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Yield line Analysis of orthotropically reinforced rectangular concrete slabs having one free edge

By

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4" stage

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الاهداء

إلى من رزقني الوجود وامرني بالســـجود

إلى الاحد المعبود ...... الى العــدل الودود

الذي لايمـــلك غيـــره الخلـــــــود....... ربي

إلى من اضناه العمر حتى رآني أكبر

إلى من كان صبري حتى عجزت ان اصبر ...... أبي

إلى التي زرعتني في الحياة بـــــذرة

وسقتني من دمها قطرة بعد قطــــرة

إلى من صبرها قبساً، ودعائها همساً .... أمي

إلى القلب الذي يهوى السلام ..... جدتي

ألى الصمت الذي يعني عن الكلام ..... خالاتي

إلى الذين واكبوا معي طريق سنين العمر بأخلاص ... أصدقائي

إلى من كانوا قنديلي في الظالام

إلى الشموع الي اضاءت لي طريق العلم

اليكم جميعاً يا مسك الختام .... أساتذتي

شكر وأمتنان

بسم الله وعلى بركته اتقدم بخالص الشكر وعظيم الامتنان الى استاذي الفاضل

الاستاذالدكتور هشام الحسني

لما خصني من جهد كبير وفضل عظيم في

الاشراف على مشروعي هذا ....

فجزاه الله عني كل الخير ....

كما واتقدم بجزيل الشكر لكل من قدم لي اي عون ولو بكلمة ....

واخص بالذكر جميع اساتذتي الكرام والسيد رئيس القسم وكل الكادر الاداري لما بذلوه من جهد كبير

بسم الله الرحمن الرحيم

"فهل جزاء الاحسان الا الاحسان "

الحمد لله اولاً واخراً حمداً يوافي نعمه وآلاءه بكل ما أنعمة علي من نعم لا تعد ولا تحصى

والسلام على سيد الرسل وخير الانام

وعلى آل بيته الميامين العز الكرام

أما بعد

أتقدم بالشكر الجزيل وعظيم الامتنان الى الاستاذ الكبير المحترم

أ.د. هشام الحسني

لما قده لي من نصح شديد ومعلومات قيمة ساعدت على إظهار هذا المشروع بهذه الصورة المتكاملة والى اساتذتي الكرام في قسم هندسة البناء والانشاءات فرع الهندسة الانشائية لجهودهم المبذولة في متابعتهم للمسيرة العلمية.

أدامهم الله لنا وللاجيال القدامة ورزقهم من نعيمة

طالب المشروع

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CHAPTER ONE

INTRODUCTION

Introduction

1.1. Yield Line theory

The inelastic behavior of reinforced concrete structures has been the subject of intensified studies for many decades. A major contribution of these studies was focused on the prediction of collapse load of such structures. Accordingly methods have been developed which take into account the conditions applied to the structure prior to failure. One of these methods is the Yield line Theory which considers the limit state of collapse for reinforced concrete slabs.

This theory was first initiated by Ingerslev[(1)] in 1923, but later developed and greatly extended by Johansen [(2)] (Denmark) in 1943. The early literature on yield line theory was mainly in Danish and in 1953 Hognestad [(3)] produced the first summary of this work in English.

In this theory (that forms part of the general theory of limit analysis), the structural elements are assumed to behave in a rigid – perfectly plastic manner, and elastic deformations, strain – hardening effects, shear stresses as well as membrane stresses are ignored.

The method is an upper bound approach and the ultimate load of the slab system is estimated by postulating a collapse mechanism which is compatible with the boundary conditions. The moment at the plastic hinge lines (yield lines) is the ultimate moment of resistance of sections, and the ultimate load (collapse load) is determined either by principle of virtual work or by the equations of equilibrium. Because it is an upper bound approach, it gives an ultimate load which is either correct or too high. Thus, all the possible collapse mechanism (yield line pattern) of the slab must be examined to ensure that the load carrying capacity of the slab is not overestimated. Furthermore, yield line theory permits the analysis of irregular as well as regular slab shapes with different kinds of supports and load conditions. This has led to an extensive use of the theory and it has been recommended in different codes of Practice[4&5].

