University of Technology

Chemical Engineering Department

Mechanic & Strength Of Materials

First Class

By

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الفصل الثاني /

Second Term : Strength of Materials

Strength of materials deals with the relations between externally applied loads and their internal effects on bodies are no longer assumed to be ideally rigid.

One of the basic problems of the engineer is to select the proper material and correctly use and proportion it so as to enable a structure or machine to do most efficiently what it is designed to do. For this purpose it is essential to determine the strength and other properties of materials.

Let us consider two bars of equal length but different materials as shown in figure. Support the loads 500 N for bar -1- and 5000 N for bar -2-. bar -1- has a cross section area of 10 mm^2 and bar -2- has an area of 1000 mm^2,

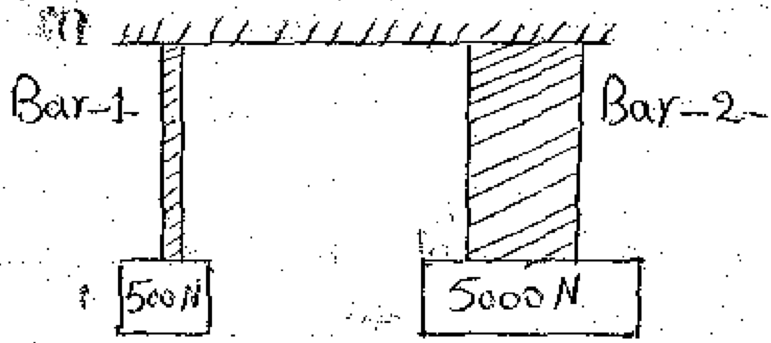
Strength is force (load) per unit area.

$$\boxed{\sigma = \frac{P}{A}}$$

$$\sigma_1 = \frac{500\ N}{10 \times 10^{-6}\ m^2} = 50 \times 10^{6}\ N/m^2$$

$$\sigma_2 = \frac{5000\ N}{1000 \times 10^{-6}\ m^2} = 5 \times 10^{6}\ N/m^2$$

Thus the material of bar-1- is ten times as strong as the material of bar-2-.



The unit strength of material is usually defined as the stress in material.

stress : $\sigma = \frac{P}{A}$

stress Unit : SI Unit (N/m^2) (Pascal)
where A : cross section area.

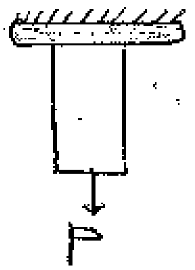
Analysis of Internal forces :

In strength of materials, we make an additional investigation of the internal distribution of the forces.
Effect of simple loads are divided into axial load and shear load.

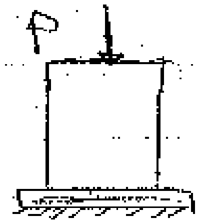
Load: External force affect on a body.

1) Normal stress

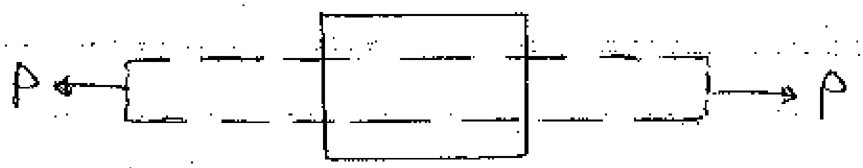
Tensil stress



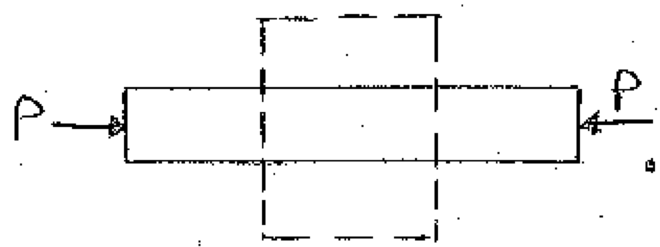
Compressive stress



a) Tensil force (Tension) tends to elongate the body.

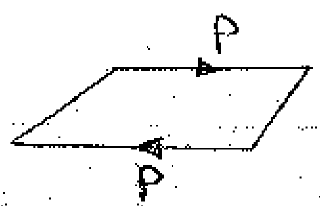


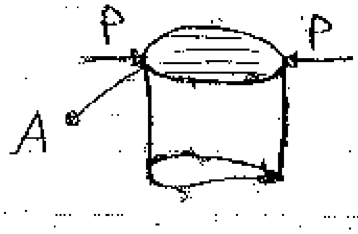
b) Compressive force (Compression) tends to shorten the body.



2) Shear Stress (shear load):

These are components of the total resistance to sliding the portion to one side of the exploratory section past the other.





$$\boxed{\tau = \frac{V}{A_s}}$$

where:

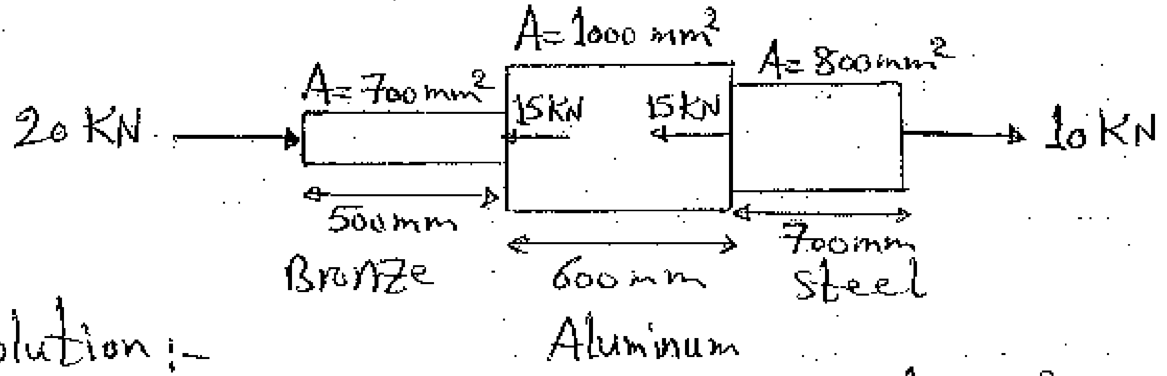
τ : shear stress (N/m^2)

V : shearing force

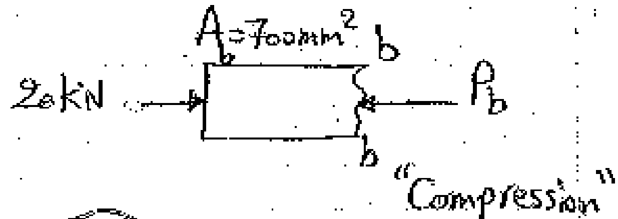
A_s : surface area being sheared.

Ex. (1): Pag. (9)

An aluminum tube is rigidly fastened between a bronze rod and a steel rod as shown in figure. Axial loads are applied at the positions indicated, determine the stress in each material.



Solution :-



$$\sigma_B = \frac{P_b}{A_b}$$

$$= \frac{20 \times 10^3 \ N}{700 \times 10^{-6} \ m^2} = 28.6 \times 10^6 \ N/m^2$$

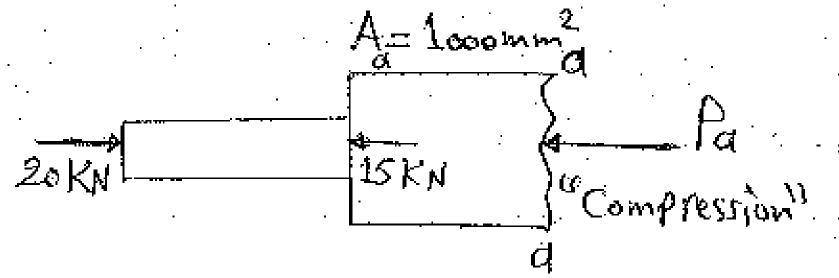
$$= 28.6 \ M \ N/m^2$$

$$= 28.6 \ M \ Pa.$$

$$\sigma_A = \frac{P_a}{A_a}$$

$$= \frac{5 \times 10^{3} \ N}{1000 \times 10^{-6} \ m^2}$$

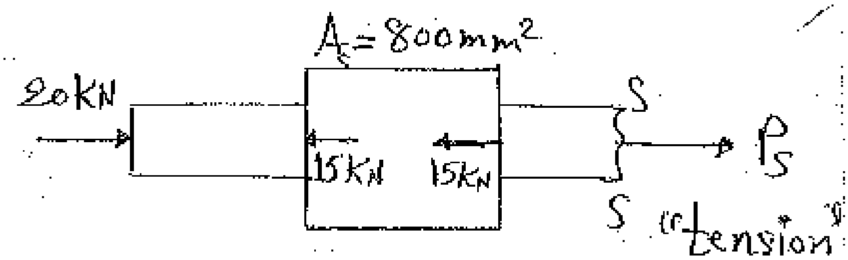
$$= 5 \ MPa.$$



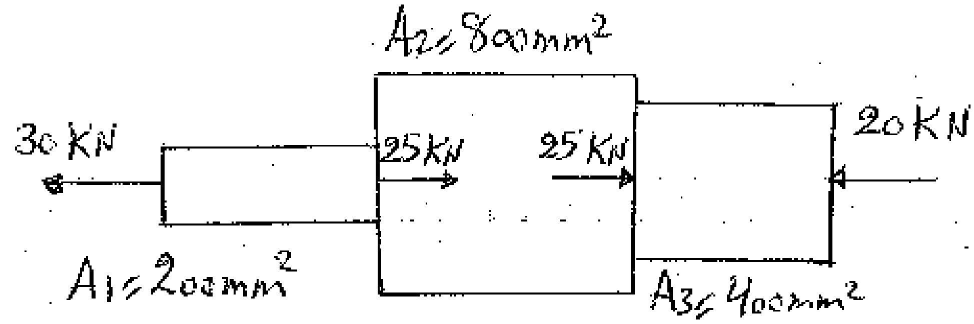
$$\sigma_S = \frac{P_s}{A_s}$$

$$= \frac{10 \times 10^{3} \ N}{800 \times 10^{-6} \ m^2}$$

$$= 12.5 \ MPa.$$

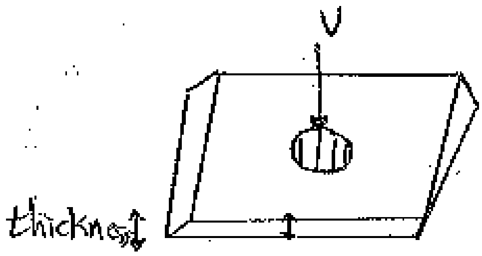


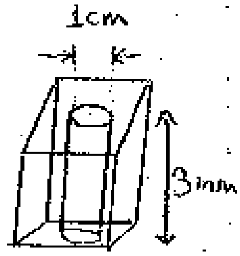
<u>H.W.</u> Determine the stress in each bar in the fig.



Ex. (2) Ordinary mild steel rupture where a shear stress of about $(3.5 \times 10^{8}\ N/m^2)$ is applied, find the force needed to punch at (1 cm) diameter hole in a steel sheet (3 mm) thickness?

Solution:-





$$\tau = \frac{V}{A_s}$$

$$A_s = \pi \cdot d \cdot L$$
$$= \pi \cdot (1 \times 10^{-2})\,m \cdot (3 \times 10^{-3})\,m$$
$$= 9.4 \times 10^{-5}\ m^2.$$

$$V = \tau \cdot A_s$$
$$= 3.5 \times 10^{8}\ \frac{N}{\cancel{m^2}} \cdot 9.4 \times 10^{-5}\ \cancel{m^2}$$
$$= 3.3 \times 10^{4}\ N.$$

Tutorial Sheet No. (1)

Problems Pag. (12)

in Strength of Material Book

Singer

P. 103.
P. 106.
P. 110,
P. 111,
P. 113,

Strain (ϵ) : Changing in shape or dimensions of the body because of the external loads.

$$\boxed{\epsilon = \frac{\delta}{L}}$$

where,

δ : change in length (elongation) or (deformation).

L : original length.

