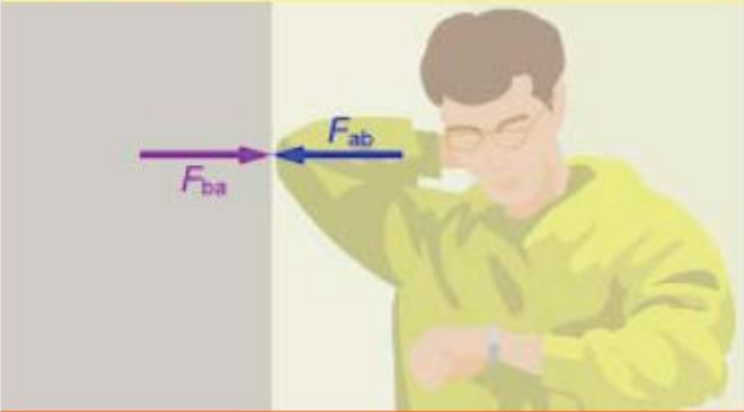
Newton's third law

Forces come in pairs

Equal in strength, opposite in direction

The forces act on different objects

equation 1



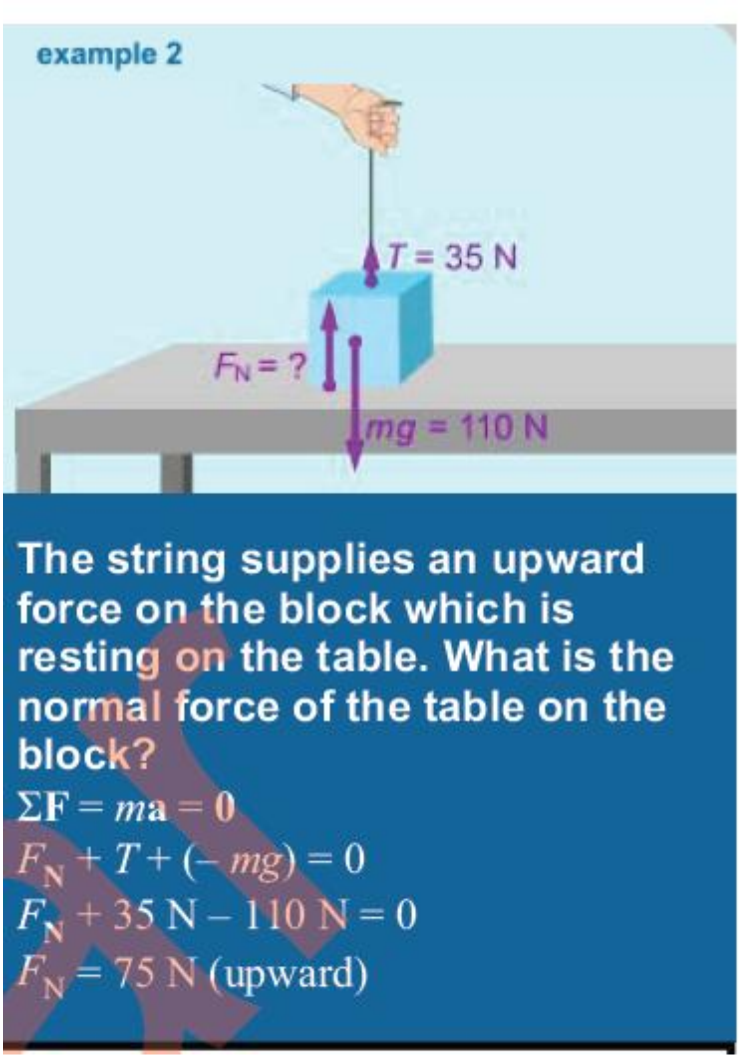
Newton's third law

$$\mathbf{F}_{ab} = -\mathbf{F}_{ba}$$

Newton's third law: "To any action there is always an opposite and equal reaction; in other words, the actions of two bodies upon each other are always equal and always opposite in direction."

Newton's third law states that forces come in pairs and that those forces are equal in magnitude and opposite in direction. When one object exerts a force on another, the second object exerts a force equal in magnitude but opposite in direction on the first.

For instance, if you push a button, it pushes back on you with the same amount of force. When someone leans on a wall, it pushes back, as shown in the illustration above.



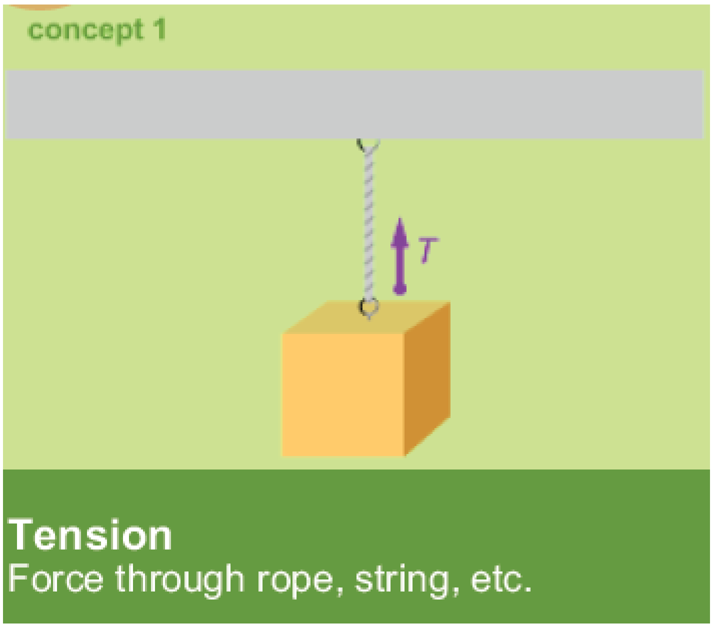
Tension: Force exerted by a string, cord, twine, rope, chain, cable, etc.

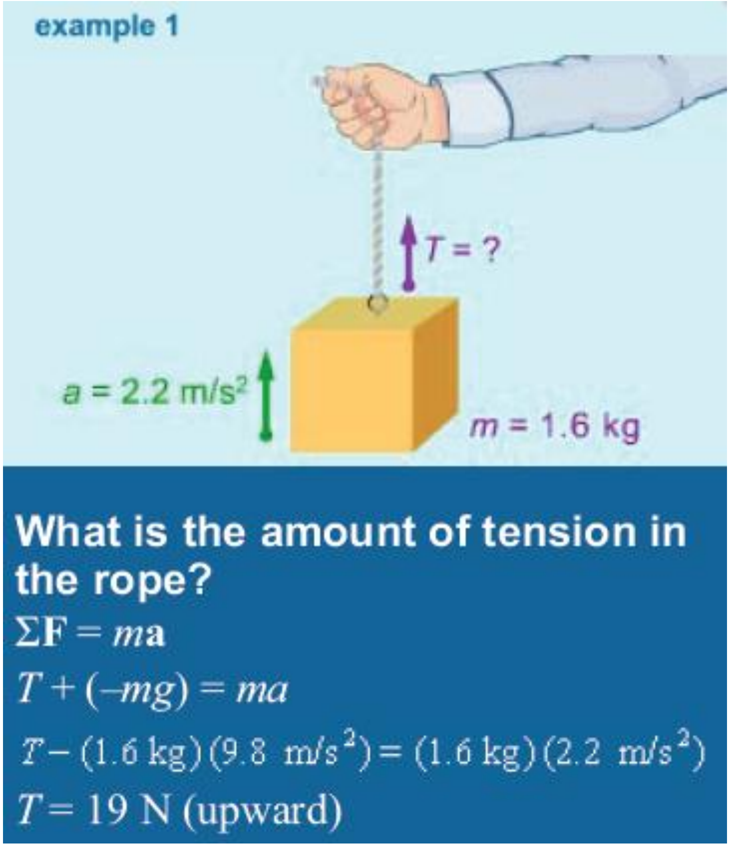
In physics textbooks, tension means the pulling force conveyed by a string, rope, chain, tow-bar, or other form of connection. In this section, we will use a rope to illustrate the concept of tension.

The rope in Concept 1 is shown exerting a force on the block; that force is called tension. This definition differs slightly from the everyday use of the word tension, which often refers to forces within a material or object − or a human brain before exams.

In physics problems, two assumptions are usually made about the nature of tension. First, the force is transmitted unchanged by the rope. The rope does not stretch or otherwise diminish the force. Second, the rope is treated as having no mass (it is massless). This means that when calculating the acceleration of a system, the mass of the rope can be ignored.

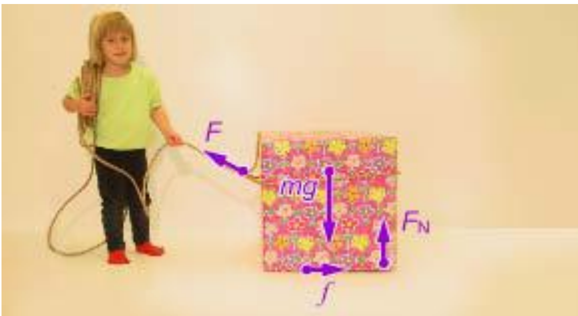
Example 1 shows how tension forces can be calculated using Newton's second law. There are two forces acting on the block: its weight and the tension. The vector sum of those forces, the net force, equals the product of its mass and acceleration. Since the mass and acceleration are stated, the problem solution shows how the tension can be determined.



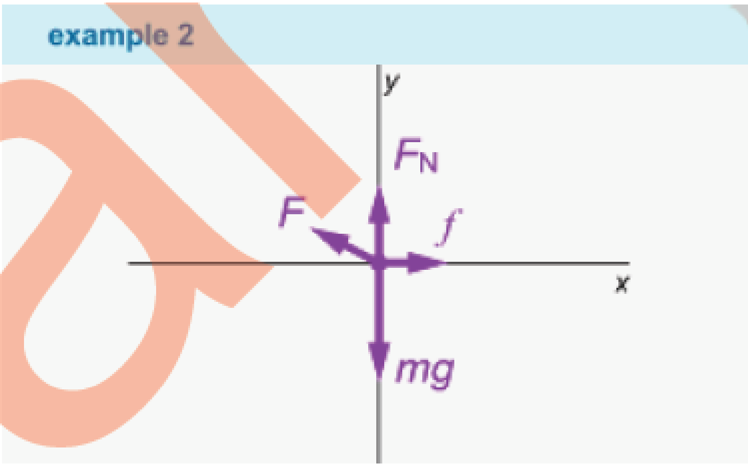


Newton's second and third laws

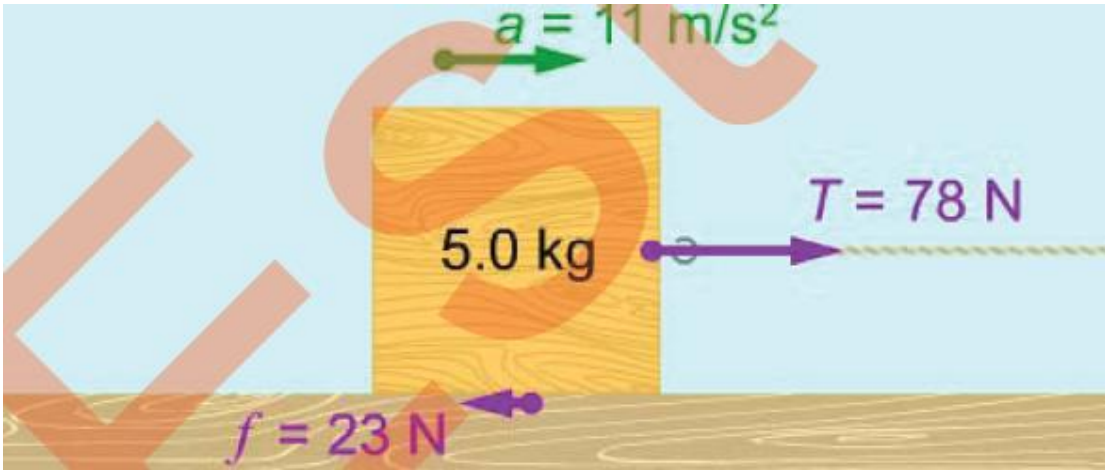
It might seem that Newton’s third law could lead to the conclusion that forces do **not** cause acceleration, because for every force there is an equal but opposite force. If for every force there is an equal but opposite force, how can there be a net non-zero force? The answer lies in the fact that the forces do not act on the same object. The pair of forces in an action-reaction pair acts on **different** objects. In this section, we illustrate this often confusing concept with an example. balance as well. By considering the forces acting in both the horizontal and vertical directions, the tensions of the ropes can be determined. In Example 1, one of the forces shown is friction, *f*. Friction acts to oppose motion when two objects are in contact.



Draw a free-body diagram of the forces on the box.

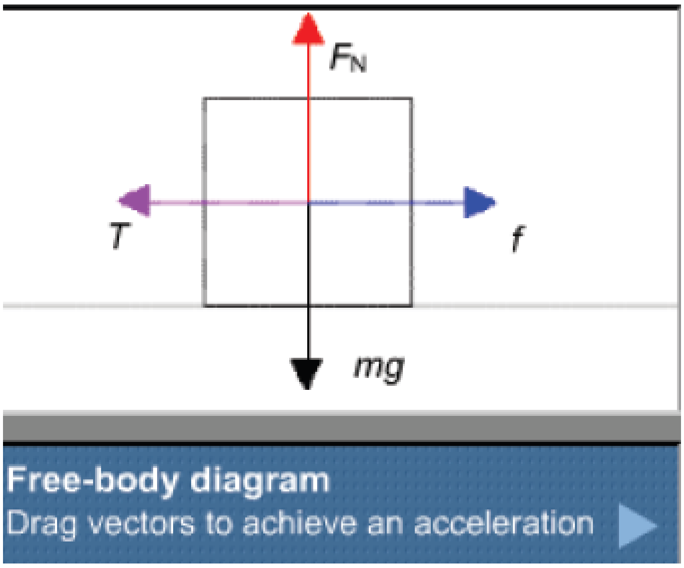


Free-body diagram of forces on box



A rope pulls the block against friction. Draw a free-body diagram. The block accelerates at 11 m/s^2 if the diagram is correct.

In this section, you practice drawing a free-body diagram. Above, you see the situation: A block is being pulled horizontally by a rope. It accelerates to the right at 11 m/s^2. In the simulation on the right, the force vectors on the block are drawn, but each one points in the wrong direction, has the wrong magnitude, or both. We ignore the force of air resistance in this simulation.



Your job is to fix the force vectors. You do this by clicking on the heads of the vectors and dragging them to point in the correct direction. (To simplify your work, they "snap" to vertical and horizontal orientations, but you do need to drag them close before they will snap.) You change both their lengths (which determine their magnitudes) and their directions with the mouse.

The mass of the block is 5.0 kg. The tension force *T* is 78 N and the force of friction *f* is 23 N. The friction force acts opposite to the direction of the motion. Calculate the magnitudes of the weight *mg* and the normal force *F*N to the nearest newton, and then drag the heads of the vectors to the correct positions, or click on the up and down arrow buttons, and press GO. If you are correct, the block will accelerate to the right at 11 m/s2. If not, the block will move based on the net force as determined by your vectors as well as its mass. Press RESET to try again. There is more than one way to arrange the vectors to create the same acceleration, but there is only one arrangement that agrees with all the information given. If you have difficulty solving this problem, review the sections on weight and normal force, and the section on free-body diagrams

Step-by-step solution

As noted, we use the convention that forces to the right are positive and those to the left are negative. A more rigorous approach would be to calculate the vector

components of these forces using the cosine of 0° for the frictional force and the cosine of 180° for the pushing force. The result would be *x* components of 18.0 N and □34.0 N (the same conclusion we reached via inspection and convention). Many instructors prefer this approach. It does not change the answer to the problem, but the component method is more rigorous, and is required to solve more difficult problems.

Step	Reason
1. $\Sigma\mathbf{F} = \mathbf{F}_{push} + \mathbf{F}_{friction}$	net horizontal force
2. $\Sigma F = -34.0\text{ N} + 18.0\text{ N}$ $\Sigma F = -16.0\text{ N}$	enter values and add
3. $\Sigma F = ma$	Newton's second law
4. $a = \Sigma F/m$	solve for *a*
5. $a = (-16.0\text{ N}) / (15.0\text{ kg})$	enter values
6. $a = -1.07\text{ m/s}^2$	division

The helicopter on the right is being used as a scale, making it one of the more expensive scales in the world, we suspect. This simulation includes three crates; each has a slightly different mass. Your assignment is to find the crate with a mass of 661 kg. Do this by lifting each crate with the helicopter and noting the acceleration. The helicopter lifts each crate with a force of 10,748 N via the tension in the cable. The resulting acceleration of each crate will let you calculate its mass.

Click on the graphic to start the simulation. To determine the answer, drag the helicopter to each of the three crates and press GO to make the helicopter lift the crate. Record the acceleration of each crate and use the acceleration to calculate the mass. When you have found the crate with a mass of 661 kg, select it by clicking on it. The simulation will tell you whether you clicked on the correct one.

If you cannot solve the problem, review Newton's second law and the section on weight.

Friction: A force that resists the motion of one object sliding past another.

Friction between the buffalo's back and the tree scratches an itch.

If you push a cardboard box along a wooden floor, you have to push to overcome the force of friction. This force makes it harder for you to slide the box. The force of friction opposes any force that can cause one object to slide past another. There are two types of friction: static and kinetic. These forces are discussed in more depth in other sections. In this section, we discuss some general properties of friction

The amount of friction depends on the materials in contact. For example, the box would slide more easily over ice than wood. Friction is also proportional to the normal force. For a box on the floor, the greater its weight, the greater the normal force, which increases the force of friction. Humans expend many resources to combat friction. Motor oil, Teflon™, WD-40™, Tri-Flo™ and many other products are designed to reduce this force. However, friction can be very useful. Without it, a nail would slip out of a board, the tires of a car would not be able to "grip" the road, and you would not be able to walk. Friction exists even between seemingly smooth surfaces. Although a surface may appear smooth, when magnified sufficiently, any surface will look bumpy or rough, as the illustration in Concept 2 on the right shows. The magnified picture of the "smooth" crystal reveals its microscopic "rough" texture. Friction is a force caused by the interaction of molecules in two surfaces. maximum amount of static friction is constant. Why? With the greater contact area, the normal and frictional forces per unit area diminish proportionally.

Static friction

$$f_{s,max} = \mu_s F_N$$

$f_{s,max}$ = maximum static friction
μ_s = coefficient of static friction
F_N = normal force

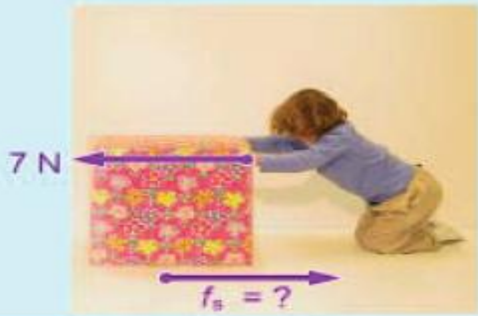
equation 2

Coefficient of static friction

Tires on dry pavement	0.90
Tires on wet pavement	0.42
Glass on glass	0.94
Steel on steel	0.78
Oak on oak	0.54
Waxed ski on dry snow	0.04
Teflon™ on Teflon™	0.04

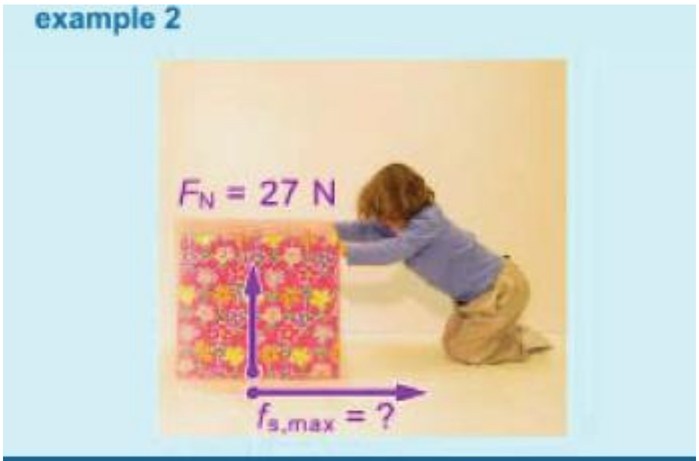
Coefficients of static friction

example 1

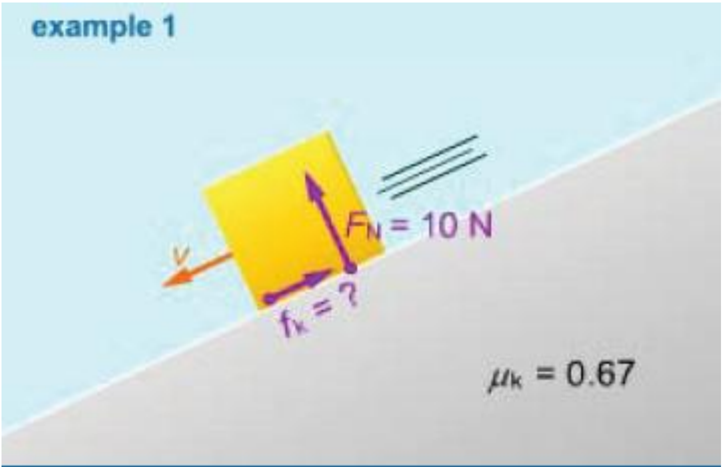


Anna is pushing but the box does not move. What is the force of static friction?

f_s = 7 N to the right



What is the maximum static

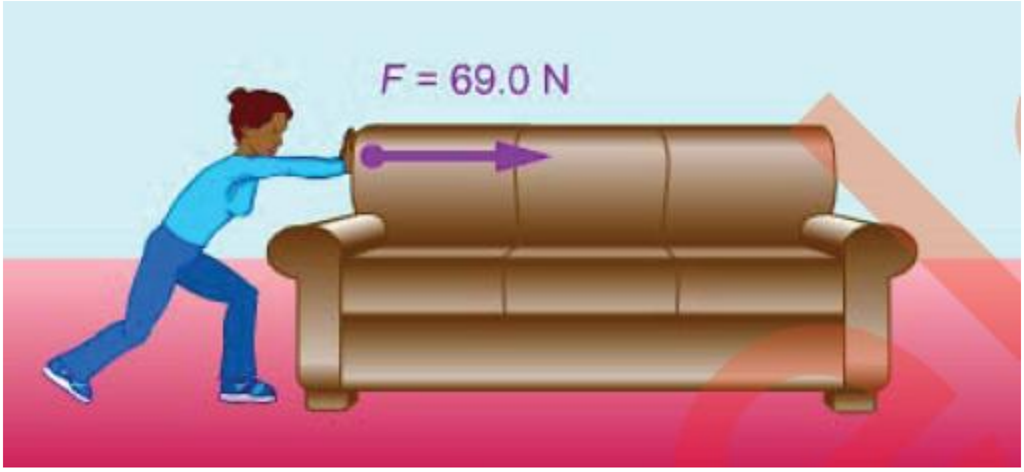


What is the force of friction?

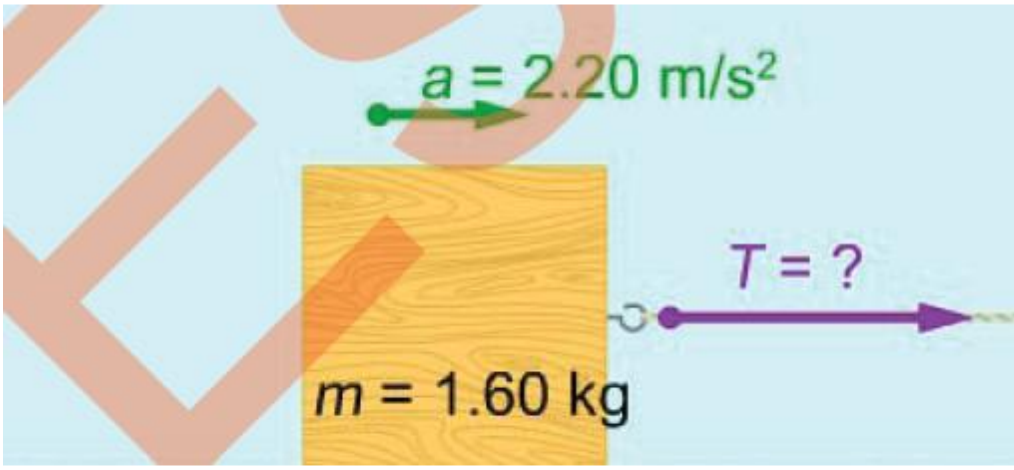
$f_k = \mu_k F_N$

$f_k = (0.67)(10\ \text{N})$

$f_k = 6.7$ N (pointing up the ramp)



While rearranging your living room, you push your couch across the floor at a constant speed with a horizontal force of 69.0 N. You are using specia pads on the couch legs that help it slide easier. If the couch has a mass of 59.5 kg, what is the coefficient of kinetic friction between the pads and the floor?

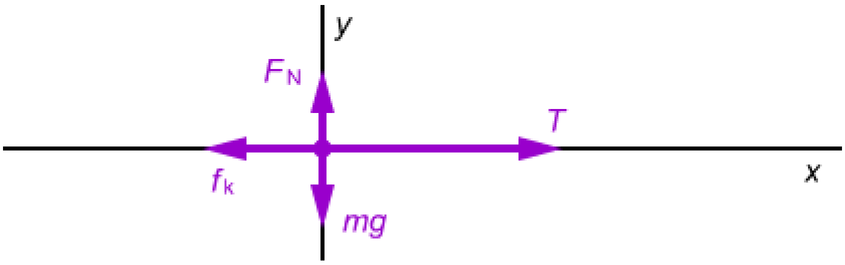


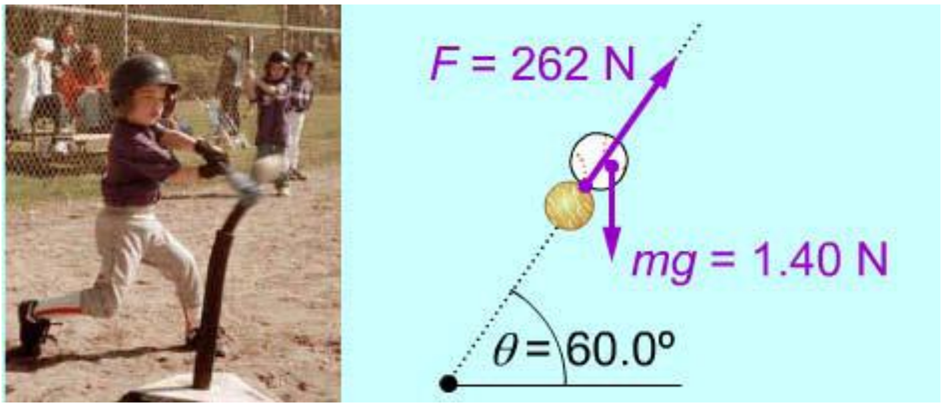
The coefficient of kinetic friction is 0.200. What is the magnitude of the tension force in the rope?

Above, you see a block accelerating to the right due to the tension force applied by a rope. What is the magnitude of tension the rope applies to the block?

Starting this type of problem with a free-body diagram usually proves helpful.

Draw a free-body diagram

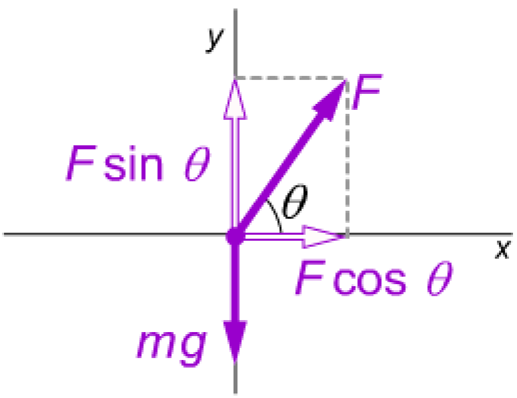




What is the magnitude of the net force on the ball along each axis, and what is the ball's acceleration along each axis?

Above, you see a bat hitting a ball at an angle. You are asked to find the net force and the acceleration of the ball along the x and y axes.

Draw a free-body diagram



The forces on the ball are its weight down and the force of the bat at the angle □ to the *x* axis.

Variables

	x component	y component
weight	0	$mg \sin 270° = -1.40$ N
force	$F \cos \theta$	$F \sin \theta$
acceleration	a_x	a_y
force	$F = 262$ N	
angle	$\theta = 60.0°$	
mass	$m = mg/g = (1.40 \text{ N}) / (9.80 \text{ m/s}^2) = 0.143$ kg	

What is the strategy?

1. Draw a free-body diagram.
2. Use trigonometry to calculate the net force on the ball along each axis.
3. Use Newton's second law to find the acceleration of the ball along each axis. The mass of the ball is not given, but you can determine it because you are told its weight. We do this in the variables table.

Physics principles and equations

Newton's second law

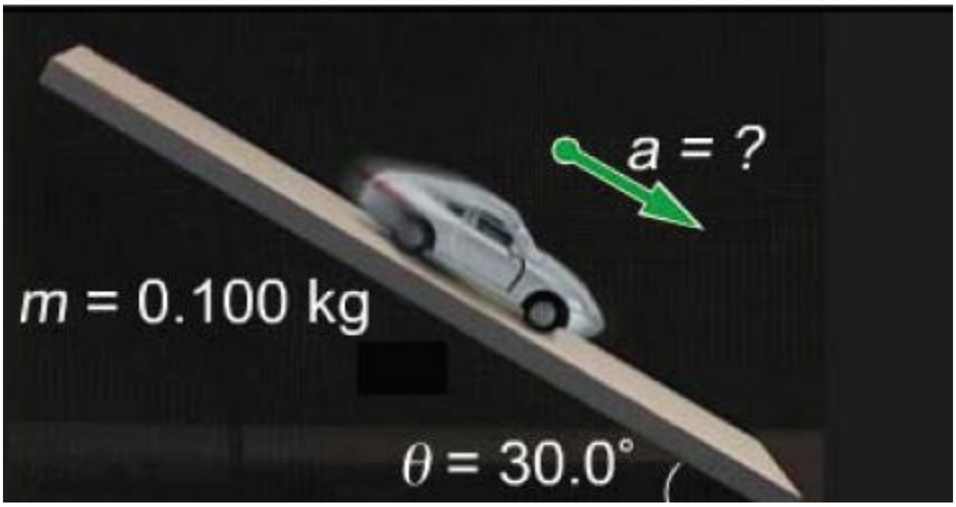
$$\Sigma \mathbf{F} = m\mathbf{a}$$

Step-by-step solution

We begin by calculating the net force along the *x* axis.

Step	Reason
1. $\Sigma F_x = F\cos\theta$	net force along x axis
2. $\Sigma F_x = (262\text{ N})(\cos 60.0°)$	x component of force
3. $\Sigma F_x = 131\text{ N}$	evaluate

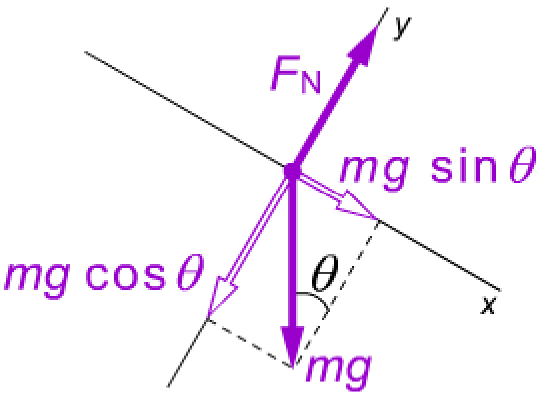
Sample problem: moving down a frictionless plane

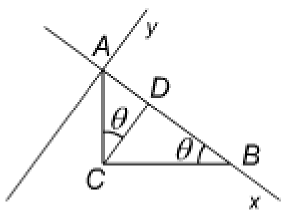


What is the car's acceleration?

Above, you see a toy car going down an inclined plane. The diagram shows the mass of the car and the angle the plane makes with the horizontal. You are asked to calculate the car’s acceleration. In this problem, ignore any friction or air resistance, as well as any energy consumed by the rotation of the wheels.

Draw a free-body diagram





By rotating the axes so that the *x* axis is parallel to the car's motion down the ramp, we make the forces along the *y* axis sum to zero. (These two forces are the *y* component of the car's weight and the normal force from the ramp.) Rotating the axes means there is a net force only along the *x* axis, and this reduces the steps required to solve the problem. It may be a little difficult to see why □, the angle that the plane makes with the horizontal, is the same as the angle □ in the free-body diagram. The drawing to the right of the free-body diagram uses two similar right triangles to show why this is true. The triangle ABC has one leg (AC) that is the weight vector, and its hypotenuse (AB) lies along the *x* axis. The hypotenuse of the smaller triangle ACD is the weight vector. These are both right triangles and share a common angle at A, so they are similar. It is often useful to check this angle with the situation shown. At a 30° angle, the *y* component of the weight is larger than the *x* component (the cosine of 30° is greater than the sine of 30°). Looking at the picture above, this is what you would expect. The component of the weight down the plane is less than the component on the plane. It may help to push it to the extreme: What would you expect at a 0° angle? At 90°?

Variables

With the axes rotated and □ as shown, the *x* component of the weight is computed using the sine, and the *y* component with the cosine. (Without the rotation, the *x* component would be calculated with the cosine, and the *y* component with the sine.)

	x component	*y* component
weight	$mg \sin \theta$	$-mg \cos \theta$
normal force	0 N	F_N
acceleration	a	0 m/s^2
mass	$m = 0.100$ kg	
angle	$\theta = 30.0°$	

What is the strategy?

1. Draw a free-body diagram, rotating the axes so the *x* axis is parallel to the motion of the car.

2. Use trigonometry to calculate the net force on the car.

3. Use Newton's second law to determine the acceleration of the car.

Physics principles and equations

Newton's second law

□**F** = *m***a**

What is the strategy?

1. Draw a free-body diagram.

2. Calculate the net force on the ball along each axis by finding the components of the two forces using trigonometry.

3. Use Newton's second law to find the acceleration of the ball along each axis.

Physics principles and equations

Newton's second law

□**F** = *m***a**

Step-by-step solution

We begin by calculating the net force along the *x* axis.

Step	Reason
1. $\Sigma F_x = F_1 \cos\theta_1 + F_2 \cos\theta_2$	net force along *x* axis
2. $\Sigma F_x = 162\cos(171°)\text{N} + 215\cos(285°)\text{N}$	enter values
3. $\Sigma F_x = -104\ \text{N}$	evaluate

Next we calculate the net force along the *y* axis.

Step	Reason
4. $\Sigma F_y = F_1 \sin\theta_1 + F_2 \sin\theta_2$	net force along *y* axis
5. $\Sigma F_y = 162\sin(171°)\text{N} + 215\sin(285°)\text{N}$	enter values
6. $\Sigma F_y = -182\ \text{N}$	evaluate

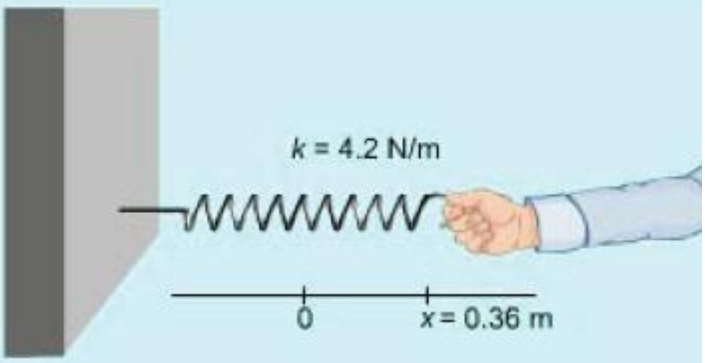
Using Newton's second law and the net force in the x dimension, calculated above, we find the acceleration in the x dim

Step	Reason
7. $\Sigma F_x = ma_x$	Newton's second law
8. $a_x = \Sigma F_x / m$	solve for a_x
9. $a_x = (-104\ \text{N}) / (0.420\ \text{kg})$	enter values
10. $a_x = -248\ \text{m/s}^2$	division

And finally we calculate the acceleration in the *y* dimension.

Step	Reason
11. $\Sigma F_y = ma_y$	Newton's second law
12. $a_y = \Sigma F_y / m$	solve for a_y
13. $a_y = (-182\ \text{N}) / (0.420\ \text{kg})$	enter values
14. $a_y = -433\ \text{m/s}^2$	division

example 1



What is the force exerted by the spring?

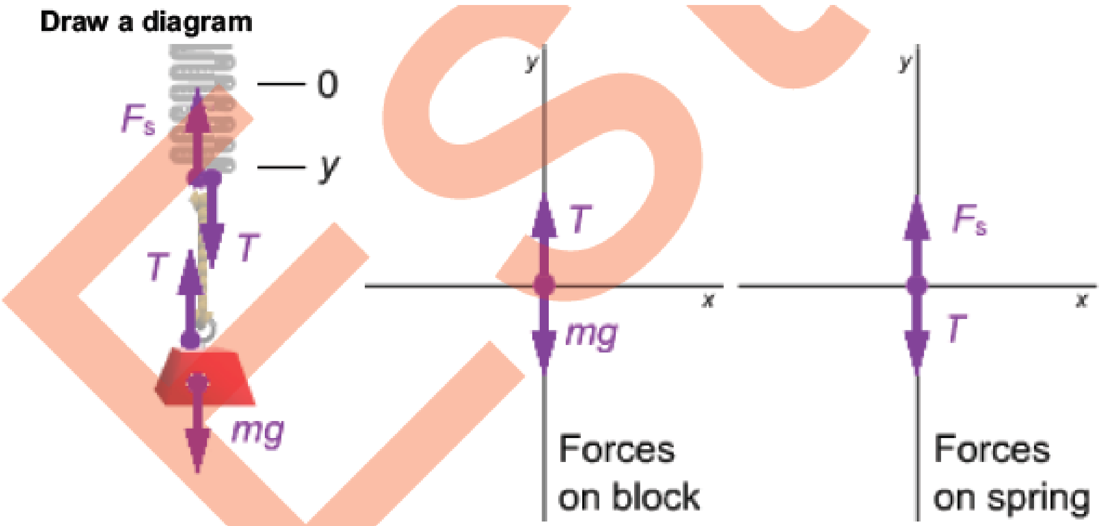
$F_s = -kx$

$F_s = -(4.2\ \text{N/m})(0.36\ \text{m})$

$F_s = -1.5\ \text{N}$ (to the left)

What is the amount of tension in the rope? What is the position of the end of the spring away from its rest position?