1.2. Aim Of The Projet

Reinforced concrete rectangular slabs supported on three edges only with the fourth edge free can be categorized into six cases as shown in fig (1). Coefficient for determining the bending moments in these slabs are not available similar to that of methods (2) and (3) of the 1963 ACI code concerning slabs resting on all supports. Therefore, the yield line theory is an alternative approach for analyzing and/or designing such slabs. It is decided in this project to analyze slab case (3) under the action of uniformity distributed load covering the slab full area, by the yield line theory. Based on the results of such analysis an example of a typical slabs of case (3) is also to be given which shows a full detailed design of such slab by the "Ultimate Strength Design Method"

These are cases for rectangular concrete slabs having one free edge

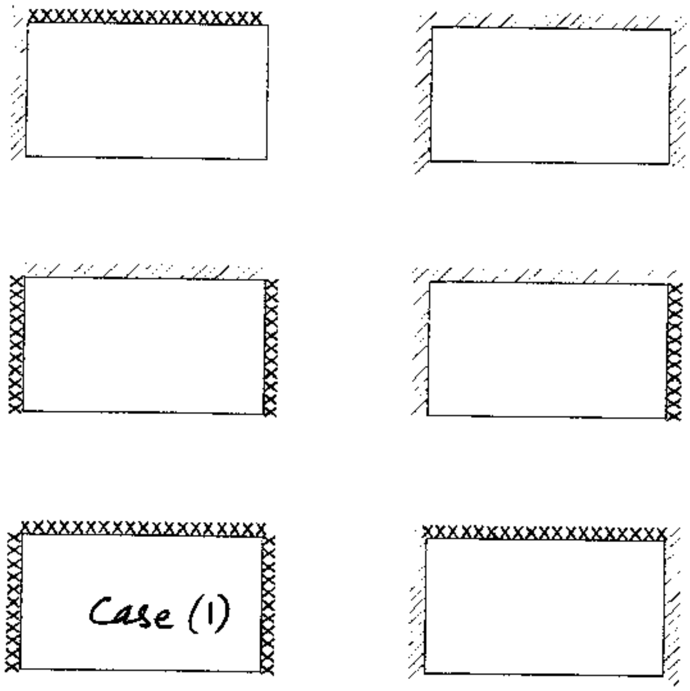
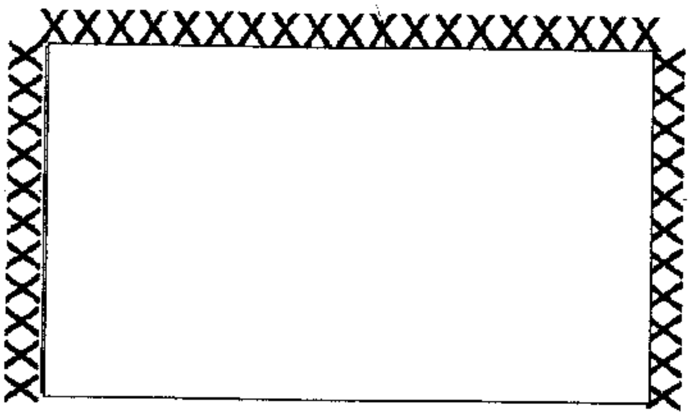


Fig (1) Types of Rc rectangular slabs

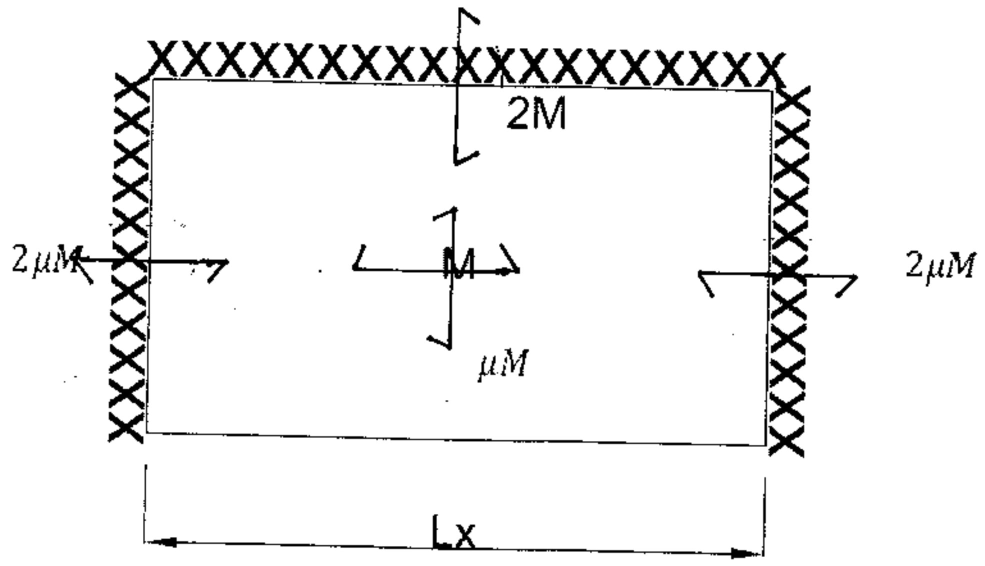
Slab case five has been chosen for such analysis