Strain
- Compressive strain (deformation)
- Tensile strain (elongation)

Stress _ Strain Diagram

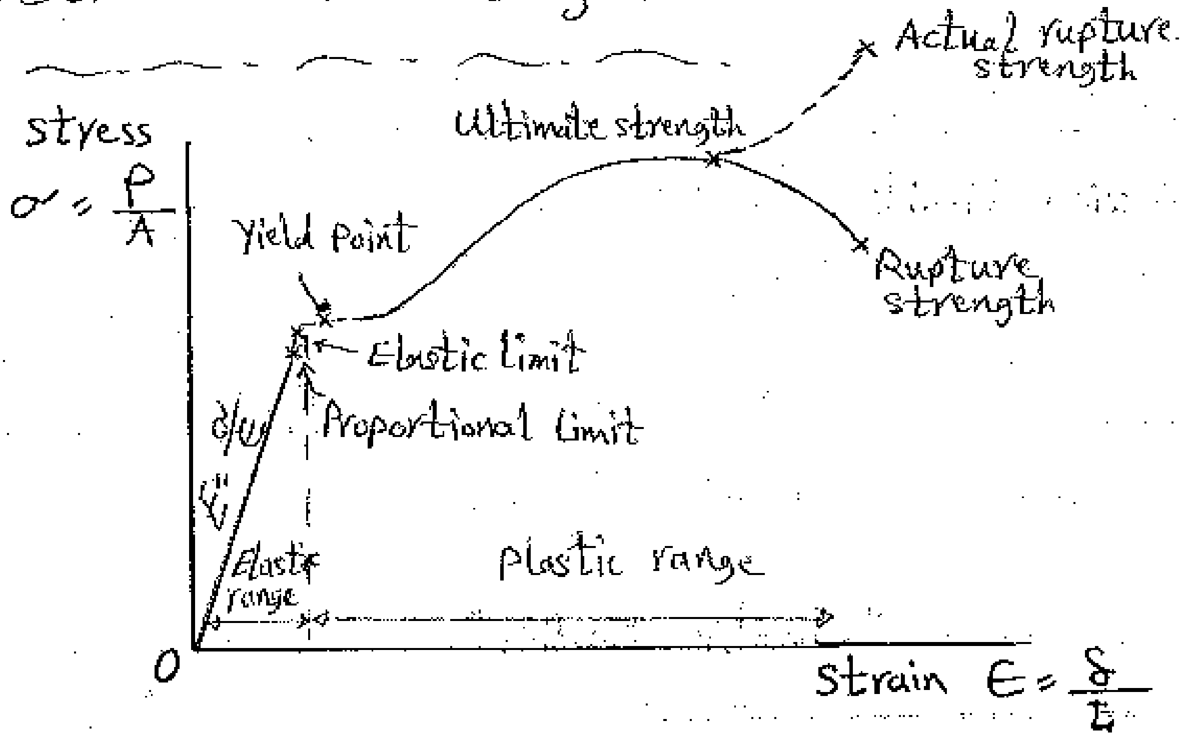


Figure-1: Stress _ Strain Diagram

Working stress :

$$\boxed{\sigma_w \leq \text{Proportion limit}}$$

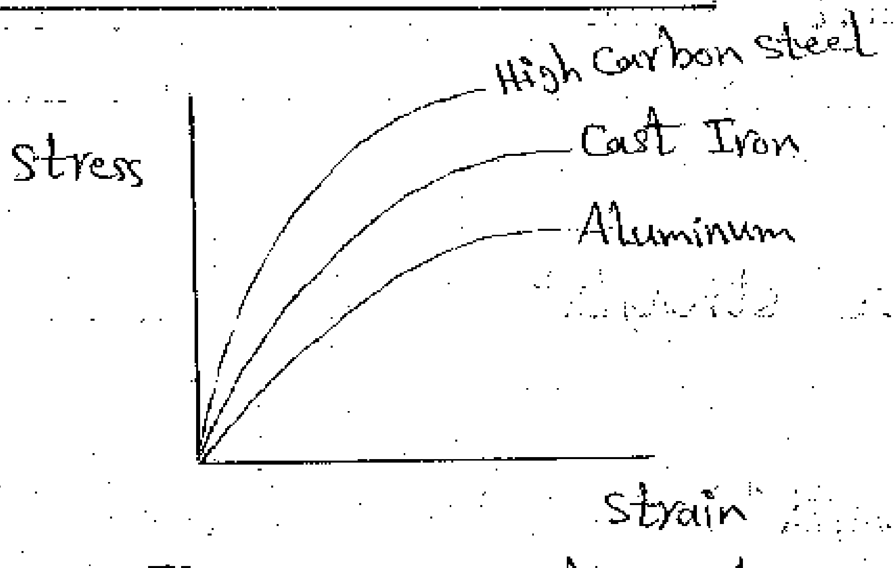


Figure-2: Comparative stress - strain diagram for different materials

In figure 1- :

1- From the origion point "0" to a point called the Proportion limit, the stress - strain diagram to be a straight line.

2- Beyond the proportion limit the stress is no longer Proportional to strength.

3- Behaviour of "elastic bodies" is based on stress - strain proportionality.

4- The "elastic limit" is the stress beyond which the material will not returne to it's origional shape when unloaded.

5- The "Yield Point" is the point at which the appreciable elongation of the material without any increase of the load.

6- The "Ultimate strength" is a highest point on the stress - strain diagram.

7- "Rupture strength" is the stress of failure

$$\sigma_w = \frac{\sigma_{y.p}}{N} \quad \text{or} \quad \frac{\sigma_{u.p}}{N}$$

where:
$\sigma_{y.p}$: stress at yield point.
$\sigma_{u.p}$: stress at ultimate point.
N : Safty factor.

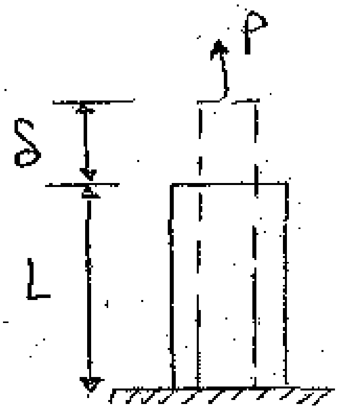
Hook's Law :-

Applicable for the strength portion of the stress-strain diagram. The ratio of stress to strain is called the "modulus of elasticity" and ~~is~~ denoted by E :- معامل المرونة

$$\boxed{\underset{(N/m^2)}{E} = \frac{\sigma}{\epsilon} = \frac{P/A}{\delta/L}}$$ ---- Hook's law

"E" also called [Young's modulus].

$$\boxed{\underset{(mm)}{\delta} = \frac{P \cdot L}{A \cdot E} = \frac{\sigma \cdot L}{E}}$$



where:
δ : is elongation

Conditions using Hook's law :-

1- The load must be axial.
2- The bar must have a constant cross-section area.
3- The stress must not exceed the proportion limit.

Shearing Deformation

Shearing forces cause a shearing deformation, an element subject to shear does not ~~change~~ change the length of it's sides, but undergoes a change in shape from a rectangle to parallelogram, an shown in figure.

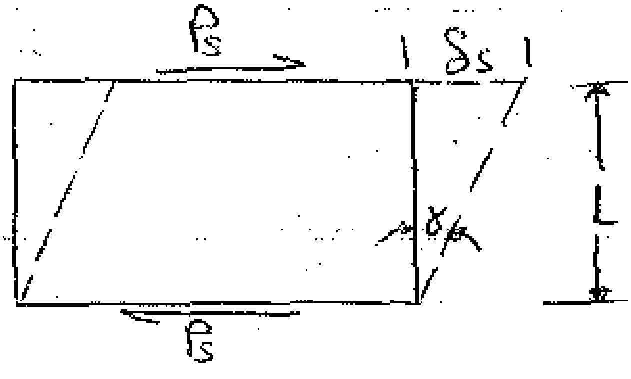


Figure: shear deformation

Shearing stress : $\tau = \frac{V}{A_s}$

Shearing strain : $\gamma = \frac{\delta_s}{L}$

The relation between shearing stress and shearing strain assuming Hooke's law to apply to shear

$$\tau = G \cdot \gamma$$

where: G: Modulus of rigidity. معامل الصلادة

$$\frac{V}{A_s} = G \cdot \frac{\delta_s}{L}$$

$$\therefore \boxed{\underset{(mm)}{\delta_s} = \frac{V \cdot L}{A_s \cdot G} = \frac{\tau \cdot L}{G}}$$ — Hook's law for shear force.

Example (1): A load of (400 N) is to be raised at the end of a (8 m) Long steel wire. If the stress in the wire is not to exceed ($5 N/mm^2$) what should be the:

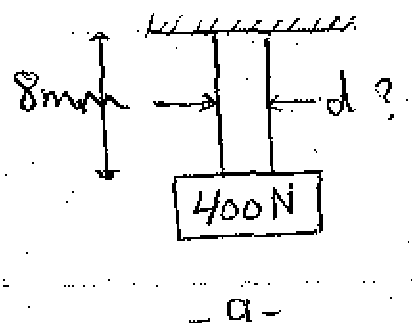
a) Minimum diameter of the wire ?

b) Elongation of the wire ?

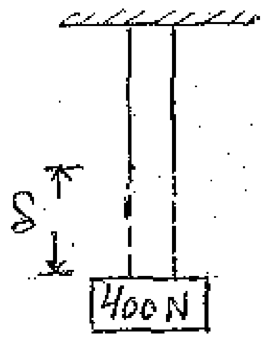
Take "E" for steel = 20,000 N/mm^2.

Solution:

$\sigma = 5 \frac{N}{mm^2}$



-a-



-b-

$$\sigma = \frac{P}{A} \Rightarrow 5\,\frac{N}{mm^2} = \frac{400\,N}{A}$$

$$\therefore A = 80\ mm^2$$

$$\underset{(c.s.a)}{A} = \frac{\pi}{4}\cdot d^2 \Rightarrow 80 = \frac{\pi}{4}(d)^2.$$

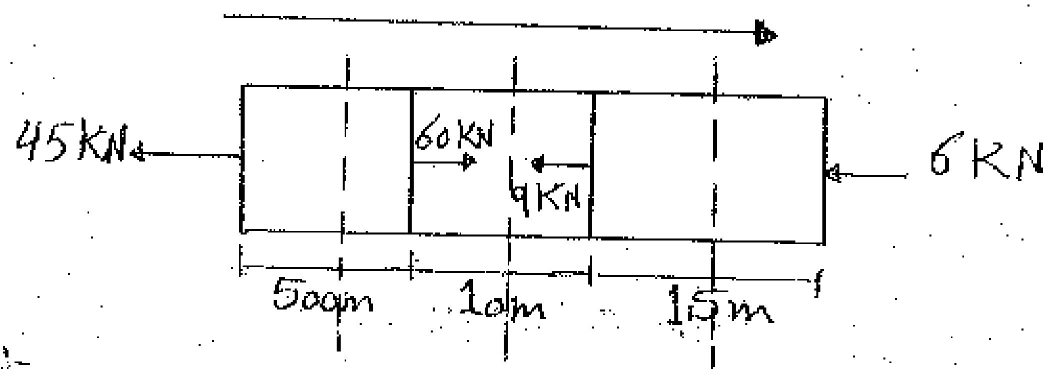
$$\therefore d = 10.09\ mm$$ (1) و.ه.م

$$\delta = \frac{P.L}{A.E} \Rightarrow \delta = \frac{400\,N \times 8\ mm}{80\ mm^2 \times 20{,}000\ \frac{N}{mm^2}}$$

$$\therefore \delta = 2\ mm$$ (2) و.ه.م

Example (2):

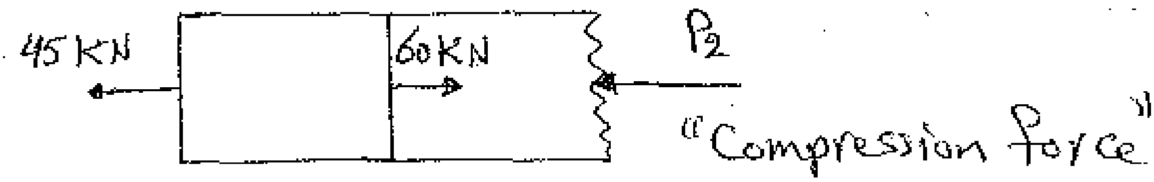
Determine the elongation in each Portion (shown in figure). $E = 90\ G\ N/m^2$, $A = 10^3\ mm^2$



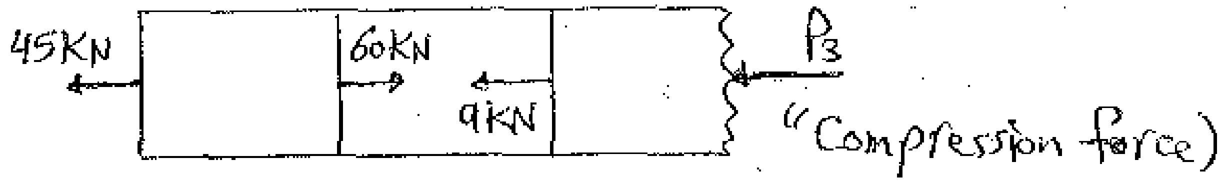
Solution:-

$$\delta_1 = \frac{P.L}{A.E} \Rightarrow = \frac{(45\times10^3)\,N \times (500\times10^3)\,mm}{(10^3\times10^{-6})\,m^2 \times (90\times10^9)\,\frac{N}{m^2}} = 250\,mm$$

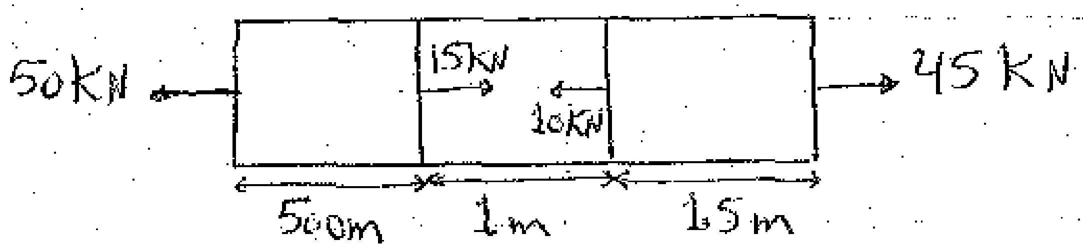
$$\delta_2 = \frac{(15\times10^{3})\,\cancel{N}\times(10\times10^{3})\,mm}{(10^{3}\times10^{-6})\,\cancel{m^2}\times(90\times10^{9})\,\frac{\cancel{N}}{\cancel{m^2}}} = 1.67\,mm$$



$$\delta_3 = \frac{(6\times10^{3})\,\cancel{N}\times(1.5\times10^{3})\,mm}{(10^{3}\times10^{-6})\,\cancel{m^2}\times(90\times10^{9})\,\frac{\cancel{N}}{\cancel{m^2}}} = 0.1\,mm$$

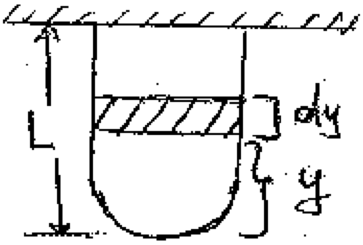


H.W A bar of $(500\ mm^2)$ c.s.a is loaded as shown in figure, calculate the total elongatio $E = 200\ GN/m^2$.