You see a block hanging from a rope attached to a spring. The block is stationary. You are asked to determine the tension in the rope and the position of the end of the spring relative to its rest point



Above on the left, we draw the forces: the weight of the block, the tension forces in the rope, and the spring force. The tension forces are equal in magnitude, so we use the same variable T for each of them. Then we draw two free-body diagrams, one for the block and one for the lower end of the spring.

Variables

weight	$-mg = -98.0\ \text{N}$
tension	T
spring force	F_s
spring constant	$k = 535\ \text{N/m}$
displacement	y

What is the strategy?

1. Draw a free-body diagram.

2. Apply Newton's second law to calculate the tension force in the rope.

3. Then apply Newton's second law with Hooke's law to find the position of the end of the spring.

Research has actually determined that cats reach terminal velocity after falling six stories. In fact, they tend to slow down after six stories. Here's why this occurs: The cat achieves terminal velocity and then relaxes a little, which expands its cross sectional area and increases its drag force. As a result, it slows down. One has to admire the cat for relaxing in such a precarious situation (or perhaps doubt its intelligence). If you think this may be an urban legend, consult the *Journal of the American Veterinary Association*, volume 191, page 1399.

Air resistance

$$F_D = \tfrac{1}{2} C \rho A v^2$$

F_D = drag force
C = drag coefficient for object
ρ = air density
A = cross-sectional area
v = velocity

Terminal velocity

$$v_T = \sqrt{\frac{2mg}{C\rho A}}$$

v_T = terminal velocity
mg = weight

Introduction

Now, you will get some additional practice applying Newton's laws. More specifically, you will use them in situations where multiple forces are acting on a single object. If the application of multiple forces results in a net force acting on an object, it accelerates. On the other hand, if the forces acting on it sum to zero in every dimension, the result is equilibrium. The object does not accelerate; it either maintains a constant velocity, or remains stationary. (Forces can also cause an object to rotate,

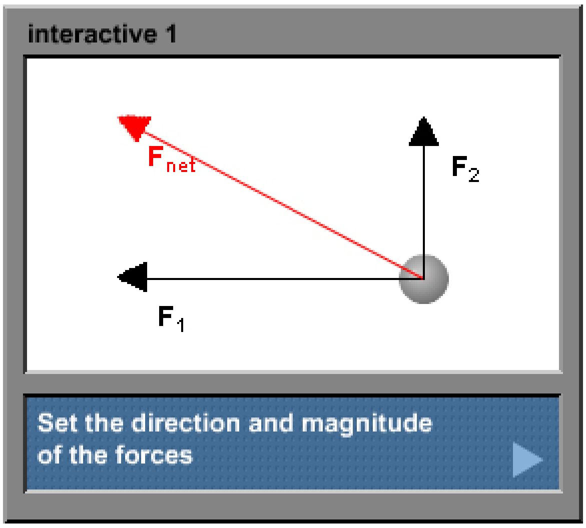
but rotational motion is a later topic in mechanics.) Equilibrium is an important topic in engineering. The school buildings you study in, the bridges you travel across all such structures require careful design to ensure that they remain in equilibrium. The simulation on the right will help you develop an understanding for how forces in different directions combine when applied to an object. The 5.0 kg ball has two forces acting on it, **F**1 and **F**2. They act on it as long as the ball is on the screen. You control the direction and magnitude of each force. In the simulation, you set a force vector's direction and magnitude by dragging its arrowhead; You will notice the angles are restricted to multiples of 90°. You can also adjust the magnitude of each vector with a controller in the control panel. The net force is shown in the simulation; it is the vector sum of **F**1 and **F**2.

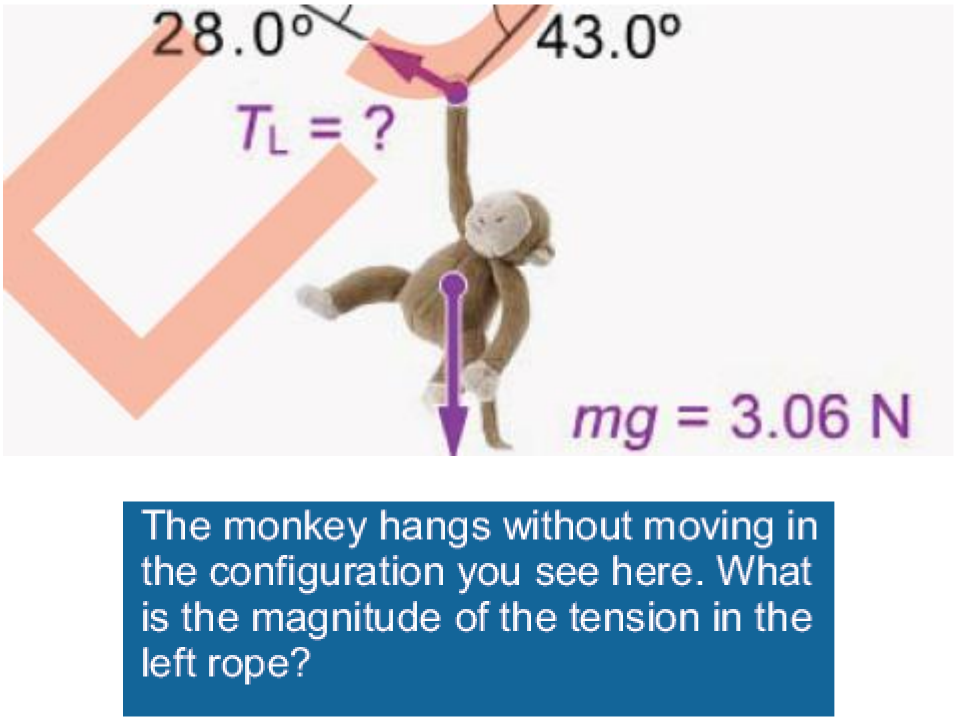
You can check the box "Display vectors head to tail" if you would like to see them graphically combined in that fashion. Press GO to start the simulation and set the ball moving in response to the forces on it. Here are some challenges for you. First, set the forces so that the ball does not move at all when you press GO. The individual forces must be at least 10 newtons, so setting them both to zero is not an option! Next, hit each of the three animated targets. The center of one is directly to the right of the ball and the center of another is at a 45° angle above the horizontal from the ball. Set the individual vectors and press GO to hit the center of each target in turn. The target to the left is at a 150° angle. It is the "extra credit" target. Determining the correct ratio of vectors will require a little thought. We allow for rounding with this target; if you set one of the vectors to 10 N, you can solve the problem by setting the other one to the appropriate closest integer value.

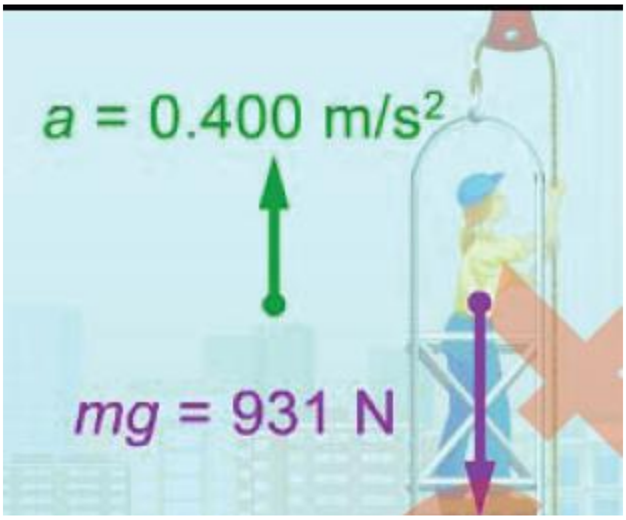
Sample problem: a mass on ropes

Since it is stationary, the monkey is not accelerating, which means no net force is acting on it. This section shows you a useful technique for solving problems that involve multiple forces acting on a single object. To calculate the overall force on an object like the rope, the *x* and *y* components of each force need to be determined. Since there is no acceleration in any direction, there is no net force along any dimension. Two equations can be developed: The sum of the *x* components equals zero, and the sum of the *y* components equals zero. We will use a consistent approach to solving multi-force problems. First we draw a free-body diagram to help us identify the variables and the force components. Then we state the variables and their values when they are known. Next, we use Newton's second law, relating the net force to the

acceleration and the mass of objects in the problem. Finally, we perform the algebraic and mathematical steps needed to solve the problem.







What is the magnitude of the tension in the rope above the window washer's hands?

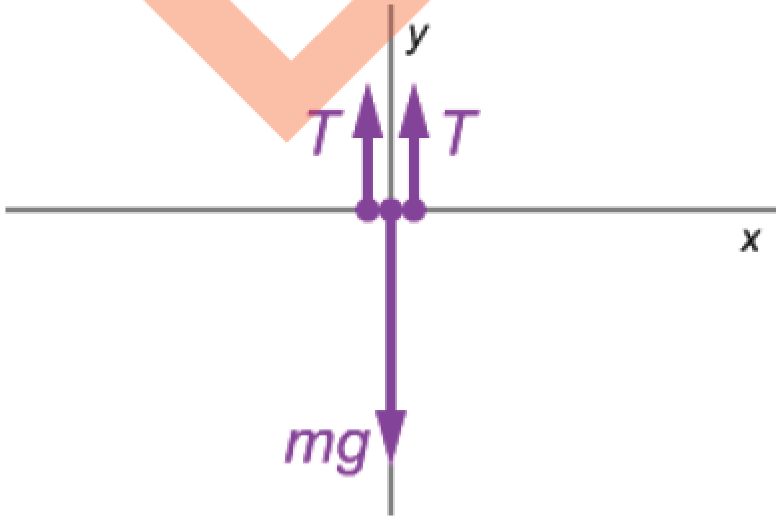
If you have ever worked in a skyscraper, you have probably seen window washers raising and lowering themselves and their scaffolding. Let's say a particularly energetic window washer is accelerating herself upward, as shown above. You are asked to find the amount of tension in one of the two ropes.

Ropes and pulleys are fairly common in mechanics problems. We will always assume that tension is transmitted with a change in direction but no change in magnitude in a rope that goes around a pulley. You may often be told to draw these conclusions when a problem states that the pulley is assumed to be frictionless and massless, or the rope is massless and does not stretch.

The drawing above shows the weight of the combination of the scaffolding and the window washer, which is 931 N. To apply Newton's second law, we need to compute the mass of the system (window washer plus scaffolding) from its weight.

Draw a free-body diagram

Draw a free-body diagram



Variables

weight	$-mg = -931\ \text{N}$
tension in left rope	T
tension in right rope	T
acceleration	$a = 0.400\ \text{m/s}^2$
mass	$m = 931\ \text{N} / 9.80\ \text{m/s}^2 = 95.0\ \text{kg}$

Variables

Block A on table	x component	y component
tension	T	0 N
weight	0 N	$-m_A g$
normal force	0 N	F_N
acceleration	a	0 m/s
mass	$m_A = 4.20$ kg	
Falling block B		
tension	0 N	T
weight	0 N	$-m_B g$
acceleration	0 m/s	$-a$
mass	$m_B = 5.70$ kg	

What is the strategy?

1. Draw a free-body diagram for each block.

2. Calculate the net force on each block. Block A moves only in the horizontal direction, so we can ignore the vertical forces on it. Block B moves only vertically, and there are no horizontal forces on it.

3. Use Newton's second law for each block to find two expressions for the tension force in the rope, and set the expressions equal to find the acceleration.

Physics principles and equations

Newton's second law relates the net force and acceleration. Block A moves only horizontally, so we will consider only the *x* direction for it; similarly, we consider only the *y* direction for block B.

$\square\mathbf{F} = m\mathbf{a}$

Step-by-step solution

We start by considering block A, and use Newton's second law to find an equation that gives an expression for the tension force in the rope.

Step	Reason
1. $\Sigma \mathbf{F}_x = m_A \mathbf{a}_x$	Newton's second law applied to A
2. $T = m_A a$	tension is net force on A

Then we apply Newton's second law to block B to find another expression for the tension. Since block B falls, we assign its acceleration a negative value.

Step	Reason
3. $\Sigma \mathbf{F}_y = m_B \mathbf{a}_y$	Newton's second law applied to B
4. $T + (-m_B g) = m_B(-a)$	net force is sum of tension and weight
5. $T = m_B g - m_B a$	solve for T

We set the two expressions for the tension equal. The rest is algebra.

Step	Reason
6. $m_A a = m_B g - m_B a$	set tensions equal, from 2 and 5
7. $a = (m_B g)/(m_A + m_B)$	solve for a
8. $a = \dfrac{(5.70\ \text{kg})(9.80\ \text{m/s}^2)}{4.20\ \text{kg} + 5.70\ \text{kg}}$	enter values
9. $a = 5.64\ \text{m/s}^2$	evaluate

The steps above determine the magnitude of the acceleration. Since the question asked for the acceleration of the block on the table, the full answer is 5.64 m/s2 to the right.

Variables

	x component	y component
weight	$-mg \sin \theta$	$-mg \cos \theta$
thrust	F_T	0 N
lift	0 N	F_L
air resistance	F_R	0 N
weight	$mg = 2.60 \times 10^5 \text{ N}$	
thrust	$F_T = 3.00 \times 10^5 \text{ N}$	
flight angle	$\theta = 60.0°$	

What is the strategy?

1. Draw a free-body diagram of the forces on the plane, rotating the axes so three of the forces lie along an axis.

2. Calculate the net force on the plane along the axis of the lift force, and solve for the lift force.

Physics principles and equations

Newton’s second law

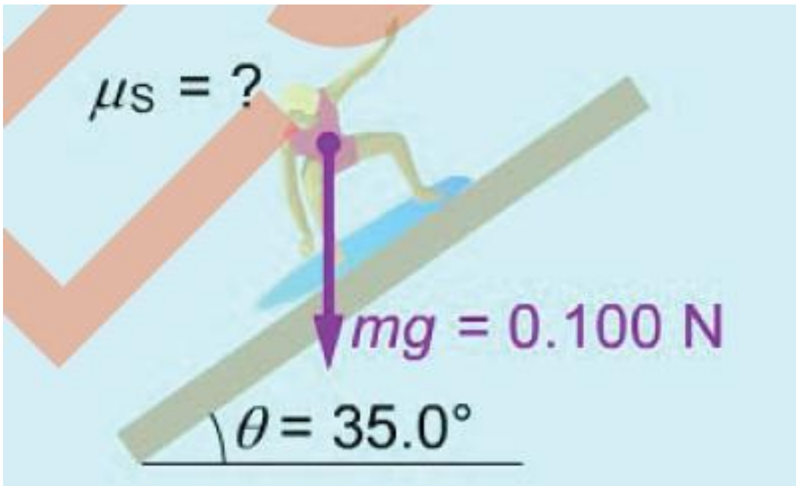
□$\mathbf{F} = m\mathbf{a}$

When the acceleration is zero, the forces along every dimension must sum to zero.

Step-by-step solution

The lift force acts in the *y* dimension. In the *y* column of the variables table, all the values are known except the lift force. So, we need only apply equilibrium in the *y* dimension to solve this problem.

Step		Reason
1.	$\Sigma \mathbf{F}_y = 0$	no acceleration in y dimension
2.	$F_L + (-mg \cos \theta) = 0$	lift and y component of weight
3.	$F_L = (2.60 \times 10^5 \text{ N}) \cos 60.0°$	rearrange and enter values
4.	$F_L = 1.30 \times 10^5 \text{ N}$	evaluate



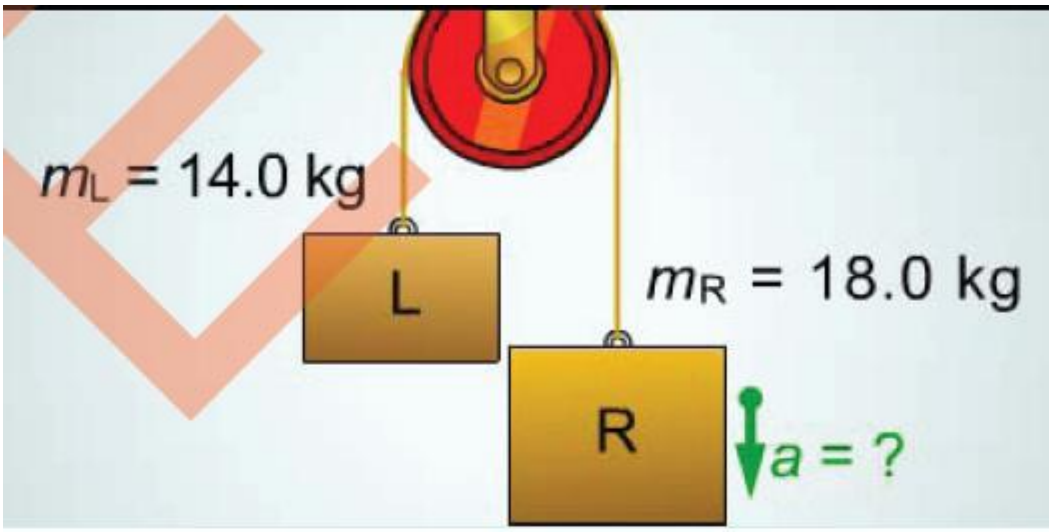
The Surfer Bob toy is just about to slide. What is the coefficient of static friction?

A classic physics lab exercise asks you to use a block on a plane to calculate the coefficient of static friction for two materials. You see that configuration shown above, although instead of a block, we are using Surfer Bob. You are given Bob's weight and the angle the plane makes with the horizontal just before Bob begins to slide. From this information, you are asked to calculate the coefficient of static friction. Since Bob is not accelerating, no net force is acting on him.

You may have performed a lab experiment like this at some point during your studies. You incline a plane until the force of gravity just overcomes friction and causes a block on the plane to slide. You then incline it a little less until the plane is at the angle at which the force of static friction balances the force of gravity down the plane. At this point, the static friction is at its maximum and you can calculate the coefficient. As you see below, we can solve this problem in fewer steps by rotating the axes. Two of the forces are acting along the inclined plane. By rotating the axes so the x axis is parallel to the plane, we can reduce the amount of trigonometry required. The rotation means that two of the forces act solely along an axis. If we did not do the rotation, each force we analyzed would have to be decomposed into its x and y components with the use of sines and cosines in order for us to solve the problem. If you do not like this axis rotation "trick," then you can always solve the problem by keeping the axes horizontal and vertical and using components. Now we substitute the expression for the normal force from step 6 into the equation of step 3 and find the coefficient of static friction.

Step	Reason
7. $\mu_s = (mg \sin \theta) / (mg \cos \theta)$	substitute step 6 into step 3
8. $\mu_s = \sin \theta / \cos \theta$	simplify
9. $\mu_s = \tan \theta$	trigonometric identity
10. $\mu_s = \tan 35.0°$	enter value
11. $\mu_s = 0.700$	evaluate

Even though we were given the weight of the dude, it turns out the coefficient of static friction does not depend on weight, but solely on the angle of the plane.



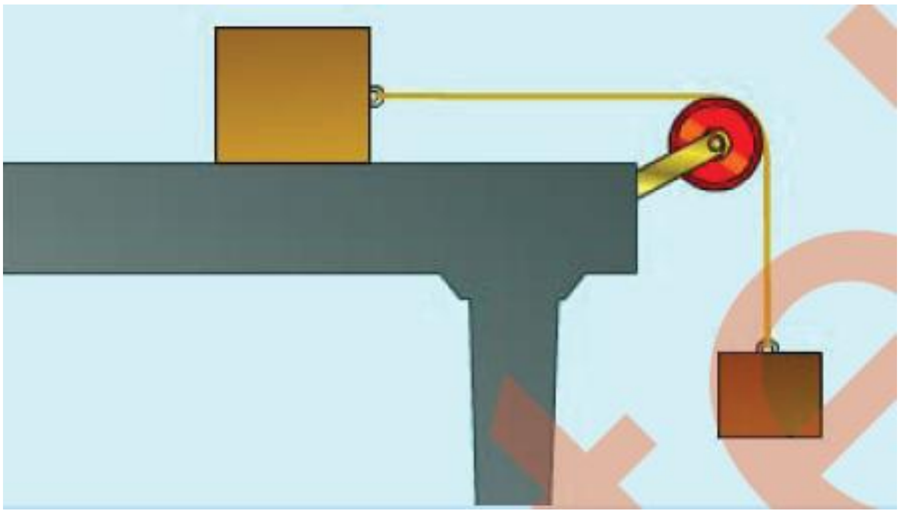
What is the magnitude of acceleration of the blocks?

The illustration above shows two blocks connected by a rope passing over a pulley. Because the blocks partially counterbalance each other, the force required to lift either of them is less than its weight. This type of system is called an *Atwood machine*. An application can be found in elevators, where a massive block partly counterbalances the weight of the elevator car to reduce the force required from the motor that lifts the car. In this sample problem, you are asked to find the rate at which the blocks accelerate. As usual, the rope and pulley are massless, the rope does not stretch, and the pulley has no friction. The rope exerts an equal tension force on both blocks. The blocks' accelerations are equal in magnitude but opposite in direction.

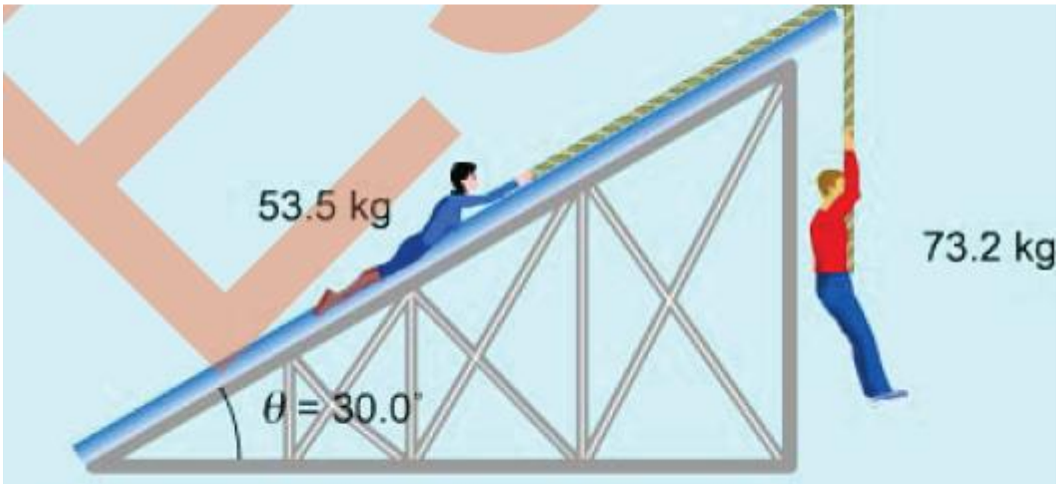
We set the two expressions for tension equal and solve for the acceleration.

Step	Reason
7. $m_L g + m_L a = m_R g - m_R a$	set tensions equal, from steps 3 and 6
8. $a = \dfrac{(m_R - m_L)(g)}{m_R + m_L}$	solve for a
9. $a = \dfrac{(18.0 - 14.0\ \text{kg})(9.80\ \text{m/s}^2)}{18.0\ \text{kg} + 14.0\ \text{kg}}$	enter values
10. $a = 1.23\ \text{m/s}^2$	arithmetic

Equation 8 can be profitably analyzed by considering a couple of special cases. If mL equals zero, equation 8 states that the acceleration equals g. This makes sense: The block on the right would be in free fall, since no force would be opposing the force of its weight. Also, if mR = mL, the acceleration would be zero, since there would be no net force. If we let mR go to infinity, equation 8 states that the acceleration equals g. This too makes sense. If mR is very much bigger than mL, then mL will hardly slow down mR as it falls in free fall.



A 15.8 kg block sits on a frictionless horizontal table. The block is attached to a horizontal string that goes over a pulley and is connected to another block that hangs freely. The string is massless and does not stretch. The acceleration of the block on the table is 3.89 m/s². What is the mass of the hanging block?



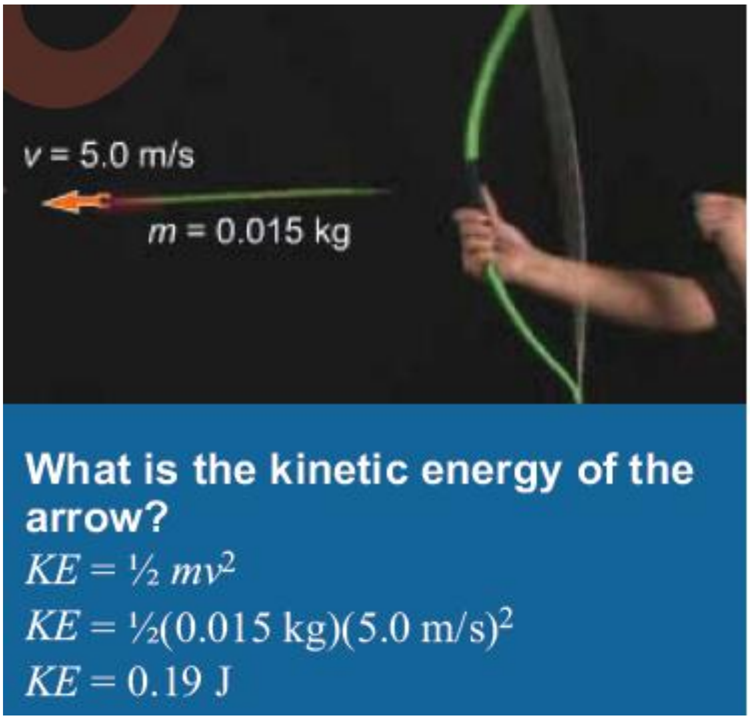
Frances, a 53.5 kg woman, slides on a frictionless, icy ramp that is inclined at 30.0° to the horizontal. An unstretchable rope connects her to Andre, who has a mass of 73.2 kg and is accelerating at the same rate, but parallel to the vertical wall of the ramp. What is the magnitude of their acceleration?

equation 1

Kinetic energy

$$KE = \tfrac{1}{2} mv^2$$

KE = kinetic energy
m = mass
v = speed
Unit: joule (J)



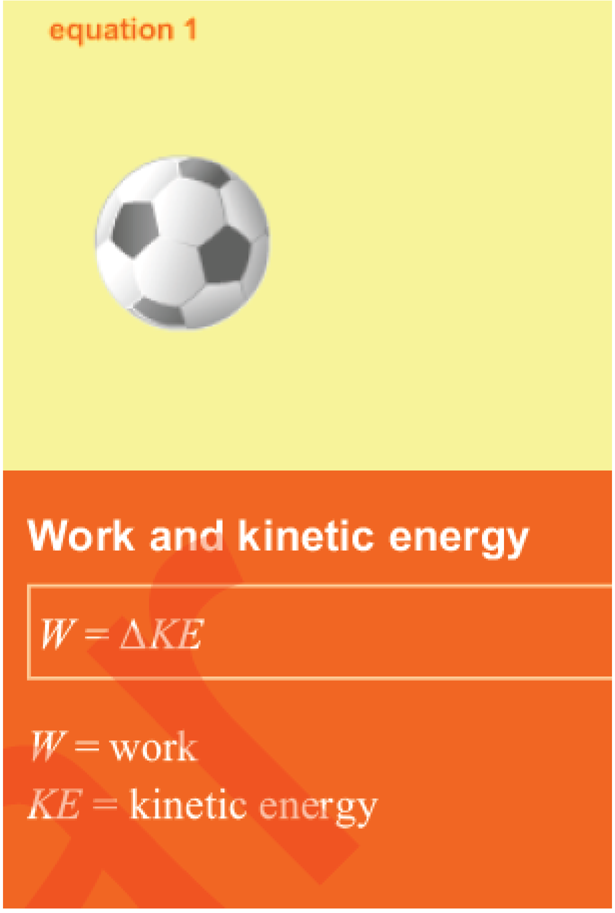
Work-kinetic energy theorem: The net work done on a particle equals its change in kinetic energy.

Consider the foot kicking the soccer ball in Concept 1. We want to relate the work done by the force exerted by the foot on the ball to the ball's change in kinetic energy. To focus solely on the work done by the foot, we ignore other forces acting on the ball, such as friction.

Initially, the ball is stationary. It has zero kinetic energy because it has zero speed. The foot applies a force to the ball as it moves through a short displacement. This force accelerates the ball. The ball now has a speed greater than zero, which means it has kinetic energy. The work-kinetic energy theorem states that the work done by the foot on the ball equals the change in the ball's kinetic energy. In this example, the work is positive (the force is in the direction of the displacement) so the work increases the kinetic energy of the ball. As shown in Concept 2, a goalie catches a ball kicked directly at her. The goalie's hands apply a force to the ball, slowing it. The force on the ball is opposite the ball's displacement, which means the work is negative. The negative work done on the ball slows and then stops it, reducing its kinetic energy to zero. Again, the work equals the change in energy; in this case, negative work on the ball decreases its energy. In the scenarios described here, the ball is the object to which a force is applied. But you can also think of the soccer ball doing work. The ball applies a force on the goalie, causing the goalie's hands to move backward. The ball does positive work on the goalie because the force it applies is in the direction of the displacement of the goalie's hands.

Derivation: work-kinetic energy theorem

In this section, we show that the net work done on an object and its change in kinetic energy are equal by using the definition of work and Newton's second law. We will again use the illustration of a soccer ball being kicked and model the ball as a particle. The ball starts at rest and we assume the force applied by the foot equals the net force on the ball, and that the ball moves without rotating.



Variables

Variables

work	W
force	F
displacement of object	Δx
mass of object	m
acceleration of object	a
initial speed of object	v_i
final speed of object	v
kinetic energy	KE

Strategy

1. Start with the definition of work.

2. Use Newton's second law to replace the net force in the definition of work by mass times the acceleration.

3. Use a motion equation from the study of kinematics to replace acceleration times displacement with one-half the speed squared.

Physics principles and equations

We will use the definition of work for when the force is in the direction of displacement.

W = *F*□*x*

Newton's second law

F = *ma*

Linear motion equation

*v*2 = *v*i

2 + 2*a*□*x*

The definition of kinetic energy

KE = ½*mv*2

Step-by-step derivation

State the definition of work and use Newton's second law to substitute *ma* for force.

Step	Reason
1. $W = F\Delta x$	definition of work
2. $W = ma\Delta x$	Newton's second law

We need to replace the acceleration and displacement terms with speed squared to end up with the definition of kinetic energy. We use a

motion equation to make this substitution.

Step	Reason
3. $v^2 = v_i^2 + 2a\Delta x$	motion equation
4. $a\Delta x = ½\, v^2$	set $v_i = 0$ and rearrange
5. $W = ½\, mv^2$	substitute equation 4 into equation 2
6. $W = KE$	definition of kinetic energy
7. $W = \Delta KE$	no initial kinetic energy

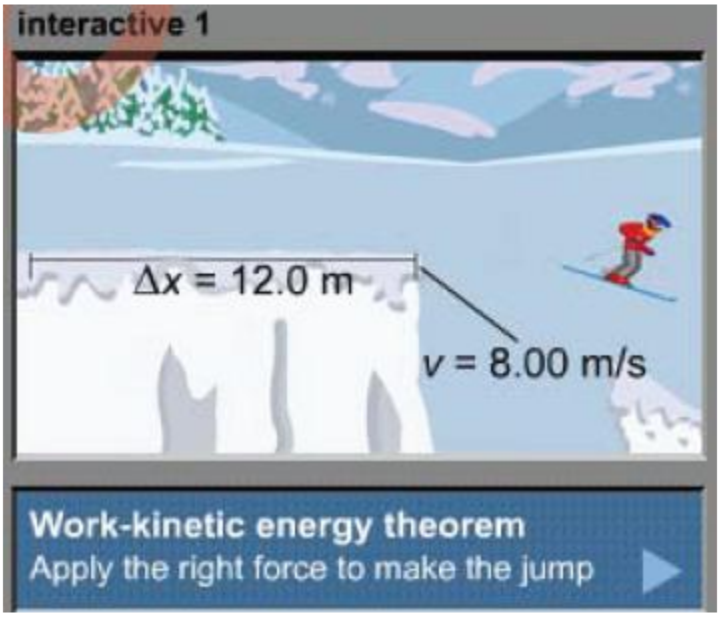
Use the work-kinetic energy theorem to find the work done on the sled. Then, use the definition of work to determine how much force was
exerted on the sled.

Step	Reason
6. $W = \Delta KE$	work-kinetic energy theorem
7. $W = (F \cos \theta)\Delta x$	definition of work
8. $(F \cos \theta)\Delta x = \Delta KE$	set two work equations equal
9. $F \cos \theta = \Delta KE/\Delta x$	rearrange
10. $F = \Delta KE/\Delta x$	force in direction of displacement
11. $F = 11{,}800\text{ J}/50.0\text{ m}$	enter values
12. $F = 236\text{ N}$	solve

Interactive problem: work-kinetic energy theorem

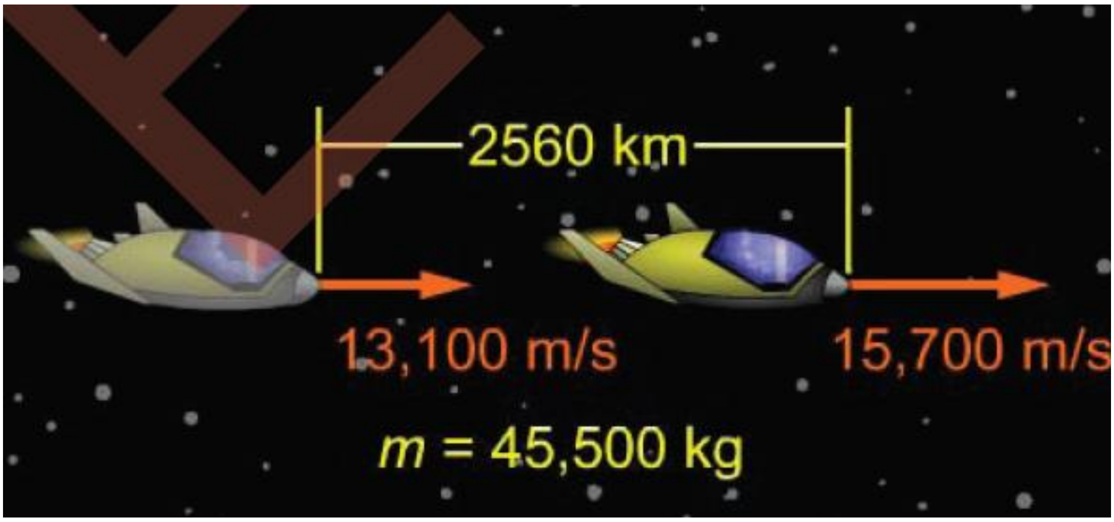
In this simulation, you are a skier and your challenge is to do the correct amount of work to build up enough energy to soar over the canyon and land near the lip of the

slope on the right. You, a 50.0 kg skier, have a flat 12.0 meter long runway leading up to the lip of the canyon. In that stretch, you must apply a force such that at the end of the straightaway, you are traveling with a speed of 8.00 m/s. Any slower, and your jump will fall short. Any faster, and you will overshoot. How much force must you apply, in newtons, over the 12.0 meter flat stretch? Ignore other forces like friction and air resistance



Enter the force, to the nearest newton, in the entry box and press GO to check your result.

If you have trouble with this problem, review the section on the work-kinetic energy theorem. (If you want to, you can check your answer using a linear motion equation and Newton's second law.)



A 45,500 kg spaceship is far from any significant source of gravity. It accelerates at a constant rate from 13,100 m/s to 15,700 m/s over a distance of 2560 km. What is the magnitude of the force on the ship due to the action of its engines? Use equations involving work and energy to solve the problem, and assume that the mass is constant.

example 1

Applying a force of 2.0×10^5 N, the tugboat moves the log boom 1.0 kilometer in 15 minutes. What is the tugboat's average power?

$$W = F\Delta x$$

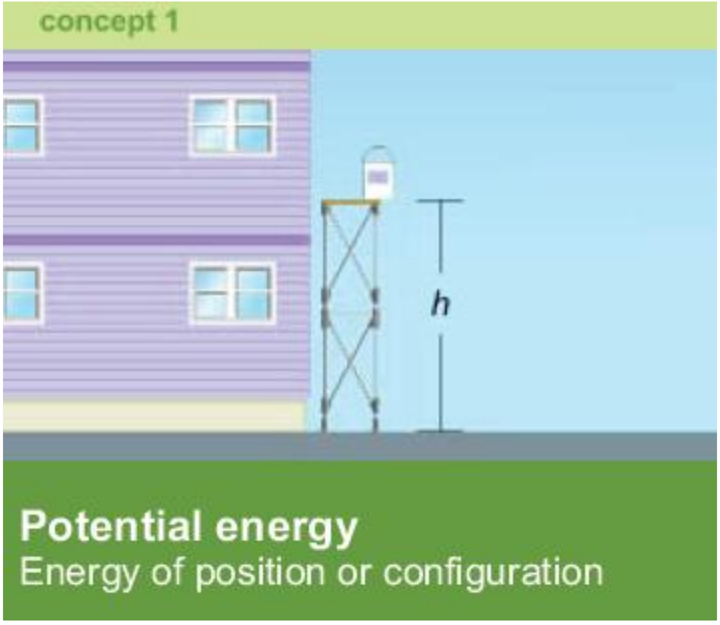
$$W = F\Delta x = (2.0 \times 10^5 \text{ N})(1.0 \times 10^3 \text{ m})$$

$$W = 2.0 \times 10^8 \text{ J}$$

$$\bar{P} = \frac{W}{\Delta t} = \frac{2.0 \times 10^8 \text{ J}}{9.0 \times 10^2 \text{ s}}$$

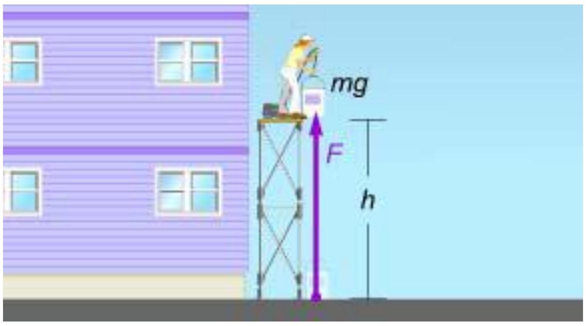
$$\bar{P} = 2.2 \times 10^5 \text{ W}$$

Potential energy: Energy related to the positions of and forces between the objects that make up a system.