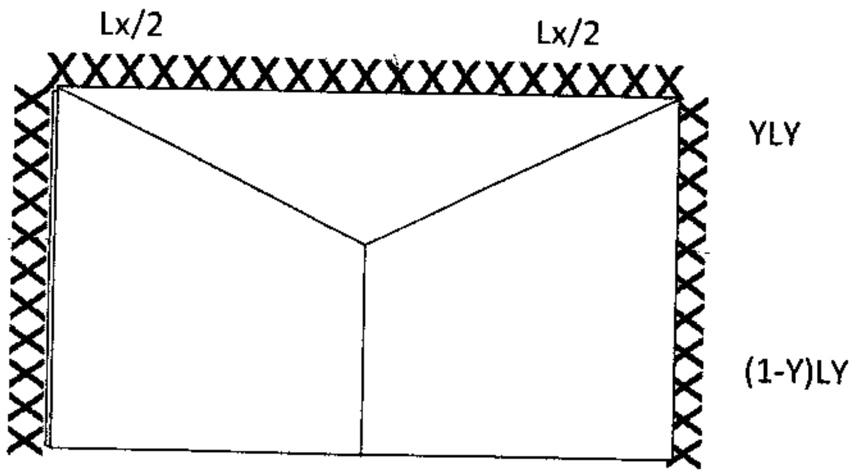


Slab case (1)

CHAPTER TWO

ANALYSIS AND DERIVATION





Mode 1

$$EW = W_u\left[\left\{(1-y)L_y\frac{L_x}{2}.\frac{1}{2}\right\}2+\frac{L_x}{2}.\frac{yL_y}{2}.\frac{1}{3}*4\right]$$

$$= W_uL_xL_y\left[\frac{1}{2}-\frac{y}{2}+\frac{y}{3}\right]$$

$$= W_uL_xL_y\left[\frac{1}{2}-\frac{y}{6}\right]$$

$$EW = \frac{W_uL_xL_y}{6}[3-y]$$

$$IW = (M+2M)L_x.\frac{1}{yL_y}+2(\mu M+2\mu M)L_y\ .\ \frac{2}{L_x}$$

$$= 3M\frac{L_x}{yL_y}+12\mu M\frac{L_y}{L_x}$$

$$= 3M\left[\frac{L_x}{yL_y}+4\mu\ \frac{L_y}{L_x}\right]$$

$$W_u\frac{L_xL_y}{6}[3-y] = 3M\left[\frac{L_x}{yL_y}+4\mu\frac{L_y}{L_x}\right]$$

$$W_u = \frac{18\ M}{L_xL_y}\ .\frac{1}{3-y}\ .\frac{L_x^2+4\ \mu\ y\ L_y^2}{y\ L_xL_y}$$

$$W_u = 18\ M\ \frac{L_x^2+4\ \mu\ y\ L_x^2}{y(3-y)L_x^2L_y^2}$$

$$= 18\ M\ \frac{\left[\frac{1}{L_y^2}+\frac{4\mu}{L_x^2}y\right]}{y(3-y)}\ .\frac{L_x^2}{L_y^2}$$

$$W_u = \frac{18\ M}{L_x^2}\ \frac{\left[\left(\frac{L_x}{L_y}\right)^2+4\mu y\right]}{(3y-y^2)}$$

$$\text{let } \frac{L_y}{L_x} = k$$

$$\frac{W_u L_x^2}{M} = 18\,\frac{\left[\frac{1}{k^2} + 4\mu y\right]}{3y - y^2}$$

$$\frac{\partial W_u}{\partial y} = \frac{18\,M}{L^2} \cdot \frac{(3y - y^2)(4\mu) - \left[\left(\frac{L_x}{L_y}\right)^2 + 4\,\mu\,y\right][3 - 2y]}{(3y - y^2)} = 0$$

$$4\mu(3y - y^2) - (3 - 2y)\left[\left(\frac{L_x}{L_y}\right)^2 + 4\mu y\right] = 0$$

$$12\mu y - 4\mu y^2 - \left[3\left(\frac{L_x}{L_y}\right)^2 + 12\mu y - 2\left(\frac{L_x}{L_y}\right)^2 y - 8\mu y^2\right] = 0$$