Example (3): Calculate total elongation caused by the weight of the body.

الاستطالة الناشئة عن وزن الجسم



$$d\delta = \frac{A \cdot y \cdot \rho \cdot dy}{A \cdot E}$$

$$\delta = \int_0^L d\delta = \int_0^L \frac{A \cdot y \cdot \rho \cdot dy}{A \cdot E}$$

$$= \frac{A \cdot \rho}{A \cdot E} \int_0^L y \cdot dy$$

$$= \frac{A \cdot \rho}{A \cdot E} \times \left[\frac{y^2}{2}\right]_0^L$$

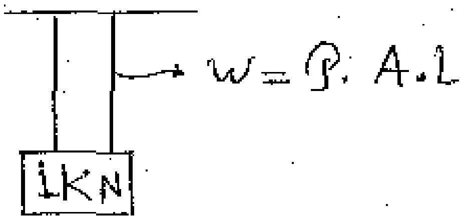
$$= \frac{A \cdot \rho}{A \cdot E} \times \frac{L^2}{2} = \frac{(A \cdot \rho \cdot L) \times L}{2 \cdot A \cdot E}$$

$$\boxed{\therefore \delta = \frac{W \cdot L}{2 \cdot A \cdot E}}$$

where : W : weight

Example (4): A solid wire of (6 mm) diameter and length (150 m) supporting a load of (1 KN) to lifted from the bottom end of the wire. The density of wire material is $(7.7 \times 10^{4}\ N/m^3)$. $E = 200\ GN/m^2$. Calculate the total elongation.

solution :-



$$\delta_1 = \frac{P.L}{A.E} \quad \text{.... Hook's law for (1KN)}$$

$$= \frac{(1\times10^{3})\,\cancel{N} \times (150\times10^{3})\,mm}{[\frac{\pi}{4}(6)^2\times10^{-6}]\,\cancel{m^2} \times (200\times10^{9})\,\cancel{\frac{N}{m^2}}} = 26.53\ mm$$

$$\delta_2 = \frac{W.L}{2.A.E} \quad \text{.... for wire weigth}$$

$$= \frac{(7.7\times10^{4}\times10^{-6})\,\cancel{N}/mm^3 \times \frac{\pi}{4}(6)^2\,\cancel{mm^2} \times (150\times10^{3})\,mm \times (150\times10^{3})\,mm}{2\times\frac{\pi}{4}(6)^2\,\cancel{mm^2} \times (200\times10^{9}\times10^{-6})\,\cancel{\frac{N}{mm^2}}}$$

$$= 4331.25\ mm$$

$$\delta_{total} = \delta_1 + \delta_2$$
$$= 26.53 + 4331.25$$
$$= 4357.78\ mm$$

Example (5) :- Calculate the yield point stress and the ultimate stress from the following results of a steel tensile test.

Cylinder bar diameter = 20 mm.

bar length = 200 mm.

Load at yield point = 10,000 N.

Maximum load = 160,000 N.

Solution:
$$A = \frac{\pi}{4} d^2$$
$$= \frac{\pi}{4} (20)^2 = 314.15 \, mm^2$$
$$\sigma_{y.p} = \frac{P_{y.p}}{A}$$
$$= \frac{10,000 \, N}{314.15 \, \cancel{mm^2} \times 10^{-6} \frac{m^2}{\cancel{mm^2}}} = 31.83 \text{ MPas.}$$
$$\sigma_{u.p} = \frac{\text{Max. load}}{A}$$
$$= \frac{160,000 \, N}{314.15 \times 10^{-6} \, m^2} = 509.31 \text{ M Pas.}$$
$$\therefore \sigma_{u.p} > \sigma_{y.p}$$

Poisson's ratio, composite stresses

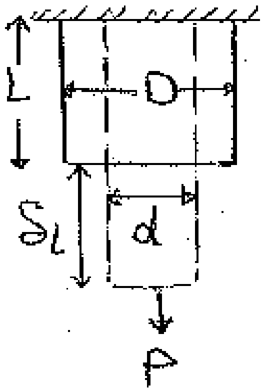
Another type of "elastic deformation" accompaying axil tension or compression.

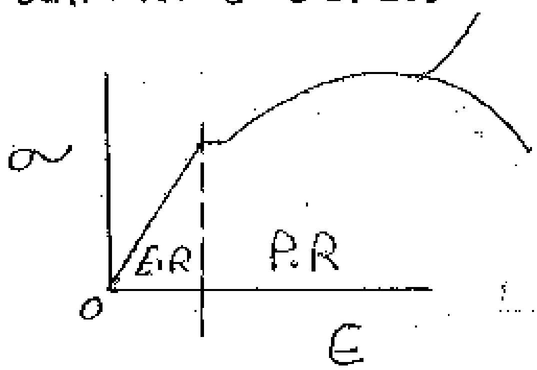
جانبي

Poisson's ratio within the elastic limit, the lateral strain is a constant ratio to the long strain and is a function of it.

$$\boxed{\text{Poisson's ratio} = \frac{\overset{\text{جانبي}}{\text{lateral strain}}}{\text{طولي longitudenal strain}}}$$

Lateral strain : Uniaxial stress





longitudinal strain $\epsilon_L = \frac{\delta_L}{L}$

Lateral strain $\epsilon_D = \frac{\delta_D}{D}$

$$\boxed{\nu = -\frac{\delta_D/D}{\delta_L/L}}$$

الإشارة السالبة :- الزيادة في اتجاه معين يصاحبه نقصان بالاتجاه الأخر،

$$\boxed{\delta_D = -\vartheta \cdot \epsilon_L \cdot D}$$ for uniaxial stress

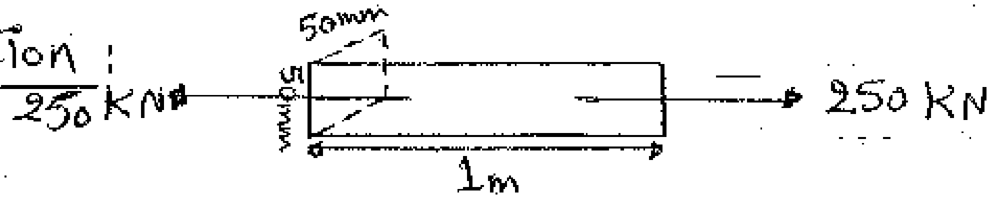
where: δ_L calculate from Hook's law

$$\boxed{\delta_L = \frac{P \cdot L}{A \cdot E}}$$

Example ①: A (50 mm) square bar of (1m) length is subjected to an axial tensile load of (250 KN), compute the total elongation in the lateral diameter. $E = 200\ GN/m^2$, $\vartheta = 0.3$.

التغير بالقطر δD

Solution:



$$\vartheta = -\frac{\epsilon_D}{\epsilon_L}$$

$$\underset{\text{Strain}}{\epsilon_L} = \frac{\overset{\text{Stress}}{\sigma_L}}{E}$$ معامل المرونة

$$\underset{\text{Longitud stress}}{\sigma_L} = \frac{P}{A} \Rightarrow = \frac{250 \times 10^3\ N}{(50 \times 50) \times 10^{-6}\ m^2}$$

$$= 100\ MPa$$

$\therefore \epsilon_L = \dfrac{100 \times 10^6 \ \cancel{N/m^2}}{200 \times 10^9 \ \cancel{N/m^2}}$

longitude strain

$= 5 \times 10^{-4}$

$\epsilon_D = -0.3 \times (5 \times 10^{-4})$

lateral strain

$= -1.5 \times 10^{-4}$

$\delta_D = D \times \epsilon_D$

$= (-)50 \text{ mm} \times 1.5 \times 10^{-4}$

$= -7.5 \times 10^{-3} \text{ mm}$

وهام

كلية الأوسمة

For Biaxial stress : الناتج من تأثير قوتين (P , $\bar{P}$) بأتجاهين مختلفين

$$\epsilon_L = \frac{\sigma_L}{E} - \nu \cdot \frac{\sigma_D}{E}$$

معامل المرونة $E = \frac{\sigma \text{ stress}}{\epsilon \text{ strain}}$

for force (P) direct — for force ($\bar{P}$) indirect

$$\epsilon_{L\,\text{strain}} = \frac{\sigma_L}{E}$$

$$\boxed{\epsilon_L = \frac{1}{E} \cdot \left[\sigma_L - \nu \cdot \sigma_D\right]}$$

$$\nu = \frac{\epsilon_d}{\epsilon_L}$$

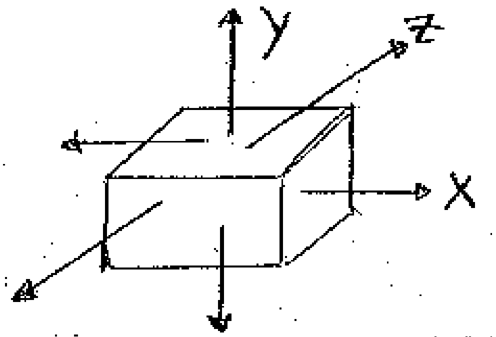
$$\boxed{\epsilon_D = \frac{1}{E} \left[\sigma_D - \nu \cdot \sigma_L\right]}$$

$$\epsilon_d = \nu \cdot \epsilon_L$$

$$\boxed{\epsilon_d = \nu \cdot \frac{\sigma_D}{E}}$$

Traiaxial stress

وتستعمل لحساب أبعاد الخزان



$$\epsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right].$$

$$\epsilon_y = \frac{1}{E}\left[\sigma_y - \nu\left(\sigma_x + \sigma_z\right)\right].$$

$$\epsilon_z = \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right].$$

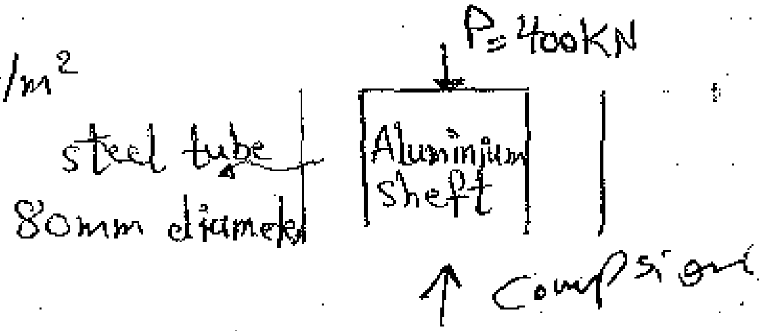
$\longrightarrow$ d_{steel}

Example 2:- Compute the minimum internal diameter of the steel tube shown in figure :-

$\nu = 0.3$

$E = 70 \times 10^{9}\ N/m^2$

Aluminium



Solution :-

$$\sigma_L = \frac{P}{A} \Longrightarrow = \frac{\ominus\ 400 \times 10^{3}\ N}{\frac{\pi}{4}(0.08)^2\ m^2}$$

compression force

$= \ominus 79.6\ MPa$

$$\nu = -\frac{\epsilon_D}{\epsilon_L} \longrightarrow \boxed{\text{معامل يونج}\ E = \frac{\sigma_L}{\epsilon_L}}$$

$\therefore \epsilon_D = -0.3 \times \epsilon_L$

$$= -0.3 \times \boxed{\frac{(-79.6 \times 10^{6}\ N/m^2)}{70 \times 10^{9}\ N/m^2}} \quad \epsilon_L = \frac{\sigma_L}{E}$$

$= +379 \times 10^{-6}$

$\delta_D = \epsilon_D \times D$

$= 379 \times 10^{-6} \times 80\ mm$

$= 0.0303\ mm$

$\therefore$ minimum diameter of steel $= D + \delta_D$

$= 80 + 0.0303$

$= 80.0303\ mm$

Example (3): A welded steel cylindrical drum made of 10mm plate has an internal diameter of 1.20m. By how much will the diameter be changed by an internal pressure of 1.5 Mpa? Assume that Poisson's ratio is 0.3 and E = 200 GPa.