Although the paint bucket in Concept 1 is not moving, it makes up part of a system that has a form of energy called potential energy. In general, potential energy is the energy due to the configuration of objects that exert forces on one other. In this section, we focus on one form of potential energy, *gravitational potential energy*. The paint bucket and Earth make up a system that has this form of potential energy. A *system* is some "chunk" of the universe that you wish to study, such as the bucket and the Earth. You can imagine a boundary like a bubble surrounding the system, separating it from the rest of the universe. The particles within a system can interact with one another via internal forces or fields. Particles outside the system can interact with the system via external forces or fields. Gravitational potential energy is due to the gravitational force between the bucket and Earth. As the bucket is raised or lowered, its **change** in potential energy (ΔPE) equals the magnitude of its weight, mg, times its vertical displacement, Δh. (We follow the common convention of using Δh for change in height, instead of Δy.) The weight is the amount of force exerted on the bucket by the Earth (and vice versa). This formula is shown in Equation 1. A change in PE can be positive or negative. The magnitude of weight is a positive value, but change in height can be positive (when the bucket moves up) or negative (when it

moves down). To define a system's *PE*, we must define a configuration at which the system has zero *PE*. Unlike kinetic energy, where zero *KE* has a natural value (when an object's speed is zero), the configuration with zero *PE* is defined by you, the physicist. In the diagrams to the right, it is convenient to say the system has zero *PE* when the bucket is on the Earth's surface. This convention means its *PE* equals its weight times its height above the ground, *mgh*. Only the bucket's distance above the Earth, *h*, matters here; if the bucket moves left or right, its *PE* does not change. In Example 1, we calculate the paint bucket's gravitational potential energy as it sits on the scaffolding, four meters above the ground. There are other types of potential energy. One you will frequently encounter is *elastic potential energy*, which is the energy stored in a compressed or stretched object such as a spring. As you may recall, this form of energy was present in the bow that was used to fire an arrow.



Change in gravitational potential energy

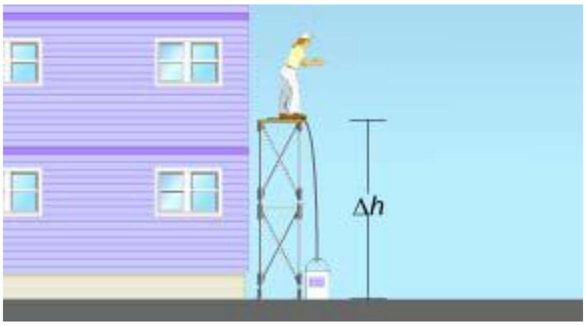
$$\Delta PE = mg\Delta h$$

PE = potential energy
mg = object's weight
Δh = vertical displacement

energy, as seen in Equation 2. Imagine that the painter drops the bucket from the scaffolding. Only the force of gravity does work on the bucket as it falls. The system has more potential energy when the bucket is at the top of the scaffolding than when it is at the bottom, so the work done by gravity has lowered the system's *PE*: the change in *PE* due to the work done by gravity is negative.

$$W = \Delta PE$$

W = work done against gravity
PE = potential energy of system



Work done by gravity

$$W = -\Delta PE$$

W = work done by gravity
PE = potential energy of system

Sample problem: potential energy and Niagara Falls

In its natural state, an average of 5.71×10^6 kg of water flowed per second over Niagara Falls, falling 51.0 m. If all the work done by gravity could be converted into electric power as the water fell to the bottom, how much power would the falls generate?

Variables

height of falls	$h = 51.0$ m
magnitude of acceleration due to gravity	$g = 9.80$ m/s^2
potential energy	PE
mass of water over falls per unit time	$m/t = 5.71 \times 10^6$ kg/s
power	P
work done by gravity	W

What is the strategy?

1. Use the definition of power as the rate of work done to define an equation for the power of the falls.

2. Use the fact that work done by gravity equals the negative of the change in gravitational potential energy to solve for the power.

Physics principles and equations

Power is the rate at which work is performed.

$$P = \frac{W}{\Delta t}$$

Change in gravitational *PE*

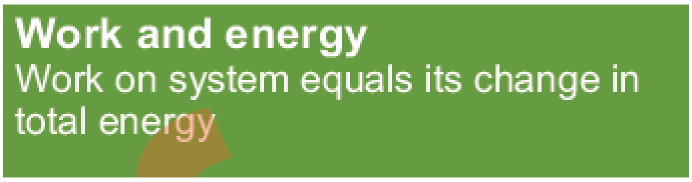
□*PE* = *mg*□*h*

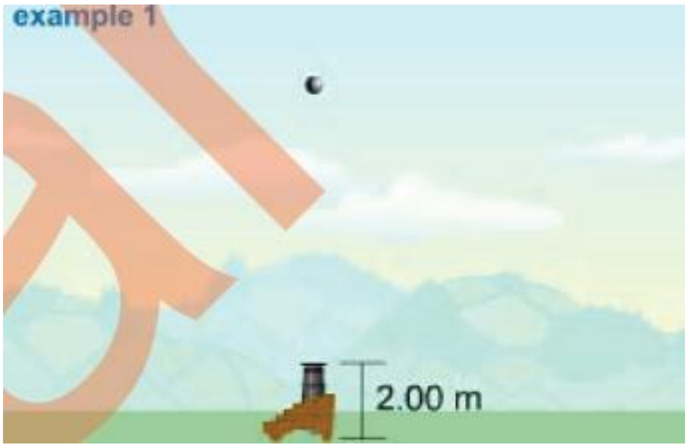
Work done by gravity

W = □□*PE*

kg cannonball straight up. The barrel of the cannon is 2.00 m long, and it exerts an average force of 6,250 N while the cannonball is in the cannon. We will ignore air

resistance. Can we determine the cannonball's velocity when it has traveled 125 meters upward? As you may suspect, the answer is "yes". The cannon does 12,500 J of work on the cannonball, the product of the force (6,250 N) and the displacement (2.00 m). (We assume the cannon does no work on the cannonball after it leaves the cannon.) At a height of 125 meters, the cannonball's increase in *PE* equals $mg\Delta h$, or 3,920 J. Since a total of 12,500 J of work was done on the ball, the rest of the work must have gone into raising the cannonball's *KE*: The change in *KE* is 8,580 J. Applying the definition of kinetic energy, we determine that its velocity at 125 m is 73.2 m/s. We could further analyze the cannonball's trip if we were so inclined. At the peak of its trip, all of its energy is potential since its velocity (and *KE*) there are zero. The *PE* at the top is 12,500 J. Again applying the formula $mg\Delta h$, we can determine that its peak height above the cannon is about 399 m.

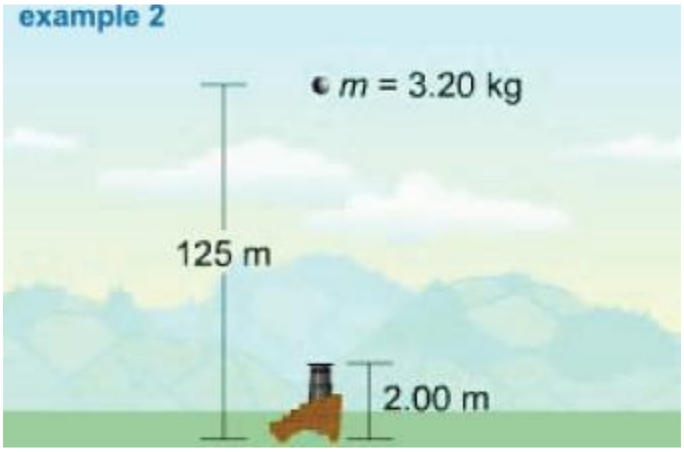




The cannon supplies 6,250 N of force along its 2.00 m barrel. How much work does the cannon do on the cannonball?

$W = (F \cos \theta)\Delta x = F\Delta x$

$W = (6250 \text{ N})(2.00 \text{ m}) = 12{,}500 \text{ J}$



What is the cannonball's velocity at 125 m? Its mass is 3.20 kg.

$W = \Delta PE + \Delta KE$

$W = mg\Delta h + \Delta KE$

$12{,}500 \text{ J} = (3.20 \text{ kg})(9.80 \text{ m/s}^2)(125 \text{ m})$
$+ \Delta KE$

$\Delta KE = 8{,}580 \text{ J}$

$\frac{1}{2} mv^2 = 8{,}580 \text{ J}$

$v^2 = 2(8{,}580 \text{ J})/(3.20 \text{ kg})$

$v = 73.2 \text{ m/s}$

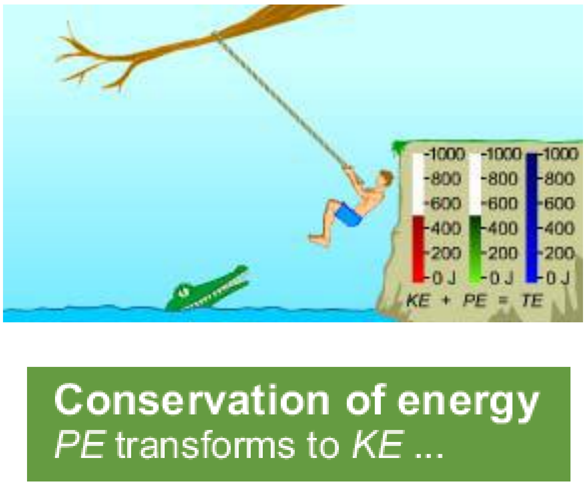
Conservative and non-conservative forces Earlier, when discussing potential energy, we mentioned that we would explain conservative forces later. The concept of potential energy only applies to conservative forces. Gravity is an example of a *conservative force*. It is conservative because the total work it does on an object that starts and finishes at the same point is zero. For example, if a 20 kg barbell is raised 2.0 meters, gravity does □40 J of work, and when the barbell is lowered 2.0 meters back to its initial position, gravity does +40 J of work. When the barbell is returned to

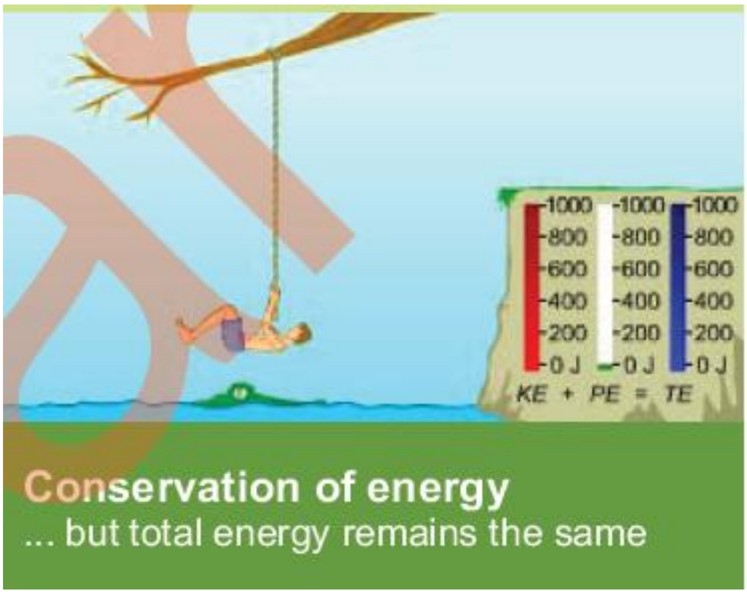
its initial position, the sum of the work done by gravity on the one that has no interactions with its environment. The particles within the system may interact with one another, but no net external force or field acts on an isolated system. Only external forces can change the total energy of a system. If a giant spring lifts a car, you can say the spring has increased the energy of the car. In this case, you are considering the spring as supplying an external force and not as part of the system. If you include the spring in the system, the increase in the energy of the car is matched by a decrease in the potential energy contained of the spring, and the total energy of the system remains the same. For the law of conservation of energy to apply, there can be no non-conservative forces like friction within the system. The law of conservation of energy can be expressed mathematically, as shown in Equation 1. The equation states that an isolated system's total energy at any final point in time is the same as its total energy at an initial point in time. When considering mechanical energy, we can state that the sum of the kinetic and potential energies at some final moment equals the sum of the kinetic and potential energies at an initial moment.

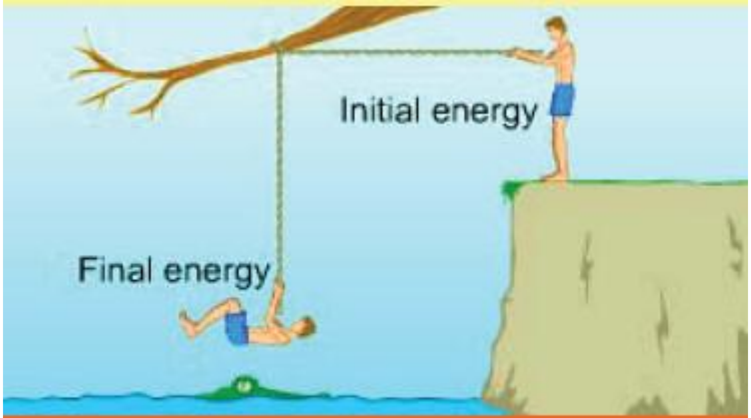
In the case of the boy on the rope, if you know his mass and height on the riverbank, you can calculate his gravitational potential energy. In this example, rather than saying his *PE* equals zero on the ground, we say it equals zero at the bottom of the arc. This simplifies matters. Using the law of conservation of energy, you can then determine what his kinetic energy, and therefore his speed, will be when he reaches the bottom of the arc, nearest to the water, since at that point all his energy is kinetic.

Let's leave the boy swinging for a while and switch to another example: You drop a weight. When the weight hits the ground it will stop moving. At this point, the weight has neither kinetic energy nor potential energy because it has no motion and its height off the Earth's surface is zero. Does the law of conservation of energy still hold true? Yes, it does, although we need to broaden the forms of energy included in the discussion. With careful observation you might note that the ground shakes as the weight hits it (more energy of motion). The weight and the ground heat up a bit (thermal energy). The list can continue: energy of the motion of flying dirt, the energy of sound and so on. The amount of mechanical energy does decline, but when you include all forms of energy, the overall energy stays constant. There is a caveat to the law of conservation of energy. Albert Einstein demonstrated that there is a relationship between mass and energy. Mass can be converted into energy, as it is inside the Sun or a nuclear reactor, and energy can be converted into mass. It is the

sum of mass and energy that remains constant. Our current focus is on much less extreme situations. Using the principle of conservation of energy can have many practical benefits, as automotive engineers are now demonstrating. When it comes to energy and cars, the focus is often on how to cause the car to accelerate, how fast they will reach say a speed of 100 km/h. Of course, cars also need to slow down, a task assigned to the brakes. As conventional cars brake, the energy is typically dissipated as heat as the brake pads rub on the rotors. Innovative new cars, called hybrids, now capture some of the kinetic energy and convert it to chemical energy stored in batteries or mechanical energy stored in flywheels. The engine then recycles that energy back into kinetic energy when the car needs to accelerate, saving gasoline







Conservation of energy

$$E_f = E_i$$

$$PE_f + KE_f = PE_i + KE_i$$

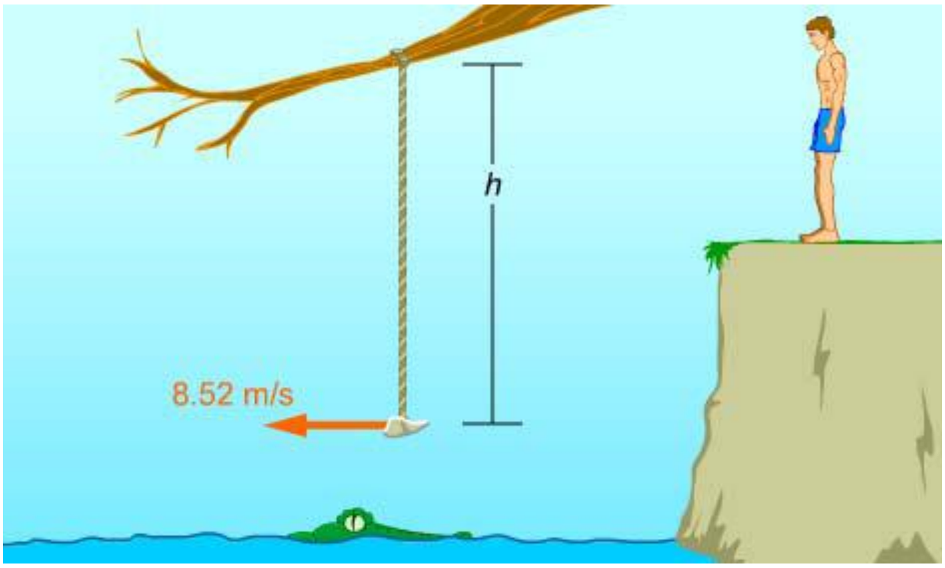
E = total energy

KE = kinetic energy

PE = potential energy

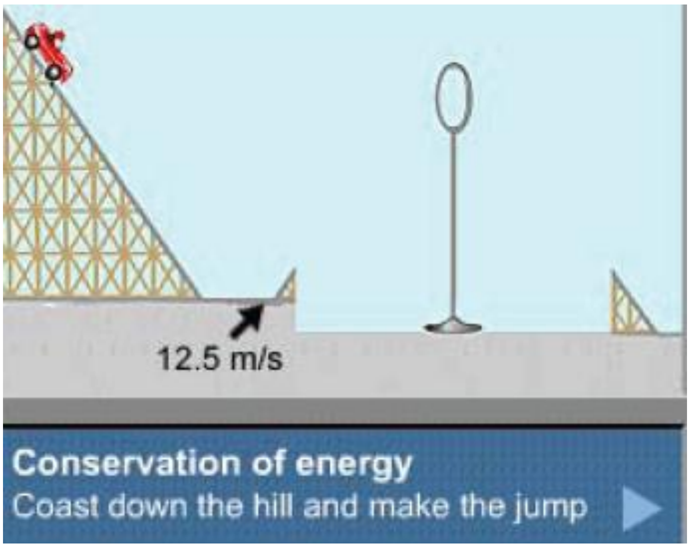
Sam is at the peak of his jump. Calculate Sam's speed when he reaches the trampoline's surface.

Sam is jumping up and down on a trampoline. He bounces to a maximum height of 0.25 m above the surface of the trampoline. How fast will he be traveling when he hits the trampoline? We define Sam's potential energy at the surface of the trampoline to be zero.



A boy releases a pork chop on a rope. The chop is moving at a speed of 8.52 m/s at the bottom of its swing. How much higher than this point is the point from which the pork chop is released? Assume that it has no initial speed when starting its swing.

The law of conservation of energy states that the total energy in an isolated system remains constant. In the simulation on the right, you can use this law and your knowledge of potential and kinetic energies to help a soapbox derby car make a jump. A soapbox derby car has no engine. It gains speed as it rolls down a hill. You can drag the car to any point on the hill. A gauge will display the car's height above the ground. Release the mouse button and the car will fly down the hill. In this interactive, if the car is traveling 12.5 m/s at the bottom of the ramp, it will successfully make the jump through the hoop. Too slow and it will fall short; too fast and it will overshoot. You can use the law of conservation of energy to figure out the vertical position needed for the car to nail the jump.



Friction and conservation of energy

In this section, we show how two principles we have discussed can be combined to solve a typical problem. We will use the principle of conservation of energy and how work done by an external force affects the total energy of a system to determine the effect of friction on a block sliding down a plane. Suppose the 1.00 kg block shown to the right slides down an inclined wooden plane. Since the block is released from rest, it has no initial velocity. It loses 2.00 meters in height as it slides, and it slides 6.00 meters along the surface of the inclined plane. The force of kinetic friction is 2.00 N. You want to know the block's speed when it reaches the bottom position.

To solve this problem, we start by applying the principle of conservation of energy. The block's initial energy is all potential, equal to the product of its mass, *g* and its height (*mgh*). At a height of 2.00 meters, the block's *PE* equals 19.6 J. The potential energy will be zero when the block reaches the bottom of the plane. Ignoring friction, the *PE* of the block at the top equals its *KE* at the bottom.

Now we will factor in friction. The force of friction opposes the block's motion down the inclined plane. The work it does is negative, and that work reduces the energy of the block. We calculate the work done by friction on the block as the force of friction times the displacement along the plane, which equals −12.0 J. The block's energy at the top (19.6 J) plus the −12.0 J means the block has 7.6 J of kinetic energy at the bottom. Using the definition of kinetic energy, we can conclude that the 1.00 kg block is moving at 3.90 m/s. You can also calculate the effect of friction by determining how fast the block would be traveling if there were no friction. All 19.6 J of *PE* would

convert to *KE*, yielding a speed of 6.26 m/s. Friction reduces the speed of the block by approximately 38%.

Review of forces, work and energy

In this chapter we have discussed the work done when a force is exerted on a particle (the work-kinetic energy theorem). We have discussed work and energy with respect to a system of objects (potential energy). We have also covered conservative and non-conservative forces.

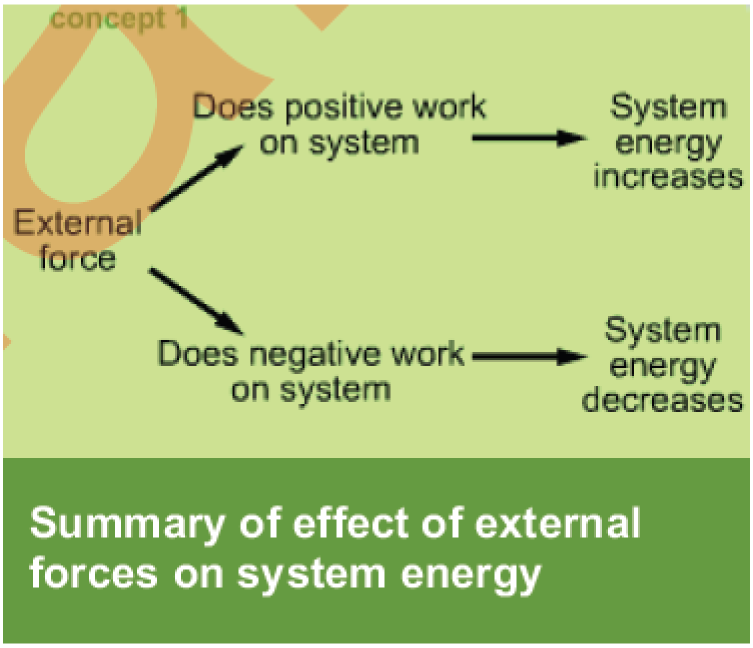
We further categorized forces by stating that some are *external forces*, forces from a source outside the objects that make up the system. For example, we talked about a foot applying force to a soccer ball, and a painter hoisting up a paint bucket. In both these examples, the foot and painter are considered external to the system, which consists either of a single particle (the ball) or multiple objects (the bucket and the Earth). In contrast, other forces are *internal forces* in a system. In the bucket/Earth system, for example, the force of gravity is an internal force. It arises from the objects that make up the system.

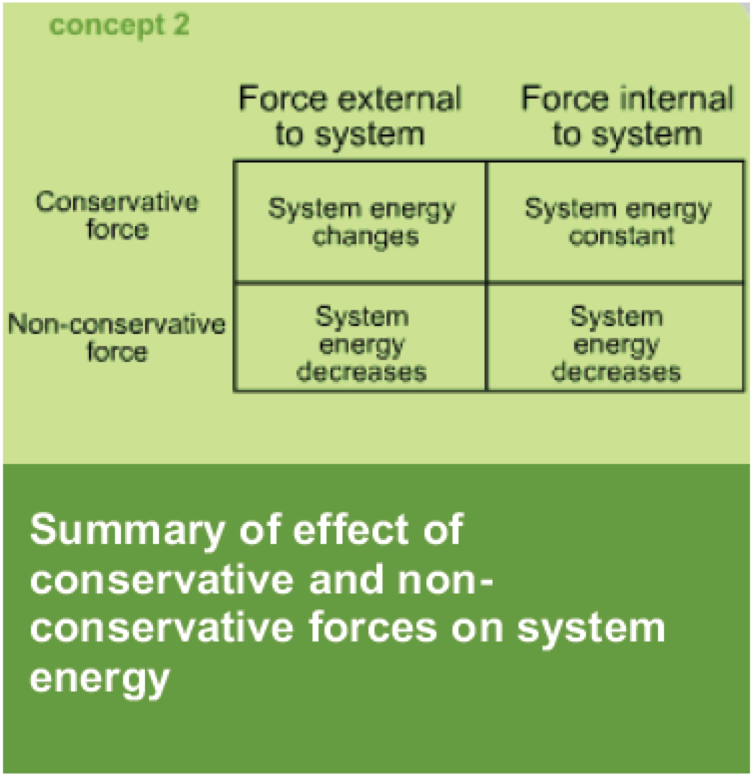
In this section, we review and summarize the effect on mechanical energy from all these types of forces: external and internal, conservative and non-conservative. We want to consider how the work done by these various types of forces affects the mechanical energy of a system. We will start with external forces, and consider the effect of the work done by an external force on the total energy of a system. Any net

external force acting on a particle or system changes the system's energy. Positive work done on a system by an external force increases the system's total energy, and negative work done on a system by an external force decreases its total energy. This is illustrated in the diagram in Concept 1. If the system consists of one particle, then the work equals the change in kinetic energy. This is the work-kinetic energy theorem. A single particle cannot have potential energy, so positive work on the particle increases its *KE*, and negative work done on the particle (or work done by the particle) decreases its *KE*. The work done on any system by an external force changes the system's total mechanical energy. Let's consider a system that consists of an apple and the Earth. Positive work may increase the *PE* (you lift the apple upwards at a constant rate), or *KE* (you run faster and faster with the apple held at a constant height), or both (you throw an initially stationary apple skyward).

You need to be careful of the sign of the work done by considering whether the force is in the direction of the displacement (positive work) or the opposite direction (negative work). If you throw a ball, you increase its energy, and when you catch it, you decrease its energy. Non-conservative forces decrease the mechanical energy of a system. (There are scenarios where they can be considered as increasing the mechanical energy, but we will ignore them here.) If you slide a block down a plane, the non-conservative forces of kinetic friction and air resistance act in the opposite direction of the block's displacement. This means they do negative work, and reduce the mechanical energy of the system.

Now let's consider the effect of internal forces on the energy of a system. We will start with internal conservative forces. These forces do not change the total mechanical energy of a system. Consider a system consisting of a block, an inclined plane, and the Earth. The block is sliding down the plane. The force of gravity is conservative, and the decrease in gravitational potential energy as the block slides down the plane is matched by an increase in kinetic energy. This is the law of conservation of energy. The conservative force of gravity does not change the total mechanical energy of the system. We will foreshadow thermodynamics here. The force of friction will increase the temperature of the block and plane. It increases the *internal*

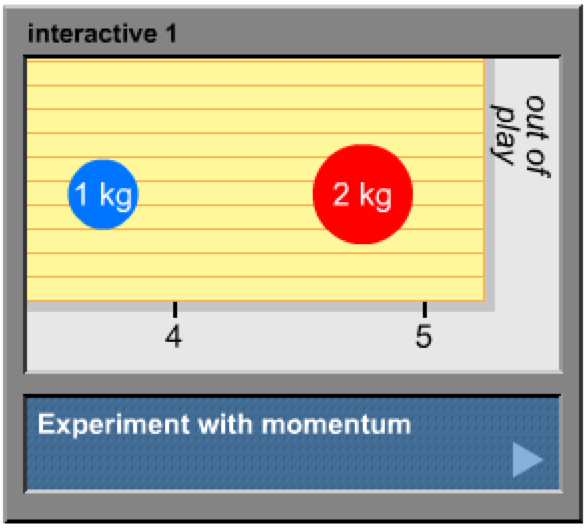




Introduction

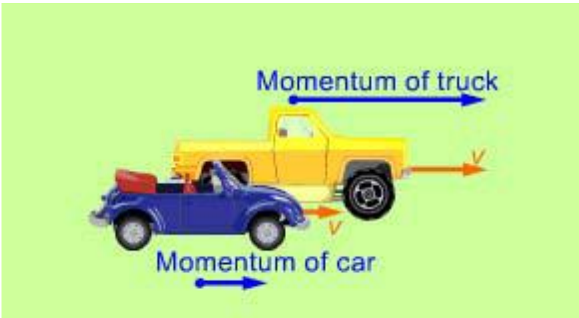
"The more things change, the more they stay the same" is a well-known French saying. However, though witty and perhaps true for many matters on which the French have great expertise, this saying is simply not good physics. Instead, a physicist would say: "Things stay the same, period. That is, unless acted upon by a net force." Perhaps a little less *joie de vivre* than your average Frenchman, but

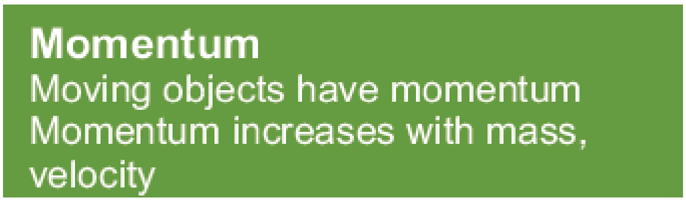
nonetheless the key to understanding momentum. What we now call momentum, Newton referred to as “quantity of motion.” The linear momentum of an object equals the product of its mass and velocity. (In this chapter, we focus on linear momentum. Angular momentum, or momentum due to rotation, is a topic in another chapter.) Momentum is a useful concept when applied to collisions, a subject that can be a lot of fun. In a collision, two or more objects exert forces on each other for a brief instant of time, and these forces are significantly greater than any other forces they may experience during the collision. At the right is a simulation a variation of shuffleboard that you can use to begin your study of momentum and collisions. You can set the initial velocity for both the blue and the red pucks and use these velocity settings to cause them to collide. The blue puck has a mass of 1.0 kg, and the red puck a mass of 2.0 kg. The shuffleboard has no friction, but the pucks stop moving when they fall off the edge. Their momenta and velocities are displayed in output gauges. Using the simulation, answer these questions. First, is it possible to have negative momentum? If so, how can you achieve it? Second, does the collision of the pucks affect the sum of their velocities? In other words, does the sum of their velocities remain constant? Third, does the collision affect the sum of their momenta? Remember to consider positive and negative signs when summing these values. Press PAUSE before and after the collisions so you can read the necessary data. For an optional challenge: Does the collision conserve the total kinetic energy of the pucks? If so, the collision is called an elastic collision. If it reduces the kinetic energy, the collision is called an inelastic collision.

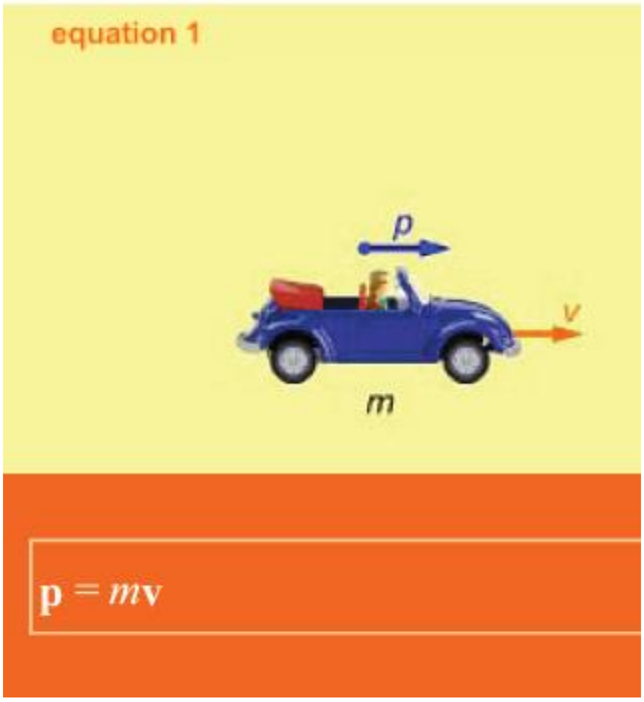


Momentum *Momentum (linear):* Mass times velocity.

An object's linear momentum equals the product of its mass and its velocity. A fast moving locomotive has greater momentum than a slowly moving ping-pong ball. The units for momentum are kilogram·meters/second (kg·m/s). A ping-pong ball with a mass of 2.5 grams moving at 1.0 m/s has a momentum of 0.0025 kg·m/s. A 100,000 kg locomotive moving at 5 m/s has a momentum of 5×105 kg·m/s. Momentum is a vector quantity. The momentum vector points in the same direction as the velocity vector. This means that if two identical locomotives are moving at the same speed and one is heading east and the other west, they will have equal but **opposite** momenta, since they have equal but oppositely directed velocities.

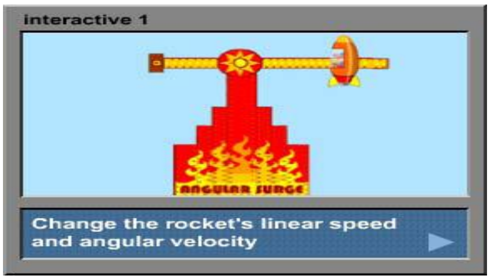






If you feel as though you spend your life spinning around in circles, you may be pleased to know that an entire branch of physics is dedicated to studying that kind of motion. This chapter is for you! More seriously, this chapter discusses motion that consists of rotation about a fixed axis. This is called *pure rotational motion*. There are many examples of pure rotational motion: a spinning Ferris wheel, a roulette wheel, or a music CD are three instances of this type of motion. In this chapter, you will learn about rotational displacement, rotational velocity, and rotational acceleration: the fundamental elements of what is called *rotational kinematics*. You will also learn how to relate these quantities using equations quite similar to those used in the study of linear motion. The simulation on the right features the “Angular Surge,” an amusement park ride you will be asked to operate in order to gain insight into rotational kinematics. The ride has a rotating arm with a “rocket” where passengers sit. You can move the rocket closer to or farther from the center by setting the distance in the simulation. You can also change the rocket’s period, which is the amount of time it takes to complete one revolution. By changing these parameters, you affect two values you see displayed in gauges: the rocket’s angular velocity and its linear speed. The rocket’s angular velocity is the change per second in the angle of the ride’s arm, measured from its initial position. Its units are radians per second. For instance, if the rocket completes one revolution in one second, its angular velocity is 2□ radians (360°) per second. This simulation has no specific goal for you to achieve, although you may notice that you can definitely have an impact on the passengers!

What you should observe is this: How do changes in the period affect the angular velocity? The linear speed? And how does a change in the distance from the center (the radius of the rocket’s motion) affect those values, if at all? Can you determine how to maximize the linear speed of the rocket? To run the ride, you start the simulation, set the values mentioned above, and press GO. You can change the settings while the ride is in motion.



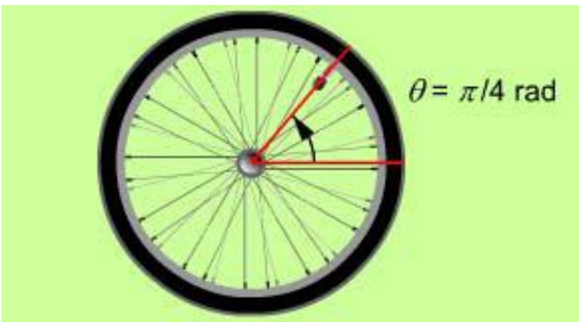
Angular position *Angular position:* The amount of rotation from a reference position, described with a positive or negative angle.

When an object such as a bicycle wheel rotates about its axis, it is useful to describe this motion using the concept of angular position. Instead of being specified with a linear coordinate such as *x*, as linear position is, angular position is stated as an angle.

In Concept 1, we use the location of a bicycle wheel’s valve to illustrate angular position. The valve starts at the 3 o’clock position (on the positive *x* axis), which is zero radians by convention. As the illustration shows, the wheel has rotated one-eighth of a turn, or □/4 radians (45°), in a counterclockwise direction away from the reference position. In other words, angular position is measured from the positive *x* axis. Note that this description of the wheel’s position used radians, not degrees; this is because radians are typically used to describe angular position. The two lines we use to measure the angle radiate from the point about which the wheel rotates.

The *axis of rotation* is a line also used to describe an object’s rotation. It passes through the wheel’s center, since the wheel rotates about that point, and it is perpendicular to the wheel. The axis is assumed to be stationary, and the wheel is assumed to be rigid and to maintain a constant shape. Analyzing an object that changes shape as it rotates, such as a piece of soft clay, is beyond the scope of this textbook. We are concerned with the wheel’s rotational motion here: its motion around a fixed axis. Its linear motion when moving along the ground is another topic.