$$12\mu y - 4\mu y^2 - 3\left(\frac{L_x}{L_y}\right)^2 - 12\mu y + 2\left(\frac{L_x}{L_y}\right)^2 . y + 8\mu y^2 = 0$$

$$4\mu y^2 + 2\left(\frac{L_x}{L_y}\right)^2 . y - 3\left(\frac{L_x}{L_y}\right)^2 = 0 \qquad] \div 4\mu$$

$$y^2 + \frac{1}{2\mu}\left(\frac{L_x}{L_y}\right)^2 . y - \frac{3}{4\mu}\left(\frac{L_x}{L_y}\right)^2 = 0$$

$$a = 1 \qquad b = \frac{1}{2\mu}\left(\frac{L_x}{L_y}\right)^2 \qquad c = -\frac{3}{4\mu}\left(\frac{L_x}{L_y}\right)^2$$

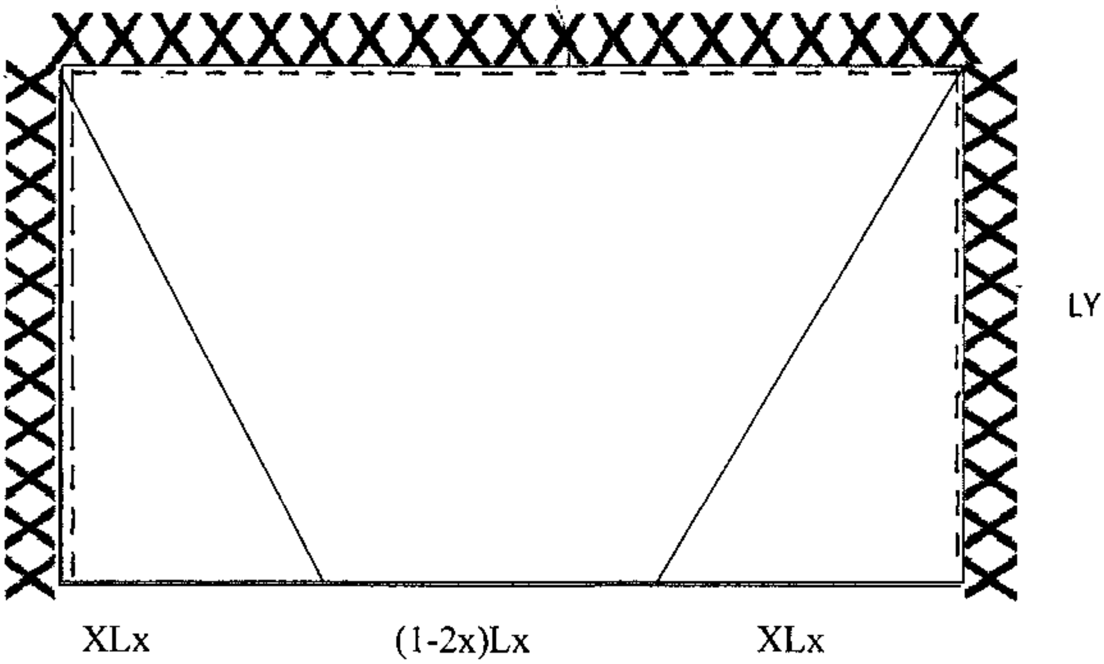
$$y = \frac{-\frac{1}{2\mu}\left(\frac{L_x}{L_y}\right)^2 + \sqrt{\left[\frac{1}{2\mu}\left(\frac{L_x}{L_y}\right)^2\right] + 4 * \frac{3}{4\mu}\left(\frac{L_x}{L_y}\right)^2 \left[\frac{1}{2\mu}\left(\frac{L_x}{L_y}\right)^2\right]^2 * \left[\frac{2\mu}{\left(\frac{L_x}{L_y}\right)^2}\right]^2}}{2}$$

$$y = \frac{-\frac{1}{2\mu}\left(\frac{L_x}{L_y}\right)^2 + \frac{1}{2\mu}\left(\frac{L_x}{L_y}\right)^2 \sqrt{1 + 12\mu\left(\frac{L_x}{L_y}\right)^2}}{2}$$

$$y = \frac{1}{4\mu}\left(\frac{L_x}{L_y}\right)^2 \left[\sqrt{1 + 12\mu\left(\frac{L_x}{L_y}\right)^2} - 1\right]$$

let $k = \frac{L_y}{L_x}$

$$\therefore y = \frac{1}{4\mu} \cdot \frac{1}{k^2} \left[\sqrt{1 + 12\mu k^2} - 1\right]$$