P_L

Solution: $E = \frac{\sigma_L}{\epsilon_L} = \frac{\text{stress}}{\text{strain}}$

$\epsilon_L = \frac{\sigma_L}{E} \Longrightarrow = \frac{-1.5 \times 10^{6}\ N/m^2}{200 \times 10^{9}\ N/m^2}$

$= -0.75 \times 10^{-5}$ Compression stress

$\nu = -\frac{\epsilon_D}{\epsilon_L}$

$\epsilon_D = -\nu \cdot \epsilon_L$

$= -0.3 \times (-0.75 \times 10^{-5})$

$= +0.225 \times 10^{-5}$ increase of diameter

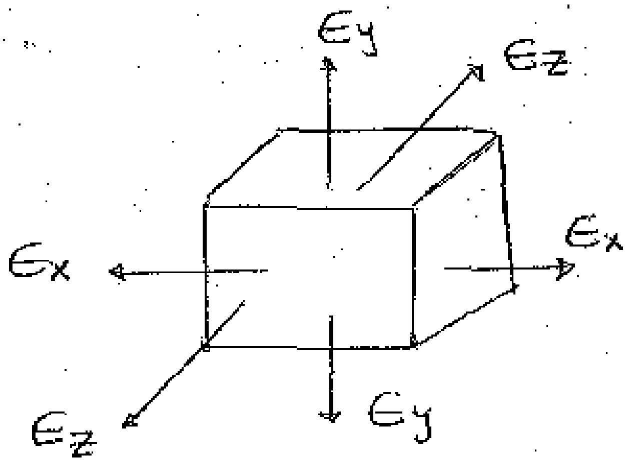
$\therefore \delta D = \epsilon_D \times D$

$= 0.225 \times 10^{-5} \times 1.20\ \cancel{m} \times 10^{+3}\ \frac{mm}{\cancel{m}}$

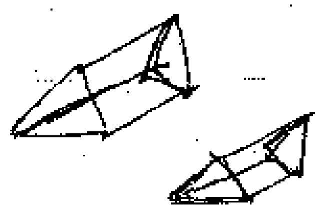
$= 0.27 \times 10^{-2}$ mm   P.Q.J

Volumetric Strain

Case 1: ~~sets~~ Cube



for ~~Prism~~ of dimensions x, y, z.



original volume $= (X \cdot Y \cdot Z)$

after elongation
$$X \longrightarrow [X + \epsilon_x \cdot X]$$
$$Y \longrightarrow [y + \epsilon_y \cdot \cancel{Y}]$$
$$Z \longrightarrow [z + \epsilon_z \cdot z]$$

New Volume $= (x + \epsilon_x \cdot X) * (y + \epsilon_y \cdot y) * (z + \epsilon_z \cdot z)$

$= X \cdot y \cdot z + X \cdot y \cdot z\,(\epsilon_x + \epsilon_y + \epsilon_z)$.

Change in volume $=$ New Volume $-$ Original volume
$= x \cdot y \cdot z\,(\epsilon_x + \epsilon_y + \epsilon_z)$.

$$\boxed{\text{Volumetric strain} = \frac{\text{change in volume}}{\text{Original volume}}}$$

$$\epsilon_v = (\epsilon_x + \epsilon_y + \epsilon_z)$$

where :

$$\epsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right].$$

$$\epsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right].$$

$$\epsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right].$$

$$\therefore \boxed{\epsilon_v = \frac{1}{E}\left[(\sigma_x + \sigma_y + \sigma_z)(1 - 2\nu)\right]}.$$

apply ① for three unequal forces.

② A cube.

<u>For</u> a <u>cube</u> subjected to equal forces in three dimensions :-

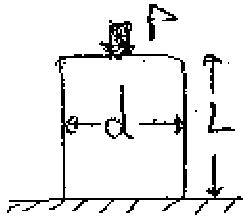
$$\sigma_x = \sigma_y = \sigma_z = \sigma$$

$$\boxed{\epsilon_v = \frac{3 \cdot \sigma}{E}(1 - 2\nu)}$$

apply ① for a cube

② equal forces

Case 2: For a cylindrical bar of diameter (d) and length (L), subjected to an axial compression Load (P).



uni-axial force

original volume $= \frac{\pi}{4} \cdot d^2 \cdot L$

New volume $= \frac{\pi}{4} (d + \delta_D)^2 \cdot (L + \delta_L)$

$\delta_V =$ New − original

$$= \frac{\pi}{4} \left(\underbrace{d^2 \cdot \delta_L}_{\times L/L} + \underbrace{2 \cdot d \cdot L \cdot \delta_D}_{\times d/d} \right)$$

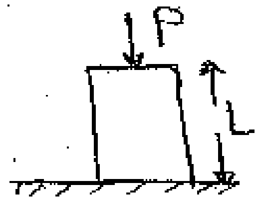
$$= \overbrace{\frac{\pi}{4} d^2 \cdot L}^{V=} \left(\frac{\delta_L}{L} + 2 \cdot \frac{\delta_D}{d} \right)$$

$$\boxed{\epsilon_V = \frac{\delta_V}{V} = (\ominus \epsilon_L + 2 \cdot \epsilon_D)}$$

for ① compress
② cylinder
③ uniaxial force

Example :- A cylindrical bar of (20 cm) length and (10 cm) diameter is subjected to an axial Compressive load of (40,000 Kg) if $E = 2 \times 10^{6}$ Kg/cm^2, $\nu = 0.4$, Calculate the change in volume of the bar.

Compression force



Solution :-

$$\frac{\delta V}{V} = \epsilon_V = (\epsilon_L + 2 \cdot \epsilon_d)$$

$$\sigma_L = \frac{P}{A} \Longrightarrow = \frac{40{,}000}{\frac{\pi}{4}(10)^2}$$

$$= \frac{40{,}000}{25 \cdot \pi} \; (\text{Kg/cm}^2)$$

$$\epsilon_L = \frac{\sigma_L}{E} \Longrightarrow = (-)\, \frac{40{,}000 \; \cancel{\text{Kg/cm}^2}}{25 \cdot \pi \times (2 \times 10^{6}) \; \cancel{\text{Kg/cm}^2}}$$

$$= (-)\, 0.00025 \quad \text{Compressive}$$

$$\epsilon_D = -\nu \cdot \epsilon_L$$

$$= -0.4 \times (-0.00025)$$

$$= 10^{-6}$$

$$\delta V = V \times \epsilon_V$$

$$= V \times (\epsilon_L + 2 \cdot \epsilon_D)$$

$$= \frac{\pi}{4} \times (10)^2 \times (20) \times (-0.00025 + 2 \times 10^{-6})$$

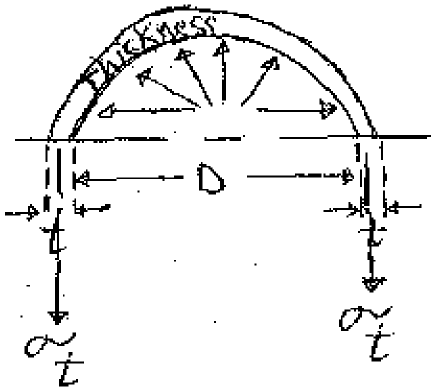
$$= \underline{0.4 \text{ cm}^3}$$

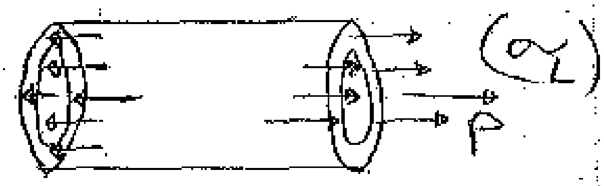
Thin-Walled Cylinders

In thin shells, the thickness of metal is very small compared to the diameter of shell. Thin cylindrical shell such as air vessels, boilers, water or gas pipes, etc. are subjected to fluid pressures. The fluid pressure is uniformly distributed over the internal surface of the shell and it produces two principle stresses in the material of the shell.

Thin walled vessels [thickness / diameter < 1/10].

<u>Case 1:</u> Cylindrical vessels.





1) σ_t : Circumferential stress is the stress acts tangenti to the circumference.

$$\boxed{\sigma_t = \frac{P \cdot D}{2 \cdot t}}$$

where:

P: Applied pressure.

D: inside diameter.

t: thickness.

2) σ_l : Longitudinal stress is the stress parallel to the longitudinal axis of the cylinder.

$$\boxed{\sigma_l = \frac{P.D}{4.t}}$$

$$\boxed{\sigma_t = 2.\sigma_l}$$

Example : A close cylindrical tank used to store compressed air with internal diameter of (600mm). The pressure of the air inside the tank is (3.5 Mpa) if $\sigma_{y.p} = 250$ Mpa , $N_{saft\,y.p} = 3.5$, Calculate the thickness of the wall of the tank.

Solution:

$$\sigma_t = \frac{P.D}{2.t}$$

$$\frac{250}{3.5} = \frac{3.5 \times 600}{2.t} \Longrightarrow t = 14.7 \text{ mm}$$

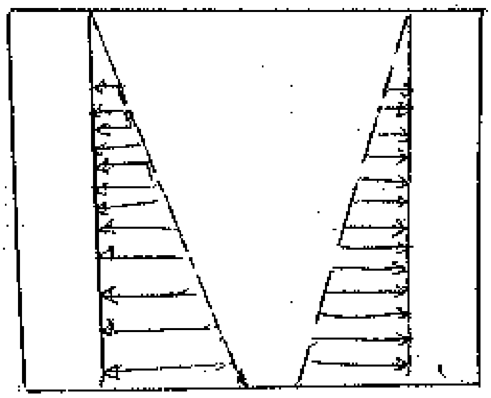
$$\sigma_l = \frac{P.D}{4.t}$$

$$\frac{250}{3.5} = \frac{3.5 \times 600}{4.t} \Longrightarrow t = 7.32 \text{ mm}$$

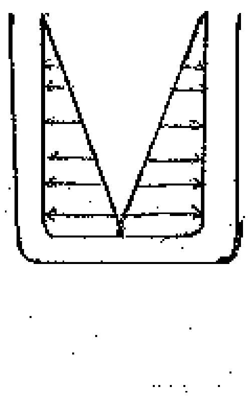
$\therefore$ Required thickness should be = 14.7 mm.

For open vessels :- $[\sigma_L = 0]$

في الخزانات المفتوحة لا يوجد اجهاد طولي لان قمة الخزان مفتوحة . ويبقى الاجهاد الجانبي فقط والذي يتسبب من تأثير الضغط الهيدروستاتيكي الذي يؤثر نصف قطرياً في أتجاه الجدار الداخلي للخزان ويتناقص كلما أتجهنا الى قمة الخزان وتقع القيمة القصوى عند القاعدة وهي المنطقة التي يجب اعتبارها عند التصميم .



Example: An open cylindrical tank with internal diameter of (3m) and height of (25m) filled with water if $\sigma_{y.p} = 250$ Mpa and $N_{y.p} = 2$, Calculate the thickness of the base of the tank if the welding efficiency $= 75\%$. $\rho = 10^4\ N/m^3$



Solution:

$$P = \rho . g . h$$

P: Pressure .
ρ: density of the liquid .
h: height of the liquid .

$\therefore \rho \times g = 10^4 \ N/m^3$

$\therefore P = 10^4 \ N/\cancel{m^3}^{2} \times 25 \ \cancel{m}$

$= 25 \times 10^4 \ N/m^2$

$= 0.25 \ Mpa$

$$\sigma_t = \frac{P \cdot D}{2 \cdot t}$$

$$\frac{\sigma_{y.p}}{N_{y.p}} = \frac{250}{2} = \sigma_t$$

$$\therefore \frac{250 \ Mpa}{2} = \frac{0.25 \ Mpa \times 3\cancel{m} \times 10^3 \ \frac{mm}{\cancel{m}}}{\cancel{2} \times t}$$

$t = 3 \ mm$ (of eff = 100%)

$\because$ eff = 75%

$\therefore t = 3/0.75$

$= \underline{4 \ mm}$ Required thickness.

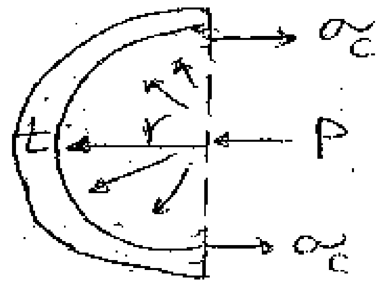
تضاف زيادة بالسمك لأغراض التآكل.

Case 2 :- Spheres

تستعمل لخزن الغازات تحت الضغوط العالية

$\sigma_t = \sigma_L = \sigma_c$

$$\boxed{\sigma_c = \frac{P \cdot D}{4 \cdot t}}$$



Example :-

Spherical storage tank of (20m) diameter is used to store gas, if the thickness of the thin wall is (10mm) and the working stress is (125 Mpa). Calculate the maximum allowable pressure of the gas.

