University of Technology

Chemical Engineering Department

Mechanic & Strength Of Materials

First Class

By

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Mechanics and Strength of Materials

A- Mechanics

1- Principles of Static.
2- Resultants of Force Systems.
3- Equilibrium of Force Systems.
4- Friction.
5- Centroids and Centers of Gravity.
6- Moment of Inertia.

B- Strength of Materials

1- Internal Forces in Rigid Bodies.
2- Definition of Stress and Strain.
3- Hook Law.
4- Poissons ratio, Composite Stresses.
5- Volumetric Stress, Bulk Modules.
6- Thin Walled Cylinders.
7- Thermal Stress.
8- Shear and Moments in Beam.

REFRENCES

1- Engineering mechanics / Statics and dynamics by "Higdon and Stiles".
2- Engineering mechanics by "Ferdinand".

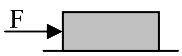
Mechanics

Principles of Statics:

Mechanics is that branch of physical science which considers the motion of bodies, with rest being considered a special case of motion.

The external effect of a force on a body is either:

1. Acceleration of the body.
2. Development of resisting forces.



(1)

(2)

Rigid Bodies:

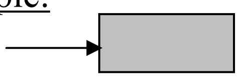
The bodies in which all particles remain at fixed distances from each other.

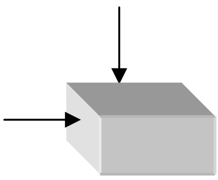
Mechanics deals only with rigid bodies.

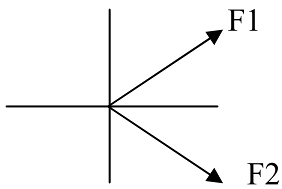
Force Systems:

A force system is any arrangement where two or more forces act on body or on a group of related bodies.

Example:







Static's (Equilibrium):

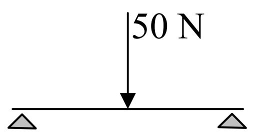
When the force system acting on a body has a resultant equal to " zero", the body is in equilibrium and the problem is one of statics.

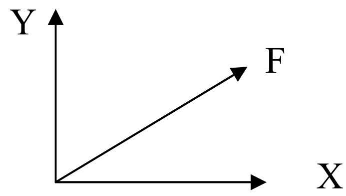
Dynamics:

When the resultant is different from zero , the problem is one of dynamics.

Scalar and Vector Quentities:

- Scalar quantity is one which has only magnitude. For example (mass, volume, time,…).
- Vector quantity has both magnitude and direction. For example (velocity, acceleration, force and momentum).

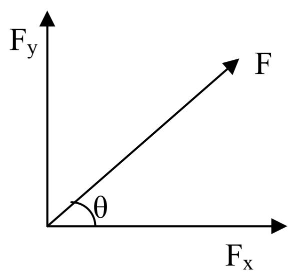




Results of Force System:

Resultant: The resultant of a force system is the simplest force system which can replace the original system without changing its external effect on a rigid body.

Composition and Resolution of Forces:



Principle of Projection:

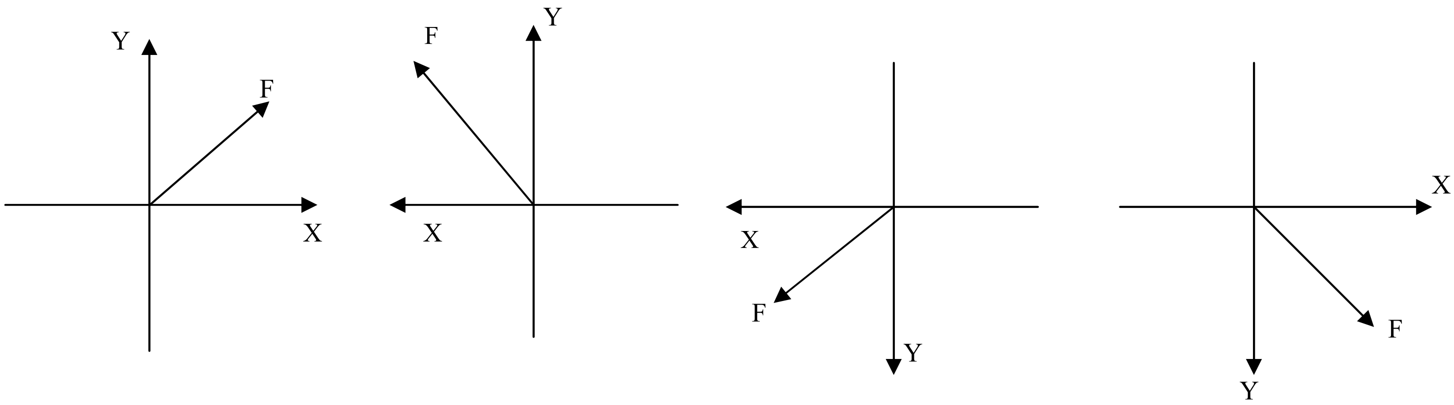
$$\cos\theta = \frac{Fx}{F} \longrightarrow F_x = F * \cos\theta$$

$$\sin\theta = \frac{Fy}{F} \longrightarrow F_y = F * \sin\theta$$

$$\tan\theta = \frac{Fy}{Fx}$$

R= $\sqrt{(Fx)^\wedge 2+(Fy)^\wedge 2}$ =F

F= $\sqrt{(F*\cos\theta)^\wedge 2+(F*\sin\theta)^\wedge 2}$



Examples

Q1: Force equal 200N and make angle θ=20^{o} with the positive direction for x, calculate the composition of force in the direction x&y.

Solution:

F_x =F * Cosθ

=200 *cos20

=187.9N

Fy =F * sinθ

=200 *sin20

=68.4N

Q2:If know that two compositions perpendicular for the specific force are Fx=400N & Fy = 300N, calculate the force quantity and direction, the angle that make with the line.

Solution:

$\tan\theta = \frac{Fy}{Fx}$

$=\frac{300N}{400N}$ =0.75

$\theta=\tan\theta^{-1}$ (0.75)

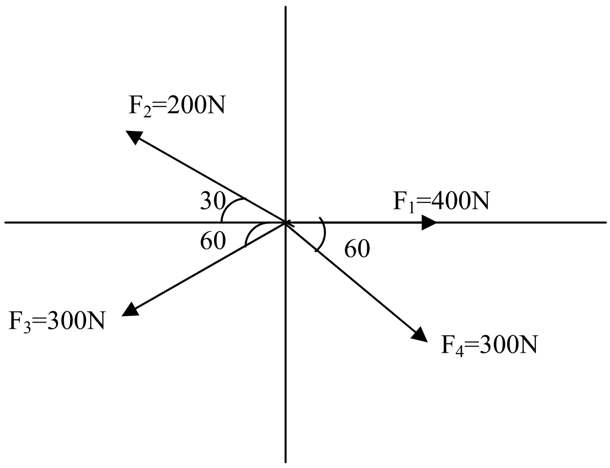
=36.87°

F_x =F * Cosθ

400N=F * Cos (36.87)

F= $\frac{400N}{0.8}$ =500N

Q3: Determine the resultant for multi force and its direction.