Mode 2

$$EW = W_u\left[\frac{L_y(xL_x)}{2}\cdot\frac{1}{3}\right]4 + W_u(1-2x)L_xL_y.\frac{1}{2}$$

$$= W_uL_yL_x\left[\frac{2}{3}x + \frac{1}{2} - x\right]$$

$$= W_uL_yL_x\left(\frac{1}{2} - \frac{x}{3}\right)$$

$$EW = \frac{W_uL_yL_x}{6}(3-2x)$$

$$IW = \left[2\mu ML_y.\frac{1}{xL_x} + \mu ML_x.\frac{1}{xL_x}\right]2 + M.2xL_x.\frac{1}{L_y} + 2ML_x.\frac{1}{L_y}$$

$$= 6\mu M.\frac{L_y}{xL_x} + 2ML_x\,\frac{(1+x)}{L_y}$$

$$= 2M\left[\frac{3\mu L_y}{xL_x} + \frac{L_x(1+x)}{L_y}\right]$$

$$= 2M\left[\frac{3\mu}{x\left(\frac{L_x}{L_y}\right)} + \left(\frac{L_x}{L_y}\right)(1+x)\right]$$

$$IW = 2M\frac{3\mu + \left(\frac{L_x}{L_y}\right)^2(x+x^2)}{X\left(\frac{L_x}{L_y}\right)}$$

$$EW = IW$$

$$\frac{W_uL_yL_x}{6}(3-2x) = 2M\frac{3\mu + \left(\frac{L_x}{L_y}\right)^2(x+x^2)}{X\left(\frac{L_x}{L_y}\right)}$$

$$W_u = \frac{12M}{L_y L_x \frac{L_x}{L_y}} \left[\frac{3\mu + \left(\frac{L_x}{L_y}\right)^2 (x + x^2)}{(3x - 2x^2)} \right]$$

$$W_u = \frac{12M}{L_x^2} \left[\frac{3\mu + \left(\frac{L_x}{L_y}\right)^2 (X + X^2)}{(3X - 2X^2)} \right]$$

$$\frac{\partial W_u}{\partial x} = 0$$

$$= \frac{12M}{L_x^2} \cdot \frac{(3x - 2x^2).\left(\frac{L_x}{L_y}\right)^2 (1 + 2x) - \left[3\mu + \left(\frac{L_x}{L_y}\right)^2 (x + x^2\right](3 - 4x)}{(3X - 2X^2)}$$

$$3x\left(\frac{L_x}{L_y}\right)^2 + 6x^2\left(\frac{L_x}{L_y}\right)^2 - 2x^2\left(\frac{L_x}{L_y}\right)^2 - 4x^3\left(\frac{L_x}{L_y}\right)^2 - \left[9\mu - 12\mu x + 3\left(\frac{L_x}{L_y}\right)^2 (x + x^2) - 4\left(\frac{L_x}{L_y}\right)^2 x(x + x^2)\right] = 0$$

$$3x\left(\frac{L_x}{L_y}\right)^2 + 6x^2\left(\frac{L_x}{L_y}\right)^2 - 2x^2\left(\frac{L_x}{L_y}\right)^2 - 4x^3\left(\frac{L_x}{L_y}\right)^2 - 9\mu + 12\mu x - 3x\left(\frac{L_x}{L_y}\right)^2 - 3x^2\left(\frac{L_x}{L_y}\right)^2 + 4x^2\left(\frac{L_x}{L_y}\right)^2 + 4x^3\left(\frac{L_x}{L_y}\right)^2 = 0$$