As mentioned, angular position is typically measured with *radians* (rad) instead of degrees. The formula that defines the radian measure of an angle is shown in Equation 1. The angle in radians equals the arc length *s* divided by the radius *r*. As you may recall, 2□ radians equals one revolution around a circle, or 360°. One radian equals about 57.3°. To convert radians to degrees, multiply by the conversion factor 360°/2□. To convert degrees to radians, multiply by the reciprocal: 2□/360°. The Greek letter □

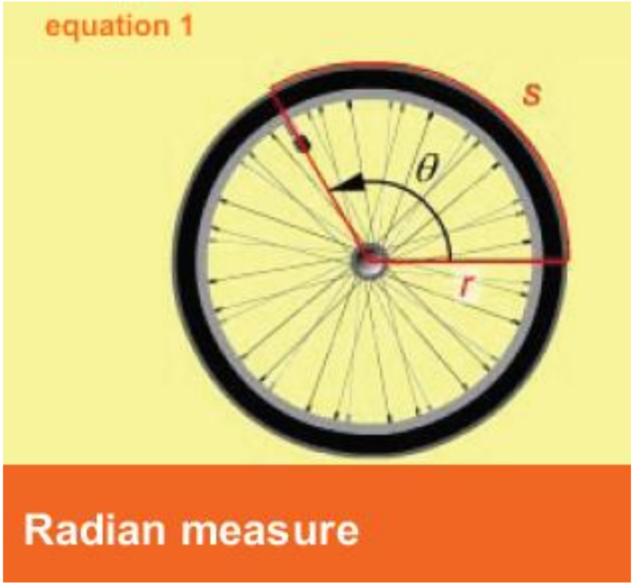


Angular position

Rotation from 3 o'clock position

- Counterclockwise rotation: positive
- Clockwise rotation: negative

Units are radians

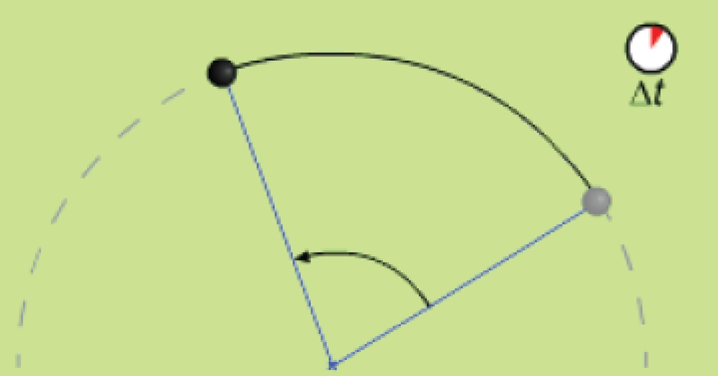


Angular velocity: Angular displacement per unit time.

In Concept 1, a ball attached to a string is shown moving counterclockwise around a circle. Every four seconds, it completes one revolution of the circle. Its angular velocity is the angular displacement 2□ radians (one revolution) divided by four seconds, or □/2 rad/s. The Greek letter □ (omega) represents angular velocity. As is the case with linear velocity, angular velocity can be discussed in terms of average and instantaneous velocity. *Average angular velocity* equals the total angular displacement divided by the elapsed time. This is shown in the first equation in Equation

1. *Instantaneous angular velocity* refers to the angular velocity at a precise moment in time. It equals the limit of the average velocity as the increment of time approaches zero. This is shown in the second equation in Equation 1. The sign of angular velocity follows that of angular displacement: positive for counterclockwise rotation and negative for clockwise rotation. The magnitude (absolute value) of angular velocity is *angular speed*.

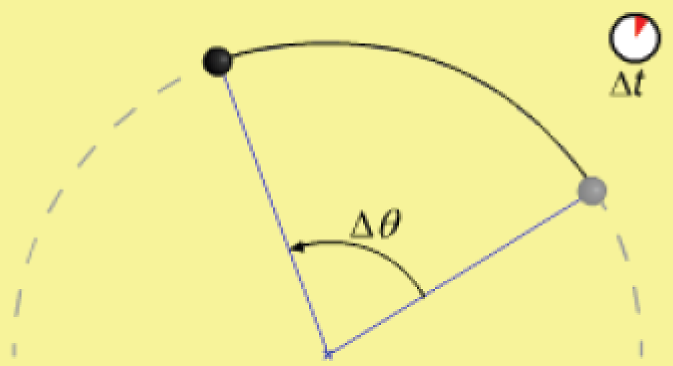
concept 1



Angular velocity

Angular displacement per unit time

equation 1



Angular velocity

$$\overline{\omega} = \frac{\Delta\theta}{\Delta t}$$

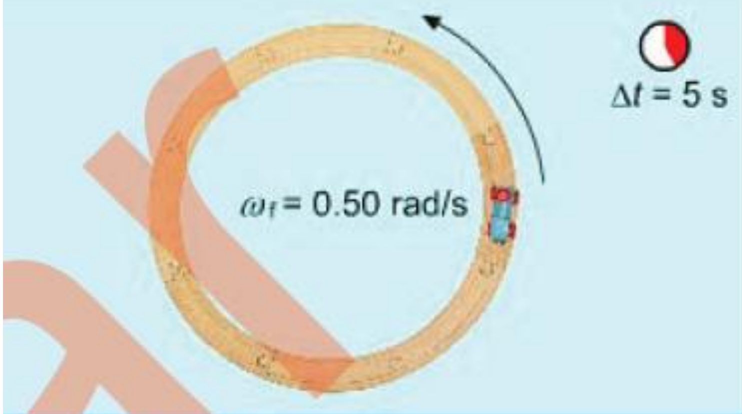
$\overline{\omega}$ = average angular velocity

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}$$

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t}$$

α = instantaneous angular acceleration
ω = angular velocity
Δt = elapsed time
Units: rad/s^2

example 1



The toy train starts from rest and reaches the angular velocity shown in 5.0 seconds. What is its average angular acceleration?

$$\bar{\alpha} = \frac{\Delta\omega}{\Delta t}$$

$$\bar{\alpha} = \frac{0.50\ \text{rad/s} - 0.00\ \text{rad/s}}{5.0\ \text{s}}$$

$$\bar{\alpha} = 0.10\ \text{rad/s}^2$$

Over the course of 1.00 hour, what is (a) the angular displacement, (b) the angular velocity and (c) the angular acceleration of the minute hand?

Think about the movement of the minute hand over the course of an hour. Be sure to consider the direction!

Variables

elapsed time	$\Delta t = 1.00$ h
angular displacement	$\Delta\theta$
angular velocity	ω
angular acceleration	α

What is the strategy?

1. Calculate the angular displacement.

2. Convert the elapsed time to seconds.

3. Use the angular displacement and time to determine the angular velocity and angular acceleration.

Physics principles and equations

Definition of angular velocity

for angular displacement, angular velocity and angular acceleration instead of linear displacement, velocity and acceleration. As with the linear motion equations, these

equations hold true when there is constant acceleration. We also show these equations below along with their linear counterparts. To apply the equations in physics problems, the first step is to identify the known values and which values are being asked for. Sketching a diagram of the situation may help you with this. The next step is to find an equation that includes both the known and the unknown (asked-for) values. Your goal is to find an equation, if possible, that has only one unknown value: the one you want to find. When applying the rotational equations, remember that positive displacement and velocity represent counterclockwise motion, and negative displacement and velocity indicate clockwise motion. Let's now work an example problem. Imagine you have just turned on the blender shown on the right. You let it run for 5.0 seconds. During this time period its blade has a constant angular acceleration of 44 radians per second squared. What is the angular displacement of the blade during this time? This problem implicitly tells you that the initial angular velocity is zero, since the blender has just been turned on. The second equation above includes time, initial angular velocity and acceleration. It also contains the value you seek to calculate: the angular displacement. This makes it the right equation to use. It does not include the value for final angular velocity, which is fine because you are not told that value, nor are you asked to calculate it. The details of the calculation appear on the right. The angular displacement is 550 radians. Because the value is positive, the motion is counterclockwise. Here is a table of the rotational motion variables and the equations that relate them, along with their linear counterparts.

The blender is turned on and runs for 5.0 seconds with a constant angular acceleration of 44 rad/s^2. What is the angular displacement of a blade?

$$\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2$$

$\omega_i = 0$ rad/s

$\Delta\theta = 0 \text{ rad} + \frac{1}{2}(44 \text{ rad/s}^2)(5.0 \text{ s})^2$

$\Delta\theta = 0 + (22)(25)$ rad

$\Delta\theta = 550$ rad

	linear	rotational
position	x	θ
displacement	Δx	$\Delta\theta$
velocity	$v = \Delta x/\Delta t$	$\omega = \Delta\theta/\Delta t$
acceleration	$a = \Delta v/\Delta t$	$\alpha = \Delta\omega/\Delta t$
	$v_f = v_i + at$	$\omega_f = \omega_i + \alpha t$
	$\Delta x = v_i t + \frac{1}{2}at^2$	$\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2$
	$v_f^2 = v_i^2 + 2a\Delta x$	$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$
	$\Delta x = \frac{1}{2}(v_i + v_f)t$	$\Delta\theta = \frac{1}{2}(\omega_i + \omega_f)t$

The carousel accelerates from rest for two revolutions at a constant angular acceleration of $0.11\ rad/s^2$. What is its final angular velocity?

Variables

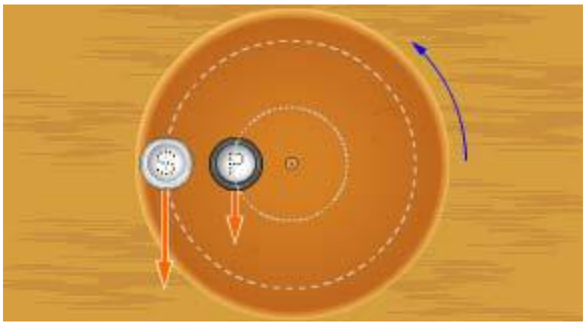
Since the carousel is starting up, the initial angular velocity must be zero. The angular displacement is given in revolutions, which must be converted to radians.

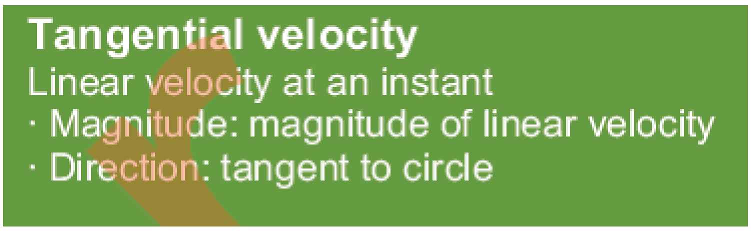
initial angular velocity	$\omega_i = 0$ rad/s
final angular velocity	ω_f
angular acceleration	$\alpha = 0.11$ rad/s^2
angular displacement	$\Delta\theta = (2.0)(2\pi$ rad)

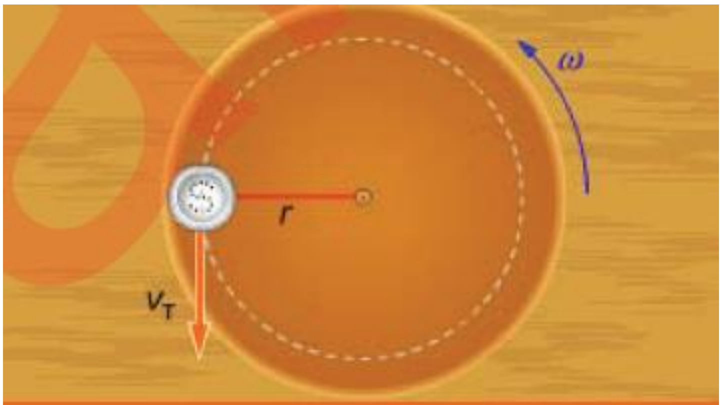
Tangential velocity: The instantaneous linear velocity of a point on a rotating object.

Concepts such as angular displacement and angular velocity are useful tools for analyzing rotational motion. However, they do not provide the complete picture. Consider the salt and pepper shakers rotating on the lazy Susan shown to the right. The containers have the same angular velocity because they are on the same rotating surface and complete a revolution in the same amount of time. However, at any instant, they have different **linear** speeds and velocities. Why? They are located at different distances from the axis of rotation (the center of the lazy Susan), which means they move along circular paths with different radii. The circular path of the outer shaker is longer, so it moves farther than the inner one in the same amount of time. At any instant, its linear speed is greater. Because the direction of motion of an object moving in a circle is always tangent to the circle, the object's linear velocity is called its tangential velocity. To reinforce the distinction between linear and angular velocity, consider what happens if you decide to run around a track. Let's say you are asked to run one lap around a circular track in one minute flat. Your angular velocity is 2□ radians per minute. Could you do this if the track had a radius of 10 meters? The answer is yes. The circumference of that track is 2□*r*, which equals approximately 63 meters. Your pace would be that distance divided by 60 seconds,

which works out to an easy stroll of about 1.05 m/s (3.78 km/h). What if the track had a radius of 100 meters? In this case, the one-minute accomplishment would require the speed of a world-class sprinter capable of averaging more than 10 m/s. (If the math ran right past you, note that we are again multiplying the radius by 2□ to calculate the circumference and dividing by 60 seconds to calculate the tangential velocity.) Even though the angular velocity is the same in both cases, 2□ radians per minute, the tangential speed changes with the radius. As you see in Equation 1, tangential speed equals the product of the distance to the axis of rotation, *r*, and the angular velocity, □. The units for tangential velocity are meters per second. The direction of the velocity is always tangent to the path of the object. Confirming the direction of tangential velocity can be accomplished using an easy home experiment. Let's say you put a dish on a lazy Susan and then spin the lazy Susan faster and faster. Initially, the dish moves in a circle, constrained by static friction. At some point, though, it will fly off. The dish will always depart in a straight line, tangent to the circle at its point of departure. The tangential speed equation can also be used to restate the equation for centripetal acceleration in terms of angular velocity. Centripetal acceleration equals *v*2/*r*. Since *v* = *r*□, centripetal acceleration also equals □2*r*. We derive the equation for tangential speed using the diagram below. To understand the derivation, you must recall that the arc length □*s* (the distance along the circular path) equals the angular displacement □□ in radians times the radius *r*. Also recall that the instantaneous speed *v*T equals the displacement divided by the elapsed time for a very small increment of time.





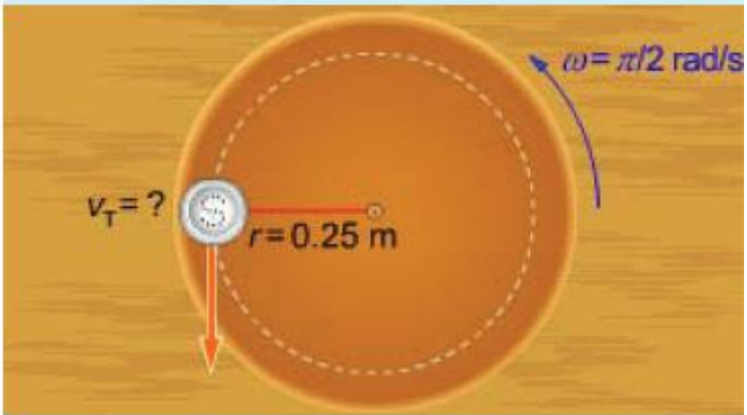


Tangential velocity

$$v_T = r\omega$$

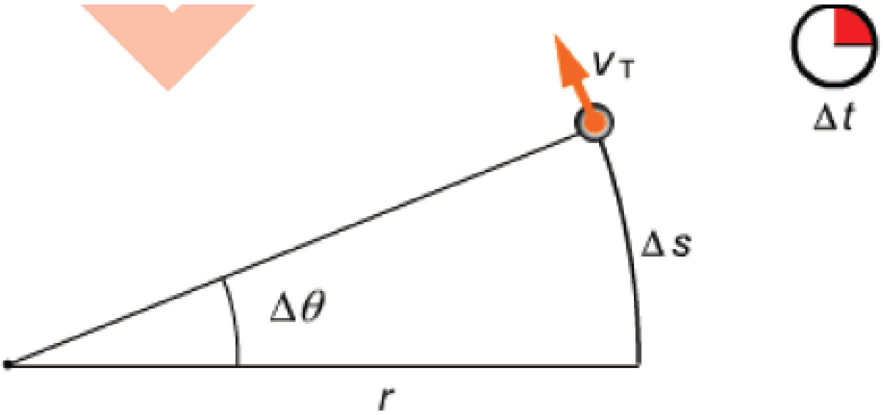
v_T = tangential speed
r = distance to axis
ω = angular velocity
Direction: tangent to circle

example 1

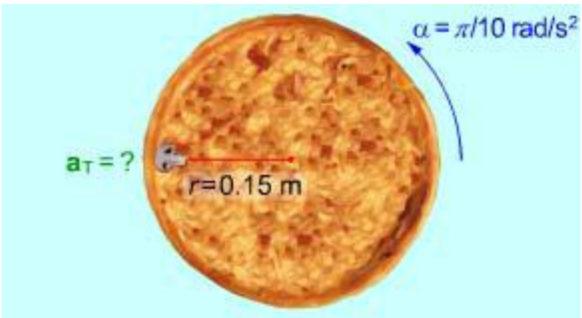


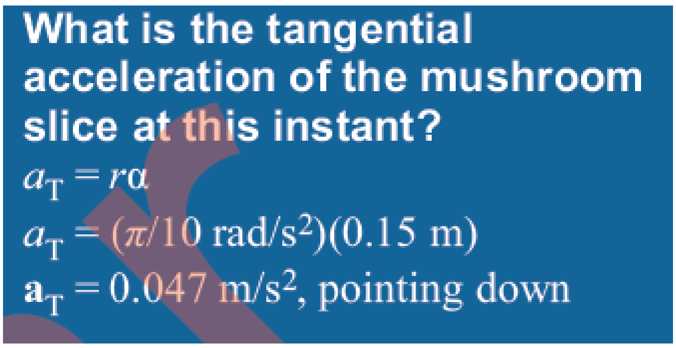
At the instant shown, what is the salt shaker's tangential velocity?

$v_T = r\omega$
$v_T = (0.25\text{ m})(\pi/2\text{ rad/s})$
$\mathbf{v}_T = 0.39\text{ m/s}$, pointing down



always toward the center of the circle. Now imagine that the car speeds up as it circles the track. It now completes a lap more quickly, so its angular velocity is increasing, which means it has positive angular acceleration (when it is moving counterclockwise; it is negative in the other direction). The car now has tangential acceleration (its linear speed is changing), and this can be calculated by multiplying its angular acceleration by the track's radius. The equation for tangential acceleration is derived below from the equations for tangential velocity and angular acceleration. We begin with the basic definition of linear acceleration and substitute the tangential velocity equation. The result is an expression which contains the definition of angular acceleration. We replace this expression with □, angular acceleration, which yields the equation we desire.





Step	Reason
1. $a_T = \lim\limits_{\Delta t \to 0} \frac{\Delta v_T}{\Delta t}$	definition of linear acceleration
2. $\Delta v_T = r\Delta\omega$	tangential velocity equation
3. $a_T = \lim\limits_{\Delta t \to 0} \frac{r\Delta\omega}{\Delta t} = r\left(\lim\limits_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t}\right)$	substitute equation 2 into equation 1
4. $a_T = r\alpha$	definition of angular acceleration

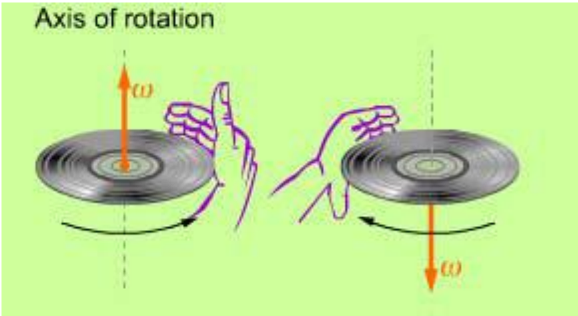
Tangential and centripetal acceleration

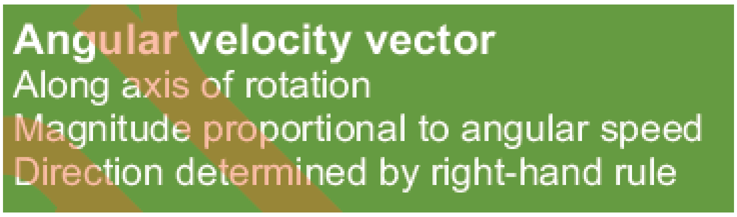
In Concept 1, a toy train is shown going around a circular track at steadily increasing speed. How can we calculate its overall acceleration at any moment? The train has both centripetal and tangential acceleration. The overall acceleration can be broken into these two components. The acceleration perpendicular to the direction of motion, directed toward the center of the circle, is the centripetal acceleration. Its magnitude at any instant is calculated using the equation for centripetal acceleration from a previous chapter: speed squared divided by the radius. The acceleration parallel to the velocity vector is the tangential acceleration, which is perpendicular to the centripetal acceleration. Since the train is increasing in speed, it has non-zero tangential acceleration. (This is not uniform circular motion.) The overall acceleration equals the vector sum of the centripetal and tangential accelerations. The two vectors are perpendicular, so they form two legs of a right triangle. The Pythagorean theorem can be used to calculate the magnitude of the overall acceleration, as the first formula in Equation 1 shows. The direction of the overall acceleration, measured from the centripetal acceleration vector (or the radius line), can be calculated using trigonometry. You see that formula in Equation 1 as well.

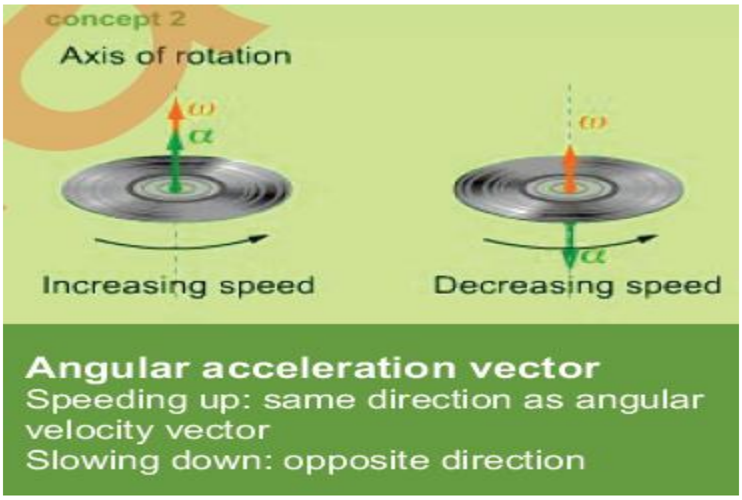
Vectors and angular motion

Although we have not stressed this fact, angular velocity and angular acceleration are

both vectors. In this section, we discuss the direction in which they point, using the *right-hand rule* to determine their direction. To apply this rule to angular velocity, curl your right hand around the axis of rotation, wrapping your fingers in the direction of the motion. This is illustrated to the right, where the hand wraps around the axis that passes through the center of the record. Your thumb then points in the direction of the angular velocity vector, which lies along the axis of rotation. The direction of the angular acceleration vector depends on whether the object in question is speeding up or slowing down. When an object speeds up, the angular acceleration vector points in the same direction as the angular velocity vector, reflecting the change in the velocity vector. When an object slows down, the angular acceleration vector points in the direction **opposite** to the angular velocity vector, again reflecting the change in the angular velocity vector. You may have noticed that we have not mentioned angular displacement. This is because it is not treated as a vector.







example 1

The motorcycle rider speeds up as she starts her ride. What is the direction of the angular velocity vector? The angular acceleration vector?

Angular velocity vector: up

Angular acceleration vector: up

Interactive summary problem: 11.6 seconds to liftoff

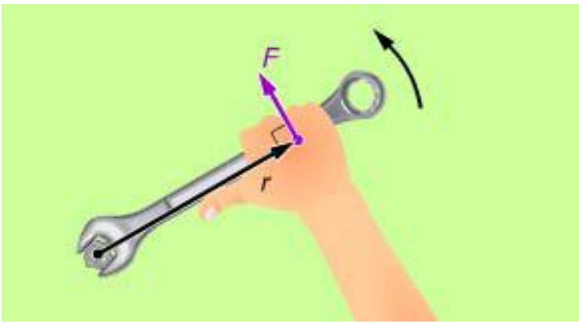
You are again operating the Angular Surge ride at a local amusement park. The ride begins with the arm in the launch position for the rocket. The motor starts the ride by providing a constant positive angular acceleration for the first 11.6 seconds. The ride has a rocket on a rotating arm, and you can control the arm's angular acceleration. You can also control the distance of the rocket from the axis of rotation. Your goal is to set both these values so that 11.6 seconds after startup, the rocket has completed one or more complete revolutions **and** has a tangential velocity of 13.0 m/s. If you do this correctly, the rocket will blast off. You can position the rocket from four to 10 meters from the Torque

Torque: A force that causes or opposes rotation.

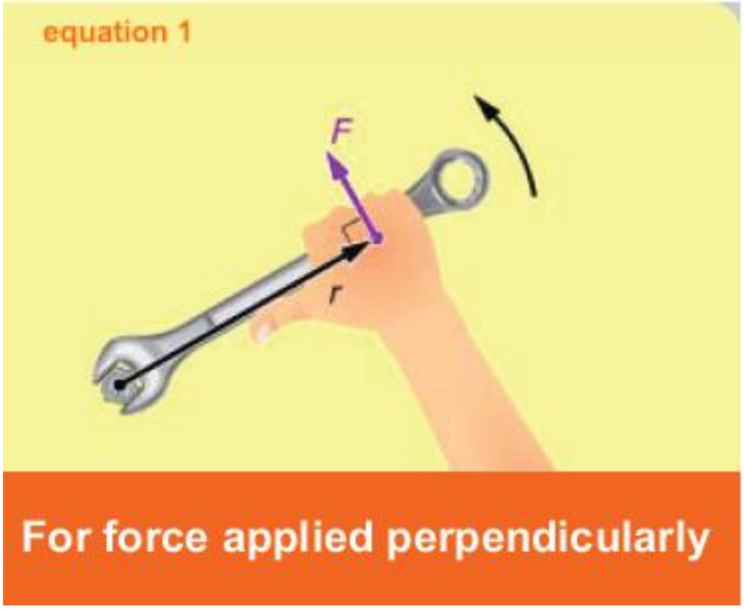
A net force causes linear acceleration: a change in the linear velocity of an object. A net torque causes angular acceleration: a change in the angular velocity. For instance, if you push hard on a wrench like the one shown in Concept 1, you will start it and the nut rotating. We will use a wrench that is loosening a nut as our setting to explain the concept of torque in more detail. In this section, we discuss two of the factors that

determine the amount of torque. One factor is how much force *F* is exerted and the other is the distance *r* between the axis of rotation and the location where the force is applied. We assume in this section that the force is applied perpendicularly to the line from the axis of rotation and the location where the force is applied. (If this description seems cryptic, look at Concept 1, where the force is being applied in this manner.) When the force is applied as stated above, the torque equals the product of the force *F* and the distance *r*. In Equation 1, we state this as an equation. The Greek letter □ (tau) represents torque. Your practical experience should confirm that the torque increases with the amount of force and the distance from the axis of rotation. If you are trying to remove a “frozen” nut, you either push harder or you get a longer wrench so you can apply the force at a greater distance.

The location of a doorknob is another classic example of factoring in where force is applied. A torque is required to start a door rotating. The doorknob is placed far from the axis of rotation at the hinges so that the force applied to opening the door results in as much torque as possible. If you doubt this, try opening a door by pushing near its hinges. The wrench and nut scenario demonstrates another aspect of torque. The angular acceleration of the nut is due to a **net** torque. Let's say the nut in Concept 1 is stuck: the force of static friction between it and the bolt creates a torque that opposes the torque caused by the force of the hand. If the hand pushes hard enough and at a great enough distance from the nut, the torque it causes will exceed that caused by the force of static



Torque
Causes or opposes rotation
Increases with:
· amount of force
· distance from axis to point of force

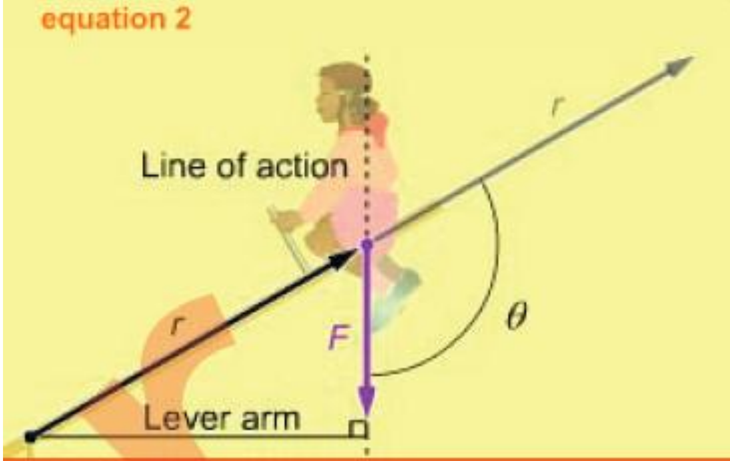


product in Equation 1.

Children are sophisticated about torques, whether they know it or not. They understand that torques can be added. For example, if two children sit on the same side of a seesaw, their torques combine to create a larger net torque than that supplied by one child alone. If they sit on opposite sides, the net torque is less than either child's torque alone. Children also learn that they can adjust the amount of torque they apply by moving toward or away from the axis of rotation. This means two children with different weights can balance each other, since both torques are a function of their weights and their distances from the axis of rotation. The heavier child slides closer to the axis, and the net torque is zero.

$\mathbf{r}$ = position vector
$\mathbf{F}$ = force
θ = angle between $\mathbf{r}$ and $\mathbf{F}$
Units: newton-meters (N·m)

equation 2

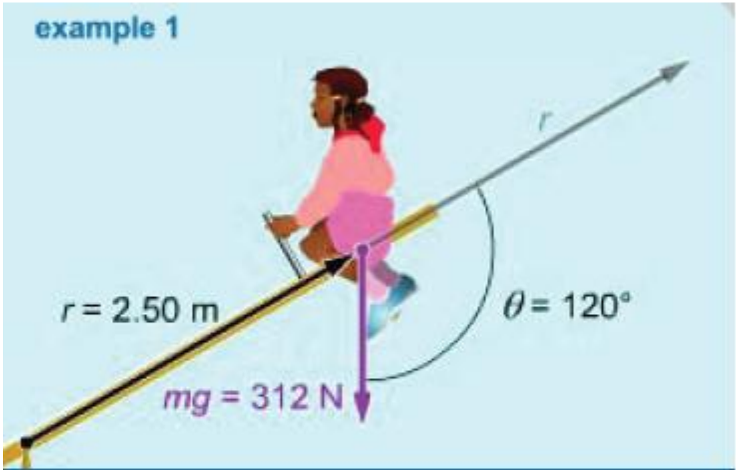


Calculating torque using lever arm

$$\tau = F\,(r \sin \theta)$$

$r \sin \theta$ = lever arm

example 1



What is the torque exerted by the girl?

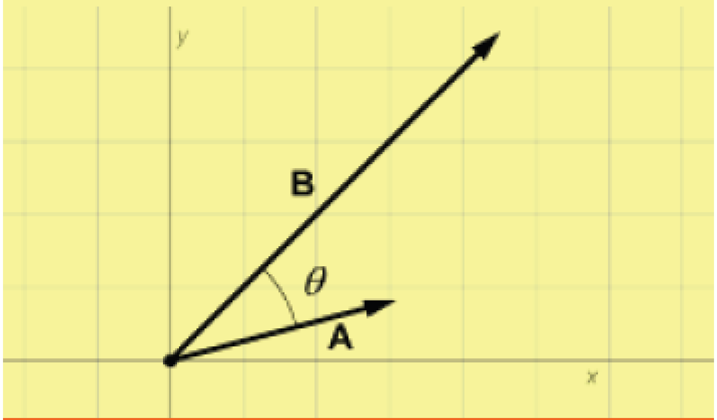
$\tau = rF \sin \theta$

$\tau = (2.50\text{ m})(312\text{ N})(\sin 120°)$

$\tau = -675\text{ N·m}$ (clockwise)

Cross product: A vector whose magnitude equals the product of the magnitudes of two vectors and the sine of the smaller angle between them. Its direction is determined by the right-hand rule.

equation 1



Magnitude of A × B

$$AB \sin \theta$$

A = magnitude of vector **A**
B = magnitude of vector **B**
θ = smaller angle between **A** and **B**

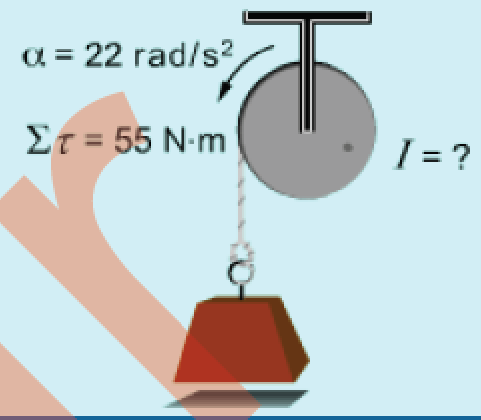
Several physics properties, including torque, are calculated using the cross product. The cross product is a way to multiply two vectors. The result is a vector that is sometimes called their *vector product*. To determine the magnitude of the cross product, multiply the product of the magnitudes of the two vectors by the sine of the angle between them. This formula is shown on the right. As the diagram shows, placing vectors tail-to-tail will allow you to determine the correct angle. The angle used is the smaller angle between the two vectors. A technique called the *right-hand rule* will help you determine the direction of the vector that results from the cross product. (Right-hand rules are also frequently used in the study of electricity and

magnetism.) How to apply the right-hand rule is shown on the right (you and your classmates may

$$\Sigma\tau = I\alpha$$

$\Sigma\tau$ = net torque
I = moment of inertia
α = angular acceleration
Units for I: kg·m^2

example 1



What is the moment of inertia of the pulley?

$\Sigma\tau = I\alpha$

$I = \Sigma\tau/\alpha$

$I = (55\ \text{N·m})/(22\ \text{rad/s}^2)$

$I = 2.5\ \text{kg·m}^2$

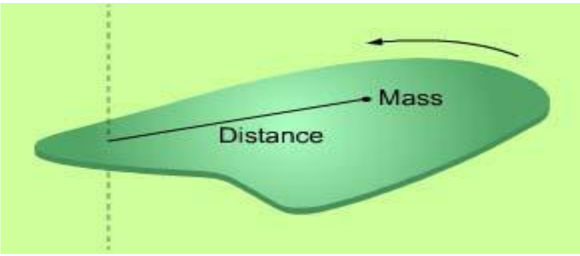
Calculating the moment of inertia

If you were asked whether the same amount of torque would cause a greater angular acceleration with a Ferris wheel or a bicycle wheel, you would likely answer: the bicycle wheel. The greater mass of the Ferris wheel means it has a greater moment of inertia. It accelerates less with a given torque. But more than the amount of mass is required to determine the moment of inertia; the distribution of the mass also matters.

Consider the case of a boy sitting on a seesaw. When he sits close to the axis of rotation, it takes a certain amount of torque to cause him to have a given rate of angular acceleration. When he sits farther away, it takes more torque to create the same rate of acceleration. Even though the boy’s (and the seesaw's) mass stays constant, he can increase the system’s moment of inertia by sitting farther away from the axis. When a rigid object or system of particles rotates about a fixed axis, each particle in the object contributes to its moment of inertia. The formula in Equation 1 to the right shows how to calculate the moment of inertia. The moment equals the sum of each particle’s mass times the square of its distance from the axis of rotation.

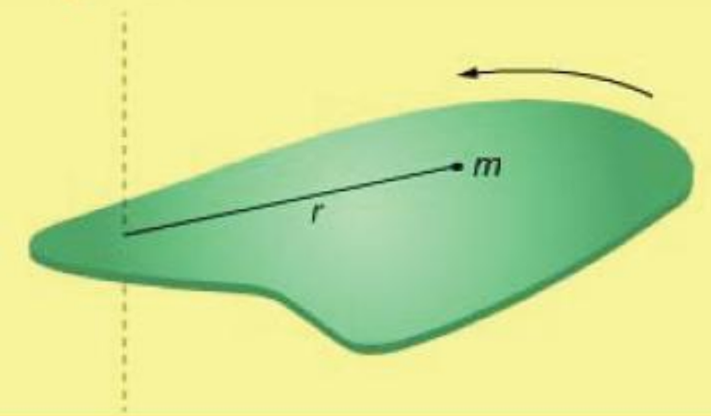
A single object often has a different moment of inertia when its axis of rotation changes. For instance, if you rotate a baton around its center, it has a smaller moment of inertia than if you rotate it around one of its ends. The baton is harder to accelerate when rotated around an end. Why is this the case? When the baton rotates around an end, more of its mass on average is farther away from the axis of rotation than when it rotates around its center. If the mass of a system is concentrated at a few points, we can calculate its moment of inertia using multiplication and addition. You see this in Example 1, where the mass of the object is concentrated in two balls at the ends of the rod. The moment of inertia of the rod is very small compared to that of the balls, and we do not include it in our calculations. We also consider each ball to be concentrated at its own center of mass when measuring its distance from the axis of rotation (marked by the ×). This is a reasonable approximation when the size of an object is small relative to its distance from the axis.