$\sum Fx$ =400-200 cos30 -300 cos60+300cos60

= 226.80 N

$\sum Fy$ =200 sin30 -300 sin60 -300 sin60

= -126.79N

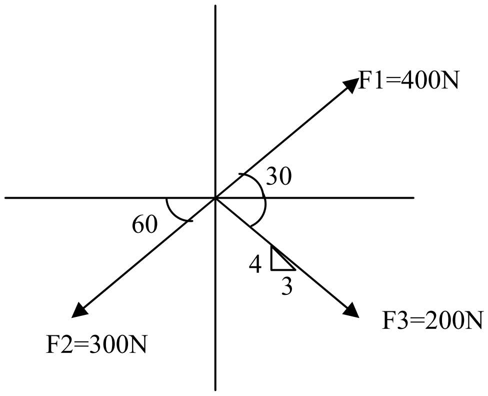
R=$\sqrt{(Fx)^2+(Fy)^2}$

=$\sqrt{(226.8)^2+(-126.79)^2}$

= 259.8N

$$\tan\theta = \frac{\sum Fy}{\sum Fx} \longrightarrow \theta = \tan\theta^{-1}\left(\frac{-126.79}{226.8}\right) = -29.2$$

<u>Q3</u>: Determine the resultant and angle for the figure below:



Solution:

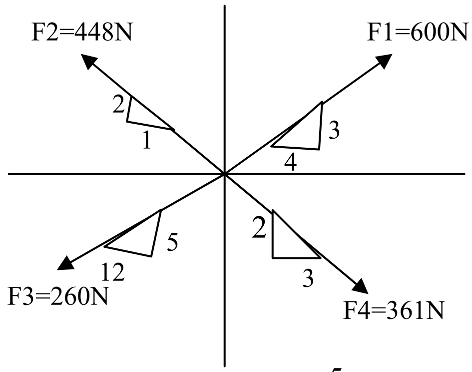
$\sum Fx$=400*cos30-300*cos60 +200*(3/5)
=346.4-150+120
=316.4N

$\sum Fy$ =400*sin30-300sin60-200*(4/5)
=200-259.8-160
=-219.8 N

R $= \sqrt{(316.4)^2 + (-219.8)^2}$
=385.25 N

$$\theta = \tan\theta^{-1}\left(\frac{\sum Fy}{\sum Fx}\right) \longrightarrow \theta = -34.78$$

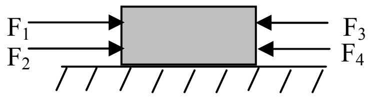
<u>H.W.</u>



Equilibrium of a Force System

In static the force system are balance that is , force system that do not cause motion when they are applied bodies at rest.

When $F_1+F_2=F_3+F_4$



Equilibrium is a term use to designate the condition where the resultant of system of force is "zero" at equilibrium.

$R=0 \longrightarrow \sqrt{(Fx)^2+(Fy)^2} =0$

Condition of Equilibrium

$$\left.\begin{array}{l}\sum Fx=0\\ \sum Fy=0\end{array}\right\}$$

Or

$$\left.\begin{array}{l}\sum Fy=0\\ \sum My=0\end{array}\right\}$$

Or

$$\left.\begin{array}{l}\sum Fx=0\\ \sum Mx=0\end{array}\right\}$$

Or

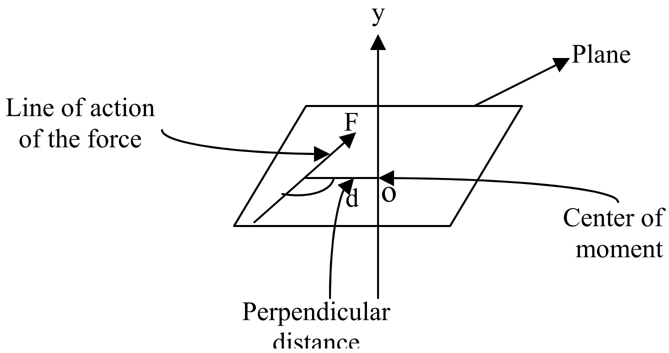
$$\left.\begin{array}{l}\sum Mx=0\\ \sum My=0\end{array}\right\}$$

Moment of a Force

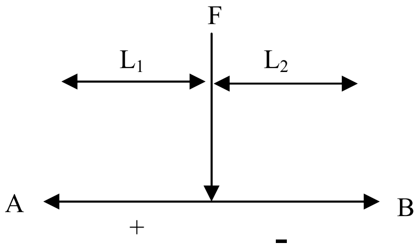
The momentum of a force about an axis or line is the measure of its ability to produce turning or twisting about the axis. The magnitude of the moment of a force about an axis which is perpendicular to a plane containing the line of action of the force is defined as the product of the force and the perpendicular distance from the axis to the line of action of the force.

<u>Example</u>

The moment of the horizontal force "F" about the vertical axis Y equal:



$+M = F * d$



$+M_A = F * L_1$

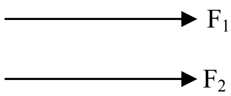
$+M_B = -\,(F * L_2)$

شروط التوازن للحركة ا طية والدورانية:

1- ا ركة ا طية (الانتقالية)

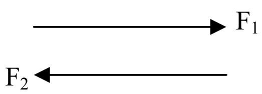
$$\sum F = 0 \longrightarrow \begin{cases} \sum Fx = 0 \\ \sum Fy = 0 \end{cases}$$

Co-current



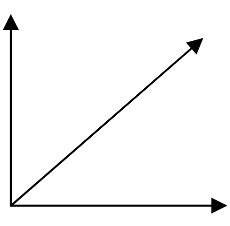
$F = F_1 + F_2$

Counter- current



$F = F_1 - F_2$

Perpendicular

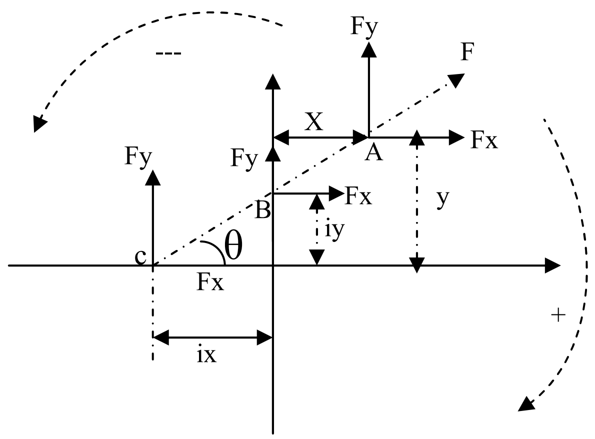


$F = \sqrt{F1^2 + F2^2}$

2-ا ركة الدورانية

$$\sum M = 0 \longrightarrow \begin{cases} \sum M+ = 0 \\ \sum M- = 0 \end{cases}$$

Suppose a force (F) ,making an angle (θ) with the x-axis, passes through a point (A) having the coordinates (x,y).