$$5x^2\left(\frac{L_x}{L_y}\right)^2 + 12\mu x - 9x = 0 \qquad \div 5$$

$$x^2\left(\frac{L_x}{L_y}\right)^2 + 2.4\mu x - 1.8\mu = 0$$

CHAPTER THREE
RESULTS

		Mode I $y = \frac{1}{4\mu} \cdot \frac{1}{k^2} \left[\sqrt{1 + 12\mu k^2} - 1\right]$ $\frac{W_u L_x^2}{M} = 18 \frac{\left[\frac{1}{k^2} + 4\mu y\right]}{3y - y^2}$		Mode II $x = 1.2\mu k^2 \left[\sqrt{1 + \frac{1.25}{\mu k^2}} - 1\right]$ $\frac{W_u L_x^2}{M} = 12 \cdot \frac{3\mu + \frac{x + x^2}{k^2}}{(3x - 2x^2)}$			
μ	$k = \frac{L_y}{L_x}$	y	$\frac{W_u L_x^2}{M}$	x	$\frac{W_u L_x^2}{M}$	True mode	True $\frac{W_u L_x^2}{M}$
1	0.2	1.353	245.698	0.225	206.759	II	206.759
	0.6	0.907	64.722	0.482	60.947	II	60.947
	1.0	0.651	42.422	0.600	44.000	I	42.422
	1.5	0.477	35.185	0.667	37.793	I	35.185
	2.0	0.375	32.000	0.700	35.330	I	32.000
	3.0	0.262	29.085	0.726	33.520	I	29.085
	4.0	0.2	27.723	0.736	32.863	I	27.723
	5.0	0.163	26.936	0.741	32.555	I	26.936

		Mode I		Mode II			
		$y = \frac{1}{4\mu} \cdot \frac{1}{k^2} \left[\sqrt{1 + 12\mu k^2} - 1\right]$ $\frac{W_u L_x^2}{M} = 18 \left[\frac{\left[\frac{1}{k^2} + 4\mu y\right]}{3y - y^2}\right]$		$x = 1.2\mu k^2 \left[\sqrt{1 + \frac{1.25}{\mu k^2}} - 1\right]$ $\frac{W_u L_x^2}{M} = 12 \left[\frac{3\mu + \frac{x + x^2}{k^2}}{(3x - 2x^2)}\right]$			
μ	$k = \frac{L_y}{L_x}$	y	$\frac{W_u L_x^2}{M}$	x	$\frac{W_u L_x^2}{M}$	True mode	True $\frac{W_u L_x^2}{M}$
0.8	0.2	1.378	236.856	0.205	193.800	II	193.800
	0.6	0.964	53.758	0.453	53.467	II	53.467
	1.0	0.705	36.222	0.577	37.295	I	36.222
	1.5	0.512	29.419	0.652	31.234	I	29.419
	2.0	0.412	26.483	0.688	28.902	I	26.483
	3.0	0.290	23.799	0.720	27.116	I	23.799
	4.0	0.223	22.552	0.733	26.470	I	22.552
	5.0	0.182	21.839	0.739	26.148	I	21.839

$$y = \frac{1}{4\mu}\cdot\frac{1}{k^2}\left[\sqrt{1+12\mu k^2}-1\right]$$

$$\frac{W_u L_x^2}{M} = 18\,\frac{\left[\frac{1}{k^2}+4\mu y\right]}{3y-y^2}$$

$$x = 1.2\mu\,k^2\left[\sqrt{1+\frac{1.25}{\mu\,k^2}}-1\right]$$

$$\frac{W_u L_x^2}{M} = 12\cdot\frac{3\mu+\frac{x+x^2}{k^2}}{(3x-2x^2)}$$

		Mode I		**Mode II**			
μ	$k = \frac{L_y}{L_x}$	y	$\frac{W_u L_x^2}{M}$	x	$\frac{W_u L_x^2}{M}$	True mode	True $\frac{W_u L_x^2}{M}$
0.6	0.2	1.405	227.888	0.181	179.724	II	179.714
	0.6	1.036	46.563	0.416	45.715	II	45.715
	1.0	0.776	29.851	0.544	30.461	I	29.851
	1.5	0.583	23.553	0.628	24.706	I	23.533
	2.0	0.364	21.078	0.672	22.436	I	21.078
	3.0	0.329	18.445	0.711	20.697	I	28.445
	4.0	0.255	17.344	0.727	20.055	I	17.344
	5.0	0.208	16.705	0.735	19.744	I	16.705

		Mode I		Mode II			
		$y = \frac{1}{4\mu}\cdot\frac{1}{k^2}\left[\sqrt{1+12\mu k^2}-1\right]$ $\frac{W_u L_x^2}{M} = 18\,\frac{\left[\frac{1}{k^2}+4\mu y\right]}{3y-y^2}$		$x = 1.2\mu k^2\left[\sqrt{1+\frac{1.25}{\mu k^2}}-1\right]$ $\frac{W_u L_x^2}{M} = 12\cdot\frac{3\mu+\frac{x+x^2}{k^2}}{(3x-2x^2)}$			
μ	$k = \frac{L_y}{L_x}$	y	$\frac{W_u L_x^2}{M}$	x	$\frac{W_u L_x^2}{M}$	True mode	True $\frac{W_u L_x^2}{M}$
0.4	0.2	1.434	218.783	0.152	163.247	II	163.247
	0.6	1.131	39.060	0.365	37.403	II	37.403
	1.0	0.880	23.228	0.495	23.397	I	23.228
	1.5	0.676	17.485	0.589	18.072	I	17.485
	2.0	0.546	15.093	0.642	15.937	I	15.093
	3.0	0.392	13.004	0.694	14.268	I	13.004
	4.0	0.305	12.055	0.717	13.645	I	12.055
	5.0	0.250	11.520	0.728	13.349	I	11.520