Not all situations lend themselves to such simplifications. For instance, let’s assume we want to calculate the moment of inertia of a CD spinning about its center. In this case the mass is uniformly distributed across the entire CD. In such a case, we need to use calculus to sum up the contribution that each particle of mass makes to the moment, or we must take advantage of a table that tells us the moment of inertia for a disk rotating



Moment of inertia
Sum of each particle's
· Mass times its
· Distance squared from the axis

equation 1

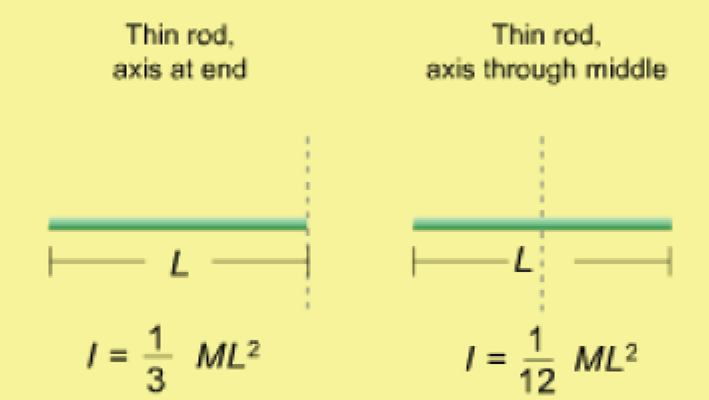


Moment of inertia

$$\boxed{I = \Sigma mr^2}$$

I = moment of inertia

equation 4

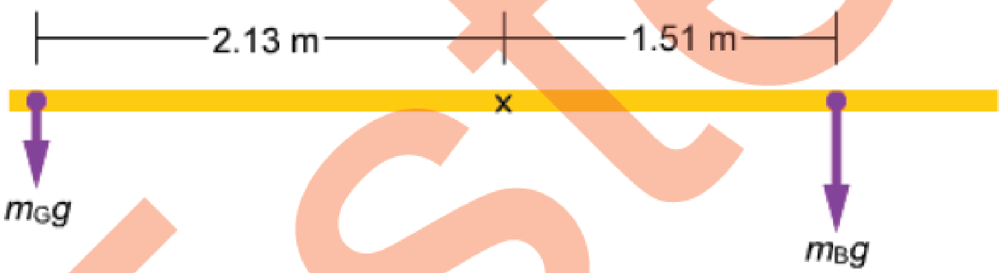


Thin rods

The seesaw plank is horizontal. Its mass is 36.5 kg, and it is 4.40 m long. What is the initial angular acceleration of this system?

The axis of rotation is the point where the fulcrum touches the midpoint of the plank. The plank itself creates no net torque since it is balanced at its middle. For every particle at a given distance from the axis that creates a clockwise torque, there is a matching particle at the same distance creating a counterclockwise torque. However, the plank does factor into the moment of inertia.

Draw a diagram



Variables

mass of seesaw plank	$m_S = 36.5$ kg
seesaw plank's moment of inertia	I_S

	girl	boy
mass	$m_G = 42.8$ kg	$m_B = 55.2$ kg
distance from axis	$r_G = 2.13$ m	$r_B = 1.51$ m
moment of inertia	I_G	I_B

What is the strategy?

1. Calculate the moment of inertia of the system: the sum of the moments for the children, and the moment of the plank.

2. Calculate the net torque by summing the torques created by each child. The torques of the left and right sides of the plank cancel, so you do not have to consider them.

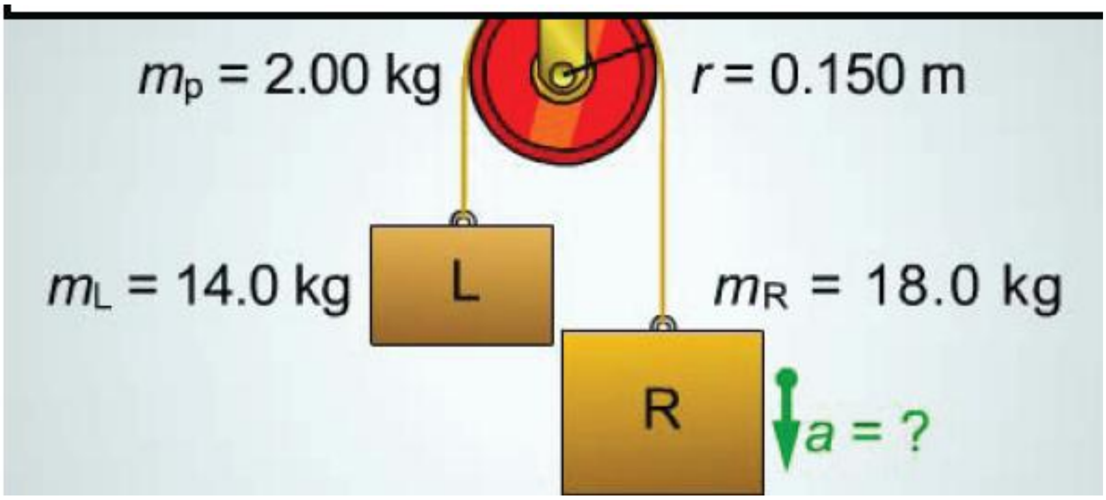
3. Divide the net torque by the moment of inertia to determine the initial angular acceleration.

Physics principles and equations

We will use the definitions of torque and moment of inertia.

□ = *rF* sin □, *I* = □*mr*2

To calculate the moments of inertia of the children, we consider the mass of each to be concentrated at one point. The plank can be considered as a slab rotating on an axis parallel to an edge through the center, with moment of inertia



What is the magnitude of the acceleration of the block on the right?

Here we consider an Atwood machine, factoring in the moment of inertia of the pulley. We will model the pulley as a uniform solid disk. We still assume that the rope is massless and does not stretch and that the pulley is frictionless. The blocks' accelerations are equal and opposite, but the tension exerted on each block by the rope is different because of the pulley’s moment of inertia. We will use the convention that downward acceleration and force are negative, and upward acceleration and force are positive. We know the block on the right will accelerate downward because it is more massive than the block on the left.

Variables

	Block L	Block R
tension	T_L	T_R
mass	$m_L = 14.0$ kg	$m_R = 18.0$ kg
weight	$-m_L g$	$-m_R g$
acceleration	a	$-a$

	Pulley
torque from left block	τ_L
torque from right block	τ_R
mass	$m_P = 2.00$ kg
radius	$r = 0.150$ m
moment of inertia	I
angular acceleration	α

What is the strategy?

1. Calculate the net force on each block. The forces are strictly vertical. Use Newton's second law for each block to find expressions for the tension force in the rope on each side of the pulley.
2. Calculate the net torque acting on the pulley. The forces that create torques on the pulley are the tensions of the rope. The tension forces are perpendicular to the radius of the pulley at the points where they act. Use Newton's second law for rotation to set $I\alpha$ equal to this net torque.
3. Use the definition of angular acceleration to introduce the linear acceleration of the masses into the previously derived equation, and solve for the linear acceleration.
4. Use the expression for the moment of inertia of a solid disk as the moment of inertia of the pulley, and use the values given in the problem to compute the acceleration.

Physics principles and equations

Newton's second law

$$\Sigma F = ma$$

Equations for torque

$$\tau = rF \sin \theta$$

$$\Sigma\tau = I\alpha$$

Moment of inertia of a solid disk

$$I = \tfrac{1}{2}mr^2$$

Angular acceleration

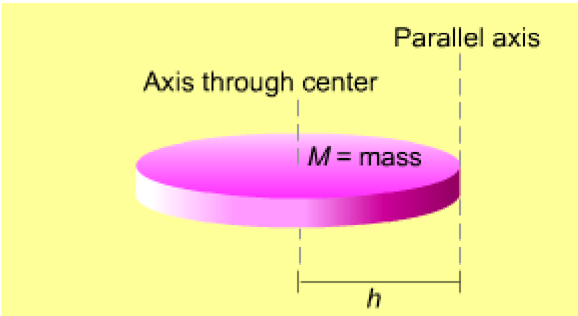
$$\alpha = a/r$$

The acceleration a is the positive magnitude of the acceleration of the blocks. Since the right block falls, this acceleration results in a clockwise

Parallel axis theorem

The parallel axis theorem is a tool for calculating the moment of inertia of an object. You use it when you know the moment of inertia for an object rotating about an axis that passes through its center of mass and want to know the moment when it rotates around a different but parallel axis of rotation.

The illustration for Equation 1 shows two such parallel axes. The axis on the left passes through the center of mass of a cylindrical disk, the other is at the edge of the disk. The theorem states that the moment of inertia when the disk rotates about the axis on its edge will be the sum of two values: the moment of inertia when the disk rotates about its center of mass, and the product of the disk's mass and the square of the distance between the two axes (shown as *h* in our diagram). This is stated as an equation to the right. The usefulness of the parallel axis theorem lies in this fact: It is usually much easier to calculate the moment of inertia of an object around an axis through its center of mass than around an off-center axis. For example, if we must use an integral to calculate the moment of inertia, doing so around the center of mass lets us more readily take advantage of any symmetry of the object. The parallel axis theorem can then be used to find the moment of inertia around another parallel axis. In sum, the parallel axis theorem lets us use an easier integral and some algebra to calculate the moment for the parallel axis. The disk to the right has a moment of inertia of ½*MR*2 when it rotates about its center. We can use this formula as the starting point in our calculation of the disk's moment of inertia when it is rotated about an axis at its edge. You see this computation worked out

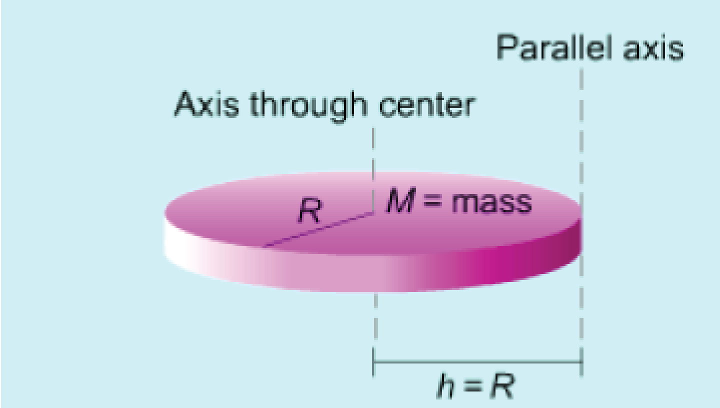


Parallel axis theorem

$$I_P = I_{CM} + Mh^2$$

I_P = moment of inertia, parallel axis
I_{CM} = moment, center of mass axis
M = mass
h = distance between axes

example 1



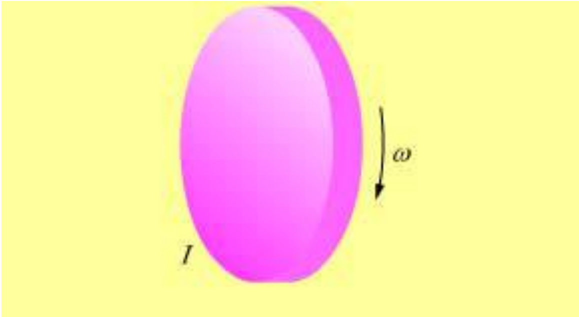
What is the moment of inertia of a disk when it rotates about the parallel axis shown above?

$I_P = I_{CM} + Mh^2$

$I_P = ½\ MR^2 + MR^2$

Rotational kinetic energy

The equation on the right enables you to calculate the rotational kinetic energy (*KE*) of a rigid, rotating object, the kinetic energy of an object due to its rotational motion. It is analogous to the equation for linear kinetic energy. The rotational *KE* equals one-half the moment of inertia times the square of the angular velocity. This equation can be derived from the definition of linear kinetic energy. The rotating object consists of a large number of individual particles, each moving at a different linear (tangential) velocity. The kinetic energies of all the particles can be added to determine the kinetic energy of the entire object. To derive the equation to the right, the key insight is to see that the distance from the axis of rotation figures both in a particle's tangential velocity and in calculating its contribution to the disk's moment of inertia. In the derivation, we start with a particle of mass *m* situated somewhere in a rigid object, as shown in the second illustration to the right. We derive the equation by first calculating the kinetic energy of the single particle. The total *KE* is the sum of the kinetic energies of all the particles.



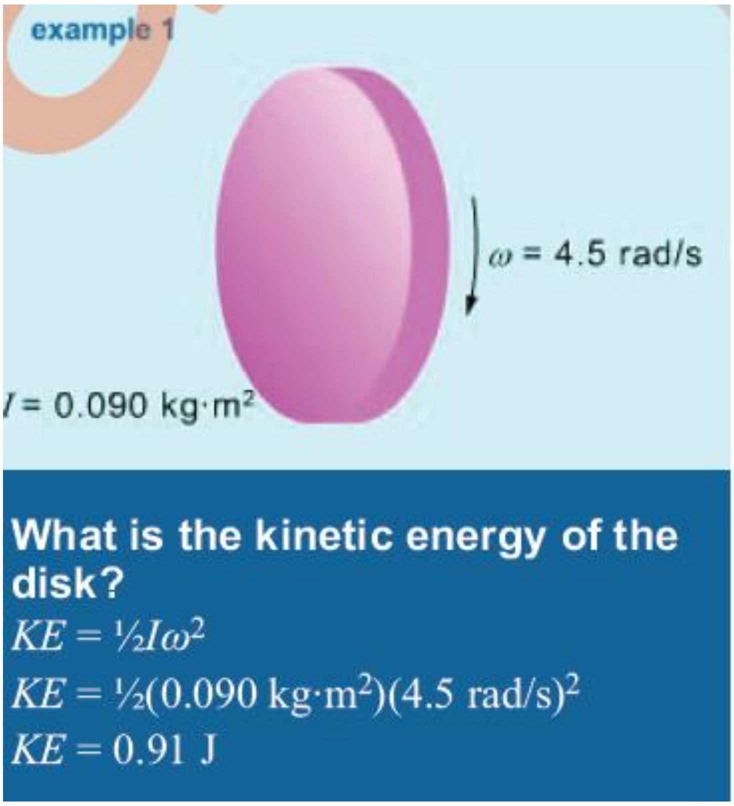
Rotational kinetic energy

$$KE = \tfrac{1}{2}I\omega^2$$

KE = rotational kinetic energy
I = moment of inertia
ω = angular velocity

Variables

mass of a particle on rotating object	m
linear speed of particle	v
distance of particle from axis of rotation	r
angular velocity of rotating object	ω
kinetic energy of particle	KE_p
kinetic energy of object	$KE = \Sigma KE_p$
moment of inertia of particle	I_p
moment of inertia of object	$I = \Sigma I_p$



Strategy

1. State the equation for the linear KE of a particle in terms of its speed. Restate the equation in rotational terms and simplify.
2. Rewrite the expression for the KE of a particle in terms of its moment of inertia.
3. Sum the individual kinetic energies of all the particles to derive the desired equation.

Physics principles and equations

The equation for the kinetic energy of a particle

$$KE = ½mv^2$$

The relationship between linear speed and angular velocity is

$$v = \omega r$$

The moment of inertia of a single particle of mass m at distance r from the axis of rotation

$$I = mr^2$$

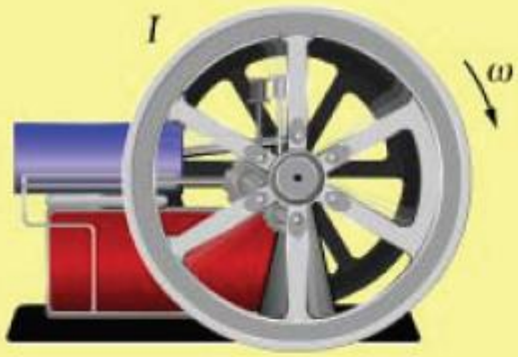
Step-by-step derivation

For a particle on a rotating object, its linear speed as it moves along a circular path is its tangential speed. We use the definition of kinetic energy and the relation between linear and angular speed to write the kinetic energy of a particle in rotational terms.

Step	Reason
1. $KE_p = \frac{1}{2}mv^2$	definition of kinetic energy
2. $v = \omega r$	linear speed and angular velocity
3. $KE_p = \frac{1}{2}m(\omega^2 r^2)$	substitute equation 2 into equation 1
4. $KE_p = \frac{1}{2}(mr^2)\omega^2$	rearrange

Serve as mechanical batteries

equation 1



Flywheel energy

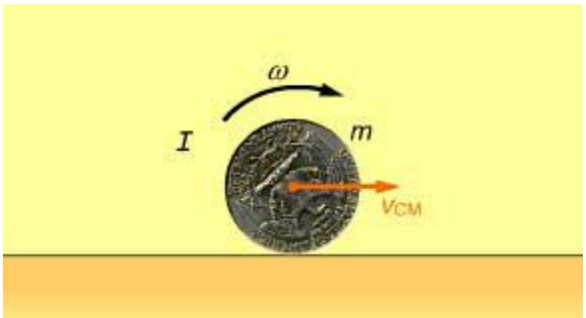
$$KE = \frac{1}{2}I\omega^2$$

KE = kinetic energy (rotational)
I = moment of inertia
ω = angular velocity

Rolling objects and kinetic energy

The coin shown in Equation 1 rolls without slipping. That is, it rotates and moves linearly as it travels to the right. Its total kinetic energy is the sum of its linear and

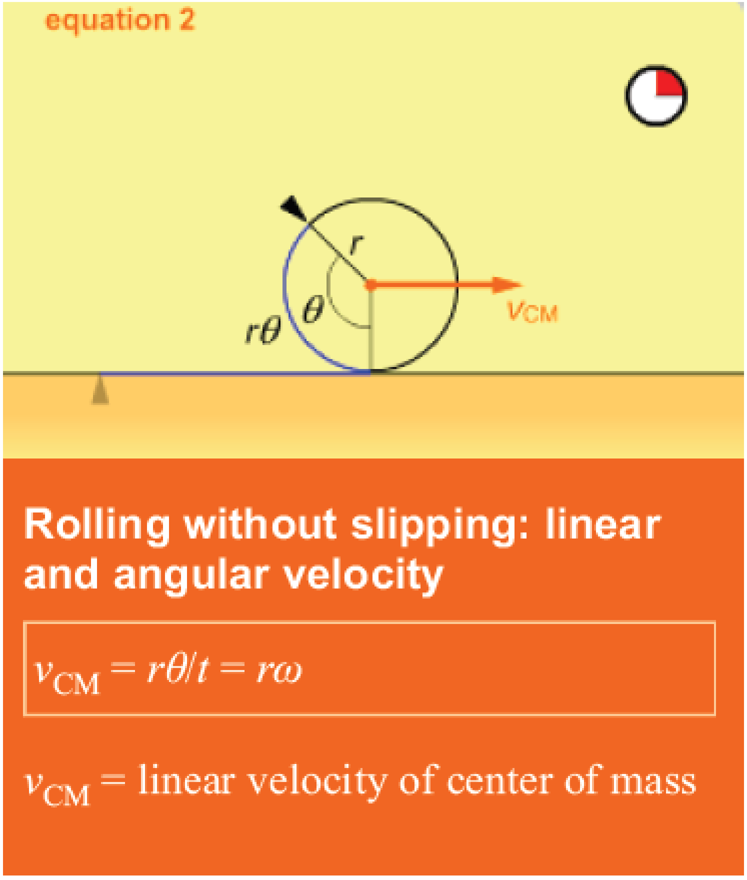
rotational kinetic energies. We can use two equations discussed earlier to determine the coin's total kinetic energy. The coin's rotational kinetic energy equals ½ *I*□2. We measure □ with respect to the coin’s axis of rotation, perpendicular to the center of the coin. Its linear kinetic energy equals ½ *mv*CM
2. The “CM” subscript indicates that the point used in calculating the linear speed is the coin's center of mass. As Equation 1 shows, the sum of these two types of kinetic energy equals the total kinetic energy. When an object rolls without slipping, it is often useful know the relationship between its linear and angular velocities. Equation 2 shows this relationship. As the rolling object with radius *r* rotates through an angle □, an arc of length *r*□ makes contact with the ground. This means the object moves linearly the same distance *r*□. Its linear speed *v*CM is that distance divided by *t*, or *r*□/*t*. Since □/*t* equals □, we can also say that *v*CM equals *r*□. The kinetic energy equation and the relationship in Equation 2 are both used to solve the example problem.



Kinetic energy of rolling object

$$\boxed{KE = \tfrac{1}{2}mv_{CM}^2 + \tfrac{1}{2}I\omega^2}$$

m = mass
v_{CM} = velocity of center of mass
I = moment of inertia
ω = angular velocity



Variables

The radius of the ball is not stated in the problem. It will cancel out and not be a factor in the answer.

initial (potential) energy	$PE = 0.75$ J
mass	$m = 0.10$ kg
radius	r
angular velocity	ω
speed of center of mass	v_{CM}
moment of inertia	I

What is the strategy?

1. Use the equation for the moment of inertia of a hollow sphere to write an expression for the rotational kinetic energy at any instant in terms of mass, radius and angular velocity.

2. Write an expression for the linear kinetic energy at any instant, also in terms of mass, radius and angular velocity.

3. Find the ratio of the two kinetic energies. This does not change as the ball moves.

4. The total energy at the top of the ramp is all potential energy. At the bottom of the ramp, all the energy is kinetic energy. Distribute the total energy between linear and rotational kinetic energy, according to the constant ratio you just calculated.

Physics principles and equations

We use the conservation of energy. In this case, all the energy at the top of the ramp is potential, and all the energy at the bottom is kinetic.

PE (top) = *KE*L + *KE*R (bottom)

The equations for linear and rotational kinetic energy

*KE*L = ½ *mv*CM 2, *KE*R = ½ *I*□2

The relationship between the linear velocity of the ball's center of mass and its angular velocity

*v*CM = *r*□

The moment of inertia of a hollow sphere

$$I = \frac{2}{3}mr^2$$

Step-by-step solution

We use the moment of inertia of a hollow sphere to find an expression for the rotational kinetic energy.

Step	Reason
1. $KE_R = ½ I\omega^2$	rotational kinetic energy
2. $I = 2mr^2/3$	moment of inertia
3. $KE_R = ½(2mr^2/3)\omega^2$	substitute equation 2 into equation 1
4. $KE_R = mr^2\omega^2/3$	simplify

Now we write an expression for the linear kinetic energy.

Step	Reason
5. $KE_L = ½\, mv_{CM}{}^2$	linear kinetic energy
6. $v_{CM} = r\omega$	rolling without slipping
7. $KE_L = ½\, m(r\omega)^2$	substitute equation 6 into equation 5
8. $KE_L = mr^2\omega^2/2$	simplify

The expressions for kinetic energy in steps 4 and 8 are the same except for a constant factor. We can write the linear kinetic energy as a constant times the rotational kinetic energy.

Step	Reason
9. $KE_L/KE_R = (mr^2\omega^2/2)/(\,mr^2\omega^2/3)$	divide equation 8 by equation 4
10. $KE_L = 1.5KE_R$	simplify

$$KE = ½mv^2 + ½I\omega^2$$

Moments of inertia

$I = mr^2$ (hollow cylinder)

$I = ½mr^2$ (solid cylinder)

The relationship of the speed and angular velocity for a rolling object

$$v_{CM} = r\omega$$

We apply the principle of the conservation of energy.

Step-by-step solution

We start by finding a general equation for the speed of an object that has rolled down the ramp, starting from rest at height *h*. We define the potential energy to be zero at the bottom of the ramp.

Step	Reason
1. $E_i = E_f$	conservation of energy
2. $E_i = PE = mgh$	initial energy is all potential
3. $E_f = KE = ½mv_{CM}^2 + ½I\omega^2$	energy at bottom of ramp is all kinetic
4. $mgh = ½mv_{CM}^2 + ½I\omega^2$	substitute equations 2 and 3 into equation 1
5. $\omega = v_{CM}/r$	relation of angular velocity and speed of center of mass
6. $mgh = ½mv_{CM}^2 + ½I(v_{CM}/r)^2$	substitute equation 5 into equation 4
7. $v_{CM} = \sqrt{\dfrac{2mgh}{\left(m + \dfrac{I}{r^2}\right)}}$	solve for v_{CM}

Solid cylinder. Now that we have a general equation for the speed of an object at the bottom of the ramp, we can apply the moment of inertia formulas to find equations for the speeds of the cylinders. We start with the solid cylinder.

Step	Reason
8. $I_s = ½mr^2$	moment of inertia for a solid cylinder
9. $v_{CM} = \sqrt{\dfrac{2mgh}{\left(m + \dfrac{\frac{1}{2}mr^2}{r^2}\right)}}$	substitute equation 8 into equation 7
10. $v_{CM} = \sqrt{\frac{4}{3}gh}$	simplify

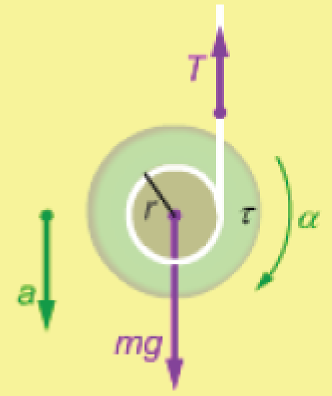
Hollow cylinder. Notice that the expression for the speed of the solid cylinder is independent of its mass and radius. This means that any solid cylinder will have the same speed as it rolls down a ramp of the same height. Next, we consider the speed of the hollow cylinder.

Step	Reason
11. $I_h = mr^2$	moment of inertia for a hollow cylinder
12. $v_{CM} = \sqrt{\dfrac{2mgh}{\left(m+\dfrac{mr^2}{r^2}\right)}}$	substitute equation 11 into equation 7
13. $v_{CM} = \sqrt{gh}$	simplify

For this reason we abandon our usual convention and take downward as the positive linear direction.

mass of yo-yo	m
acceleration of yo-yo	a
acceleration of gravity	$g = 9.80\ \text{m/s}^2$
upward force exerted by string	T
torque of string on yo-yo spindle	τ
radius of spindle	r
moment of inertia of yo-yo	I
angular acceleration of yo-yo	α

equation 1



Acceleration of descending yo-yo

$$a = \frac{g}{1 + (I/mr^2)}$$

a = acceleration of yo-yo
g = acceleration of gravity
I = moment of inertia of yo-yo
m = mass of yo-yo
r = radius of spindle

Strategy

1. Analyze the linear motion of the yo-yo using Newton's second law. The equation will contain the tension T in the yo-yo string.
2. Analyze the angular motion of the yo-yo using the rotational form of Newton's second law. This equation, too, will contain T.
3. Combine the equations for linear and rotational motion, using the common variable T, and solve for the downward acceleration of the descending yo-yo.

Physics principles and equations

We will use two versions of Newton's second law to calculate linear and angular acceleration.

$$\Sigma F = ma, \quad \Sigma\tau = I\alpha$$

For a force perpendicular to the line from the axis of rotation to the point where the force is applied, the torque is

$$\tau = rF$$

The resulting tangential acceleration is

$$a = r\alpha$$

Step-by-step derivation

In the first steps we analyze the linear motion of the yo-yo. Note that the equation in the third step contains the tension force exerted by the string.

Step	Reason
1. $\Sigma F = ma$	Newton's second law
2. $\Sigma F = mg + (-T)$	inspection
3. $mg - T = ma$	substitute equation 2 into equation 1

We now analyze the angular acceleration of the yo-yo using Newton's second law for rotation. We solve the resulting equation for the tension force exerted by the string.

Step	Reason
4. $\Sigma\tau = I\alpha$	Newton's second law for rotation
5. $\Sigma\tau = rT$	torque and tangential force
6. $\alpha = a/r$	relationship of angular and tangential acceleration
7. $rT = I(a/r)$	substitute equations 5 and 6 into equation 4
8. $T = Ia/r^2$	solve for T

Now we combine the linear and rotational analyses and solve for the linear acceleration.

Step	Reason
9. $mg - Ia/r^2 = ma$	substitute equation 8 into equation 3
10. $a = \dfrac{g}{1+(I/mr^2)}$	solve for a

Steps

First, calculate the moment of inertia of each of the cylinders making up the yo-yo. We start with the identical disks A, and then calculate the moment of the spindle B.

Step	Reason
1. $I_A = ½\, m_A r_A{}^2$	moment of inertia of solid cylinder
2. $I_A = ½(0.030 \text{ kg})(0.075 \text{ m})^2$ $I_A = 8.44 \times 10^{-5} \text{ kg} \cdot \text{m}^2$	enter values and multiply
3. $I_B = ½ m_B r_B^2$ $I_B = ½(0.0050 \text{ kg})(0.0065 \text{ m})^2$ $I_B = 1.06 \times 10^{-7} \text{ kg} \cdot \text{m}^2$	moment of inertia of spindle

Now we calculate the moment of inertia of the yo-yo.

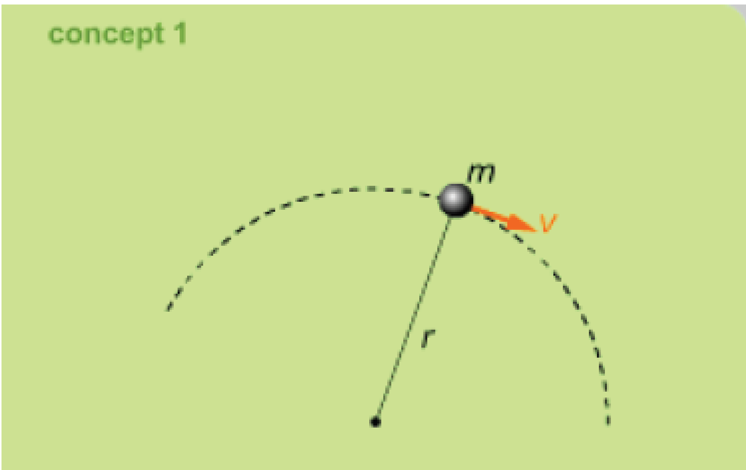
Step	Reason
4. $I = 2I_A + I_B$	yo-yo has two disks and spindle
5. $I = 2(8.44 \times 10^{-5} \text{ kg} \cdot \text{m}^2) + 1.06 \times 10^{-7} \text{ kg} \cdot \text{m}^2$ $I = 1.69 \times 10^{-4} \text{ kg} \cdot \text{m}^2$	sum of moments

Now we use the equation stated above for the acceleration of a yo-yo.

Step	Reason
6. $a = \dfrac{g}{1+(I/mr_B^2)}$	acceleration of a yo-yo
7. $m = 2m_A + m_B$ $m = 2(0.030\text{ kg}) + 0.0050\text{ kg}$ $m = 0.065\text{ kg}$	total mass
8. $a = \dfrac{9.80\text{ m/s}^2}{1+\dfrac{1.69\times10^{-4}\text{ kg}\cdot\text{m}^2}{(0.065\text{ kg})(0.0065\text{ m})^2}}$	enter values into equation 6
9. $a = 0.16\text{ m/s}^2$	evaluate

Angular momentum of particle in circle motion:

The concepts of linear momentum and conservation of linear momentum prove very useful in understanding phenomena such as collisions. *Angular momentum* is the rotational analog of linear momentum, and it too proves quite useful in certain settings. For instance, we can use the concept of angular momentum to analyze an ice skater's graceful spins.

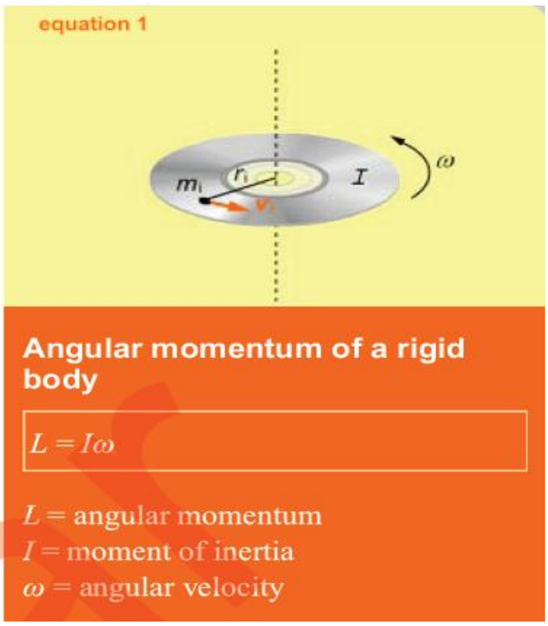


Angular momentum of a particle
Proportional to mass, speed, and distance from origin

In this section, we focus on the angular momentum of a single particle revolving in a circle. Angular momentum is always calculated using a point called the origin. With circular motion, the simple and intuitive choice for the origin is the center of the circle, and that is the point we will use here. The letter **L** represents angular momentum. As with linear momentum, angular momentum is proportional to mass and velocity. However, with rotational motion, the distance of the particle from the origin must be taken into account, as well. With circular motion, the amount of angular momentum equals the product of mass, speed and the radius of the circle: *mvr*. Another way to state the same thing is to say that the amount of angular momentum equals the linear momentum *mv* times the radius *r*. Like linear momentum, angular momentum is a vector. When the motion is counterclockwise, by convention, the vector is positive. The angular momentum of clockwise motion is negative. The units for angular momentum are kilogram-meter2 per second (kg·m2/s).

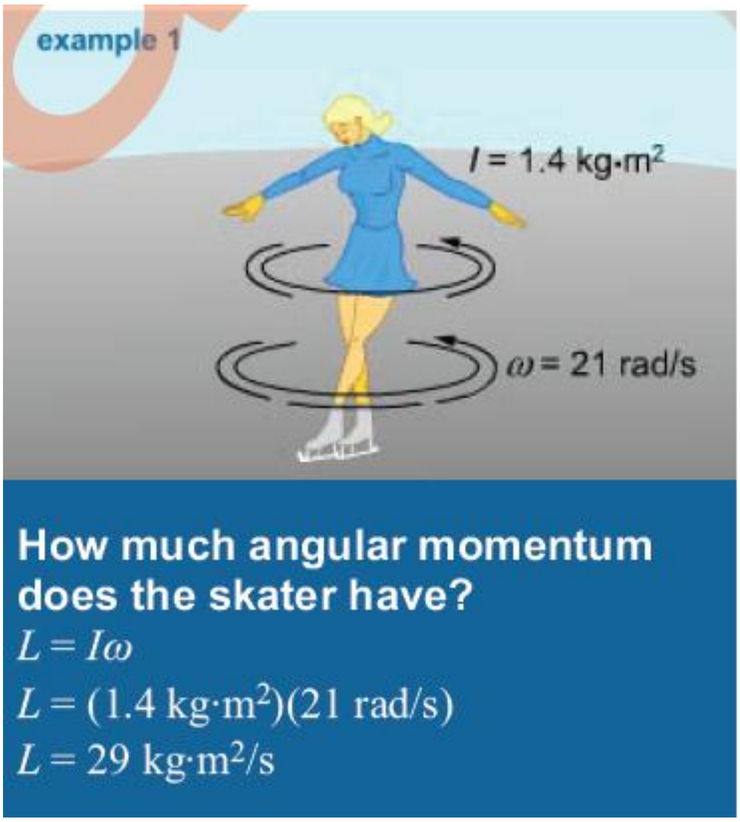
Variables

mass of a particle	m_i
tangential (linear) speed of a particle	v_i
radius of a particle	r_i
angular momentum of particle	L_i
angular momentum of CD	L
angular velocity of CD	ω
moment of inertia of CD	I



Strategy

1. Express the angular momentum of the CD as the sum of the angular momenta of all the particles of mass that compose it.
2. Replace the speed of each particle with the angular velocity of the CD times the radial distance of the particle from the axis of rotation.
3. Express the sum in concise form using the moment of inertia of the CD.



Physics principles and equations

The angular momentum of a particle in circular motion

$$L = mvr$$

We will use the equation that relates tangential speed and angular velocity.

$$v = r\omega$$

The formula for the moment of inertia of a rotating body

$$I = \Sigma m_i r_i^2$$

Step-by-step derivation

First, we express the angular momentum of the CD as the sum of the angular momenta of the particles that make it up.

Step	Reason
1. $L_i = m_i v_i r_i$	definition of angular momentum
2. $L = \Sigma m_i v_i r_i$	angular momentum of object is sum of particles

We now express the speed of the ith particle as its radius times the constant angular velocity ω, which we then factor out of the sum. The angular velocity is the same for all particles in a rigid body.

Step	Reason
3. $v_i = r_i\omega$	tangential speed and angular velocity
4. $L = \Sigma m_i(r_i\omega)r_i$	substitute equation 3 into equation 2
5. $L = (\Sigma m_i r_i^2)\omega$	factor out ω

In the final steps we express the above result concisely, replacing the sum in the last equation by the single quantity I.