In this case it is inconvenient to calculate the moment arm “d”.

By resolving the force into its components (Fx) & (Fy) at “A”, the moment arm of (Fx) about “o” is the coordinate distance “y”, and the moment arm of (Fy) about “o” is the coordinate distance “x”.

$+\sum M_o = F * d$

$= F_x .y - F_y .x$

The intercepts of the line of action of “F” with the (x&y) axes may also be computed from the principle of moments. Replacing “F” by its components at “B” &”C”, we have:

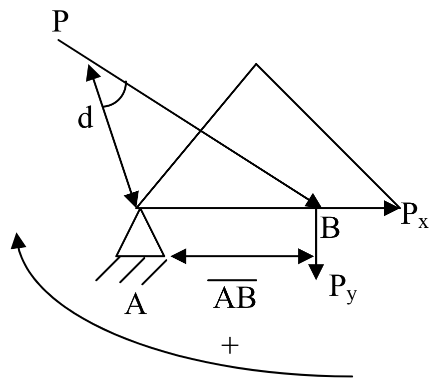
$+\sum M_o = F_x * iy + 0$ at point "B"

$+\sum M_o = F_y * ix + 0$ at point "c"

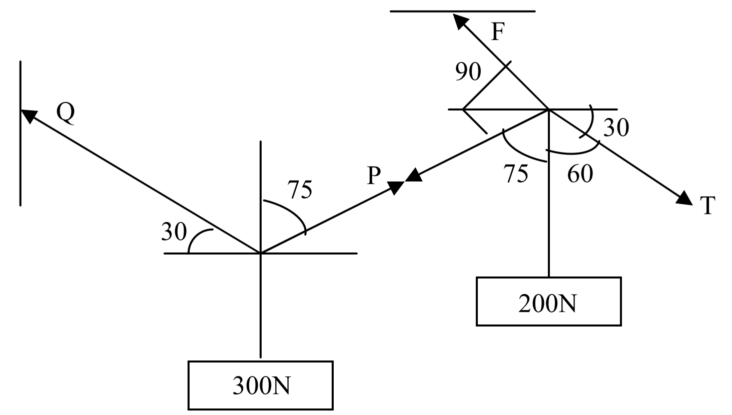
Example

$+M_A = P*d$

$= \cancel{P_x}^{0} + P_y * \overline{AB}$



Q1: Find the value of force (F, Q , P ,T) of the following figure:



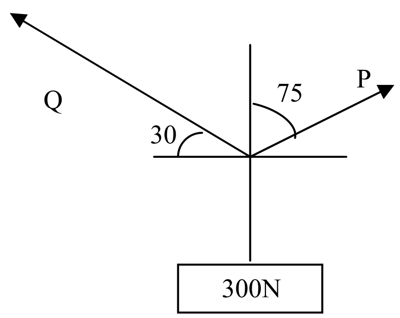
Solution:

At Equilibrium

$\sum Fx = 0$ $\sum Fy = 0$

Horizontal component: P * cos 15 – Q * cos 30 =0

Vertical component: P * sin 15 + Q * sin 30 =0



At Equilibrium

$\sum Fx = 0$

$\sum Fy = 0$

- F * cos 75- P * cos 15+ T * cos 30 = 0

F * sin 75- P * sin 15-200 - T * sin 30 = 0

P = Q *0.87/0.97 $\longrightarrow$ P=0.896 *Q …………(1)

0.5* Q + (0.896 * Q) * 0.26 – 300 = 0

Q = 411 N …………(2)

Sub. eq(2) into eq(1) :

P = 0.896*411

= 368 N

- F * 0.26 - (368 * 0.97) + T * 0.87 = 0

T=(0.26*F+357) / 0.87 …………(3)

F*0.97-(368*0.26)-200-0.5*[(0.26*F+357) / 0.87] = 0

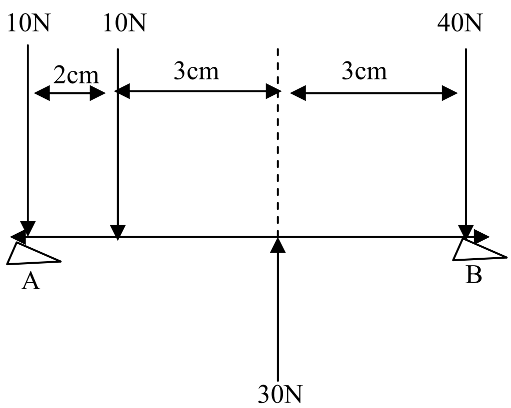
F=500.85/0.82

=611N …………(4)

Sub. eq(4) into eq(3) :

T=593N

Q2: Determined the resultant of the parallel force system acting on the bar AB.



Solution:

$F = \sum fy$

$= -10-10+30-40$

$= -30N$

$\sum M_A = 10*0+10*2-30*5+40*8$

$= 190$ N.cm

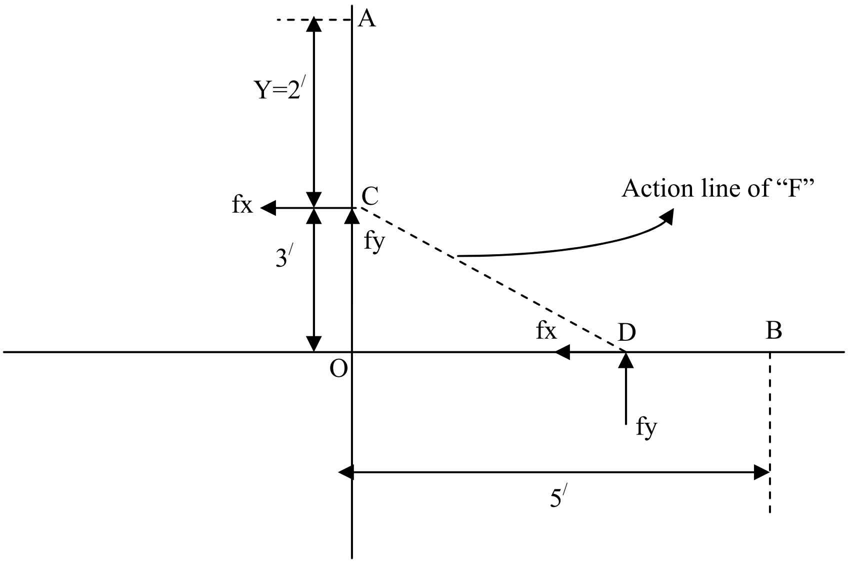
$+M_A = F*d$

190(N.cm) = 30N * d

d=6.3cm

The resultant F=30N down roda t 6.3cm from A.