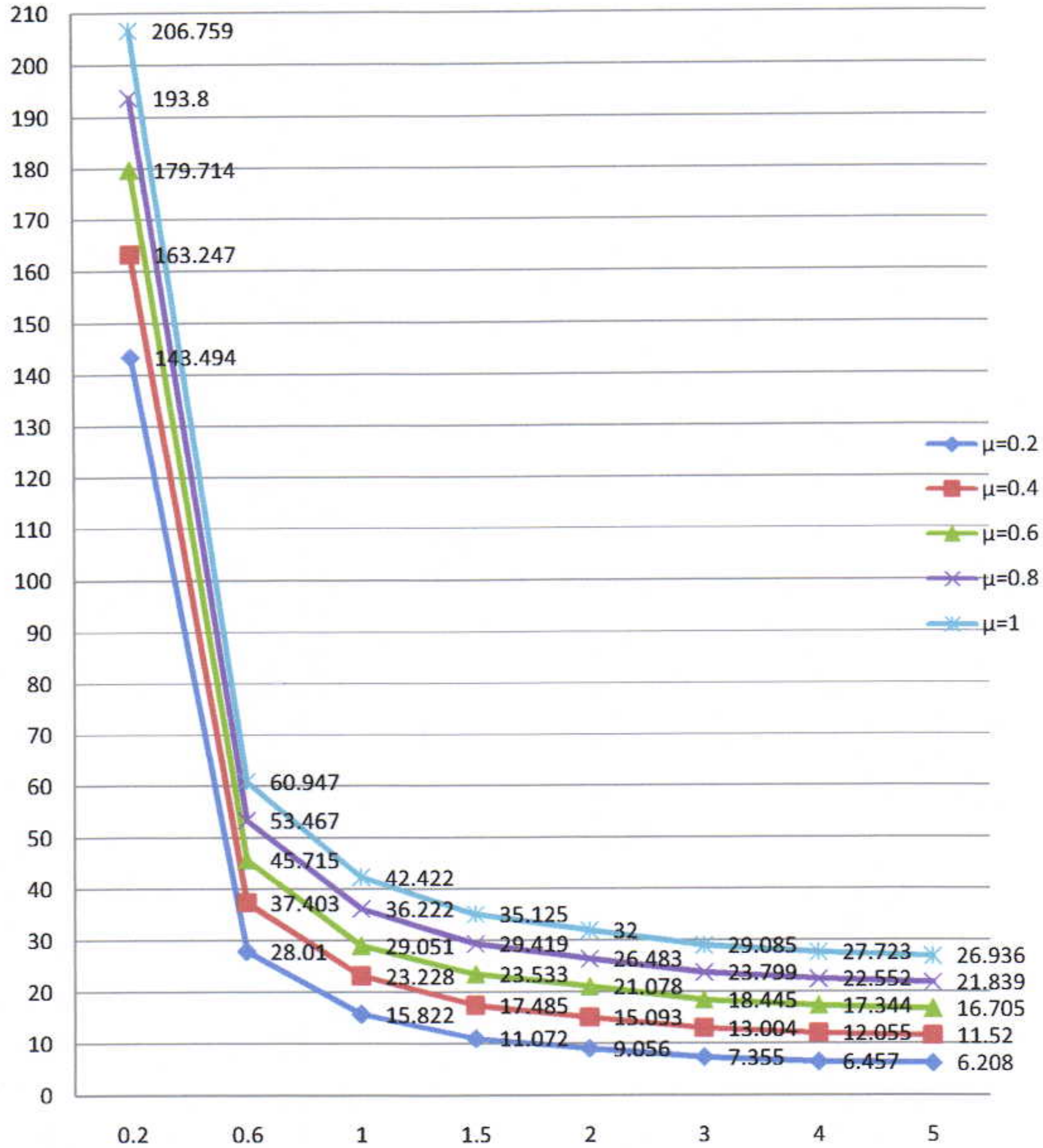
$$y = \frac{1}{4\mu} \cdot \frac{1}{k^2} \left[\sqrt{1 + 12\mu k^2} - 1\right]$$

$$\frac{W_u L_x^2}{M} = 18 \, \frac{\left[\frac{1}{k^2} + 4\mu y\right]}{3y - y^2}$$

$$x = 1.2\mu \, k^2 \left[\sqrt{1 + \frac{1.25}{\mu \, k^2}} - 1\right]$$

$$\frac{W_u L_x^2}{M} = 12 \cdot \frac{3\mu + \frac{x + x^2}{k^2}}{(3x - 2x^2)}$$