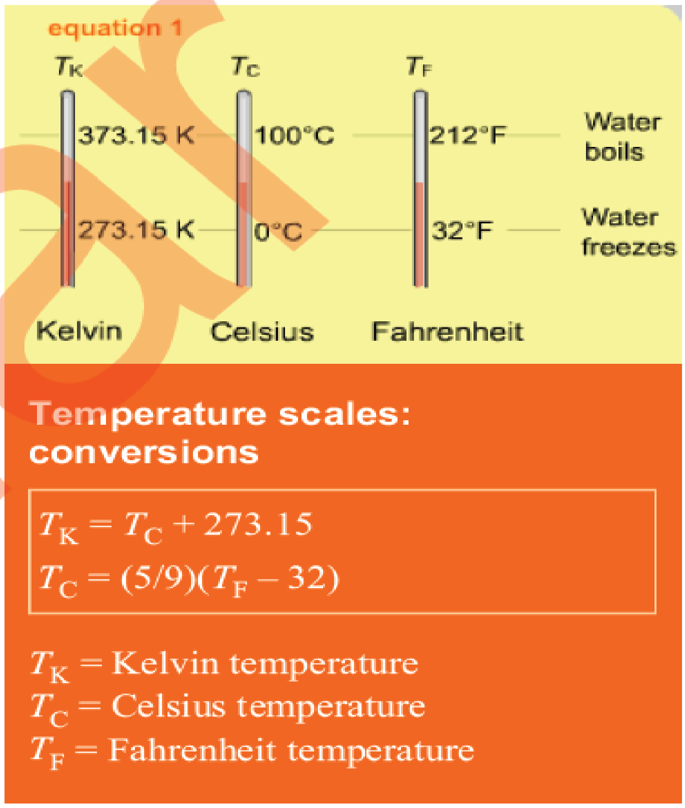
Step	Reason
6. $I = \Sigma m_i r_i^2$	moment of inertia
7. $L = I\omega$	substitute equation 6 into equation 5

Another important concept is shown in the illustration to the right: absolute zero. At this temperature, molecules (in essence) cease moving. Reaching this temperature is not theoretically possible, but temperatures quite close to this are being achieved. Absolute zero is 0 K, or □273.15°C. To standardize temperatures, scientists have agreed on a common reference point called the *triple point*. The triple point is the sole combination of pressure and temperature at which solid water (ice), liquid water, and gaseous water (water vapor) can coexist. It equals 273.16 K at a pressure of 611.73 Pa. The triple point is used to define the kelvin as an SI unit. One kelvin equals 1/273.16 of the difference between absolute zero and the triple point. If you are a sharp-eyed reader, you may have noticed the references to both 273.16 and 273.15 in this section. The freezing point of water is typically stated as 273.15 K (0°C) because

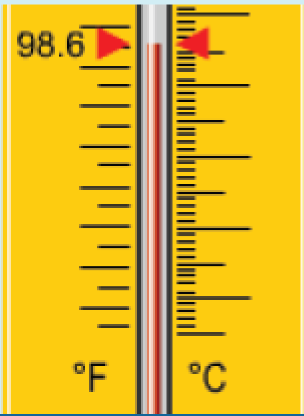
this is its value at standard atmospheric pressure, but at the triple point pressure, water freezes at 273.16 K (0.01°C).

Temperature scale conversions

Since the Celsius and Kelvin scales have the same number of units between the freezing and boiling points of water, it takes just one step to convert between the two systems, as you see in the first conversion formula in Equation 1. To convert from degrees Celsius to kelvins, add 273.15. To convert from kelvins to degrees Celsius, subtract 273.15. Since water freezes at 32° and boils at 212° in the Fahrenheit system, there are 180 degrees Fahrenheit between these points, compared to the 100 units in the Celsius and Kelvin systems. To convert from degrees Fahrenheit to degrees Celsius, first subtract 32 degrees (to establish how far the temperature is from the freezing point of water) and then multiply by 100/180, or 5/9, the ratio of the number of degrees between freezing and boiling on the two systems. That conversion is shown as the second equation in Equation 1. If you further needed to convert to kelvins, you would add 273.15. To switch from Celsius to Fahrenheit, you first multiply the number of degrees Celsius by 9/5 (the reciprocal of the ratio mentioned above) and then add 32. In Example 1, you see the conventionally normal human body temperature, 98.6°F, converted to degrees Celsius and kelvins



example 1



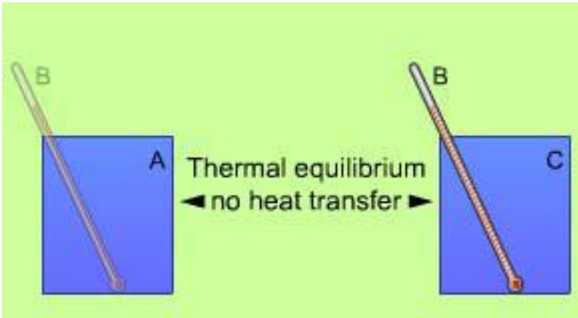
Convert 98.6°F to Celsius and Kelvin.

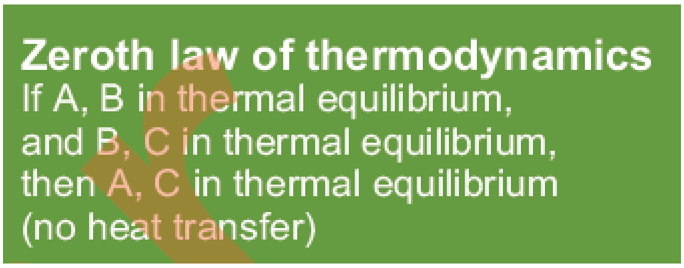
$$T_C = (5/9)(T_F - 32.0)$$
$$T_C = (5/9)(98.6°F - 32.0)$$
$$T_C = (5/9)(66.6) = 37.0°C$$
$$T_K = T_C + 273.15$$
$$T_K = 37.0 + 273.15 = 310\ K$$

Zeroth law of thermodynamics: If objects A and B are in thermal equilibrium, and objects B and C are in thermal equilibrium, then A and C will be in equilibrium as well.

When you place two objects with different temperatures next to each other, the warmer object will cool off and the cooler object will warm up. Heat will flow until the objects reach *thermal equilibrium*, meaning they have the same temperature. For instance, place a pint of ice cream in a warm car, and the result will be warmer ice cream and a cooler car. Thermometers rely on heat flowing until they reach thermal equilibrium with the substance whose temperature they are measuring. Their practical use also relies on another principle, called the zeroth law of thermodynamics. This principle states that if object A is in thermal equilibrium with object B, and object B is in equilibrium with object C, then A and C will be in equilibrium when they are placed in direct contact, and no heat will flow between them. We illustrate this law on the right. Let's say you put thermometer B in a container of water A. When the thermometer's reading stabilizes at a constant value, say 20°C, it has reached thermal equilibrium with the water. If you then place the thermometer in a second container C

and its reading remains 20°C there, you can conclude that A and C would be in thermal equilibrium when placed in direct contact with each other. They have the same temperature and heat would not flow between them. This may seem commonsense, but it is an important assumption in thermodynamics. Its importance was realized after the first and second laws of thermodynamics (which you will study later) were already codified hence it became the zeroth law, since it is an underlying assumption for the other laws.

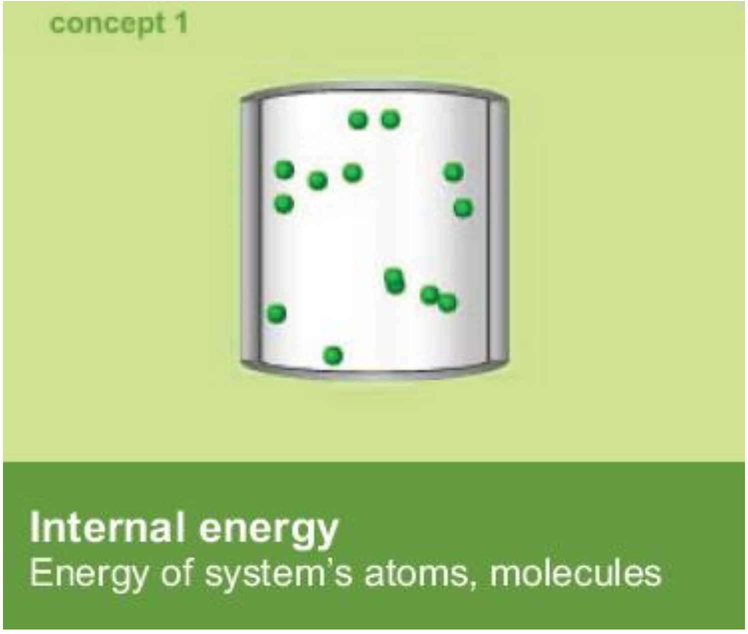




Internal energy: The energy associated with the molecules and atoms that make up a system.

In the study of mechanics, energy is an overall property of an object or system. The energy is a function of factors like how fast a car is moving, how high an object is off the ground, how fast a wheel is rotating, and so forth. In thermodynamics, the properties of the molecules and/or atoms that make up the object or system are now the focus. They also have energy, a form of energy called internal energy. The internal energy includes the rotational, translational and vibrational energy of individual molecules and atoms. It also includes the potential energy within and between molecules. To contrast the two forms of energy: If you lift a pot up from a stovetop, you will increase its gravitational potential energy. But in terms of internal energy, nothing has changed. The potential energy of the pot's molecules based on their relationship to each other has not changed. However, if you turn on the burner under the cooking pot, the flow of heat will increase the kinetic energy of its

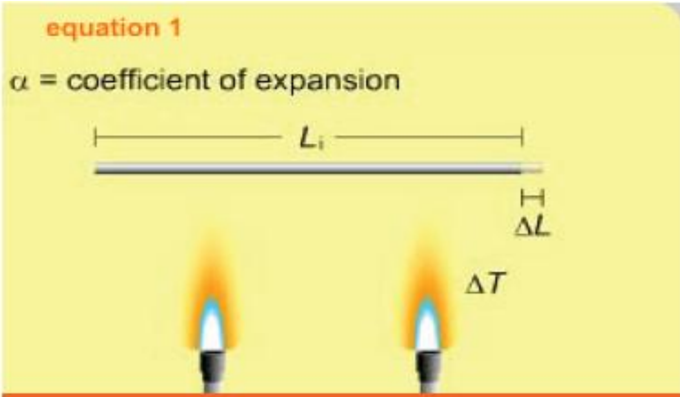
molecules. The molecules will move faster as heat flows to the pot, which means the internal energy of the molecules of the pot increases.



Thermal expansion: The increase in the length or volume of a material due to a change in its temperature.

You buy a jar of jelly at the grocery store and store it on a pantry shelf. When it comes time to open the jar, the lid refuses to budge. Fortunately, you know that placing the jar under hot water will increase your odds of being able to twist open the lid.

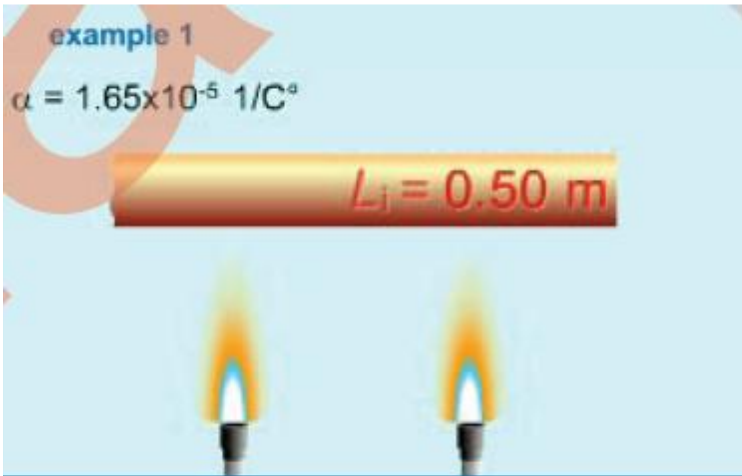
Expansion joints allow bridge sections to expand without breaking.



Linear expansion

$$\Delta L = L_i \alpha \Delta T$$

L = length
α = coefficient of linear expansion
ΔT = change in temperature
Coefficient calibrated for K or °C



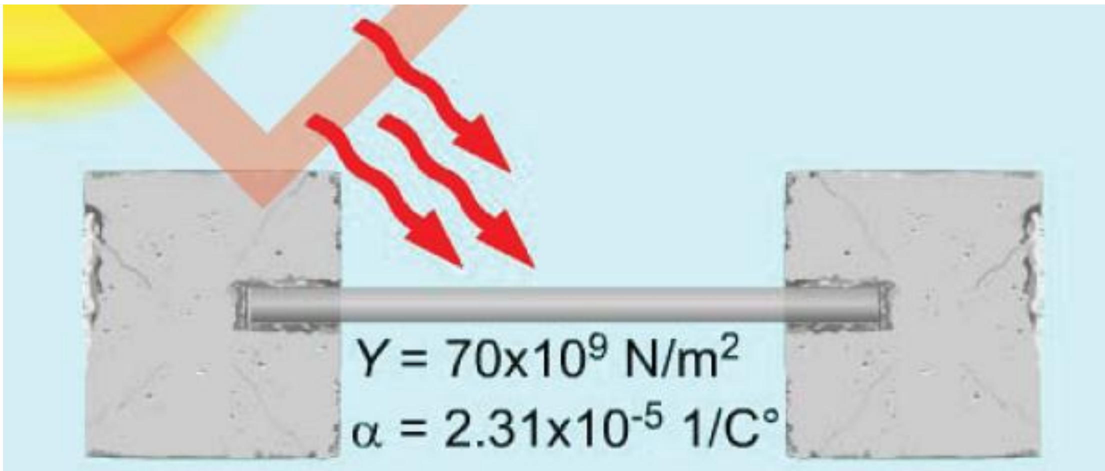
The copper rod is heated from 15°C to 95°C. What will its increase in length be?

$\Delta L = L_i \alpha \Delta T$

$\Delta T = 95°C - 15°C = 80\ C°$

$\Delta L = (0.5\ m)(1.65 \times 10^{-5}\ 1/C°)(80\ C°)$

$\Delta L = 6.6 \times 10^{-4}\ m$



What stress does the aluminum rod exert when its temperature rises 20 K?

Above, you see an aluminum rod heated by the Sun and held in place with concrete blocks. Since the rod increases in temperature, its length also increases. This exerts a force on the concrete blocks. Stress is force per unit area, and an equation for tensile stress was presented in another chapter. Young's modulus for aluminum is given; it relates the fractional increase in length (the strain) to stress. You are asked to find the stress that results from the increase in temperature.

until 4°C
From 4°C to 0°C, water expands and stays on top
Then ice forms on top and floats

of ice that insulates the water below. Water is also atypical in that its solid form, ice, is less dense than its liquid form and floats on top of it. Fish and other aquatic life can live in the relatively warm (and liquid) water below, protected by a shield of ice. If water always expanded with increasing temperature for all temperatures above 0°C, and contracted with decreasing temperature, the coldest water would sink to the bottom where it might never warm up. Water's negative coefficient of expansion in the temperature range from 0°C to 4°C is crucial to life on Earth. If ice did not float, oceans and lakes would freeze from the bottom to the top. This would increase the likelihood that they would freeze entirely, since they would not have a top layer of ice

to insulate the liquid water below and their frozen depths would not be exposed to warm air during the spring and summer.

Coefficient of volume expansion (1/C°)

Liquids		*Solids*	
Mercury	19.6×10^{-5}	Glass*	2.14×10^{-5}
Water	20.7×10^{-5}	Copper*	5.00×10^{-5}
Glycerin	50.4×10^{-5}	Silver*	5.64×10^{-5}
Olive Oil	72.0×10^{-5}	Lead*	8.37×10^{-5}
Methyl Alcohol	120×10^{-5}	Ice (-26°C)	11.3×10^{-5}
Acetone	149×10^{-5}		

* between 0 -100°C

Thermal volume expansion: Change in volume due to a change in temperature.

The equation for thermal linear expansion is used to calculate the thermally induced change in the size of an object in just one dimension. Thermal expansion or contraction also changes the volume of a material, and for liquids (and many solids) it is more useful to determine the change in volume rather than expansion along one dimension. The expansion in volume can be significant. Automobile cooling systems have tanks that capture excess coolant when the heated fluid expands so much it exceeds the radiator's capacity. A radiator and its overflow tank are shown in Concept 1 on the right. The formula in Equation 1 resembles that for linear expansion: The increase is proportional to the initial volume, a constant, and the change in temperature. The constant □ is called the *coefficient of volume expansion*. Above, you see a table of coefficients of volume expansion for some liquids and solids. The coefficients for liquids are valid for temperatures at which these substances remain liquid.

Thermal expansion: volume
Volume increases with temperature
Constant β varies by material
Increase is proportional to initial volume

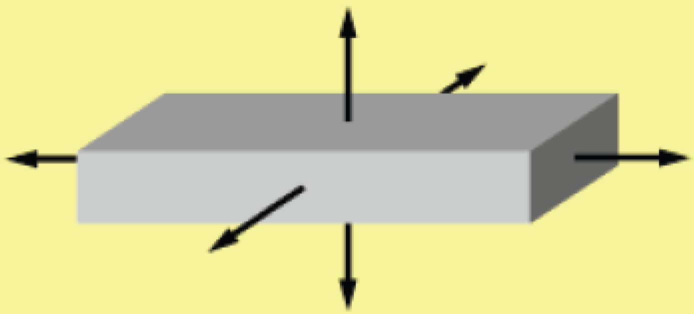
equation 1

Thermal expansion: volume

$$\Delta V = V_i \beta \Delta T$$

V = volume
β = coefficient of volume expansion
ΔT = change in temperature
Coefficient calibrated for K or °C

equation 2



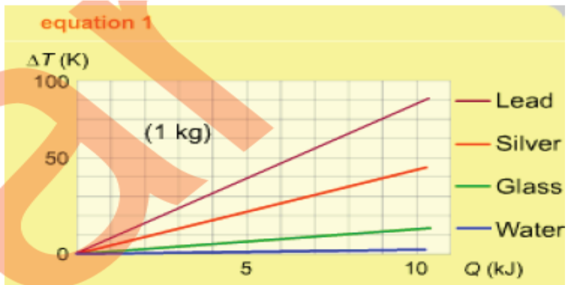
For solids

A material with a greater specific heat requires more heat per kilogram to increase its temperature a given amount than one with a lesser specific heat. In spite of its name, specific heat is not an amount of heat, but a constant relating heat, mass, and temperature change. The specific heat of a material is often used in the equation shown in Equation 1. The heat flow equals the product of a material's specific heat *c*, the mass of an object

consisting of that material, and its change in temperature. The illustration in Equation 1 shows how specific heat relates heat flow to change in temperature. As you can see from the graph, lead increases in temperature quite readily when heat flows into it, because of its low specific heat. In contrast, water, with a high specific heat, can

absorb a lot of energy without changing much in temperature. Temperatures in locations at the seaside, or having humid atmospheres, tend to change very slowly because it takes a lot of heat flow into or out of the water to accomplish a small change in temperature. Summer in the desert southwest of the United States is famous for its blazing hot days and chilly nights, while on the east coast of the country the sweltering heat of the day persists long into the night. Materials with large specific heats are sometimes informally called “heat sinks” because of their ability to store large amounts of internal energy without much temperature change. Above, you see a table of some specific heats, measured in joules per kilogram· kelvin. The specific heat of a material varies as its temperature and pressure change. The table lists specific heats for materials at 25°C to 30°C (except for ice) and 105 Pa pressure, about one atmosphere. Specific heats vary somewhat with temperature, but you can use these values over a range of temperatures you might encounter in a physics lab (or a kitchen).

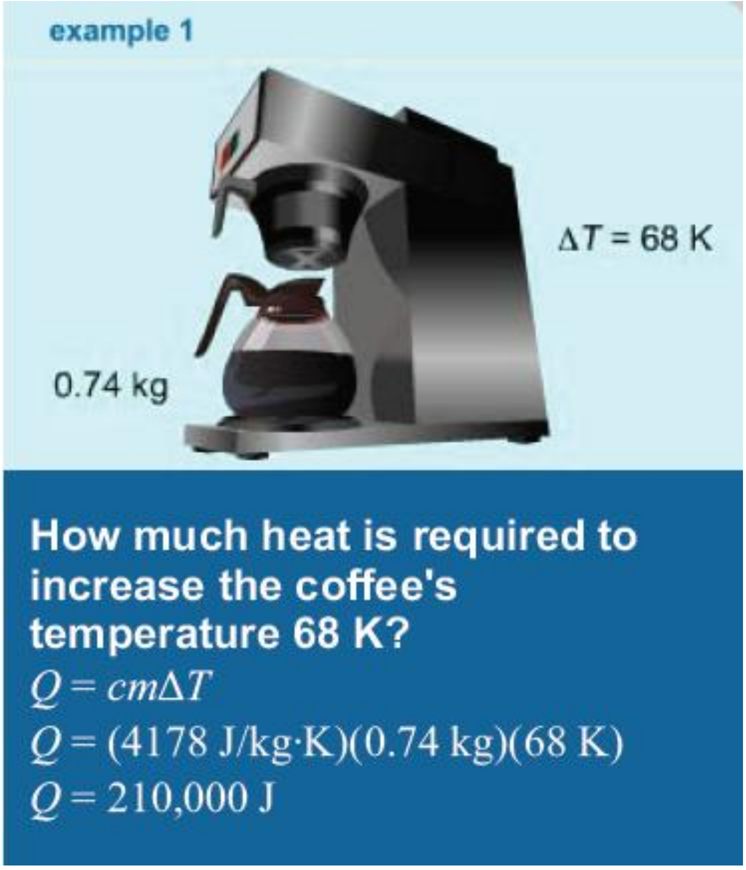
Specific heat of a material
Relates heat and temperature change, per kilogram



Specific heat of a material

$$\boxed{Q = cm\Delta T}$$

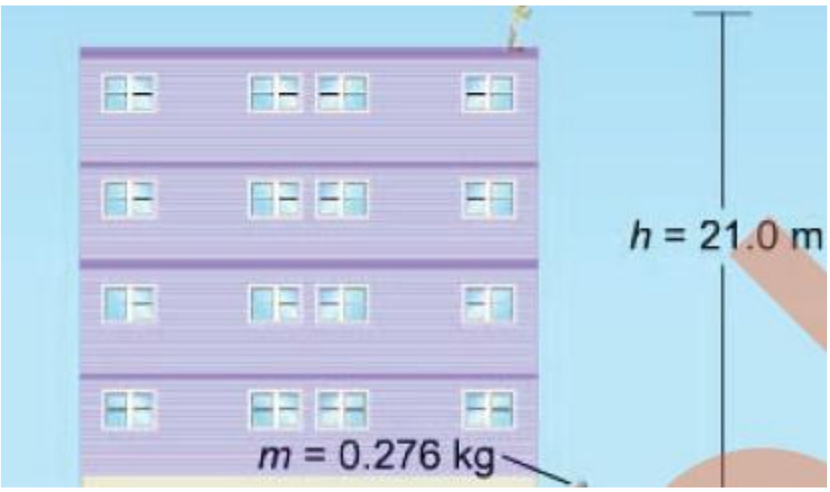
Q = heat
c = specific heat (J/kg·K)
m = mass
ΔT = temperature change in C° or K



Now we solve the equation for the unknown specific heat of the object and evaluate.

Step	Reason
6. $c_o = -\dfrac{c_w m_w (T_f - T_w)}{m_o (T_f - T_o)}$	solve for specific heat of object
7. $c_o = -\dfrac{(4178\ \mathrm{J/kg \cdot K})(0.744\ \mathrm{kg})(25.6\,°\mathrm{C} - 23.2\,°\mathrm{C})}{(0.197\ \mathrm{kg})(25.6\,°\mathrm{C} - 67.8\,°\mathrm{C})}$	substitute
8. $c_o = 897\ \mathrm{J/kg{\cdot}K}$	evaluate

Based on the values in the table of specific heats, it appears that the material may consist of aluminum.

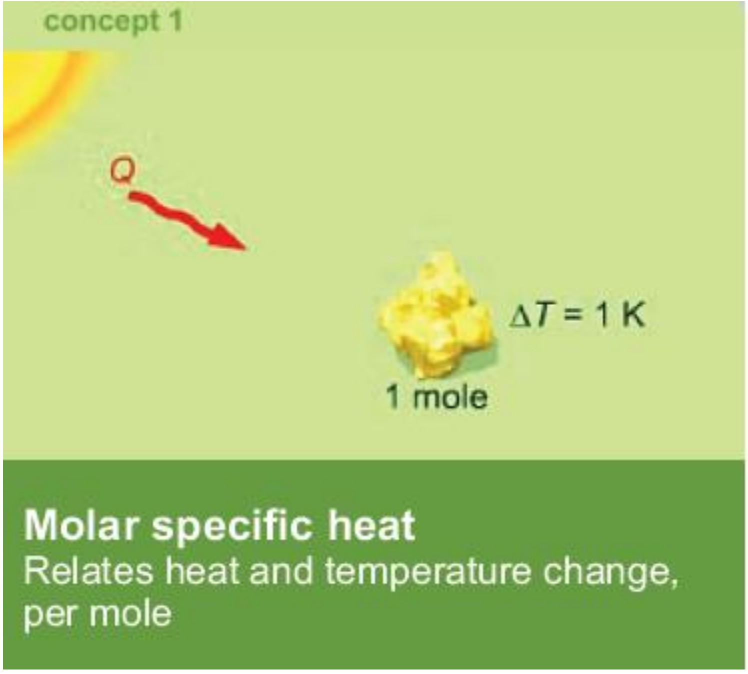


A 0.276 kg lead ball falls from a height of 21.0 m and lands on the ground without bouncing. Assume that half the energy generated by the impact of the ball with the ground becomes internal energy in the ball. The specific heat of lead is 129 J/kg·K. What is the temperature change of the ball?

Molar specific heat: A proportionality constant that relates the amount of heat flow per mole to a material's change in temperature.

	Molar specific heat (J/mol·K)
Aluminum	24.20
Copper	24.44
Iron	25.10
Silver	25.35
Lead	26.65
	at 10^5 Pa, 25°C

Scientists find it convenient at times to measure substances in terms of moles. If you have studied chemistry, you probably studied moles. Briefly, one mole of a substance contains 6.022×10^{23} particles (typically molecules; moles are explained in more depth later). Measuring in moles focuses particularly on the number of molecules in an object instead of its mass.



Specific heat and molar specific heat are both proportionality constants, relating the heat transfer per an amount of a material to the resulting change in temperature. Specific heat is stated in terms of joules per kilogram, and molar specific heat in terms of joules per mole.

A material's molar specific heat is determined by how many joules are required to heat one mole of the substance one kelvin. A material with a greater molar specific heat requires more heat per mole to produce a given change in temperature than a material with a lesser molar specific heat. This is quantified in Equation 1. The table above lists the molar specific heats of some metals at room temperature. Measuring specific heat in terms of moles reveals an interesting fact: The values do not vary much. In fact, the molar specific heats of all solids approach a value of about 25 J/mol·K as their temperatures increase. When they turn into liquids or gases, their molar specific heats change. This consistency means that the differences in specific heat values for solids (when measuring by kilograms) are due mainly to the number of molecules contained in a kilogram, rather than differences in the properties of the solids.

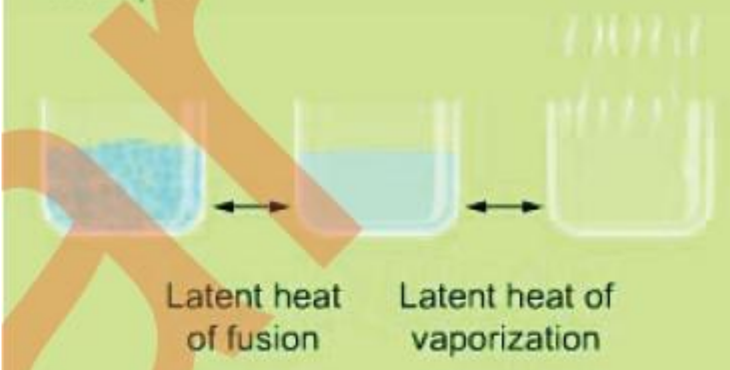
Latent heat: Energy required per kilogram to cause a phase change in a given material.

Heat flow can cause a substance to change phases by converting it between a solid and a liquid, or a liquid and a gas. Latent heat describes how much energy per

kilogram is required for a given substance to change phase. It is a proportionality constant, expressing the relationship between heat and mass as shown in Equation 1. The constant depends on the material and on the phase change. Different amounts of energy are required to transform a material between its liquid and solid states than between its liquid and gaseous states. The *latent heat of vaporization* is the amount of heat per kilogram consumed when a given substance transforms from a liquid into a gas, or released when the substance transforms from a gas back to a liquid. The *latent heat of fusion* is the heat flow per kilogram during a change in phase between a solid and a liquid. The table above shows the latent heats of fusion and vaporization for various substances. For instance, you need 3.34×105 J of energy to convert a kilogram of ice (at 0°C) to liquid water. Continued flow of heat into the water will raise its temperature until it reaches 100°C. At this temperature, it will take 2.26×106 joules of heat to turn it into a gas, about seven times as much as it took to convert it to a liquid. Salt causes ice to melt, a phenomenon called “freezing point depression.” When you add rock salt to the crushed ice in a hand-cranked ice cream freezer, you force the ice to melt. Heat flows from the resulting saltwater solution into the ice as it changes phase from solid to liquid, resulting in a slurry having a temperature far colder than 0°C. Heat then flows from the ice cream solution into this mixture, and the ice cream freezes

	Melting point (°C)	Latent heat of fusion (J/kg)	Boiling point (°C)	Latent heat of vaporization (J/kg)
Aluminum	660	3.97×10^5	2519	1.09×10^7
Carbon	4489	9.74×10^6		
Copper	1085	2.09×10^5		
Iron	1538	2.47×10^5		
Lead	327	2.30×10^4	1749	8.66×10^5
Mercury	-39	1.14×10^4	357	2.95×10^5
Nitrogen	-210	2.53×10^4	-196	1.99×10^5
Table salt	801	3.78×10^5		
Water	0	3.34×10^5	100	2.26×10^6
			at standard pressure	

concept 1



Latent heat

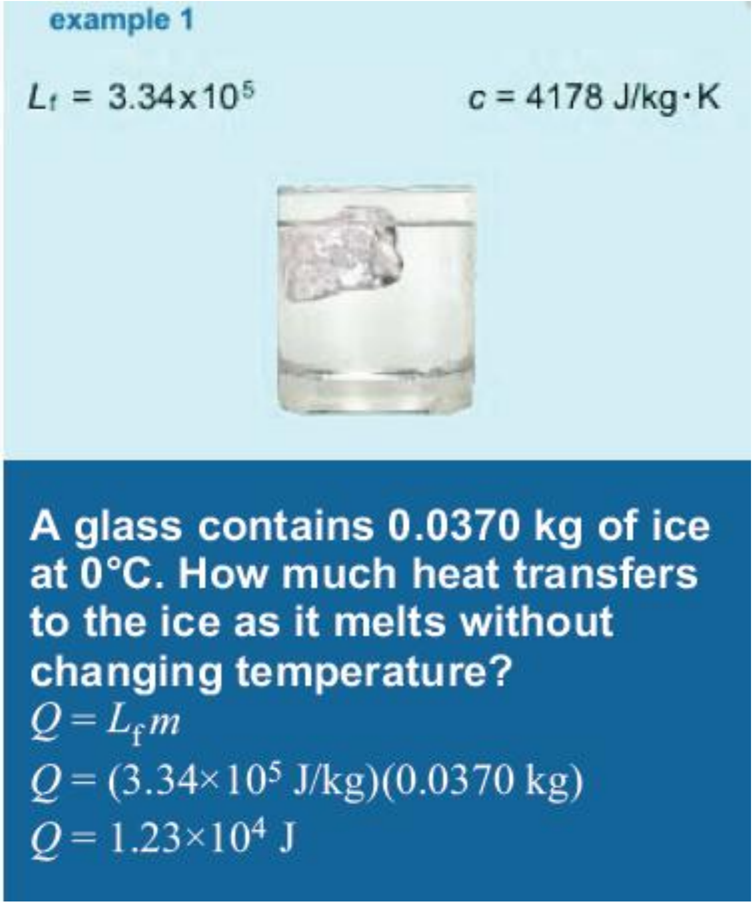
Energy required per kg to change state
Latent heat of fusion: solid to liquid
Latent heat of vaporization: liquid to gas
Amount same in either "direction"

equation 1

Heat required for phase changes

$$Q = L_f m$$
$$Q = L_v m$$

Q = heat
m = mass
L_f = latent heat of fusion (J/kg)
L_v = latent heat of vaporization (J/kg)



Step-by-step solution

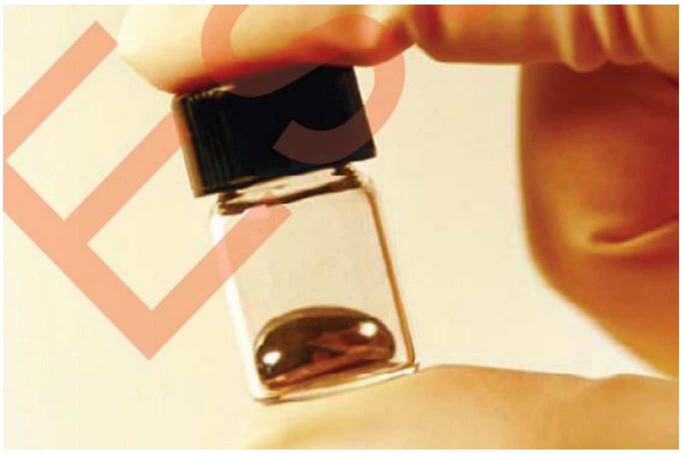
First we calculate the temperature of the liquid water after it gives up heat to melt the ice. We use the specific heat of water, 4178 J/kg·K.

Step	Reason
1. $Q = cm\Delta T$	specific heat equation
2. $-1.23 \times 10^4 \text{ J} = (4178 \text{ J/kg} \cdot \text{K})(0.160 \text{ kg})\Delta T$	substitute values
3. $\Delta T = -18.4 \text{ K} = -18.4 \text{ C°}$	solve
4. $T_{\text{Lf}} = T_{\text{Li}} + \Delta T$ $T_{\text{Lf}} = 30.0°\text{C} + (-18.4°\text{C})$ $T_{\text{Lf}} = 21.6°\text{C}$	calculate water temperature

Now we use the fact that the heat transfers sum to zero as the two masses of water reach thermal equilibrium to calculate the final temperature of the total mass of water.

Step	Reason
5. $Q_L + Q_S = 0$	equation above
6. $cm_L(T - T_{Lf}) + cm_S(T - T_{Sf}) = 0$	specific heat equation
7. $T = \dfrac{m_L T_{Lf} + m_S T_{Sf}}{m_L + m_S}$	solve for T
8. $T = \dfrac{(0.160\ \text{kg})\ (21.6\,°\text{C}) + (0.0370\ \text{kg})\ (0.00\,°\text{C})}{0.160\ \text{kg} + 0.0370\ \ \text{kg}}$	substitute values
9. $T = 17.5°\text{C}$	evaluate

Interactive checkpoint: vaporizing mercury



A vial of 0.0500 kg of mercury is at room temperature (20.0°C). What amount of heat must be transferred to the mercury in order to vaporize it? Mercury boils at 357°C, its specific heat is 140 J/kg·K (assume this is constant over the temperature range of interest) and its latent heat of vaporization is 2.95×10^5 J/kg.

Conduction: The flow of thermal energy directly through a material without motion of the material itself.

When a frying pan is placed on a burner, heat flows from the burner to the pan. The heat then spreads through the pan, soon reaching the handle even though the handle is not in direct contact with the burner. This process illustrates the flow of thermal

energy via conduction. effective insulators and can be combined with other reasonably good insulators such as wood for even greater energy efficiency. Third, materials can be combined. Double-paned windows trap a quantity of an inert gas like argon between two layers of glass. Argon has a high *R* value and considerably reduces the rate of heat transfer through the window.

$$P_c = \frac{kA\Delta T}{L}$$

k = thermal conductivity
A = area
ΔT = temperature difference
L = thickness
k units: J/s·m·K = W/m·K

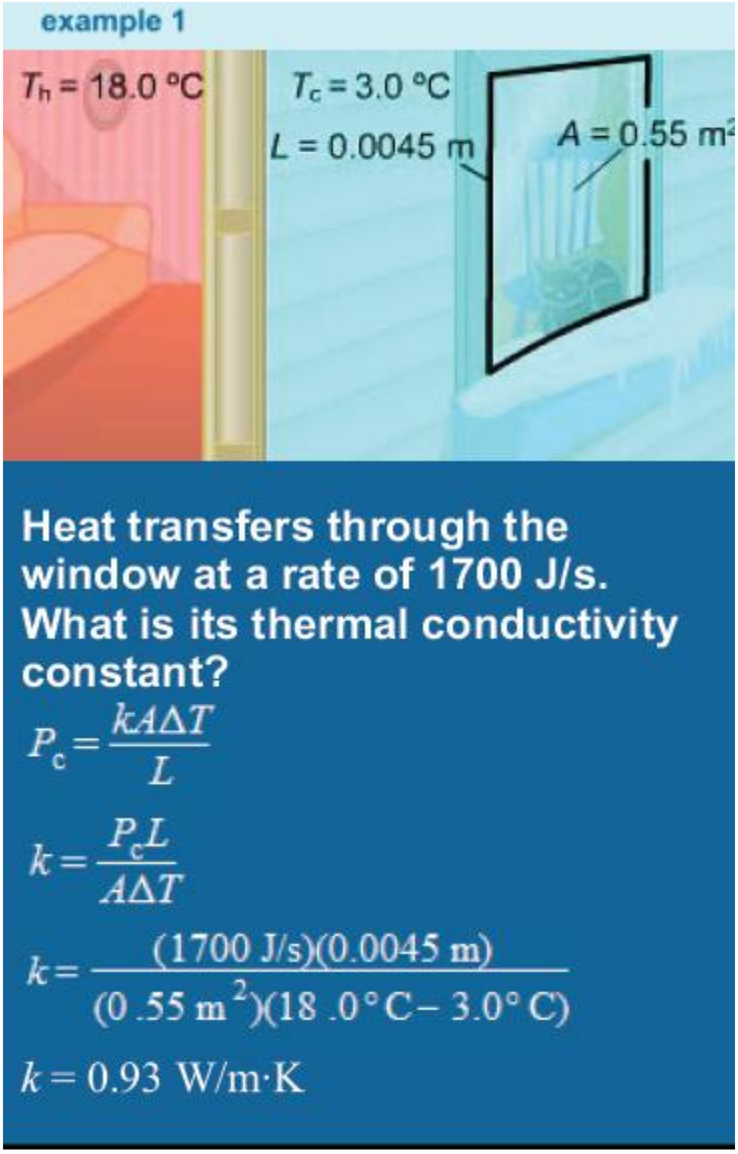
equation 3

Thermal resistance

$$R = L/k$$

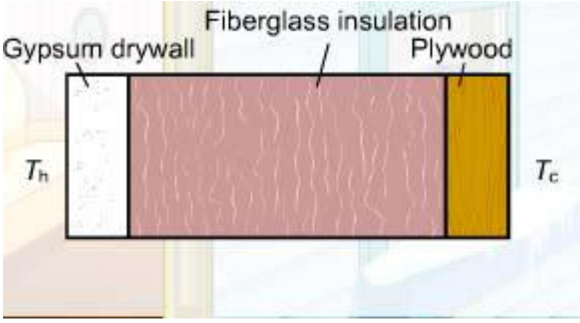
$$P_c = \frac{A\Delta T}{R}$$

R = thermal resistance
R units (SI): m^2·K/W
R units (British): ft^2·F°·h/Btu



Conduction through composite objects

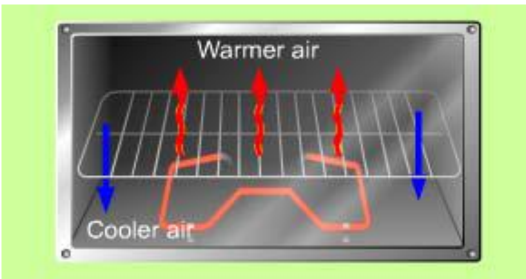
Real-world objects such as the walls of a house are often a composite of different materials. For example, a house wall may consist of gypsum drywall, fiberglass insulation and plywood. At the right, you see a schematic of a wall made of materials of varying thicknesses. To calculate the rate of heat flow through this composite object, the overall thermal resistance is calculated by summing the resistance of each object. This value can be used as the R-value of a single object in other equations. You see this in Equation 1 on the right. When designing buildings, the rate at which heat will flow through the walls is an important consideration. Example 1 shows a calculation of the rate of heat flow using R values for three common building materials



To calculate rate of heat transfer:

Convection: Heat transfer through a gas or liquid caused by movement of the fluid.