Q3: In figure below, a force "F" passing through "c" causes a clock-wise moment of 120b.ft about "A" and a clock-wise moment of 70b.ft about "B".Determine the force and its "x" intercept"ix".



Solution:

By resolving the F into its components at C,

$\sum M_A = -Fx + 0$

$120 = -F_x * 2$

Fx= - 60b (left)

Considering again the components at “C”, with respect to B, F_x causes a counter clock-wise moment, F_y must act upward in order to create the specified clock-wise moment of 70b.ft about B.

$\sum M_B = F_y.\overline{(OB)} - F_x.\overline{(CO)}$

$70 = F_y. 5' - 60 . 3'$

$= 50b$

$F = \sqrt{(Fx)^2 + (Fy)^2}$

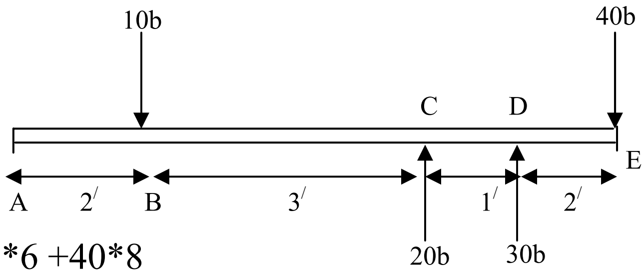
$= \sqrt{60^2 + 50^2}$

$= 78.2b$

$\theta = \tan\theta^{-1} \left(\frac{\sum Fy}{\sum Fx}\right)$

$= 39.8$

<u>Q3</u>: Take the moment sums about point C, D, E &A.



$\sum M_A = 10*2 - 20*5 - 30*6 + 40*8$

$= 60$ b.ft

$\sum M_B = -20*3 - 30*4 + 40*6$

$= 60$ b.ft

$\sum M_C = -10*3 - 30*1 + 40*3$

$= 60$ b.ft

$\sum M_D = -10*4 + 20*1 + 40*2$

$= 60$ b.ft

$\sum M_E = -10*6 + 20*3 + 30*2$

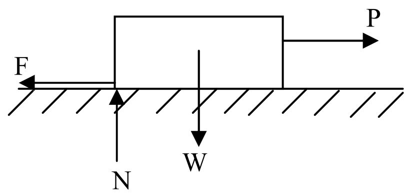
$= 60$ b.ft

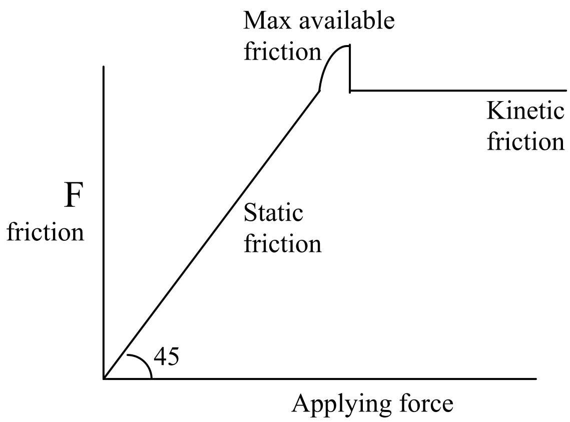
Friction

The sliding of one rigid body relative to another rigid body with which it is in contact, it always resisted by a force called a force of friction.

The force of friction is a retarding force always acting in opposite direction to the motion or tendency to move, and will be tangential to the surface of the two bodies at the point of contact. This frictional force is basically due to the roughness of the contact surfaces.

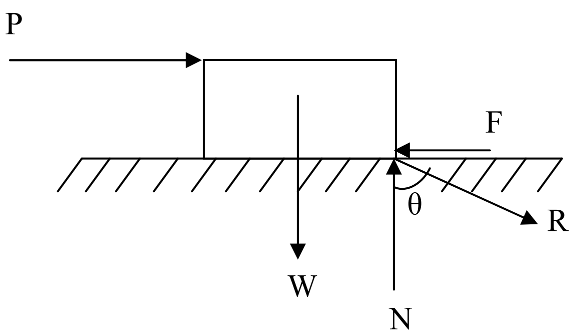
Theory of Friction





Let block of weight (W) rest on horizontal surface and (P) is horizontal force applying to block. When force (P) is given increasing values, the friction resistance also increases. When (P) is very small, the block does not move. This other force is called " the static friction ", if the applied force (P) is increased the friction force (F) also increase to oppose force (P) until it reaches a maximum value "F_{max} ".

If (P) is further increased, the frictional force (F) let balance it and the block starts sliding. The value of (F) drops a value below that acting when motion starts.



F α N **"Proportion relationship"**

$F = \mu * N$ ……………(1)

Where μ: coefficient of friction

$\tan\theta = \frac{F\max}{N}$ ……………(2)

$N = R *\cos\theta$ ……………(3)

$F_{max} = R *\sin\theta$ …………….(4)

$F_{max} = N *\tan\theta$ …………….(5) obtained from eq.(2).

By equating eq.(1) & eq.(5):

$\mu = \tan\theta$ …………….(6)

$\theta = \tan\mu^{-1}$ …………….(7)

where the body is impending motion : $F=F_{max}$

at equilibrium "static", $\mathbf{F < F_{max}}$.

Example (1):

Calculate the forces {F & N}, and is the body at equilibrium if μ =0.3?

Solution:

$\sum F_y=0$

$100 - F = 0 \longrightarrow F=100N$

$\sum F_y=0$

$N - 400 = 0 \longrightarrow N =400N$

$F_{max} = \mu * N$

$=0.3 * 400$

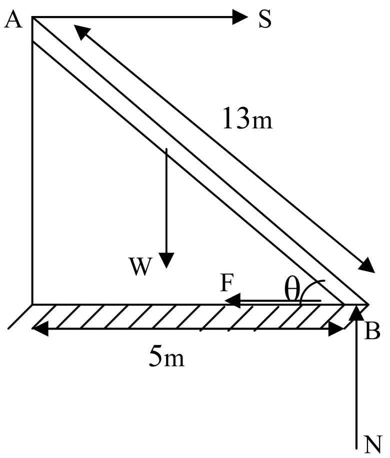
$=120N$

AT EQUILIBRIUM $\mathbf{F < F_{max}}$.