Solution :-

$$\sigma_c = \frac{P \cdot D}{4 \cdot t}$$

$$125\ Mpa = \frac{P \times 20 \times 10^{3}\ \cancel{mm}}{4 \times 10\ \cancel{mm}}$$

$\therefore P = 0.25\ Mpa.$

*Example: Show that when a thin walled spherical vessel of diameter (d) and thickness (t) is subjected to an internal fluid pressure (P), the increase in volume (δv) is equal to

$$\delta v = \boxed{\frac{\pi \cdot d^4 \cdot P}{8 \cdot t \cdot E}(1-\nu)}$$

$$\epsilon_v = \frac{\delta v}{v}$$

$$V = \frac{4}{3}\pi R^3 = \boxed{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3} \quad (1)$$

Solution:

$$\therefore \epsilon_t = \frac{1}{E}(\sigma_t - \nu \cdot \sigma_L) \quad \text{—— (2)}$$

sub (1) in (2)

$$\epsilon_t = \frac{\sigma_c}{E}(1-\nu) \qquad \because \sigma_c = \frac{Pd}{4t}$$

$$= \frac{P \cdot d}{4t \cdot E} \cdot (1-\nu)$$

$$\epsilon_L = \epsilon_t$$

$$\boxed{\epsilon_v = \epsilon_L + 2 \cdot \epsilon_t} = 3\,\epsilon_t$$

$$\cancel{\frac{P.d}{4.t.E}(1-2\nu) + \frac{2P.d}{4.t.E}(1-2\nu)}$$

$$\frac{\delta V}{V} = \frac{3.P.d}{4.t.E}(1-\nu)$$

so $V_{sph.} = \frac{4\pi}{3}\left(\frac{d}{2}\right)^3$

$\therefore \quad \delta v = \epsilon_v \times V$

$$= \frac{\cancel{3}.P.d}{\cancel{4}.t.E}(1-\nu) \times \frac{\cancel{4}\pi}{\cancel{3}} \frac{(d)^3}{8}$$

$$\delta v = \frac{\pi.d^4.P}{8.t.E}(1-\nu)$$ و.ه.م

Thermal Stresses

It is well known that changes in temperature cause bodies to expand or contract, the amount of the linear deformation, δ_T, expressed by:

$$\boxed{\delta_T = \alpha \cdot L \cdot \Delta T}$$

Unit : (mm)

α: Coefficient of linear expansion m/m.K.
L: length original.
ΔT: Temperature change.
δ_T: change in dimension due to change in temperature.

Example (1) Compute the total elongation of (30m) aluminium wire caused by a tensile load of (70 MPa). Find the change in temperature that may cause the same elongation. $E = 70\ GN/m^2$, $\alpha = 25 \times 10^{-6}\ K^{-1}$

$$\delta = \frac{PL}{AE} = \frac{\sigma L}{E}$$

Solution: $\delta = \frac{\sigma \cdot L}{E}$

$$\delta = \frac{\cancel{70} \times 10^{6}\ \cancel{N/m^2} \times 30\ \cancel{m} \times 10^{3}\ mm/\cancel{m}}{\cancel{70} \times 10^{9}\ \cancel{N/m^2}}$$

$= 30$ mm "elongation from force".

For the same elongation from change in Temperature :

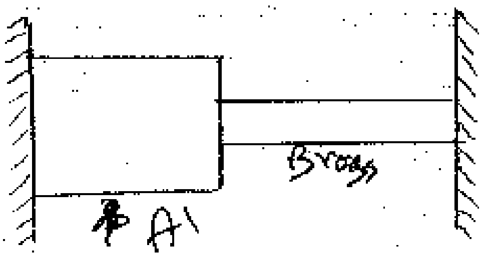
$\therefore \delta_T = \delta = 30mm$

$\therefore \delta_T = \alpha . l . (\Delta T)$

$30mm = 25 \times 10^{-6} K^{-1} \times 30 \times 10^{3} \times \Delta T$.

$\therefore \Delta T = 40 K$. P.O.9

Example 2 : the bar shown in figure is fixed at both ends at (300K) if the temperature dropped mean while the right support moved $\boxed{\text{by} (0.25mm)}$ toward the contracted part, Compute the minimum temperature that will not exceed the stresses in aluminium (more than (150 MPa). and Brass



	Brass	Aluminium
E	$120 GN/m^2$	$70 GN/m^2$
α	$20 \times 10^{-6} K^{-1}$	$25 \times 10^{-6} K^{-1}$
A	$7500 mm^2$	$2000 mm^2$
L	300 mm	200 mm

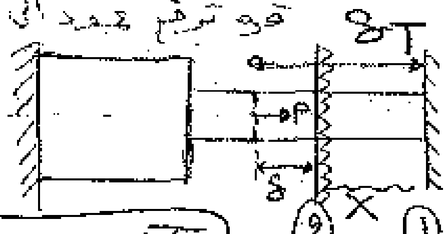
Solution : $\delta_T - x = \delta$? —— ①

$\delta_T = \delta_{T.Brass} + \delta_{T.Alumin}$

moved from right.

لو ترفع عدد الى ممدد فتحسب امتداد ... تأخذ بما فعل قوة مقاومة

$$\delta_T = (\alpha \cdot L \cdot \Delta T)_{Brass} + (\alpha \cdot L \cdot \Delta T)_{Alum.}$$



$$\boxed{\delta_T = 20\times10^{-6} \times 300 \times \Delta T + 25\times10^{-6} \times 200 \times \Delta T} \quad \text{--- (2)}$$

$$\delta_P = \delta_B + \delta_A$$

$$= \frac{P \cdot L}{A \cdot E}\Big)_B + \frac{P \cdot L}{A \cdot E}\Big)_{Alumi.}$$

$$\delta = 0.528$$

$$= \frac{P \times (300)}{7500 \times (120\times10^9 \times 10^{-6})} + \frac{P \times (200)}{2000 \times (70\times10^9 \times 10^{-6})} \quad (3)$$

sub. eq (2) + eq (3) into eq (1) :

$$\underbrace{0.006 \times (\Delta T) + 0.005 \times (\Delta T)}_{\delta_T} - 0.25 = \overset{0.528}{0.333\times10^{-4} \times P + 0.143\times10^{-5} \times P} \;---$$

$$\therefore \sigma_{\text{جاوزت خالية}} = \frac{P}{A}$$

$$\therefore P = 2000\,mm^2 \times 150\times10^6 \times 10^{-6}\, \frac{m^2}{mm^2}$$

$$= \boxed{30\times10^4\ N} \quad (\text{sub. in eq (4)})$$

$$0.006 \times \Delta T + 0.005 \times \Delta T - 0.25 = 0.333\times10^{-4} \times 30\times10^4 + 0.143\times10^{-5} \times 30\times10^4$$

$$\Delta T = 70.7\ K$$

$$\Delta T = T_1 - T_2 \quad \Rightarrow \quad T_2 = 300\,K - 70.7$$

$$= 229.3\ K$$

ص.م

H.W: Illustrative Problems Pag. 63

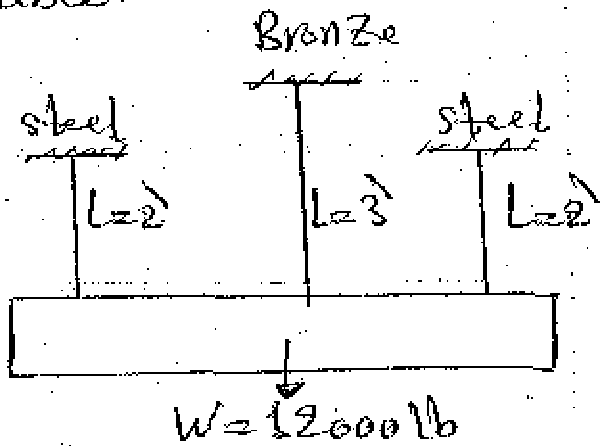
1. P. 258
2. P. 259
3. P. 260

H.W:

1- A brass bar of (2m) length and (1500 mm^2) c.s.a. is rigidly fixed at both ends. The system was at (300K) where their is no stresses. Compute the stress in the bar if temperature is droped to (285K), assuming no yield in the fixed ends. $\alpha = 20 \times 10^{-6}$, $E = 120$ GN/m^2.

2- A rigid block weighing (12000 lb) is supported by three rods symmetrically placed, as shown in figure. Assuming the block to remain horizontal, determine the stress in each rod after a temp. rise of (100 °F). The lower ends of the rods are assumed to have been at the same level before the block was attached and the temp. changed. use the data in following table:-

	steel rod	Bronze rod
A	3/4 in^2	1.5 in^2
E	30×10^6 Psi	12×10^6 psi
α	6.5×10^{-6}	10×10^{-6}



(P.258)

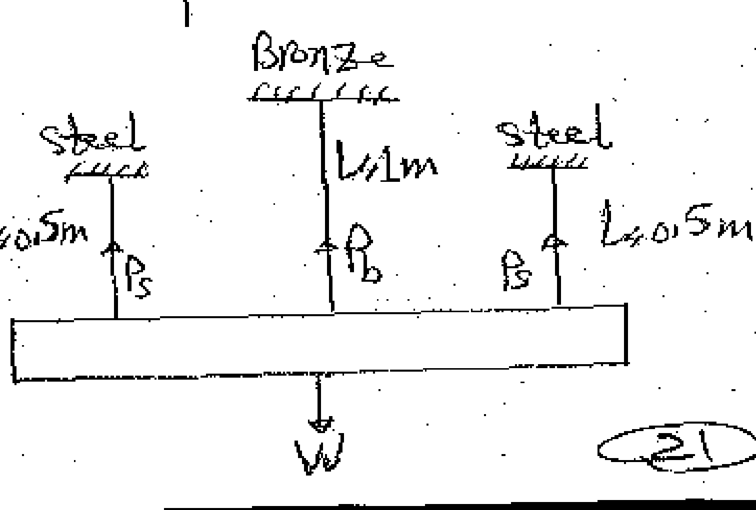
A steel rod (2.5 m) Long is secured between two walls. If the load on the rod is (zero) at (20°C), compute the stress when the temperature drops to (−20 °C). The cross sectional area of the rod is (120 mm²), $\alpha = 11.7\ \mu m/m.°c$ and $E = 200\ GN/m^2$. assuming

1) that the walls are rigid
2) that the walls spring together a total distance of (0.5) mm as the temperature drops.

(P-259) A rigid block having a mass of (5 Mgm) is support by three rods symmetrically placed, as shown in f Determine the stress in each rod after a temperatur rise of (40 °C). The lower ends of the rods are assume to have been at the same level before the block was attached and the temp. changed. Note that summetry dictates that the block will remain horizontal

H.W (2)

	Steel	Bronze
A (mm²)	500	900
E (N/m²)	200×10^9	83×10^9
α (μm/m.°c)	11.7	18.9



5) P-260

P-259

Using the data in table in (B), determine the temp rise necessary to case all the applied load to be supported by the steel rods.

5 3 1

6 4 2

Tutorial Sheet No. 2

1- A steel and copper wire of same length are joined together of their ends and a weight of (900 N) is suspended from the bottom of the system. Find the load taken by each wire, if the c.s.a of steel and copper are (2 and 4) cm^2 respectively, take $E_{copper} = 0.8 \times 10^6\ N/cm^2$ and $E_{steel} = 2 \times 10^6\ N/cm^2$.

2- A (20 mm) diameter of steel rod (3m) long, extends δ(3 mm) under a pull of (6280 N). Calculate the stress and strain in the rod. Also, find out the modulus of elasticity of the material.

3- A rigid block weighing (40,000 lb) is supported by three rods symmetrically placed, as shown in figure 1. Assuming the block to remain horizontal, determine the stress in each rod after rigid block is attached. Use the data in the following table:

	Steel rod	Bronze rod
A	1 in^2	1.5 in^2
E	30×10^6 Psi	12×10^6 Psi

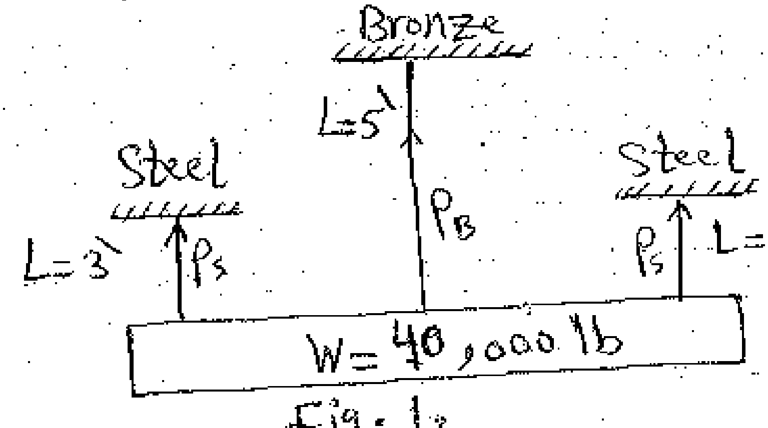
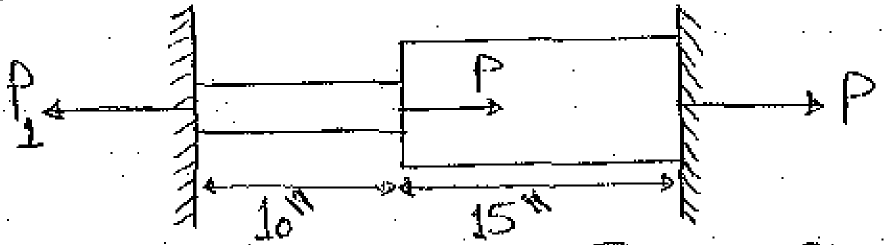


Fig. 1.