Gases and liquids usually decrease in density when they are heated (liquid water near 0°C is a notable exception). When part of a body of liquid or gas is heated, the warmed component rises because of its decreased density, while the cooler part sinks. This occurs in homes, where heat sources near the floor heat the nearby air, which rises and moves throughout the room. The warmer air displaces cooler air near the ceiling, causing it to move near the heat source, where it is heated in turn. This transfer of heat by the movement of a gas or liquid is called *convection*. All kitchen ovens, like the one shown in Concept 1, rely largely on convection for baking. The heating element at the bottom of the oven warms the air next to it, causing it to rise. The heated air then reaches the food in the oven to warm it, while the cooler air sinks to the bottom of the oven. So-called "convection ovens" speed this process with fans that cause the air to circulate more quickly. Convection occurs in liquids as well as in gases. If you stir spaghetti sauce as it heats, you are accelerating the process of convection. Again, your goal is to uniformly distribute the thermal energy. If you see a hawk soaring upward without flapping its wings, it may be riding what is called a "thermal." As the Sun warms the ground, the nearby air also becomes warmer. In the process, it becomes less dense, and is forced upward by air that is cooler and denser. A bird can ride this upward draft.



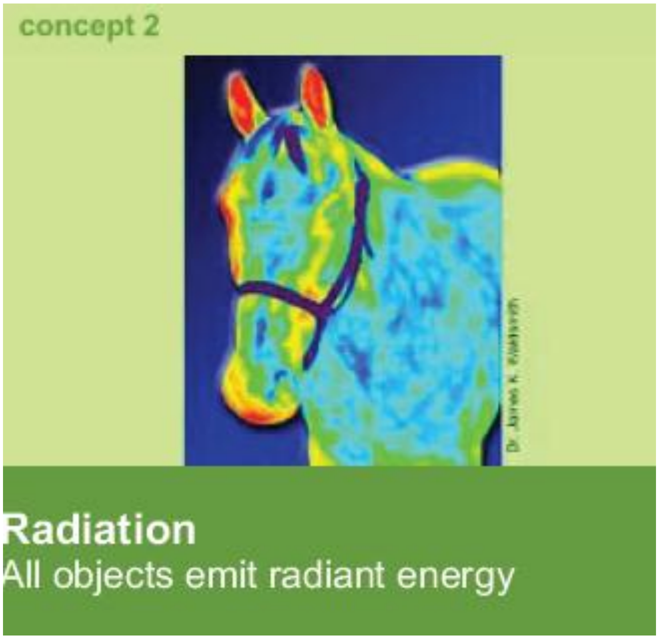
Radiation: Heat transfer by electromagnetic waves.

If you place your hand near a red-hot heating element and feel your hand warm up, you are experiencing thermal radiation: the transfer of energy by electromagnetic waves. You correctly think of objects like the heating element as radiating heat; in fact, every object with a temperature above absolute zero radiates energy. Radiation consists of electromagnetic waves, which are made up of electric and magnetic fields. Radiation needs no medium in which to travel; it can move through a vacuum. The wavelength of radiation varies. For instance, red light has a wavelength of about 700 nm, and blue light a wavelength of about 500 nm. Infrared and ultraviolet radiation are two forms of radiation whose wavelengths are, respectively, longer and shorter than those of visible light. All objects radiate electromagnetic radiation of different wavelengths. For instance, you see the red-hot stove coil because it emits some visible light. The coil also emits infrared radiation that you cannot see but do feel as heat flowing to your hand, and it emits a minimal amount of ultraviolet radiation too.

Although any particular object radiates a range of wavelengths, there is a peak in that range, a wavelength at which the power output is the greatest. This peak moves to shorter wavelengths as the temperature of the object increases. Understanding the exact form of the spectrum of thermal radiation wavelengths requires concepts from quantum physics, and its derivation was one of the early triumphs and verifications of quantum theory. Bodies with temperatures near the temperature of the surface of the Earth emit mostly infrared radiation. In the photograph in Concept 2, called an *infrared thermograph*, yousee the radiation emitted by a horse. Since areas of inflammation in the body are unusually warm, and emit extra thermal radiation, veterinarians can use photographs like this to diagnose an animal's ailments. They are created by a digital or film camera that assigns different (visible) colors to different intensities of (invisible) infrared radiation in a process called false color reproduction.

Sunlight is a form of radiation and is crucial to life on Earth. The Sun emits massive amounts of energy in the form of radiation: 3.9×1026 joules every second. Some of that strikes the Earth, where it warms the planet and supplies the energy that plants use in photosynthesis. The amount of power radiated by a body is proportional to the

fourth power of its absolute temperature, its surface area, and a factor called its emissivity. The Sun emits tremendous amounts of radiation energy because it is quite hot (about 6000 K) and vast (with a surface area of about 6×1018 m2).Only a small portion of the total power emitted by the Sun reaches the Earth. Even this fraction is an enormous amount: 1.8×1017 watts, about 100 times what human civilization consumes. The average solar power striking the Earth's atmosphere in regions directly facing the Sun is about 1370 watts per square meter. This value is called the *solar constant*.



This example serves to illustrate the role of the greenhouse effect. The Earth's atmosphere "traps" a substantial amount of the radiation emitted by the Earth's surface. Without this effect, the temperature at the surface of the Earth would be cooler.

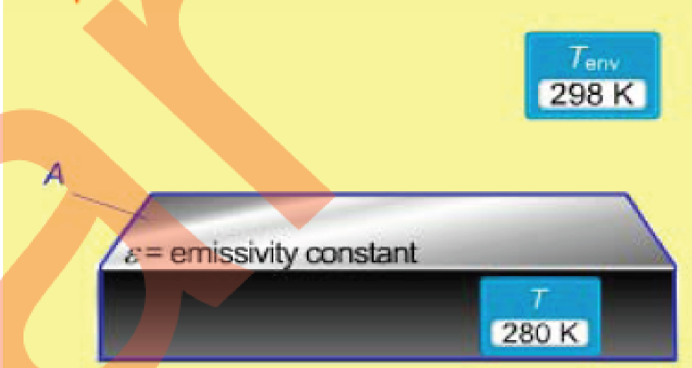
concept 2

	Emissivity
Polished silver, gold, aluminum	0.02-0.04
Mercury (the element)	0.09-0.12
Venus	0.24
Earth average	0.67
White enamel paint	0.87-0.91
Mercury (the planet)	0.9
Flat black lacquer	0.92-0.96
Candle soot	0.95

Emissivity

ε = ratio absorbed/incident radiation

equation 1



Radiated and absorbed power

$$P_{rad} = \sigma\varepsilon A T^4$$

$$P_{abs} = \sigma\varepsilon A T_{env}^{\ 4}$$

P_{rad} = power radiated

P_{abs} = power absorbed

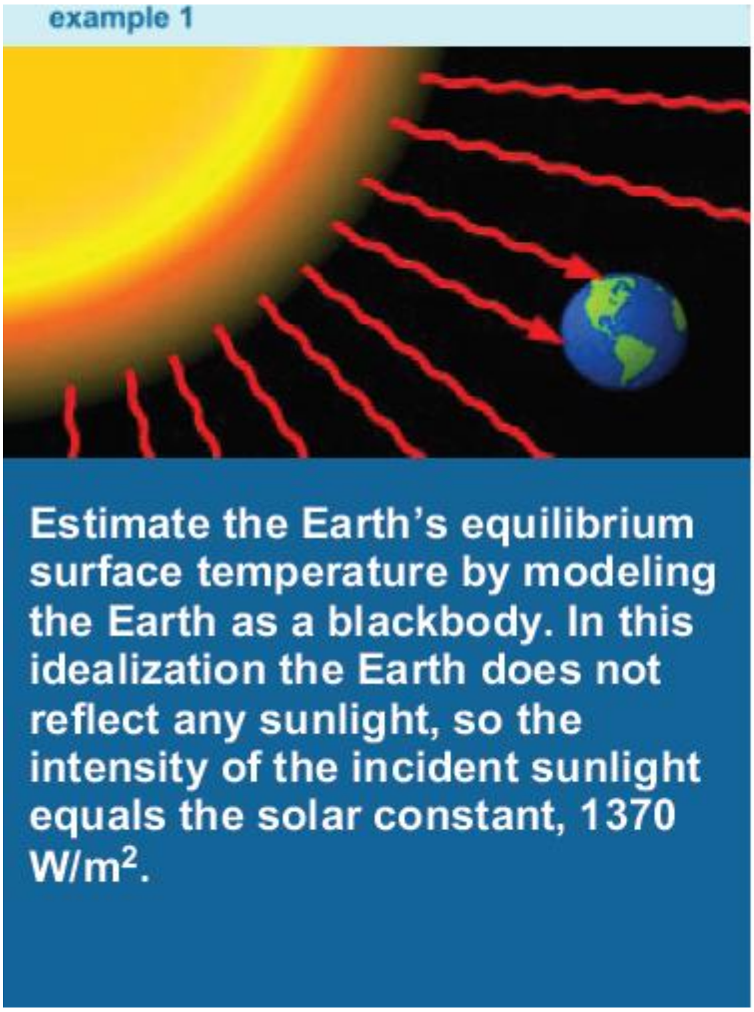
ε emissivity, A = surface area

T = temperature of object (K)

T_{env} = temperature of environment (K)

$\sigma = 5.67\times10^{-8}$ W/m^2·K^4

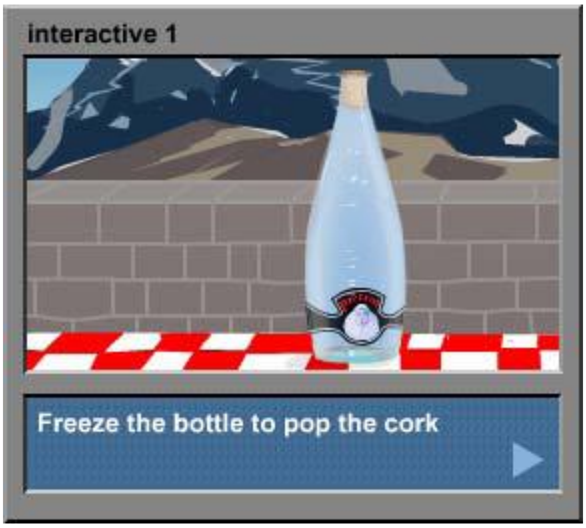
(Stefan-Boltzmann constant)



Interactive summary problem: pop the cork

You just bought a bottle of Pierrot, the water from ancient limestone caves deep in the French Alps, filtered by pure quartz crystals. But you did not realize the bottle came with a cork, and you have no corkscrew. Fortunately, your knowledge of thermal physics comes in handy. You remember that the density of ice is 9% less than the density of water. This means that water expands quite a bit when it freezes into ice. If you let the water in the bottle freeze, the expansion of the ice will push the cork out. If 89.0% of the water freezes, the expanding ice will just push the cork out. But if more than 89.0% of the water freezes, the ice will expand too much and the bottle will break. You want to remove just enough heat from the water so that exactly 89.0% of it turns to ice. In the interactive simulation on the right, you control the amount of heat removed from the water. The bottle contains 0.750 kg of water and its temperature is now 15.0°C. You need to remove enough heat to reduce the temperature of all the water to 0°C, and then remove enough additional heat to freeze 89.0% of it. To do these calculations, you will need to use the specific heat of water, 4178 J/kg·K, and

the latent heat of fusion of water, 3.34×105 J/kg. We ignore the glass bottle itself in these calculations. Heat is removed from the bottle, but much more heat (about 50 times more) is removed from the water. Also, while the volume of the glass bottle decreases slightly as it cools, the expansion of the ice is much greater. Similarly, the small air space at the top of the bottle has little effect. Set the amount of heat to be removed from the water, then press GO. If you are right, the ice will push the cork out. Press RESET to try again. If you have trouble calculating the correct amount of heat transfer, review the sections in this chapter on specific heat and latent heat, and the sample problem that combines the two concepts.



Gotchas

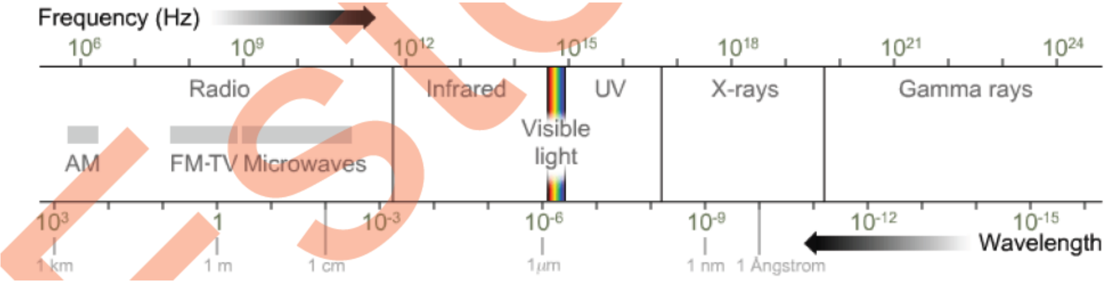
Heat is the same as temperature. No, heat is a flow of energy. Temperature is a property of an object. The flow of heat will change the temperature of an object, and a thermometer measures the object's temperature.*The Fahrenheit temperature system is the wave of the future.* If you think so, can I interest you in buying a record player? *Two rods of the same material experience the same increase in temperature, which means they must have expanded by the same amount of length.* Only if they were the same initial length. Their percentage increase would be the same in any case. *You throw a football upward. You have not increased the internal energy of the air within the football.* Correct: You have not increased the internal energy of the molecules inside the football. (You have increased its translational kinetic energy and its rotational kinetic energy and its

gravitational PE by throwing it upward, but not its internal energy.)

x-rays, microwaves

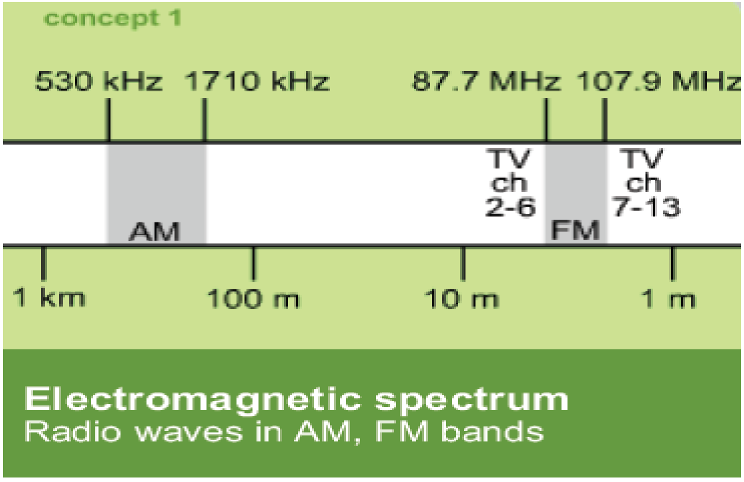
Radio and television signals, x-rays, microwaves: Each is a form of electromagnetic radiation. If steam and internal combustion engines symbolize the Industrial Revolution, and microprocessors and memory chips now power the Information Revolution, it almost seems that we have neglected to recognize the “Electromagnetic Revolution.” Think about it: Can you imagine life without television sets or cell phones? You may long for such a life, or wonder how people ever survived without these devices! These examples are from the world of engineered electromagnetic radiation. Even if you think we might all prosper without such technologies to entertain us, do our cooking, carry our messages, and diagnose our illnesses, you would be hard-pressed to survive without light. This form of electromagnetic radiation brings the Sun’s energy to the Earth, warming the planet and supplying energy to plants, and in turn to creatures like us that depend on them. There are primitive forms of life that do not depend on the Sun’s energy, but without light there would be no seeing, no room with a view, no sunsets, and no Rembrandts. Some of the electromagnetic radiation that reaches your eyes was created mere nanoseconds earlier, like the light from a lamp. Other electromagnetic radiation is still propagating at its original speed through the cosmos, ten billion years or more after its birth. An example of this is the microwave background radiation, a pervasive remnant of the creation of the universe that is widely studied by astrophysicists. Back here on Earth, this chapter covers the fundamental physical theory of electromagnetic radiation. Much of it builds on other topics, particularly the studies of waves, electric fields and magnetic fields. **Electromagnetic radiation: Rainbows and radios. Sundazzled reflections. Shadowlamps and lampshadows. Red, white, and blue.**

The electromagnetic spectrum



Electromagnetic spectrum: Electromagnetic radiation ordered by frequency or wavelength.

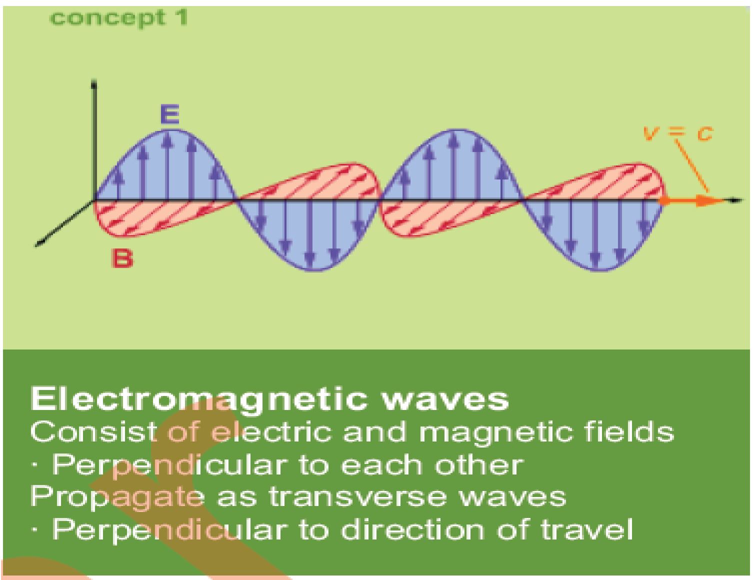
Electromagnetic radiation is a traveling wave that consists of electric and magnetic fields. Before delving into the details of such waves, we will discuss the electromagnetic spectrum, a system by which the types of electromagnetic radiation are classified. The illustration of the electromagnetic spectrum above orders electromagnetic waves by frequency and by wavelength. In the diagram, frequency **increases** and wavelength **decreases** as you move from the left to the right. The chart's scale is based on powers of 10. Wavelengths range from more than 100 meters for AM radio signals to as small as 10□16 meters for gamma rays. All electromagnetic waves travel at the same speed in a vacuum. This speed is designated by the letter *c* and is called the speed of light. (The letter *c* comes from *celeritas*, the Latin word for speed. It might be more accurate to refer to it as the speed of electromagnetic radiation.) The speed of light in a vacuum is exactly 299,792,458 m/s, and it is only slightly less in air. The unvarying nature of this speed has an important implication: The wavelength of electromagnetic radiation is inversely proportional to its frequency. As you may recall, the speed of a wave equals the product of its frequency and wavelength. This means that if you know the wavelength of the wave, you can determine its frequency (and vice versa). For instance, an electromagnetic wave with a wavelength of 300 meters, in the middle of the AM radio band, has a frequency of 1×106 Hz. This equals 3×108 m/s, the speed of light, divided by 300 m. The frequencies of electromagnetic waves range from less than one megahertz, or 106 Hz, for long radio waves to over 1024 Hz for gamma rays. We will now review some of the bands of electromagnetic radiation and their manifestations. The lowest frequencies are often utilized for radio



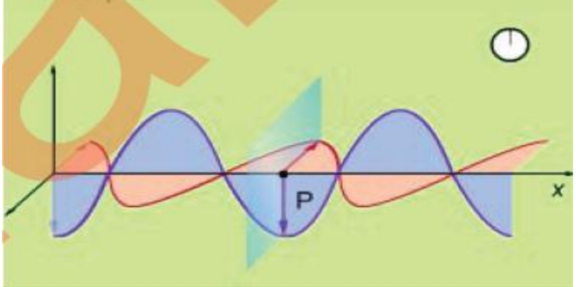
Electromagnetic wave: A wave consisting of electric and magnetic fields oscillating transversely to the direction of propagation.

Physicist James Clerk Maxwell's brilliant studies pioneered research into the nature of electromagnetic waves. He correctly concluded that oscillating electric and magnetic fields can constitute a self-propagating wave that he called electromagnetic radiation. His law of induction (a changing electric field causes a magnetic field) combined with Faraday's law (a changing magnetic field causes an electric field) supplies the basis for understanding this kind of wave. As the diagrams to the right show, the electric and magnetic fields in an electromagnetic wave are perpendicular to each other and to the direction of propagation of the wave. These illustrations also show the amplitudes of the fields varying sinusoidally as functions of position and time. Electromagnetic waves are an example of *transverse waves*. The fields can propagate outward from a source in all directions at the speed of light; for the sake of visual clarity, we have chosen to show them moving only along the x axis. The animated diagram in Concept 2 and the illustrations below are used to emphasize three points. First, the depicted wave moves away from the source. For example, if you push the "transmit" button on a walkie-talkie, a wave is initiated that travels away from the walkie-talkie. Second, at any fixed location in the path of the wave, both fields change over time. The wave below is drawn at intervals that are fractions $T/4$ of the period T. Look at the point P below, on the light blue vertical plane. The vectors from point P represent the direction and strength of the electric and magnetic fields at this point. As you can see, the vectors, and the fields they represent, change over time at P. Concept 2 shows them varying continuously with time at the point P.

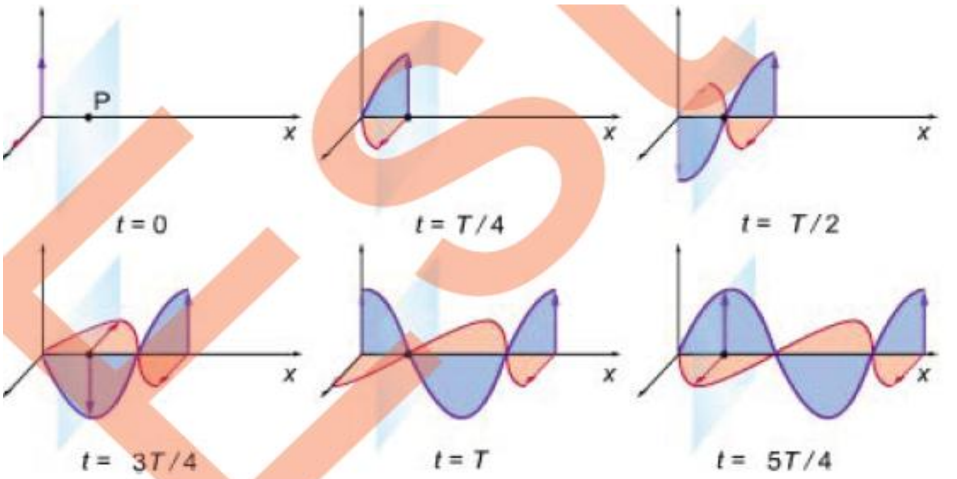
Third, the diagrams reflect an important fact: The electric and magnetic fields have the same frequency and phase. That is, they reach their peaks and troughs simultaneously. A wave on a string provides a good starting point for understanding electromagnetic waves. Both electromagnetic radiation and a wave on a string are transverse waves. The strengths of the two fields constituting the radiation can be described using sinusoidal functions, just as we can use a sinusoidal function to calculate the transverse displacement of a particle in a string through which a wave is moving. There is a crucial difference, though: Electromagnetic radiation consists of electric and magnetic fields, and does not require a medium like a string for its propagation. Electromagnetic waves can travel in a vacuum. If this is troubling to you, you are in good company. It took some brilliant physicists a great deal of hard work to convince the world that light and other electromagnetic waves do not require a medium of transmission. Furthermore, when electromagnetic waves radiate in all directions from a compact source like an antenna or a lamp, the radiation emitted at a particular instant travels outward on the surface of an expanding sphere, and its strength diminishes with distance from the source. The waves cannot be truly sinusoidal, since the amplitude of a sinusoidal function never diminishes. In the sections that follow we will analyze *plane waves*, which propagate through space, say in the positive x direction, in parallel planar wave fronts rather than expanding spherical ones. They are good approximations to physical waves over small regions that are distant from the source of the waves. Plane waves never diminish in strength; they can be accurately modeled using sinusoidal functions, and we will do so.



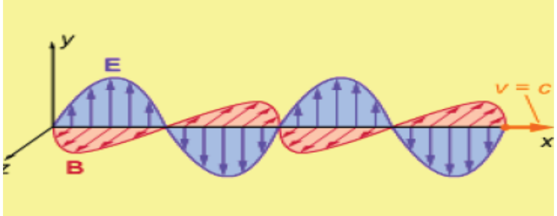
concept 2



Electric, magnetic fields
Vary in strength over time at each point
Have same frequency and are in phase
Drive each other by changing strength



equation 1

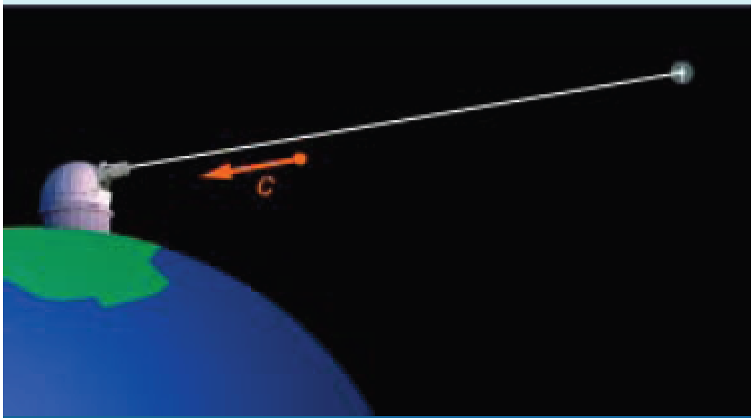


Speed of an electromagnetic wave

$$c = \frac{\omega}{k} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

c = speed of electromagnetic wave
ω = angular frequency of wave
k = angular wave number of wave
μ_0 = permeability of free space
Constant $\mu_0 = 4\pi\times10^{-7}$ T·m/A
ε_0 = permittivity of free space
Constant $\varepsilon_0 = 8.854\times10^{-12}$ C²/N·m²

example 1



What is the speed of an electromagnetic wave in a vacuum?

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

$$\mu_0 \varepsilon_0 = (4\pi \times 10^{-7})(8.854 \times 10^{-12})$$

$$\mu_0 \varepsilon_0 = 1.113 \times 10^{-17} \text{ s}^2/\text{m}^2$$

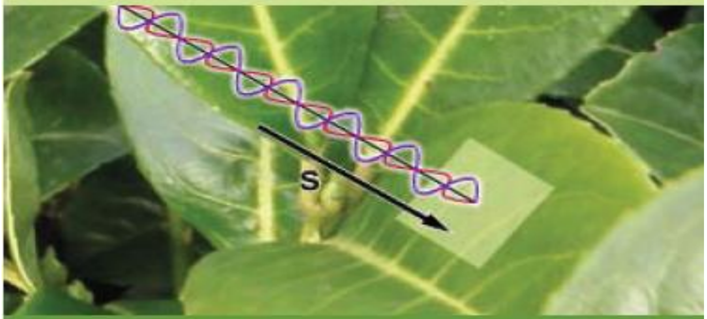
$$\sqrt{\mu_0 \varepsilon_0} = 3.336 \times 10^{-9} \text{ s/m}$$

$$c = 2.998 \times 10^8 \text{ m/s}$$

Creating electromagnetic waves: antennas Radio antennas create electromagnetic waves. A radio antenna is part of an overall system called a radio transmitter that converts the information contained in sound waves into electromagnetic waves. A radio receiver then reverses the process, converting the signals from electromagnetic waves back to sound waves. The system depicted to the right shows the fundamentals of a radio transmitter. In the illustrations, the terminals of an AC generator are connected to two rods of conducting material: an antenna. The AC generator produces an emf □ that varies sinusoidally over time. The emf drives positive

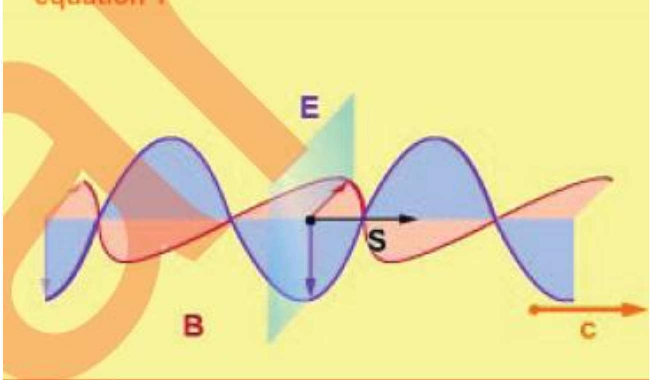
through the surface by the wave, per unit area, is called the *area power density* of the wave. The area power density is equal to the magnitude S of the Poynting vector. The surface area through which the instantaneous power density is measured is perpendicular to the direction of the wave's propagation. When radiation reaches a physical surface obliquely, the cosine of its angle with the area vector can be used to calculate the power conveyed to the surface. This is analogous to the calculation of electric or magnetic flux. As Equation 1 shows, the Poynting vector equals the cross product of the vectors representing the electric and magnetic fields of the electromagnetic radiation, divided by the permeability constant. Since these fields are always perpendicular to one another, the sine of the angle between them, used to evaluate the magnitude of the cross product, always equals one, and can be effectively ignored when calculating the instantaneous area power density S. The units of the Poynting vector are watts per square meter. The direction of **S** is determined by the right-hand rule. If you apply the rule, wrapping your fingers from **E** to **B** and noting the direction of your thumb, you can correctly determine that it is parallel to the direction of propagation of the wave. When **E** reverses its direction, so does **B**, and the direction of **S** remains the same, "pointing" (heh, heh) in the direction of the wave's motion. As an electromagnetic wave passes through a surface, the strengths of its electric and magnetic fields there change sinusoidally with time. Since the Poynting vector is the product of these fields, it changes sinusoidally over time, as well. In fact, it varies with values between zero and $E_{max}B_{max}/\mu_0$, with a frequency twice that of the fields. If you are curious why it has this frequency, recall from the field equations that E and B are both cosine functions of time at a fixed point. Then use the trigonometric identity $\cos^2 t = [1 + \cos 2t]/2$.

concept 1



Poynting vector
Power per unit surface area
Surface perpendicular to wave direction

equation 1



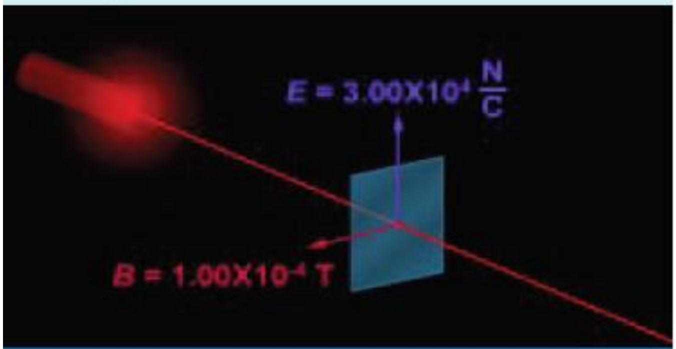
Poynting vector

$$\mathbf{S} = \frac{1}{\mu_0}\mathbf{E} \times \mathbf{B}$$

$\mathbf{S}$ = Poynting vector
S = instantaneous area power density
μ_0 = permeability of free space
$\mathbf{E}$ = electric field
$\mathbf{B}$ = magnetic field

Units: watts per square meter (W/m^2)

example 1



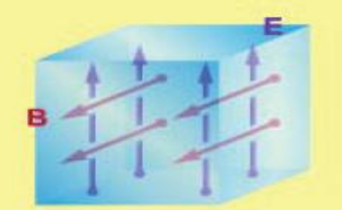
At this instant, what is the area power density of the ruby laser light?

$S = EB / \mu_0$

$$S = \frac{(3.00 \times 10^{4} \frac{\text{N}}{\text{C}})(1.00 \times 10^{-4}\ \text{T})}{4\pi \times 10^{-7} \frac{\text{T}\cdot\text{m}}{\text{A}}}$$

$S = 2.39 \times 10^{6}\ \text{W/m}^2$

equation 2



Electric, magnetic energy densities

$$u_E = \frac{\varepsilon_0 E^2}{2} \qquad u_B = \frac{B^2}{2\mu_0}$$

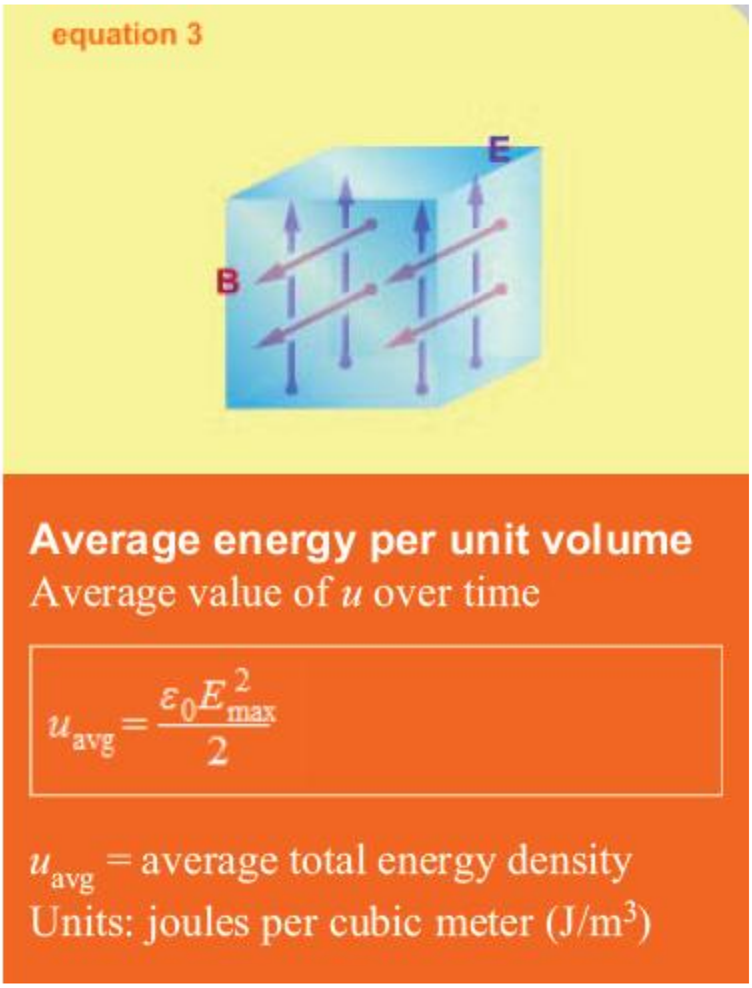
Since $E^2/B^2 = c^2 = 1/\mu_0\varepsilon_0$, then

$$u_E = u_B$$

The total energy density is

$$u = u_E + u_B = 2u_E = 2u_B$$

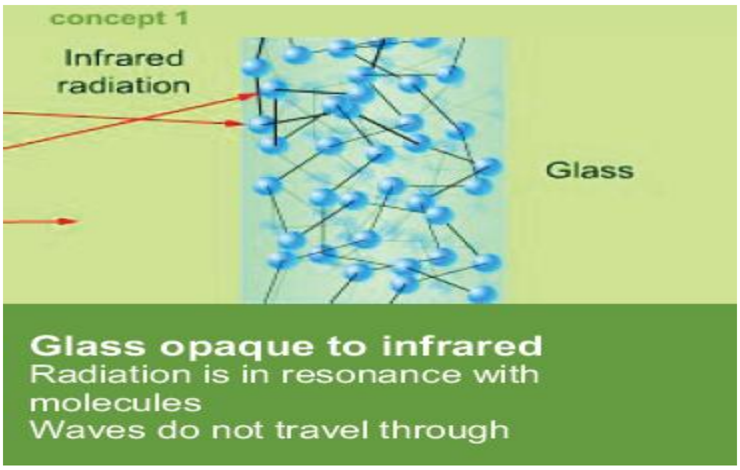
u_E = electric field energy density
u_B = magnetic field energy density
u = total energy density

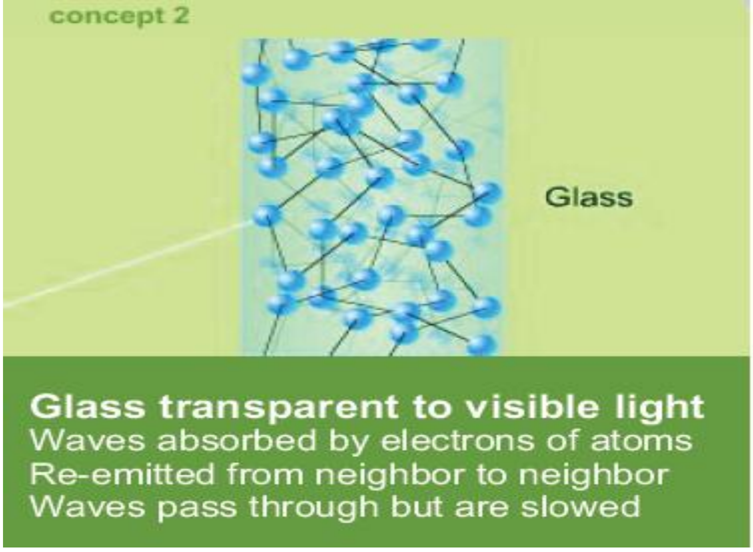


How electromagnetic waves travel through matter

Light and other forms of electromagnetic radiation can travel through a vacuum, and it is often simplest to study them in that setting. However, radiation can also pass through matter: If you look through a glass window, you are viewing light that has passed through the Earth's atmosphere and the glass. Other forms of radiation such as radio waves pass through matter, as well. This section focuses on how such transmission occurs. It relies on a classical model of electrons and atoms that predates quantum theory. In this model, electrons orbit an atom. They have a resonant frequency that depends on the kind of atom. On a larger scale, atoms themselves and the molecules composed of them also have resonant thermal frequencies at which they can vibrate or rotate. We will use the example of light striking the glass in a window to discuss how substances transmit (or do not transmit) electromagnetic radiation. When an electromagnetic wave encounters a window, it collides with the molecules that make up the glass. If the frequency of the wave is near the resonant thermal frequency of the glass molecules, which is true for infrared radiation, the amplitude of

the molecules' vibrations increases. They absorb the energy transported by the wave, and dissipate it throughout the glass by colliding with other molecules and heating up the window. Because it absorbs so much infrared energy, the glass is opaque to radiation of this frequency, preventing its transmission. Scientists in the 19th century noted a phenomenon in greenhouses caused by the opaqueness of glass to infrared radiation, which they called the *greenhouse effect*. The glass in a greenhouse admits visible light from the Sun, which is then absorbed by the soil and plants inside. They reradiate the solar energy as longer infrared waves, which cannot pass back out through the glass and so help warm up the greenhouse. The same phenomenon occurs on a vaster scale in the atmosphere as gases like methane and carbon dioxide trap solar energy near the Earth's surface. In contrast to infrared radiation, higher frequency radiation such as visible light does not resonate thermally with atoms or molecules, but may resonate with the electrons of the atoms of a substance. In glass, visible light experiences much less reduction in the amplitude of its waves than infrared radiation does, and most of its energy passes through the glass quite easily. Atoms with resonant electrons that do absorb energy from a light wave quickly pass on that energy by re-emitting it as radiation of the same frequency to other atoms, which in turn pass it on to their neighbors. This chain of absorptions and re-emissions, called *forward scattering*, follows a path close to the light's original direction of travel. A beam of light that strikes a pane of glass will reach the "last atom" on the far side of the pane in an extremely short time. We see the light after it emerges, and think of glass as transparent.