Example (2):

A(13m) ladder weighing ($25kg_f$) is place against a vertical wall with its, and (5m) from wall. The coefficient of friction to the ladder and floor is (0.3).

1. Show that the ladder will remain in equilibrium in this position.
2. what force (F)acting on the ladder at point contact between ladder and floor?



Solution:

$\sum F_x=0$ ⟶ S = F

$\sum F_y=0$ ⟶ N = W =25 kg_f

Takin moment about B :

$\sum M_B= 0$

S*(12) – 25*2.5 = 0 ⟶ S=5.21 kg_f =F

$F_{max} = \mu * N$

$= 0.3 * 25$

$= 7.5\ kg_f$

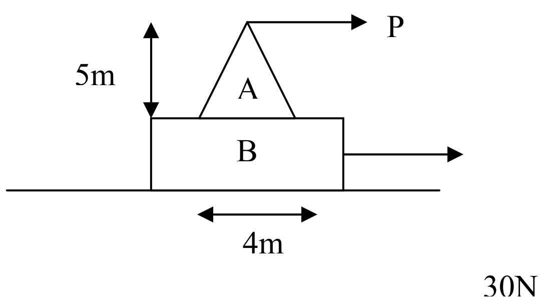
AT EQUILIBRIUM **$F < F_{max}$.**

Example (3):

Determine the force (P) of figure below, that will cause the (200N) homogeneous scale triangular, body "A"; to have impending motion. Body "B" weight (300N). The coefficient of friction between B and the plane is (0.2) and the coefficient between A & B is (0.3).

Solution:

Body B at rest & body A move to right.



$\sum F_x=0$

P – F = 0 ⟶ P = F

$\sum F_y=0$

N – W = 0 ⟶ N = 200 N

Impending motion $F = F_{max}$

$F_{max} = \mu * N$

$= 0.3 * 200$

$= 60$ N

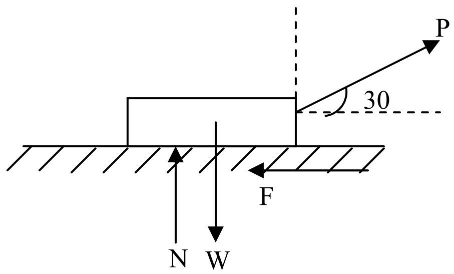
P = 60N

Example (4):

Abody resting on rough plane required a pull of (18 N) inclined at (30^o) to the plane just to moved the body. If was found the push at (22N) inclined at (30^o) to plane, determined weight and coefficient of friction.

Solution:

Case one (pull force)



$\sum F_x = 0$

$P * \cos 30 - F = 0$

$F = 18 * 0.87$

$= 15.7$ N

$F = F_{max}$

$= \mu * N$

$N = 15.7 / \mu$ ………..(1)

$\sum F_y = 0$

$N - W + 18 * \sin 30 = 0$

$N + 9 = W$ ………..(2)

Case two (push force)

$\sum F_x = 0$

$22 * \cos 30 - F = 0 \longrightarrow F = 19N$

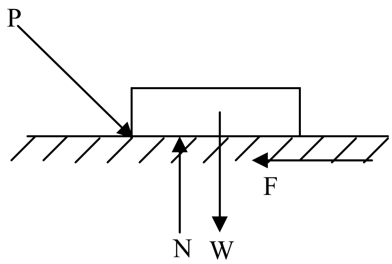
$F = F_{max}$

$= \mu * N \longrightarrow N = 19/ \mu$ …………..(3)

$\sum F_y = 0$

$N - W - 22 * \sin 30 = 0$

$N - 11 = W$ ………….(4)



From eq. (1) & (2) :

$W = 15.7/\mu + 9$ …………….(5)

From eq. (3) & (4) :

$W = 19/\mu - 11$ …………….(6)

Equating eq. (5) & (6):

$19/\mu - 11 = 15.7/\mu + 9$

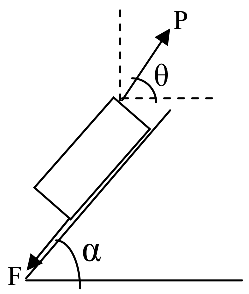
$\mu = 0.165$

$W = 104N$

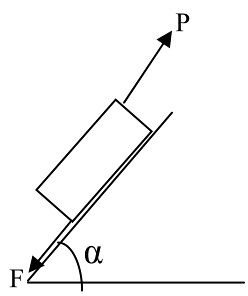
Friction on an Inclined Plane

Consider a body of weight (w) lying on an inclined plane having an angle of inclination (α) with the horizontal. If angle (α) is more than angle of repose, the body will not be in equilibrium unless an external force (P) is applied to it. As the magnitude of (P) is increased gradually initially the body will have a tendency to move downward as in figure (a). It will come in equilibrium and then it will have a tendency to move upwards as shown in figure (b).

When the force (P) is small and the body tends to move downwards, the frictional force (F) acts along the plane upwards and tries to oppose the tendency of the body to move downwards. When the magnitude of force is sufficient to pull the body upwards along the plane, the frictional force (F) would again oppose this tendency of the body to move upwards and it will act downward along the plane.



(a)



(b)

a) Body tends to move downwards.
b) Body tends to move upwards.

Example 1:

A block rests on a plane inclined to the horizontal at (15^o), μ=0.3. The least force parallel to the line of greatest slope which will just move the block down the plane is (5N). Find the weight of the block and the least force acting parallel to line of greatest slope and which will just move the block up the plane.

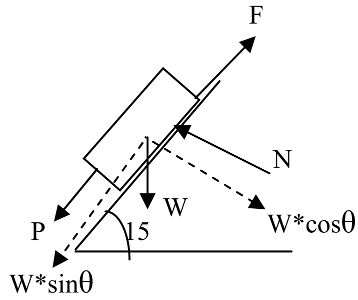
Solution:

$\sum F_x=0$

$F - 5 - W*\sin15 = 0$

$F = 5 + W*\sin15$

$F = 5 + 0.259*W$ ……………………..(1)



$\sum F_y = 0$

$N = W * \cos15$ ……………………..(2)

Assume impending motion the block :

$F = F_{max}$

$= \mu * N$

$= 0.3 * (W*\cos15)$

$F = 0.29 * W$ ……………………..(3)

Equating eq.(1) & (3):

$5 + 0.259*W = 0.29 * W$

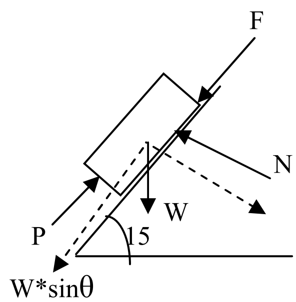
$W = 160N$

F=46N “friction force when block move downward.