4- The bar shown in fig.2. is firmly attached to unyielding supports. Find the stress caused by each material by applying an axial load (P = 44,000 lb.)



Aluminium   Steel $P_1 - P$

$E = 10 \times 10^6 \ \frac{lb}{in^2}$ $E = 30 \times 10^6 \ \frac{lb}{in^2}$

$A = 1.5 \ in^2$ $A = 2 \ in^2$

Figure-2-

5- A (3m) Long bar is made up of a (2m) Long of steel bars square in cross-section area and the remaining length is made of copper of (10mm) diameter. Determine:

a- the load that may be hung safely from the bottom of bar.

b- the cross sectional area of the steel bar

c- the total elongation of the composite bar when the load is applied.

d- Locate the minimum and maximum loads.

$E_s = 2 \times 10^6 \ N/cm^2$

$\sigma_{ws} = 1400 \ N/cm^2$

$E_c = 0.8 \times 10^6 \ N/cm^2$

$\sigma_{wc} = 560 \ N/cm^2$

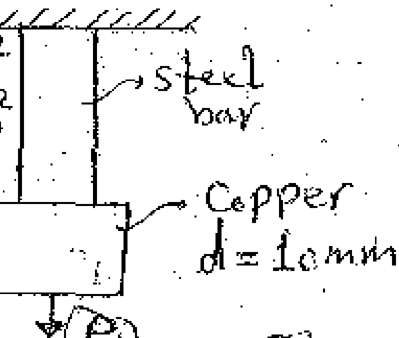


Figure-3-

6- A rigid block of mass "M" is supported by three symmetrically spaced rods as shown in fig. 4. Each copper rod has an area of (900 mm^2) and (E = 120 GPa) and the allowable stress is (70 MPa). The steel rod has an area of (1200 mm^2) and (E = 200 GPa), and the allawable stress is (140 MPa). Determine the largest mass (M) which can be supported.

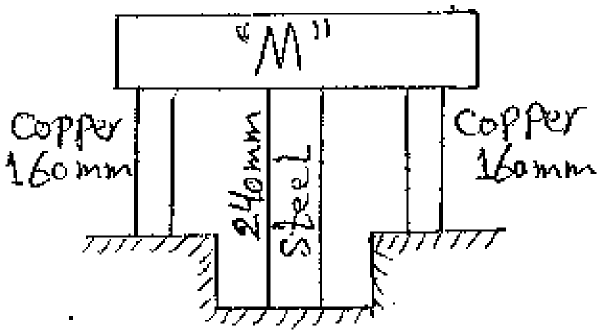


Figure - 4 -

Elements Structural { Rod = axial loads
Beam = axial & bending loads

Shear and Bending Moment in Beam

قوى القص وعزم الانحناء في الدعامة الأفقية (عوارض)

The basic Problem is strength of materials is to determin the relations between the stresses and deformations cau by loads applied to any structure.

The study of bending loads, is complicated by the fact that the loading effects vary from section to section of the beam. These loading effects take the form of a shearing force and a bending moment, sometimes ref to as shear and moment.

Methods of supporting some types of beams (Concentrated) are sho in fig. 1-:

1) A simple beam is supported by a hinged rea at one end and a roller support at the other.

ناتئ ساند

2) A cantilever beam is supported at one end only, wit a suitable restraint to prevent rotation of that end.

متدلية / مفصلة

3) An overhanging beam is supported by a hinge and a roller reaction, with either or both ends extendino beyond the supports. أحد طرفيها أو كليهما / امتدادها / تتجاوز

These beams are all statically determinate; thei reactions can be determined directly from the equati of static equilibrium.

A concentrated load is one that acts over small distance that it can be assumed to act at a point as in fig -a-.

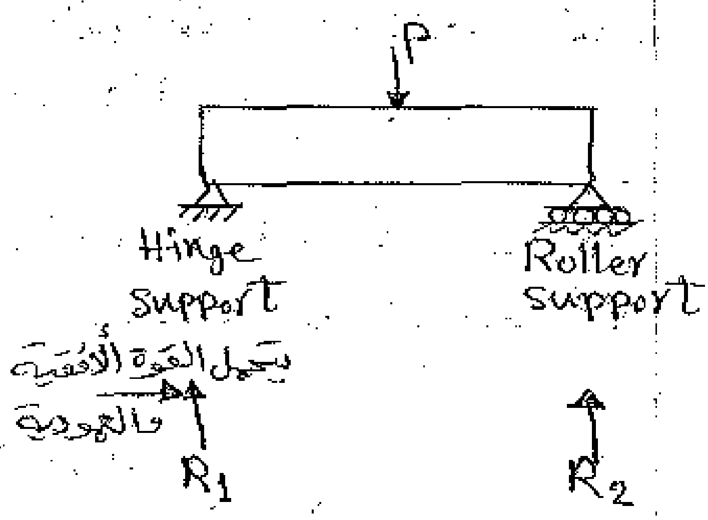
A distributed load acts over a considerable length of the beam. It may be distributed uniformaly over the entire length of the beam as in fig. -b-

or over part of the length as in fig -c-.

Shear and Bending Moment in Beam

قوى القص وعزم الأنحناء في العتبات الأفقية (عوارض)

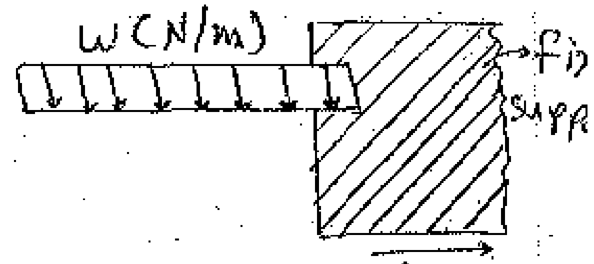
1- Concentrated Load (simple beam)



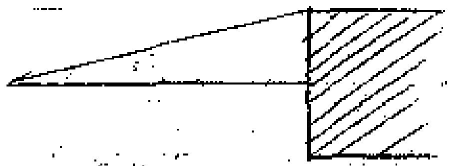
-a-

2- Distributed Load (Cantilever beam)

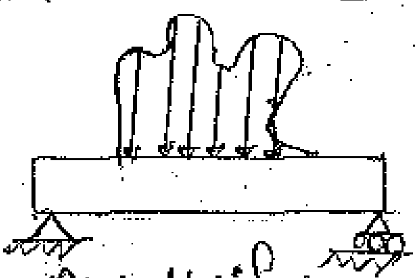
حامل جسر (دارجة)



(Beam) عزم موزع ع طول المسند



Uniform



Non-Uniform

-b-

3- Distributed load (overhanging beam)

-c-

Figure (1): Statically determinate beams

طريقة رسم عزم الانحناء وقوة القص :-

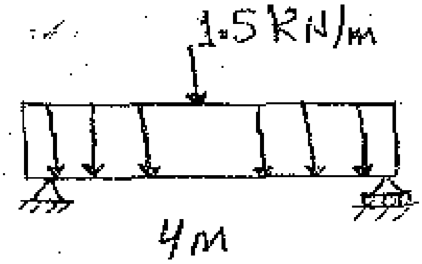
١- نوجد ردود الأفعال $\{R_2 \;\&\; R_1\}$ ان وجدت بطريقة التوازن.
٢- نرسم مخطط قوة القص بالاعتماد على القوى الموجودة على العتبة المعطاة حيث كل قوة مركزة تصبح مستطيل وكل مثلث يصبح منحني وكل مستطيل يصبح مثلث.
٣- نرسم مخطط عزم الانحناء بالاعتماد على مخطط قوة القص حيث تمثل القوى في مخطط عزم الانحناء المساحة تحت مخطط قوة القص.

٤- نستعمل معادلتين حتى نرسم الشكل :-
قوة القص = محصلة قوى الخارجية المؤثرة.

$$\boxed{\begin{array}{c} \underset{\text{قوة القص}}{V} = \sum F_y \\ M = \sum M \\ \text{Bending Moment} \end{array}}$$

ملاحظات :-
١) دائما نبدأ القطع من اليسار.
٢) بالنسبة للحمل الموزع يكون مركز الثقل للمستطيل في النصف و للمثلث في الثلث القريب من الزاوية القائمة.
٣) نقطع الدعامة قبل وبعد كل قوة ونبدأ من جهة اليسار، القيمة التي نحسب لل V و M اذا كانت (موجبة) نرسم فوق الخط الأفقي واذا كانت (سالبة) نرسم تحت الخط الأفقي.
٤) نرسم المستقيم بنقطتين ، اما المنحني فيحتاج الى ثلاث نقاط لرسمه.
٥) يكون شكل المنحني اما مقعر او محدب بالاعتماد على اشارة X^2 في معادلة (V) فاذا كانت (-) شكل محدب و (+) شكل مقعر.
٦) النقطة التي عندها تكون $V=0$ يكون $B.M$ عند قيمته العظمى.

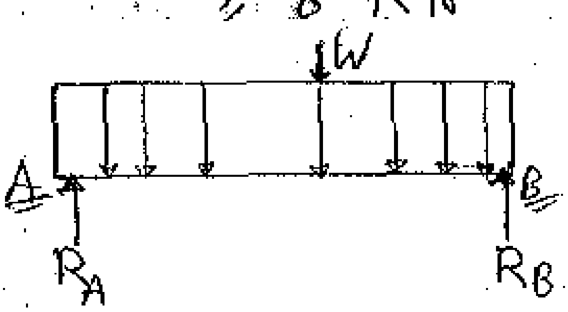
Ex. (1): Write the shear and bending moment equation for (1.5 KN/m) and Sketch the shear and bending.



solution: الحمل موزع لذلك يجب ان تضرب الحمل بالمسافة.

$W_t = 1.5 \frac{KN}{\cancel{m}} \times 4 \cancel{m}$ مساحة المستطيل = الطول × العرض

$= 6 \ KN$ مجموع اوزان (حيث من المسافة الكلية)



خطوة حساب ردود أفعال المسند

$\sum F_y = 0$

$R_A + R_B - W_t = 0 \quad \text{---} \ (1)$

نضع الوزن في منتصف الشكل (لأن الشكل مستطيل) فيكون المركز هو:

$X_c = 2 \ m$ from A & B [rectangular $= \frac{b}{2}$

$\circlearrowleft + \sum M_A = 0$

$R_B \times 4 - 6 \times 2 = 0 \implies R_B = 3 \ KN$

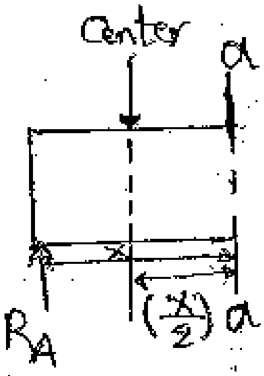
$\circlearrowleft + \sum M_B = 0$

$R_A \times 4 - 6 \times 2 = 0 \implies R_A = 3\ KN.$

For checking : $R_A + R_B = W_t$ [from eq ①]

$3 + 3 = 6$

∴ ok



Section between A & B

X_{range} $(0 - 4)\ m$

مبدأ القطع بين نقطتين يتغير فيها التحميل حيث تكون

(X) متغيرة بين القيم (4 to 0).