Radiation also can be *partially polarized*, having a few waves oscillating in all planes, but with most of its waves concentrated in a single plane. This is true of sunlight scattered by the atmosphere. As the photo above shows, the sky in certain directions is partially polarized in a vertical plane so that most of its light can pass through a pair of sunglasses whose transmission axis is vertical. Less light (but still some) passes through the rotated sunglasses. (Polarizing sunglasses are specifically intended to reduce horizontally polarized glare reflected from roadways and water, not skylight.)

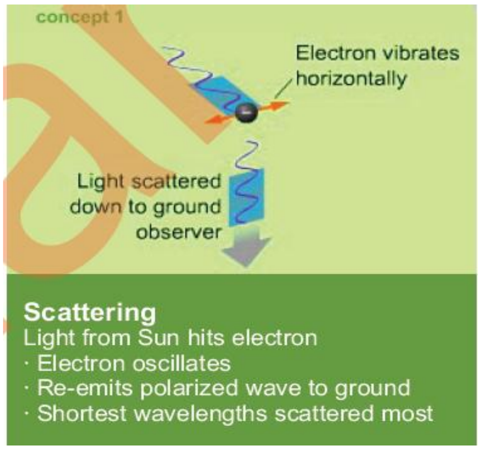
Many forms of artificial electromagnetic radiation are polarized. A radio transmitter emits polarized radiation. If the rods of its antenna are vertical, then so is the electric field of every radio wave it creates. In this case, the most efficient receiving antenna is also vertically oriented; a horizontal receiving antenna would absorb radio waves much less efficiently. You may be familiar with this fact if you have ever tried to maneuver a radio antenna wire or a set of television “rabbit ears” to get the best reception. (If you do not know what “rabbit ears” are for television, well, before there was cable television, there was….)

Scattering: Absorption and re-emission of light by electrons, resulting in dispersion and some polarization.

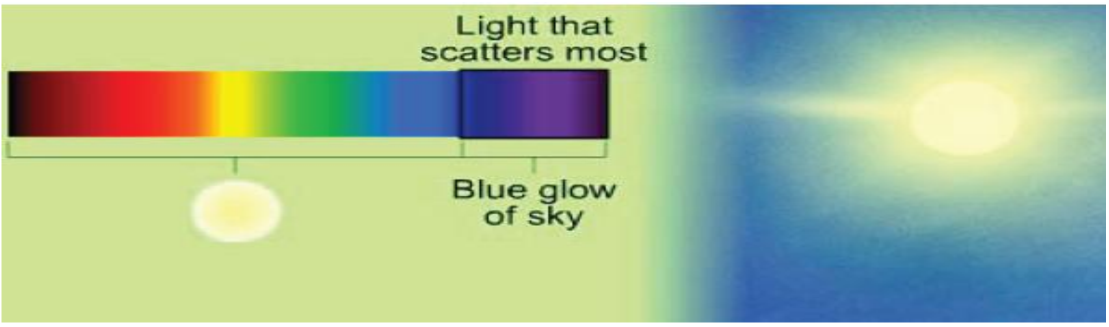
The answer to a classic question Why is the sky blue? rests in a phenomenon called scattering. In this section, we give a classical (as opposed to quantum mechanical) explanation of how scattering occurs. When light from the Sun strikes the electrons of various atoms in the Earth's atmosphere, the electrons can absorb the light's energy, oscillating and increasing their own energy. The electrons in turn re-emit this energy as light of the same wavelength. In effect, the oscillating electrons act like tiny antennas, emitting electromagnetic radiation in the frequency range of light.

An electron oscillates in a direction parallel to the electric field of the wave that energizes it, as shown in Concept 1. The electron then emits light polarized in a plane parallel to its vibration. We show a particular polarized wave that is re-emitted downward toward the ground, since we are concerned with what an observer on the surface of the Earth sees. Other light is scattered in other directions, including light scattered upward and light scattered forward in its original direction of travel. Scattering explains why we see the sky: Light passing through the atmosphere is redirected due to scattering toward the surface of the Earth. In contrast, for an astronaut observer in the vacuum of space, sunlight is not scattered at all so there is no sky glow: Except for the stars, the sky appears black. To the astronaut, the disk of the Sun, a combination of all colors, looks white. We illustrate this below: The full spectrum combines to form white light. The question remains, why is our sky blue rather than some other color? Light at the blue end of the visible spectrum, which has the shortest wavelength, is 10 times more resonant with the electrons of atmospheric

atoms than red light. This means blue light is scattered more than red, so that more of it is redirected toward the ground. Scattering also explains why we see the Sun as yellow rather than white. When you look up at the disk of the Sun from the Earth's surface, the bluest portion of its light has been scattered away to the sides. The remaining part of the Sun's direct light appears somewhat yellowish. You may also have noted how the Sun appears to change color when it sets. As the Sun's disk descends toward the horizon, its light must pass through a greater and greater thickness of atmosphere in order to reach you. Since a certain amount of sunlight is scattered aside for each kilometer of atmosphere it passes through, its position at sunset causes it to lose large amounts of light at the blue end and even toward



View from space
No scattering: sky is black
Sun appears white

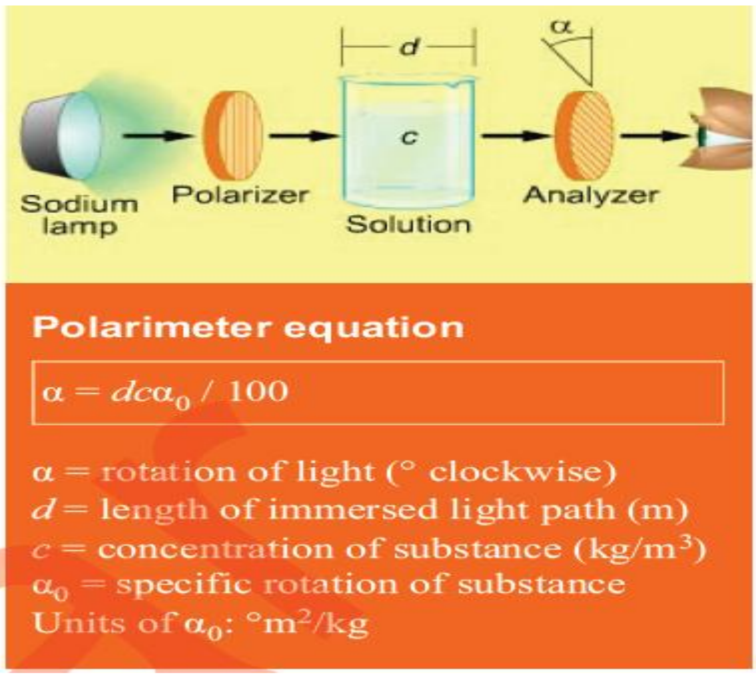


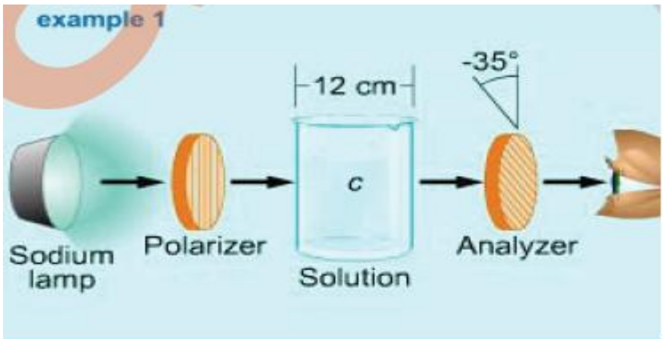
Why the sky is blue (and the Sun is yellow)
Shortest waves are scattered: sky is blue
With blue scattered, Sun appears yellowish white

the concentration of the dissolved substance. The rotation is also proportional to a constant □0 called the *specific rotation* of the substance, which reflects the rotating power of its molecules. These relationships are summarized to the right in the *polarimeter equation*. Note that (and this is unusual for a physics equation) the rotation angle □ is measured in degrees rather than radians, and the clockwise direction is considered positive. The *polarimeter* is a device that can be used to measure the net rotation of polarized light passing through an optically active solution. An experimenter directs polarized light through a container of the solution to be analyzed. The analyzer, which starts out parallel to the polarizer, does not transmit all the light from the polarizer because the light's plane of polarization has been rotated by the solution. The experimenter turns the analyzer to one side or the other until the transmitted light has maximum brightness. Then she knows that the analyzer's transmission axis matches the rotated polarized light, and she can measure the angle □ through which the analyzer has turned. The polarimeter equation gives an expression for the angle □ of the analyzer at which the transmitted light will be the brightest. If the polarized light encounters more molecules of the optically active substance, either because the solution is more concentrated or because the immersed light path is longer, the rotation will be greater. Since the amount of rotation also

depends on the wavelength of the light, the specific rotations □0 given in tables for particular dissolved substances are based on a polarimeter employing the 589 nm light that is emitted by a sodium vapor lamp. Dextrose and fructose molecules are chemically identical (they have the same atoms arranged in the same pattern) but they are mirror images of each other. Because of this they rotate polarized light by the same amount in opposite directions. Organic molecules such as *carvone* may exist in two mirror image forms; you smell carvone as caraway or spearmint, depending on which way the molecule twists. The scents are different because the smell receptors in the nose react differently to the mirror image forms. Using a polarimeter is one way to distinguish between the two forms of mirror image compounds. Also, if the specific rotation of a particular substance is known, the device can be used together with the polarimeter equation to determine the concentration of the substance in a solution. You are asked to perform such an analysis in the example problem to the right.

The diagrams below show the mirror image molecular forms of the citrus oil *limonene*, which is the essence of either orange or lemon, depending on the orientation of its molecules! (The gray spheres represent carbon atoms, and the blue spheres are hydrogen atoms.)





Carvone's specific rotation is +62.5 (caraway) or –62.5 (spearmint). What is the concentration of the carvone in this beaker?

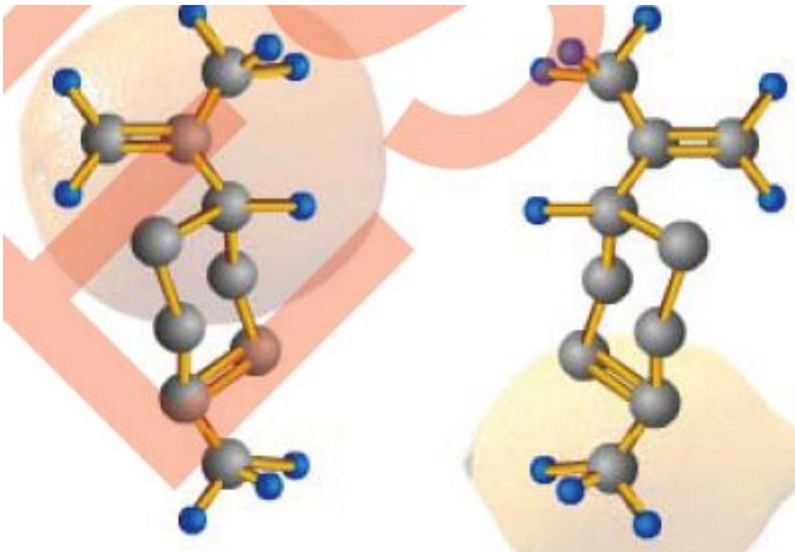
$\alpha = dc\alpha_0 / 100$

$$c = \frac{100\alpha}{d\alpha_0}$$

Counterclockwise rotation means spearmint, so we use $\alpha_0 = -62.5$:

$$c = \frac{(100)(-35°)}{(0.12\ \mathrm{m})(-62.5\ °\mathrm{m}^2/\mathrm{kg})}$$

$c = 470\ \mathrm{kg/m^3}$



Equations

Proportionality of fields

$$\frac{E}{B} = c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

Poynting vector

$$\mathbf{S} = \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B}$$

Intensity of electromagnetic radiation

$$I = S_{\text{avg}} = \frac{E_{\text{max}}^2}{2\mu_0 c} = \frac{E_{\text{rms}}^2}{\mu_0 c}$$

$$I = \frac{P}{4\pi r^2}$$

Energy density

$$u_{\text{E}} = \frac{\varepsilon_0 E^2}{2}, \quad u_{\text{B}} = \frac{B^2}{2\mu_0}$$

$$u_{\text{E}} = u_{\text{B}}$$

$$u = u_{\text{E}} + u_{\text{B}} = 2u_{\text{E}} = 2u_{\text{B}}$$

$$u_{\text{avg}} = \frac{\varepsilon_0 E_{\text{max}}^2}{2}$$

Momentum transferred by radiation absorption

$$\Delta p = \frac{\Delta U}{c} \quad \text{for a blackbody}$$

Reflection: Light "bouncing back" from a surface.

When you look at yourself in a mirror, you are seeing a reflection of yourself. When you look at the Moon at night, you are seeing sunlight reflecting off that distant body. Not all the light that reaches a surface reflects. In fact, you see an object like a tree as having different colors because its varied parts reflect some wavelengths of light and absorb others. Light can pass through a material, as it does with a glass window. It can also be absorbed by a material, as evidenced by how a black rock warms up during a sunny day. All this can happen simultaneously: Light will reflect off the surface of a lake (which is why you see the lake), penetrate the water (otherwise, it would be completely dark below the surface), and be absorbed by the water, warming it. To understand reflection, it is often useful to treat light as a stream of particles that move in a straight line and change direction only when they encounter a surface. Each light "particle" acts like a ball bouncing off of a surface, and like a ball, it reflects off the surface at a rebound angle equal to its incoming angle. You see yourself in a mirror because the light bounces back to your eyes from the mirror. The term "reflection" likely conjures up images of light and perhaps mirrors. Studying mirrors is a good way to learn about reflection because they are designed to reflect light in a way that creates a clear visual image. However, it is worth noting that reflection does not apply only to light. Some creatures use the reflection of sound (echoes) to help them perceive their surroundings and stalk their prey. For example, bats, seals and dolphins emit high frequency sound and then listen for the reflected waves. By analyzing these reflections, they can "see" with great precision. *Radar*, used to track airplanes, is based on the reflection of radio waves. A sophisticated understanding of reflection can be used to design "stealth" aircraft that are difficult to detect with radar. Stealth aircraft register on radar screens as being about as large as a BB, in part because of their ability to reflect incoming waves in "random" directions.

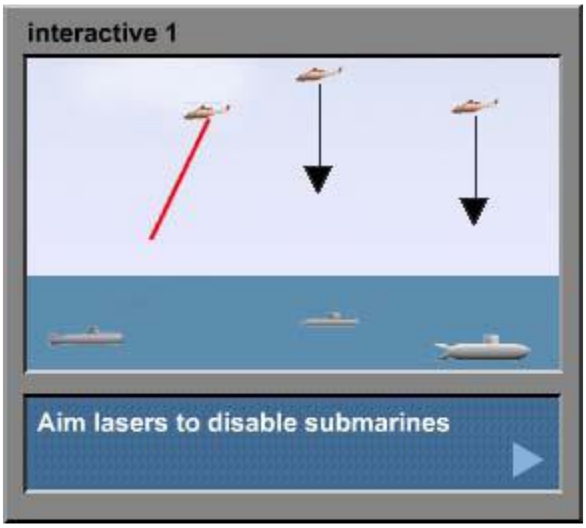
Light can refract

□ change direction □ as it moves from one medium to another. For instance, if you stand at the edge of a pool and try to poke something underwater with a stick, you may misjudge the object’s location. This is because the light from the object changes direction as it passes from the water to the air. You perceive the object to be closer to the surface than it actually is because you subconsciously assume that light travels in a straight line. Although refraction can cause errors like this, it can also serve many useful purposes. Optical microscopes, eyeglass lenses, and indeed the lenses in your eyes all use refraction to bend and focus light, forming images and causing objects to appear a different size or crisper than they otherwise would. Where a lens focuses light, and whether it magnifies an object, is determined by both the curvature of the lens and the material of which it is made. Scientists have developed quantitative tools to determine the nature of the images created by a lens. We will explore these tools thoroughly later, “focusing” first, so to speak, on the principle of refraction underlying them. To begin your study of refraction, try the simulation to the right. Each of your helicopters can fire a laser □ a sharp beam of light □ at any of three submarines lurking under the sea. The submarines have lasers, too, and will shoot back at your craft. Your mission is to disable the submarines before they disarm your helicopters. When you make a hit, you can shoot again. Otherwise, the submarines get their turn to shoot until they miss. You play by dragging the aiming arrow underneath any one of your helicopters. Press FIRE and the laser beam will follow the direction of this arrow until it reaches the water, where refraction will cause the beam to change direction. In addition to hitting the submarines before they get you, you can conduct some basic experiments concerning the nature of refraction. As with reflection, the angle of incidence is measured from a line normal (perpendicular) to a surface. In this case, the surface is the horizontal boundary between the water and the air. Observe how the light bends at the boundary when you shoot straight down, at a zero angle of incidence, or grazing the water, at a large angle of incidence. You can create a large angle of incidence by having the far right helicopter, for example, aim at the submarine on the far left. You can also observe how refraction differs when a laser beam passes from air to water (your lasers) and from water to air (the submarines’ lasers). Observe the dashed normal line at each crossover point and answer the following question: Does the laser beam bend toward or away from that line as it changes media? You should notice that the laser beams of the submarines behave differently than those of the helicopters when they change media. As a final aside:

You may see that some of the laser beams of the submarines never leave the water, but reflect back from the surface between the water and the air. This is called total internal reflection.

Refraction: The change in the direction of light as it passes from one medium to another.

A material through which light travels is called a *medium* (plural: *media*). When light traveling in one medium encounters another medium, its direction can change. It can reflect back, as it would with a mirror. It can also pass into the second medium and change direction. This phenomenon, called refraction, is shown to the right. In the photo, a beam of light from a laser refracts (bends) as it passes from the air into the water. Light refracts when its speeds in the two media are different. Light travels faster through air than in water, and it changes direction as it moves from air into water, or from water into air. Although we are primarily interested in the refraction of light, all waves, including water waves, refract. Above, you see a photograph of surf wave fronts advancing parallel to a beach. Deep-ocean swells may approach a coastline from any angle, but they slow down as they encounter the shallows near the shore. The parts of a wave that encounter the shallow water earliest slow down first, and this causes the wave to refract. Sound waves can also refract. During a medical ultrasound scan, an acoustic lens can be used to focus the sound waves. The lens is made of a material in which sound travels faster than in water or body tissues. The surface between two media, such as air and water, is called an *interface*. As with mirrors, light rays are often used to depict how light refracts when it meets an interface. Lasers are often used to demonstrate refraction because they can create thin beams of light that do not

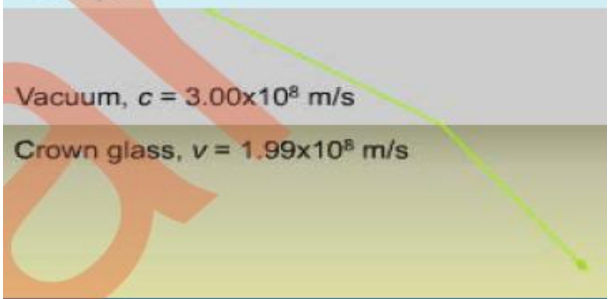


equation 2

	Index of refraction
Air	1.0003
Water	1.33
Vegetable oil	1.47
Crown glass	1.51
Salt	1.54
Flint glass	1.61
Corundum (ruby, sapphire)	1.77*
Diamond	2.42
At 20° C, λ = 589 nm	*Approximate value

Indices of refraction

example 1

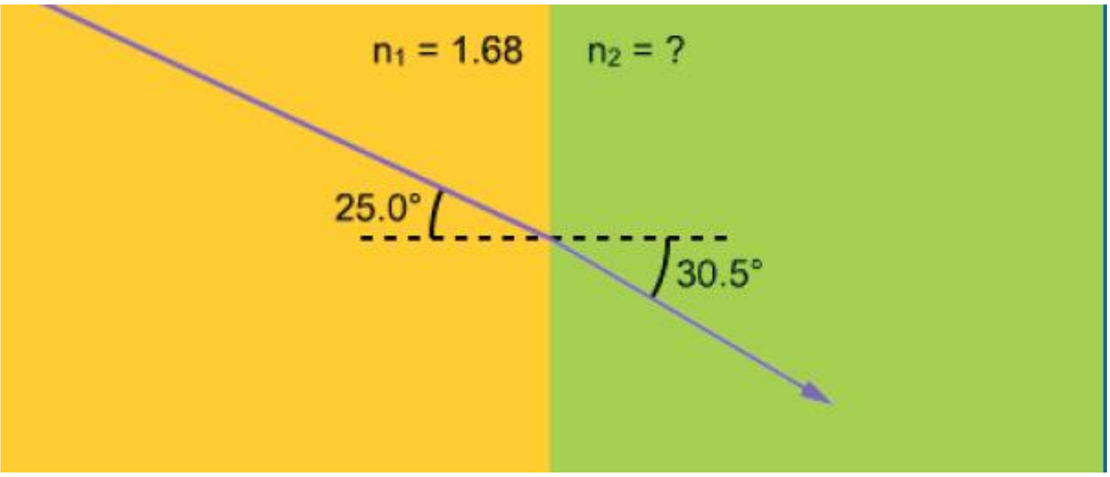


Green light travels at 1.99×10^8 m/s in crown glass. What is the index of refraction of the glass for this light?

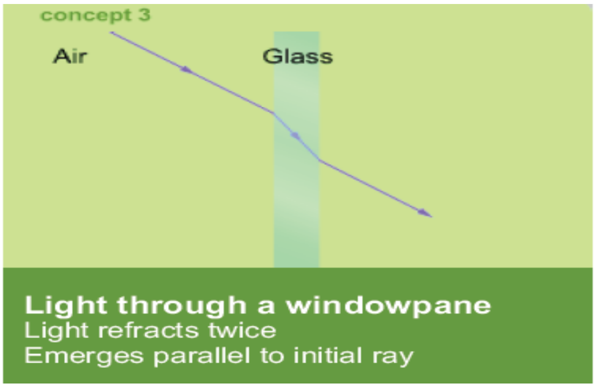
$$n = \frac{c}{v}$$

$$n = \frac{3.00\times10^8 \text{ m/s}}{1.99\times10^8 \text{ m/s}}$$

$$n = 1.51$$

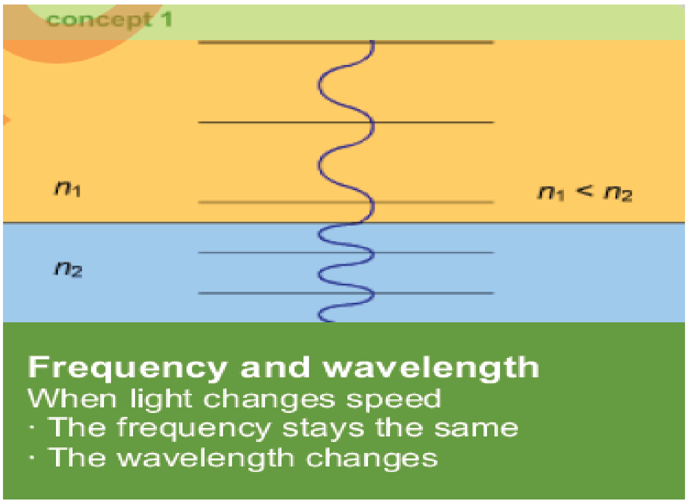


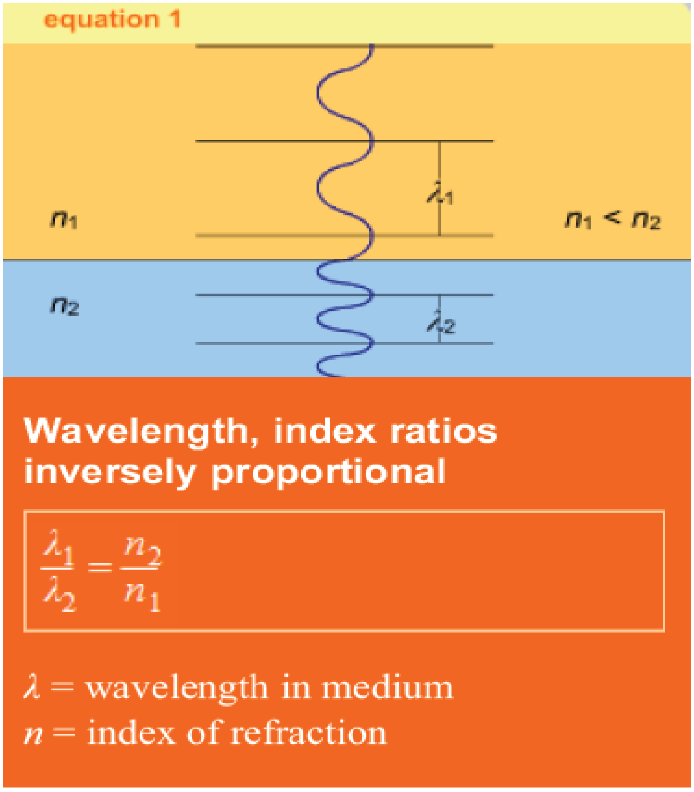
What is the index of refraction of the second material?



Wavelength of light in different media When light changes speed as it moves from one medium to another, its frequency stays the same but its wavelength changes. The

ratio of its wavelengths in the two media is the inverse of the ratio of the indices of refraction. We show this as an equation to the right and derive it below. Before deriving the equation, let's consider why the frequency stays the same, since this is an essential part of the derivation. The frequencies in the media must be the same, because if they were not, waves would either pile up at the interface or be destroyed. Neither occurs. You can witness this at the beach, where wave speed and wavelength may change as waves approach the beach, but the frequency of the waves does not change.





Variables

In this derivation, *c* represents the speed of light in a vacuum. For the other two media

we define the variables in the following table:

	medium 1	medium 2
light speed	v_1	v_2
frequency	f_1	f_2
wavelength	λ_1	λ_2
index of refraction	n_1	n_2

Strategy

1. Use the equality of frequencies in the two media together with the wave speed equation to obtain a proportionality of the light speeds and wavelengths in the media.
2. Use the definition of the index of refraction to convert the previous proportion to one involving wavelengths and indices of refraction.

Physics principles and equations

The wave speed equation states that for any wave, the speed is the product of the wavelength and the frequency:

$$v = \lambda f$$

As a wave passes from one medium to another, its speed and wavelength may change, but its frequency must remain the same. The definition of the index of refraction of a medium is

$$n = \frac{c}{v}$$

Step-by-step derivation

we explain the diagram you see above. The purple line is a light ray refracting at an interface. In the diagram, light travels more slowly in the lower medium than the upper. This could represent, for example, light passing from air into water. The gray lines perpendicular to the ray represent wave fronts. You see the wavelength labeled as □ (□i in the upper medium, □r in the lower medium). There are two right triangles in the diagram that share the hypotenuse labeled *x*. The bright yellow triangle shows elements of a wave front that has not yet entered the lower medium. The dark orange triangle shows elements of a wave front that is now traveling in the lower medium. The angles of incidence and refraction □i and □r are also shown in the diagram.

Because the wave fronts are perpendicular to the light rays, we can identify angles in each of the triangles that are equal to □i and □r. These base angles are shown in the diagram.

Variables

In this derivation, *x* represents the common hypotenuse of the two triangles in the diagram. For the incident and refractive media we define the variables in the following table.

	incident medium	refractive medium
angle	θ_i	θ_r
wavelength	λ_i	λ_r
index of refraction	n_i	n_r

Strategy

1. Consider the two triangles in the diagram. State the sines of their base angles □i and □r as trigonometric ratios of the triangles' sides.

2. Construct the ratio sin □i / sin □r. The common hypotenuse *x* will cancel out, leaving a ratio of wavelengths.

3. Restate the ratio of wavelengths as a ratio of indices of refraction to obtain Snell's law.

Physics principles and equations

The ratio of the wavelengths is inversely proportional to the ratio of the indices of refraction.

$$\frac{\lambda_i}{\lambda_r} = \frac{n_r}{n_i}$$

Step-by-step derivation

We construct the fraction sin □i / sin □r, and calculate the sines as the ratios of the sides of triangles. This leads to a ratio of wavelengths that can be replaced by a ratio of indices of refraction, yielding Snell's law.

Step	Reason
1. $\sin\theta_i = \lambda_i / x$, $\sin\theta_r = \lambda_r / x$	definition of sine
2. $\dfrac{\sin\theta_i}{\sin\theta_r} = \dfrac{\lambda_i/x}{\lambda_r/x}$	ratio using definition of sine
3. $\dfrac{\sin\theta_i}{\sin\theta_r} = \dfrac{\lambda_i}{\lambda_r}$	simplify
4. $\dfrac{\lambda_i}{\lambda_r} = \dfrac{n_r}{n_i}$	change of wavelength
5. $\dfrac{\sin\theta_i}{\sin\theta_r} = \dfrac{n_r}{n_i}$	substitute equation 4 into equation 3

Light is a particle.

Many of the great scientists of the 17th and 18th centuries who made fundamental contributions to the study of optics, including Isaac Newton, thought that light consisted of a stream of "corpuscles," or particles. In the 20th century, Albert Einstein explained the photoelectric effect. His explanation, for which he was awarded the 1921 Nobel Prize, depended on the fact that light acts like a particle. This property of light led to the coining of the term "photon" for a single particle of light by the chemist Gilbert Lewis.

Light is a wave.

Between the 18th and 20th centuries, physicists discovered many wave-like properties of light. They found that a number of phenomena they routinely observed with water waves they could also observe with light. For instance, the English scientist Thomas Young (1773-1829) showed that light could produce the same kinds of interference patterns that water waves produce. At the right, you see examples of interference patterns formed by light and by water waves. The similarities are striking. In this chapter, you will apply to light some of what you have studied about the interference of sound waves and traveling waves in strings.

Let there be light.

Is light a particle, a wave, or both? Perhaps an Early Authority had it right. Light is light. It is a combination of electric and magnetic fields. Trying to classify light as a particle or as a wave may be a fruitless effort □ better to revel in its unique properties.

In this chapter, we will revel in its wave-like properties, and discuss the topic of interference. Your prior study of electromagnetic radiation modeled as a wave phenomenon will prove useful.

Interference

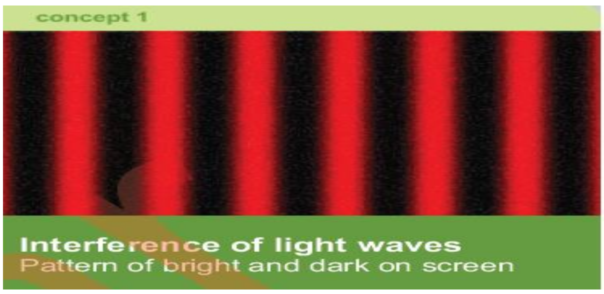
In Concept 1, you see an *interference pattern* created by causing a beam of light to pass through two parallel slits to illuminate a viewing screen. Constructive interference of light waves accounts for the bright regions (called bright *fringes*) while destructive interference causes the dark fringes. In this section, we review some of the fundamentals of interference, and discuss the conditions necessary for light to make the pattern you see to the right. You may have already studied the interference of mechanical waves; for instance, what occurs when two waves on a string interact. In this chapter, you will study what happens when electromagnetic waves meet. Some of the same principles and terminology are used in discussing both kinds of interference.

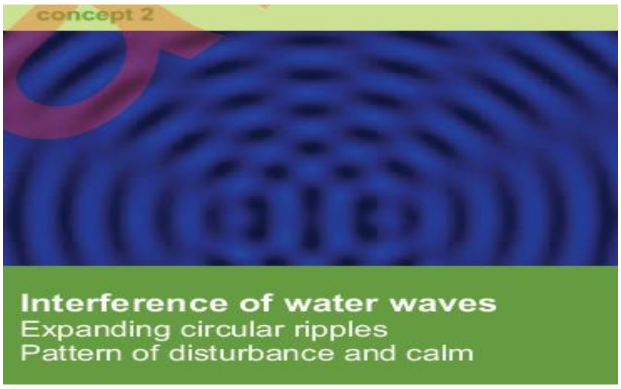
When two light waves meet, the result can be constructive or destructive interference.

In the following discussion, we assume that the waves have equal amplitude. Constructive interference creates a wave of greater amplitude and more intensity than either source wave; destructive interference results in a wave of smaller amplitude and less intensity than either source wave. At any point in a two-slit interference pattern such as that to the right, light waves from the two sources meet and interfere constructively, destructively, or partially (exhibiting a degree of interference somewhere between complete constructive and destructive interference). To create an interference pattern, a physicist needs light that is:

1. *Monochromatic*. This means light with a specific wavelength. For instance, experimenters can produce the pattern you see in Concept 1 by using pure red light.

2. *Coherent*. This means the phase difference between the light waves arriving at





The pattern of bright and dark fringes extends to both the left and the right on the screen. The light is interfering constructively at the bright fringes, and destructively at the dark fringes, because of different path lengths to these regions and the resulting phase differences. There are a few limitations to showing Young's apparatus in a compact diagram. First, the diagram is far from being drawn to scale. The screen should be much farther from the double-slit barrier than we show here, and the slits should be narrower and closer together. In actual interference experiments, the interfering rays from the two slits are practically parallel. Second, we vastly exaggerate the wavelength of the light. You may have a question about what you would see if you conducted this experiment yourself. What if, at some instant, two waves meet at the screen and are in phase, but their electric and magnetic fields both happen to be zero at that point? Would you see "flickering" as the two reinforcing waves moved from peak to trough and back again? The answer is no: The frequency of light is so great that you only perceive the average brightness of a region; the human eye does not perceive changes in intensity due to the oscillation of a light wave. You do not even perceive flicker in systems oscillating at far lower frequencies, much less than the frequency of visible light, which is on the order of 1014 Hz. For

example, a computer monitor refreshes its display 60 times a second, but you do not ordinarily perceive any flicker when you look at it.