$P = F + W * \sin15$

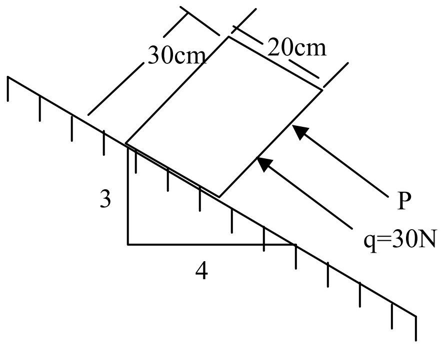
$=46 + (160 * 0.259)$

$= 87.4N$ “force which move the block upward”



Example2:

A homogeneous block of weight 500N rest upon the incline shown in the figure. Find the limits of the force P which keep the block at equilibrium. μ =0.25.



Solution:

1) Downward Move (Minimum)

$\theta= \sin^{-1} (3/5)$

$=36.86$

$\sum F_y=0$

N-500*cos36.86=0

N=400N

F= μ * N

=0.25 *400

=100N

$\sum F_x=0$

F + P +30N –W*sin36.86 =0

P=170N (minimum).

2) Upward Move (Maximum)

$\sum F_x=0$

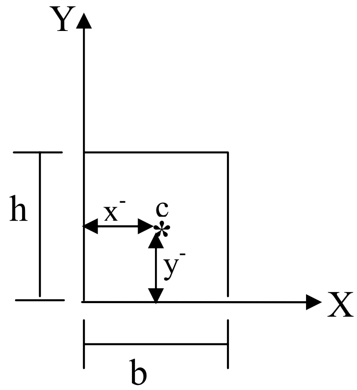
P – F +30N –500*sin36.86 =0

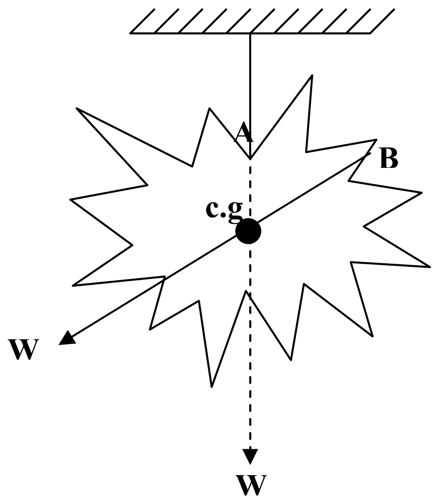
From part (1): F=100N

P =370N (maximum).

Centroid and Center of Gravity

The centroid of areas can be obtained by means of the principle of moments if the area can be determined and the moment of these quantities about any axis can also be determined, this method avoids the necessary of integration. The area can be divided into simple shapes e.g. (rectangular, triangles, circules…….) whose areas and centroid coordinates can sum of the seperat areas and the result of moment about any axis is the alegebric sum of the moment of the component areas.





Center Coordinates: ($\bar{x}$,$\bar{y}$).

$\bar{x}$: The distance between the center and Y-axis.

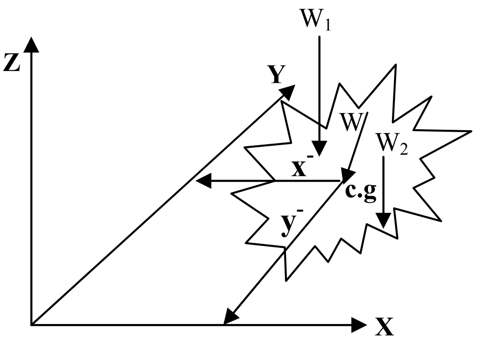
$\bar{y}$: The distance between the center and X-axis.

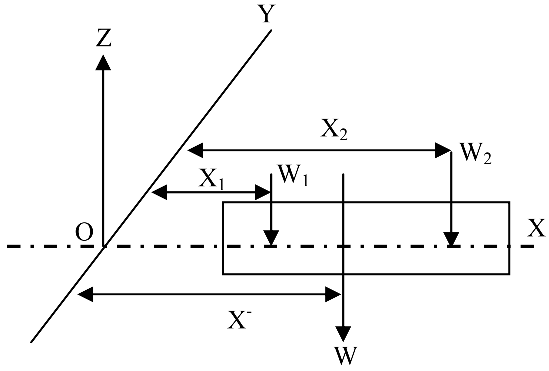
Center of gravity of body is that point through which the total weight of the body is assumed to act whatever the position of the body.

The line of action of the weight (W) can be determined by the line of action of support.

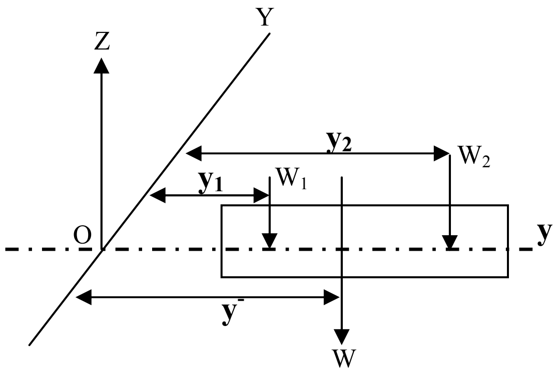
Let the body be supported in a new position by the string now attached to B. The body will shift its positions of the line of action of the weight are determined. The intersection of these positions of the line of action determines a point which is defined as the center of gravity of the body; this is the point through which the action line of the weight always passes.

The analytical location of the center of gravity is a simple variation of the principle of moments; the moment of the resultant is equal to the moment sum of its parts.





Front-View



Side-View

Let the coordinates of each elemental weight (x_1,y_1), (x_2,y_2)…………,and the coordinates of the resultant weight (x^-,y^-).

The moments of weights about y-axis:

$W*\bar{X} = W_1*X_1+W_2*X_2+$ …………….. $=\sum W*X$ ………………..(1)

With respect to the X-axis:

$W*\bar{Y} = W_1*y_1+W_2*y_2+$ …………….. $=\sum W*y$ ………………..(2)

If the material of the plate is homogeneous, the W may be expressed as product of density γ multiplied by (τ * A).

τ :thickness of plate.

A:its area.

$W= \gamma*\tau*A$ ..…………(3) total weight of plate.

or: $w= \gamma*\tau*a$ ..…………(4) weight of element.

Substitute eqs.(3)&(4) into eq.(1):

$\gamma*\tau*A*\bar{X} = \gamma*\tau*a_1*x_1+ \gamma*\tau*a_2*x_2+ .... = \gamma*\tau*\sum a*x$ ....(5)

γ & τ: are constant

$A*\bar{X} = \sum a*x$

$A*\bar{Y} = \sum a*y$

The expression $\{A*\bar{X}, A*\bar{Y}\}$ is called the moment of area. It is equivalent to the sum of the moments of the elemental areas composing the total area.