Shear Load $V = \sum F_y$

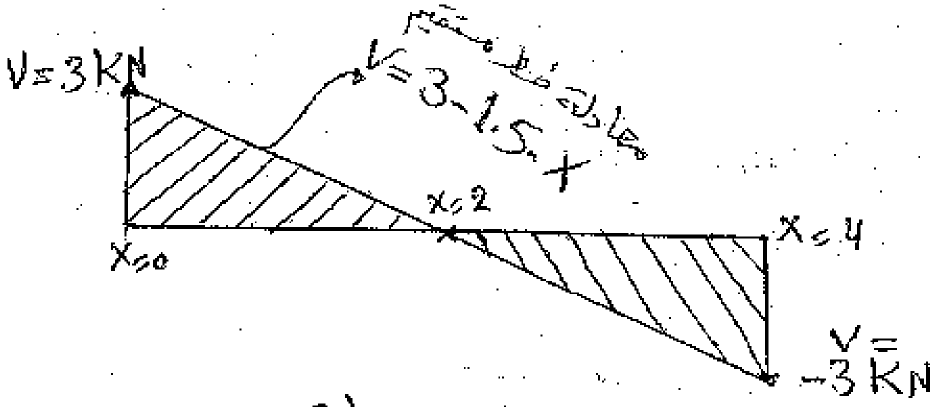
$V = R_A - W$

$$\boxed{V = 3KN - (1.5 \frac{KN}{m} \times X)}$$ -- first order equation

$M_a = 3 \times X - (1.5 \times X) \times (\frac{X}{2})$

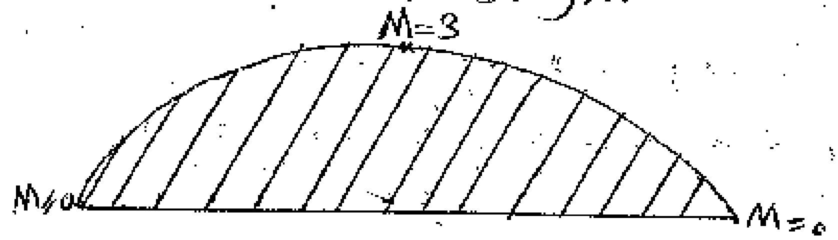
$$\boxed{M = 3 \cdot X - \frac{3}{4} \cdot X^2}$$ --- Second order equation

x	V
0	3
2	0
4	-3



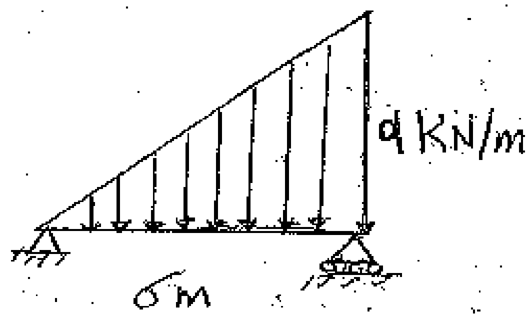
Shear Diagram

x	M
0	0
2	3
4	0



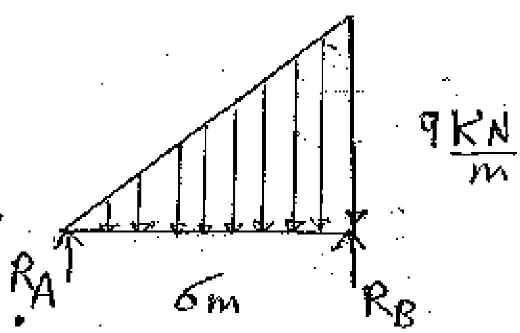
Bending Moment Diagram.

Ex.(2): Write shear and bending moment equations and draw S.&B.M. diagram.



Solution:

$R_A + R_B - W_t = 0$



$W_t = \frac{1}{2} \cdot b \cdot h$ مساحة المثلث

$= \frac{1}{2} \times 6 \times 9$

$= 27$ KN

$X_c = \frac{2}{3}$ from A & 1/3 from B.

$\circlearrowleft + \sum M_A = 0$

$R_B \times 6 - 27 \times (\frac{2}{3} \times 6) = 0$

$\therefore R_B = 18 \ KN.$

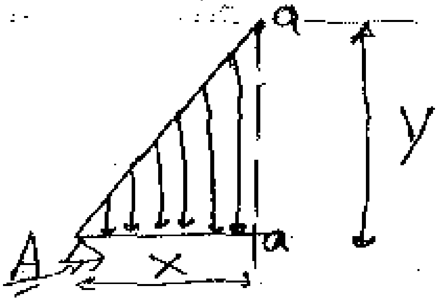
$\circlearrowleft + \sum M_B = 0$

$R_A \times 6 - 27 (\frac{1}{3} \times 6) = 0$

$\therefore R_A = 9 \ KN.$

Check : $18 + 9 = 27$
$27 = 27$ $\therefore$ OK

Section between A & B :-



$V = R_A - W$
$= 9 - W$

$W = \frac{1}{2} . X . Y$ ارتفاع يكون متغير بتغير مسافة القاعدة

$\frac{X}{Y} = \frac{6}{9}$ من تشابه المثلثات

$3X = 2Y \Longrightarrow Y = \frac{3}{2} . X$

$\therefore W = \frac{1}{2} . X . \frac{3}{2} . X$
$W = \frac{3}{4} . X^2$

$$\therefore \boxed{V = 9 - \frac{3}{4}x^2}$$ second order eq.

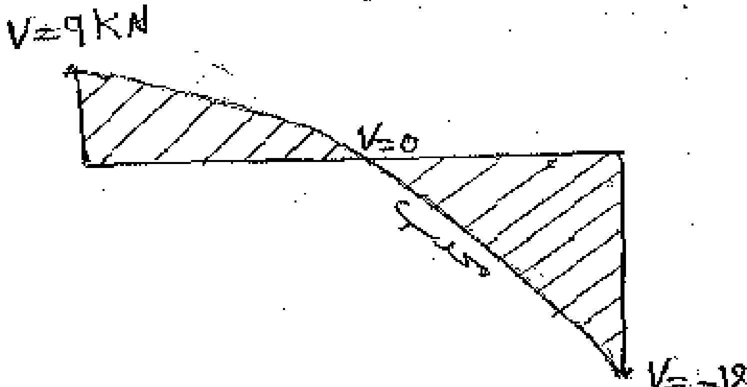
$$\circlearrowleft + \Sigma M_a = R_A \cdot x - w \cdot \left(\frac{x}{3}\right)$$

$$\therefore M = 9 \cdot x - \frac{3}{4} \cdot x^2 \cdot \frac{x}{3}$$

$$\therefore \boxed{M = 9x - 0.25x^3}$$ third order eq.

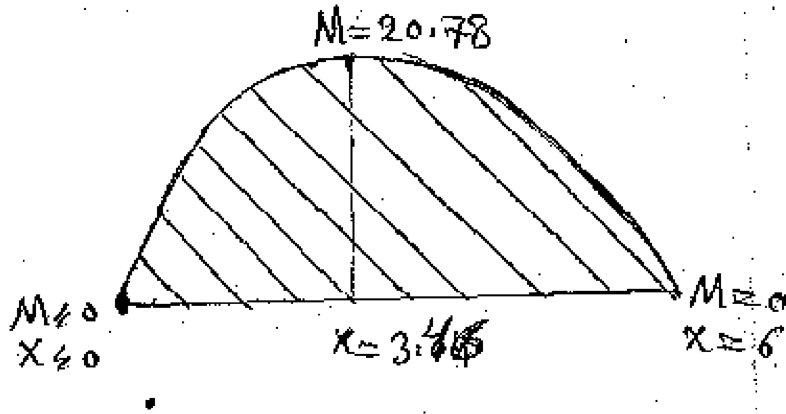
x	V
0	9
3.46	0
6	-18

$\sqrt{12}$



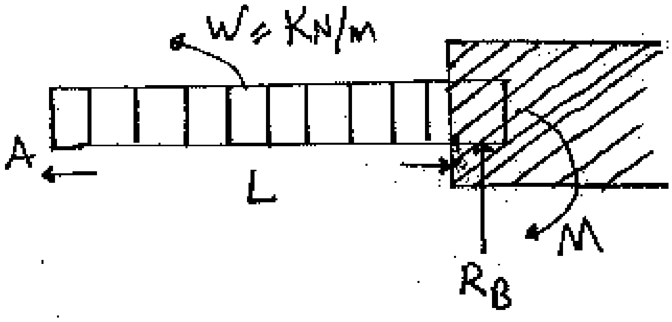
Shear diagram

x	M
0	0
3.46	20.78
6	0



Bending Moment

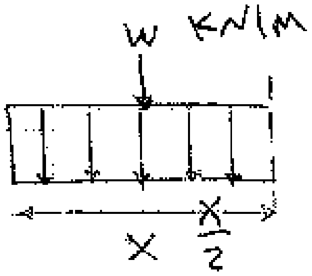
Ex. 3: Draw shear and bending moment diagrams



Solution

$V = \sum f_y$

قوة رد الفعل واقع في أقصى اليمين لا نحتاج R_B نحذف عكس القطع



$$\boxed{V = -W \cdot X}$$

$$M = -W \cdot X \left(\frac{1}{2} X\right)$$

$$\boxed{M = -\frac{W \cdot X^2}{2}}$$

X range (0 – L) :

X	V	M
0	0	0
L	$-W \cdot L$	$-W \cdot \frac{L^2}{2}$

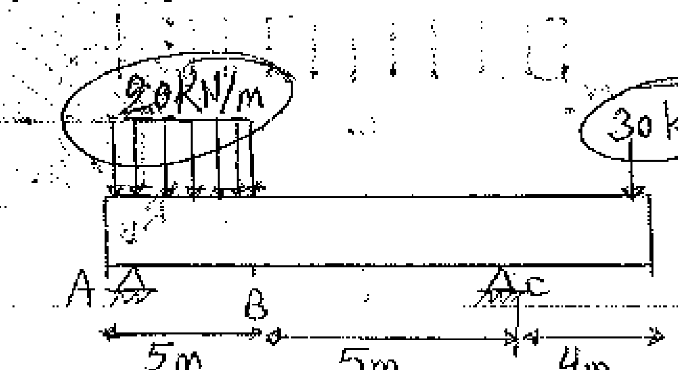
Shear diagram

Bending moment diagram

112 ص

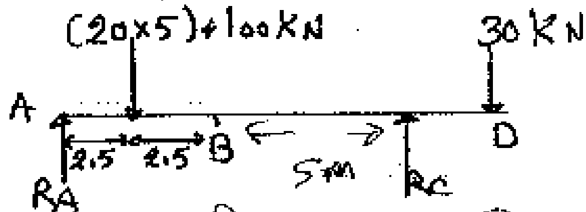
Ex. 4: Write S & B. M. eqs. and Draw S & B. M. diagram.

Two force لينا



$R_A + R_C - 100 - 30 = 0$

solution: $\boxed{R_A + R_C = 130}$



$\sum F_y = 0$

$+30\,KN + (20\,\frac{KN}{m} \times 5\,m) = R_A + R_C$ ——— ①

$\boxed{+130 = R_A + R_C}$ — — ①

$\circlearrowright + \sum M_A = 0$

$+100 \times 2.5 - R_C \times 10 + 30 \times 14 = 0$

$R_C = 67\ KN.$

$\circlearrowright + \sum M_C = 0$

$R_A \times 10 - 100 \times 7.5 + 30 \times 4 = 0$

$R_A = 63\ KN$

check: From eq ①

$100 + 30 = 67 + 63$

$\therefore 130 = 130$

ok

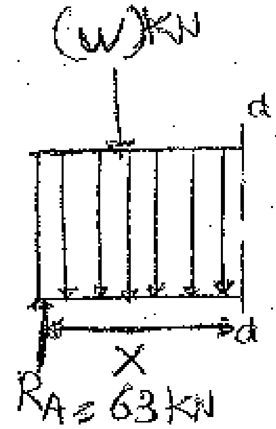
نقطع قبل ومع كل قوة

① Section (A – B), X rang (0 – 5) m

14

$V = R_A - w$

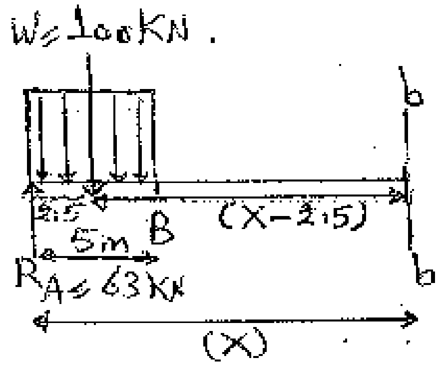
$\boxed{V = 63 - 20 \cdot X}$ —— (1)



$M_a = 63 \cdot X - 20 \cdot X \left(\frac{X}{2}\right)$

$\boxed{M = 63 \cdot X - 10 \cdot X^2}$ —— (a)

2) Section (B−C), X range (5−10)m:



$V = R_A - W$

$= 63 - 100$

$\boxed{V = -37}$ —— (2)

$M_b = 63 \cdot X - 100(X - 2.5)$

$= 63 \cdot X - 100X + 250$

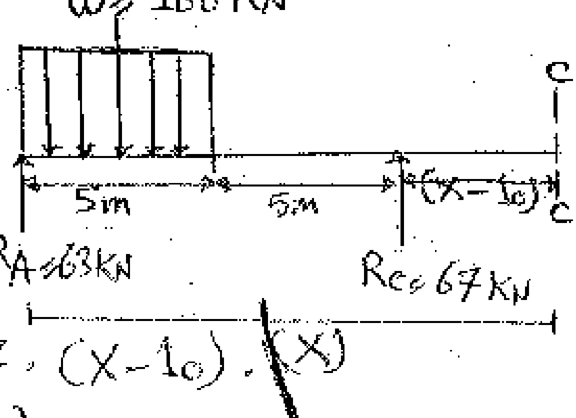
$\boxed{M = -37 \cdot X + 250}$ —— (b)