Moment of area is defined as the product of the area multiplied by the perpendicular distance from the center of area to the axis of moments.

$\bar{X} = \sum a*x/A$

$\bar{Y} = \sum a*y/A$

Where: $A=\sum a$

This gives a method of locating a point called the centroid of areas $(\bar{X}, \bar{Y})$.

The centroid of area is corresponding the center of gravity.

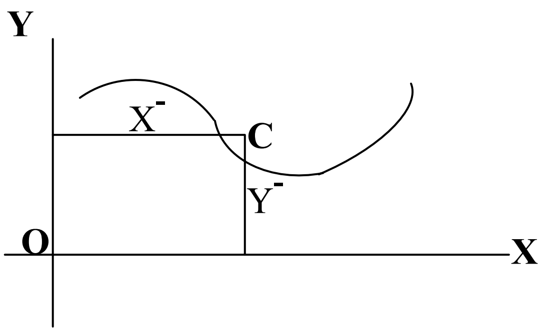


Figure represents the center line of a homogeneous wire of length (L) and constant cross sectional area (a) lying in the (X,Y) plane.

And the weight (w) of an elemental length (L) by:

$W= \gamma*L*a$

$\gamma*L*a*\bar{X} = \gamma*L_1*a*x_1+ \gamma* L_2*a*x_2+ .... = \gamma*a*\sum L*x$

$L * \bar{X} = \sum L * x$

$L * \bar{Y} = \sum L * y$

Centroids Determined by Integration:

If area of an element as dA (i.e.,a small part of the total area A).

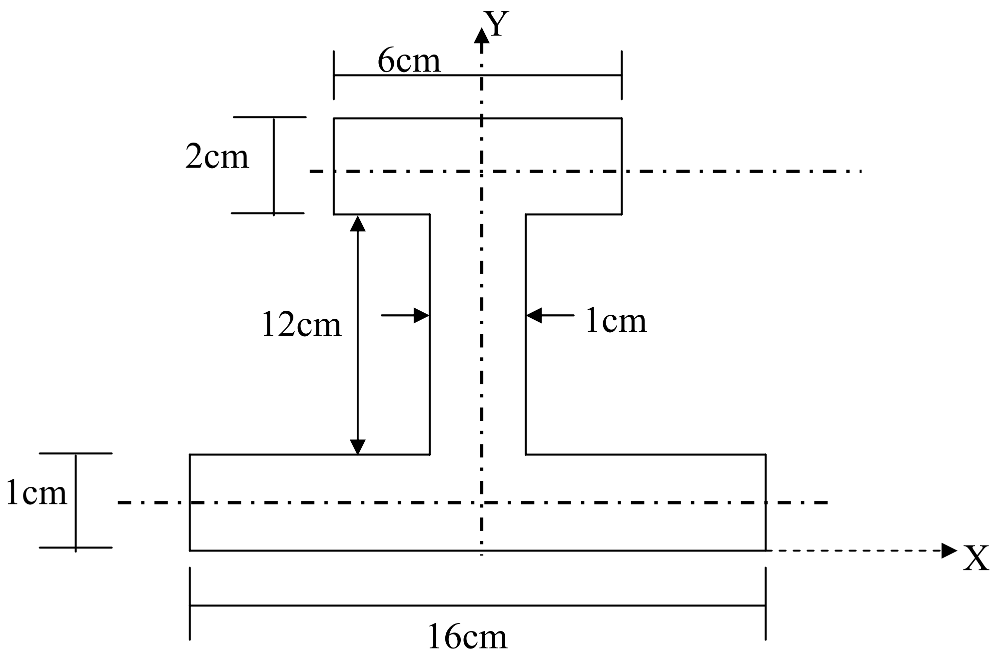
Centroid of an area: $A.\ \bar{X} = \int X * dA$

$A.\ \bar{Y} = \int Y * Da$

Centroid of an line: $L.\ \bar{X} = \int X * dL$

$L.\ \bar{Y} = \int Y * Dl$

Q1: Find the position of the (c.g.) of the section shown in figure:



$\bar{Y} = \sum a * y / A$

$A = a_1 + a_2 + a_3$

$= 6*2 + 12*1 + 16*1$

$= 40 cm^2$

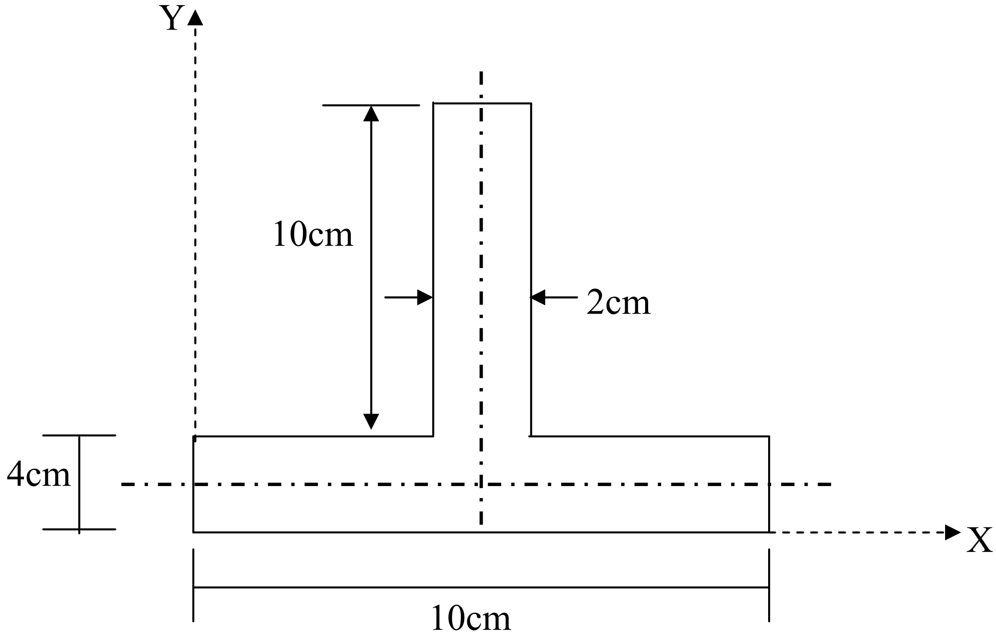
$\bar{Y} = (12*14 + 12*7 + 16*0.5)/40$

=6.5cm

$\bar{X}$=0 "symmetrical about y-axis"

C.g.=(0,6.5).

<u>Q2:</u> Find the centroid of the figure below :



$\bar{Y} = \sum a*y / \sum a$

$\bar{X} = \sum a*x / \sum a$

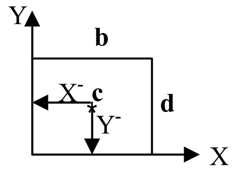
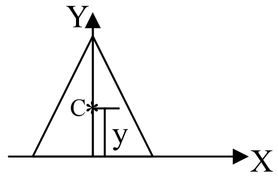
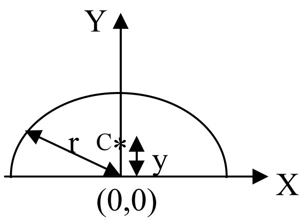
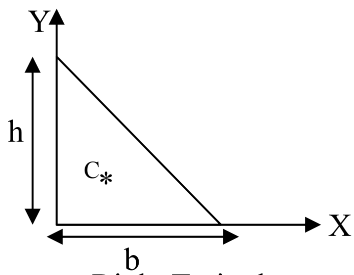
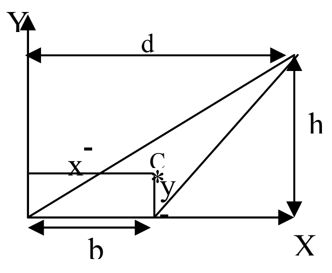
Shape	x	y	a	a.x	a.y
1) 10 2	1+4=5	5+4=9	2*10=20	20*5=100	20*9=180
2) 4 10	5	2	4*10=40	40*5=200	40*2=80
			A=60	$\sum a*x$=300	$\sum a*y$=260

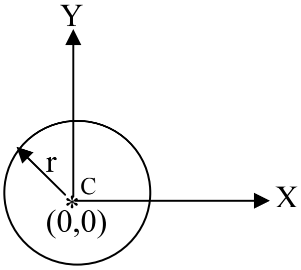
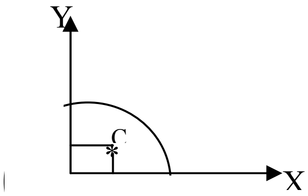
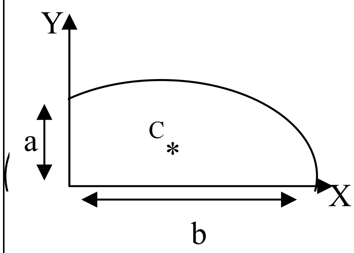
$\bar{Y}$ =260/60
=4.33cm

$\bar{X}$ =300/60
=5cm

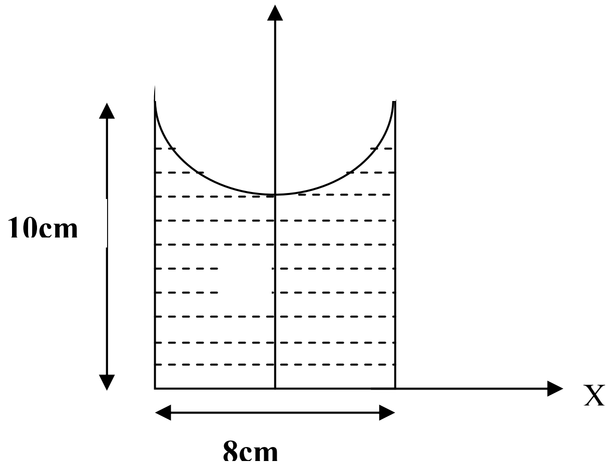
(c.g) =(5,4.33)

Table: Centroids for Common Geometric Shapes

Shape	Area	$\bar{X}$	$\bar{Y}$
	b*d	b/2	d/2
 Traingle	(1/2)*b*h	0	h/3
 Semicircle	(3.14*r^2)/2	0	(4*r)/(3*3.14)
 Right Traingle	1/2*b*h	b/3	h/3
	1/2*b*h	(b+d)/3	h/3

 Circle	3.14*r^2	0	0
 Quarter Circle	(3.14*r^2)/4	(4*r)/(3*3.14)	(4*r)/(3*3.14)
 Ellips-Quarter	(3.14/4)*a*b	(4*b)/(3*3.14)	(4*a)/(3*3.14)

<u>Q3</u>: Find the centroid for the shaded area for the figure shown :



<u>Solution:</u>

Shape	y	a	a.y
semicircle	10-(4r/3*3.14) =8.3	-(3.14/2)*r^2 =-25.1	-208.6
rectangule	10/2=5	10*8=80	400
		A=54.9cm^2	$\sum a*y$ =191.4cm^2

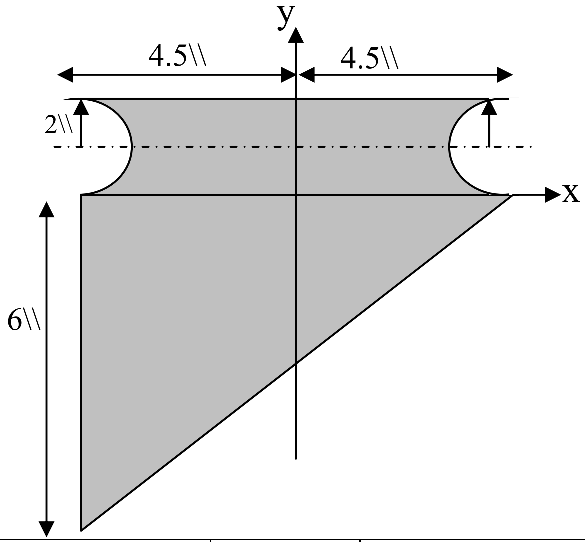
$\bar{Y}$ = 191.4 / 54.9

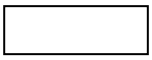
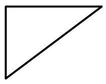
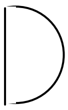
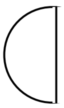
= 3.48cm

$\bar{X}$ = 0

(c.g) = (0 , 3.48)

Q4: Locate the centroid of the shaded area for the figure shown :



Shape	x	y	a	a.x	a.y
1) 	0	2	4*9=36	0	72
2) 	b/3=9/3	h/3=-2	1/2*9*6=27	-40.5	-54
3) 	4.5-[4*r/3*3.14] =3.65	2	-(3.14*r^2/2) =-2*3.14	-22.9	-4*3.14
4) 	-(4.5-[4*r/ 3*3.14]) =-3.65	2	-(3.14*r^2/2) =-2*3.14	22.9	-4*3.14
			A=50.44	$\sum a*x$=-40.5	$\sum a*y$=-7.12

$\bar{Y}$ = -7.12 / 50.44

= -0.14cm

$\bar{X}$ = -40.5 / 50.44

=-0.80

(c.g) = (-0.80 , -0.14)