3) Section (C−D), X range (10−14):

$V = RA - W + R_c$

$V = 63 - 100 + 67$

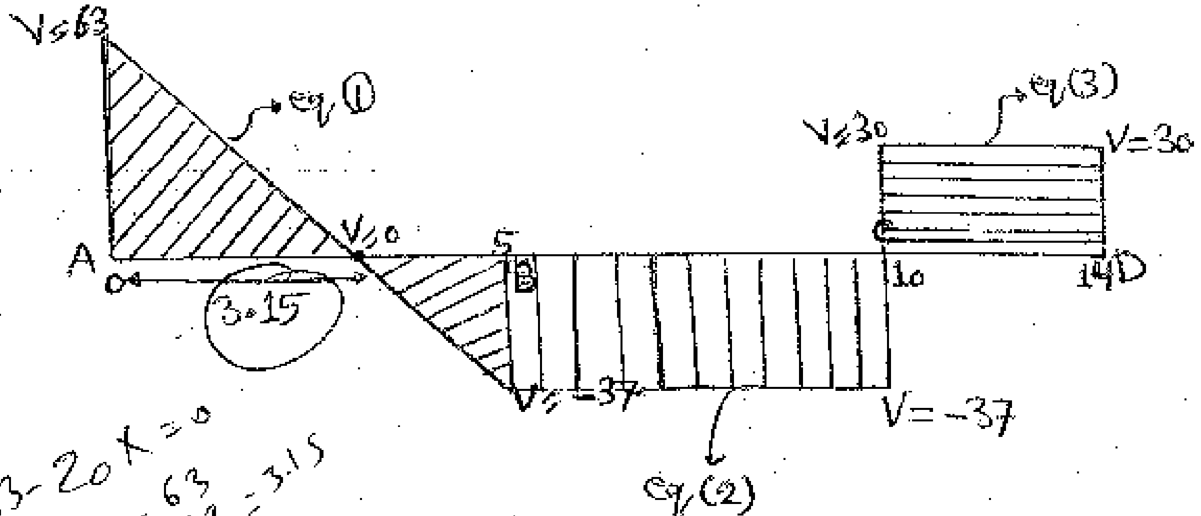
$\boxed{V = 30 \text{ KN}}$ —— (3)



$M = 63 \cdot X - 100(X - 2.5) - 67 \cdot (X - 10)$

$\boxed{M = 30X - 420}$ —— (C)

X	V	M
0	63	0
3.15	0	
5	-37	65
5	-37	65
10	-37	-120
10	30	-120
14	30	0

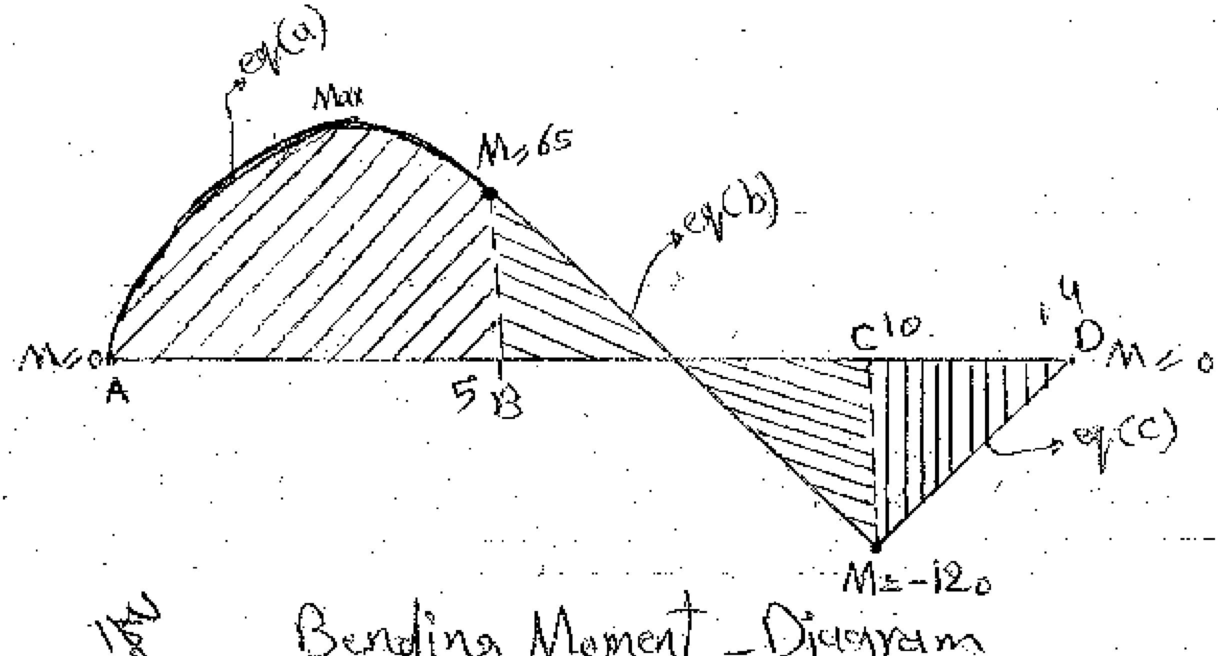


$V = 63 - 20X = 0$

$20X = 63$

$X = \frac{63}{20} = 3.15$

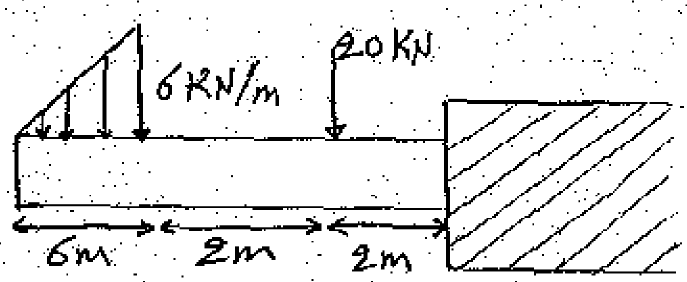
Shear_Diagram



Bending Moment_Diagram

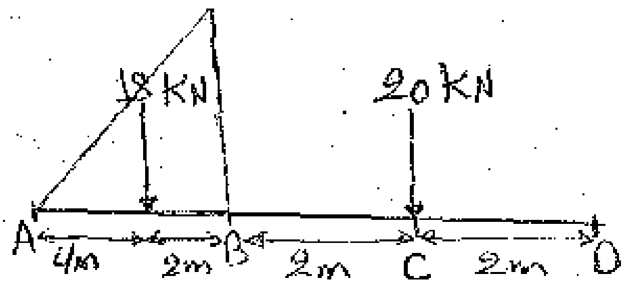
116

Ex. 5:



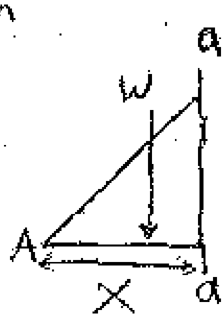
Solution:

$$W_t = \frac{1}{2}\times 6\,\tfrac{KN}{m} \times 6\,m$$
$$= 18\ KN$$



$$X_c = \frac{1}{3}\times 6$$
$$= 2m$$

Section (A-B) , Xrange (0-6) m



$$V = \Sigma F_y$$
$$= -W$$
$$V = -\left(\frac{1}{2}\cdot x \cdot y\right)$$

$$\left(\frac{x}{y}\right)_{Small} = \left(\frac{x}{y}\right)_{Large}$$

$$\frac{x}{y} = \frac{6}{6} \Longrightarrow \boxed{x = y}$$

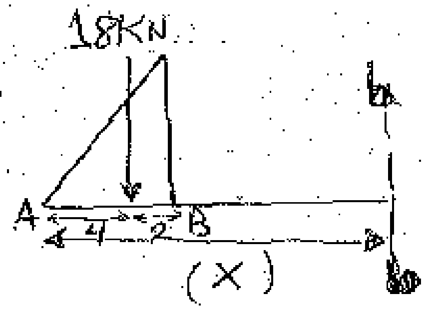
$$\therefore \boxed{V = -\frac{1}{2}x^2} \quad \text{—— (1)}$$

$$M_a = -\frac{1}{2}x^2 \cdot \frac{x}{3}$$

$$\boxed{M_a = -\frac{x^3}{6}} \quad \text{—— (a)}$$

18

Section (B–C), X range (6–8) m :



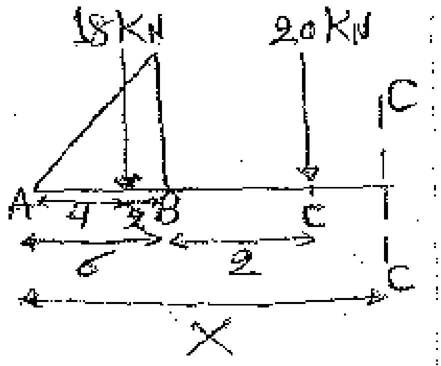
$V = -w$

$\boxed{V = -18\ KN}$ —— (2)

$M = -18(x-4)$

$\boxed{M_b = -18X + 72}$ —— (b)

Section (C–D), X range (8–10) m :



$V = -18 - 20$

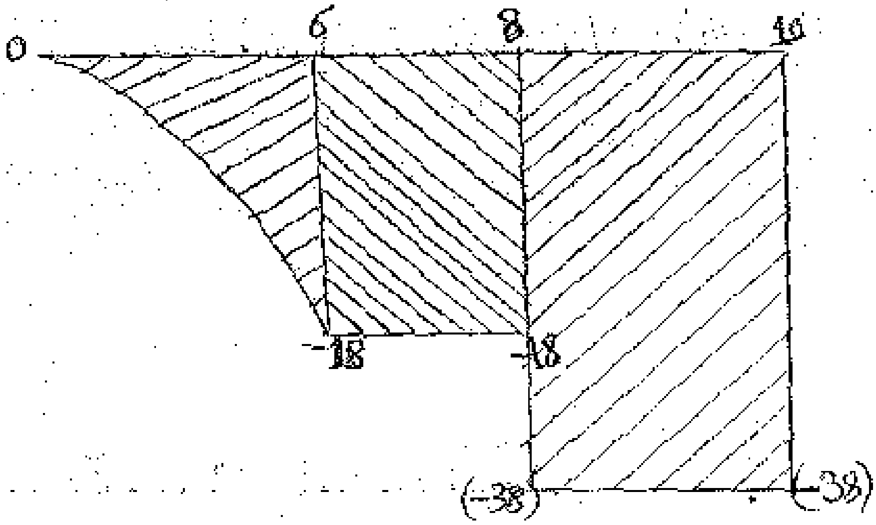
$\boxed{V = -38\ KN}$ —— (3)

$M = -18(X-4) - 20(X-8)$

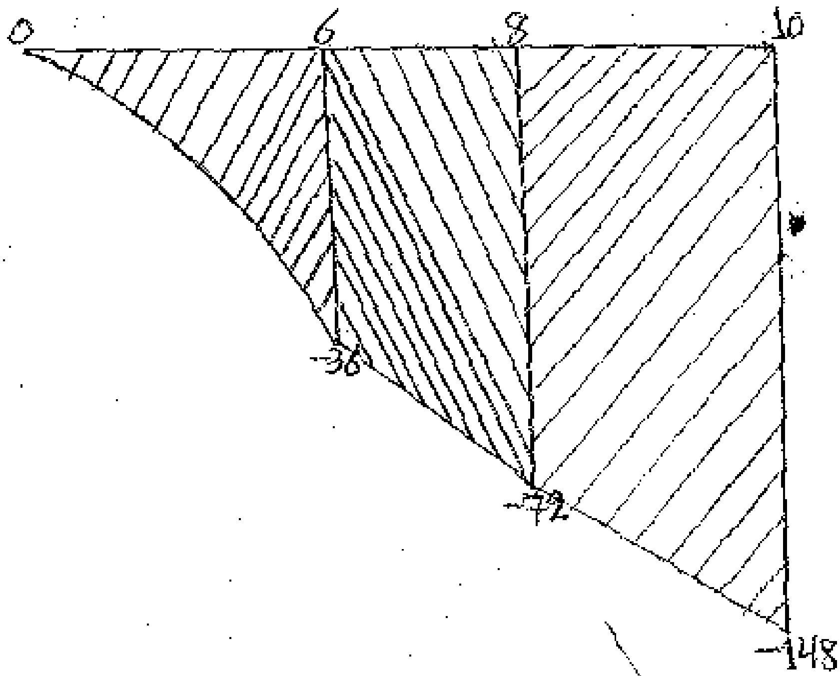
$= -18X + 72 - 20X + 160$

$\boxed{M = -38X + 232}$ —— (C)

X	V	M
0	0	0
6	-18	-36
6	-18	-36
8	-18	-72
8	-38	-72
10	-38	-148



"Shear Diagram"



"Bending Moment Diagram"

H.W: Illustrative Problem Pag. (112)

P. 401 ص 112

P. 402 ص 116

Tutorial Problem Sheet

Pag. (118)

Q1 : 403

Q2 : 407

Q3 : 418

Q4 : 419

Q5 : 420