		Mode I		**Mode II**			
μ	$k = \frac{L_y}{L_x}$	y	$\frac{W_u L_x^2}{M}$	x	$\frac{W_u L_x^2}{M}$	True mode	True $\frac{W_u L_x^2}{M}$
0.2	0.2	1.466	209.477	0.111	143.494	II	143.494
	0.6	1.268	31.083	0.284	28.010	II	28.010
	1.0	1.055	16.175	0.406	15.822	II	15.822
	1.5	0.850	11.072	0.510	11.296	I	11.072
	2.0	0.705	9.056	0.597	9.324	I	9.056
	3.0	0.521	7.355	0.652	7.808	I	7.355
	4.0	0.412	6.457	0.688	7.221	I	6.457
	5.0	0.341	6.208	0.708	6.941	I	6.208

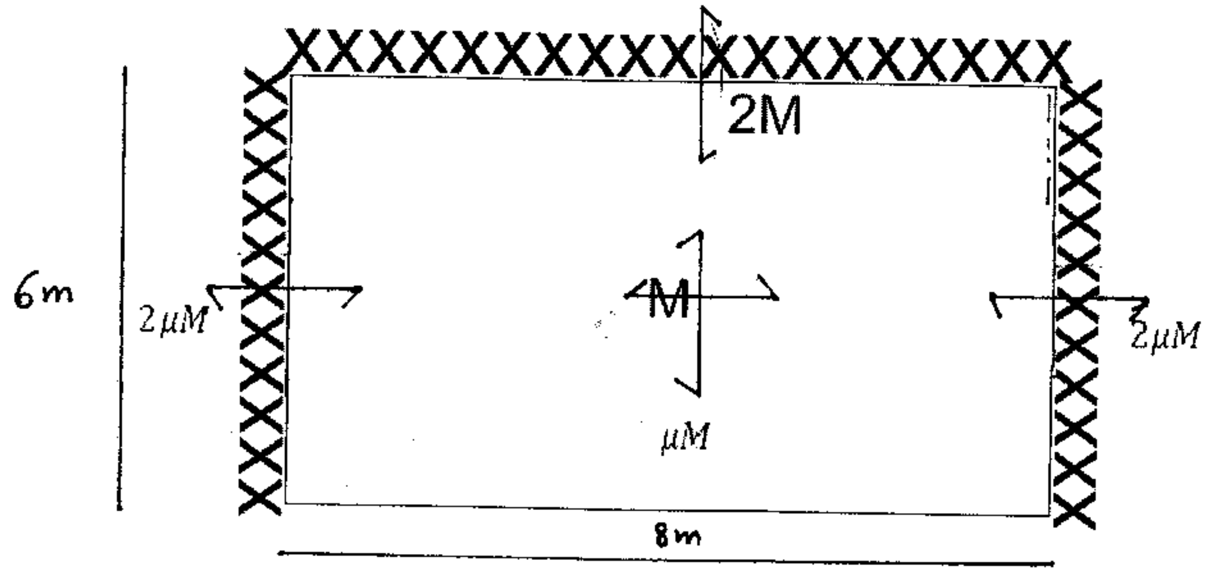


$$\frac{W_U L^2_X}{M}$$

$$k = \frac{L_Y}{L_X}$$

CHAPTER FOUR

DESIGN OF RECTANGULAR SLAB FOR EXAMPLE



Ex:- Design the rectangular RC panel using Yield Line theory as the Fig. is shown : given

$\mathbf{f_{\bar{c}} = 20\ Mpa}$

$\mathbf{f_y = 400\ Mpa}$

using $\emptyset = 12$ mm

Self weight = 0.16 m

Tiles of motar = 0.04 m

Live load = 4 Kn/m2

Let $\mu = 0.8$

$$M_u = \emptyset\, c_x \left(d - \frac{a}{2}\right)$$

$$M_u = \emptyset\, 0.85\, f_{\bar{c}}\, a\, b \left(d - \frac{a}{2}\right)$$

$$\frac{M_n}{0.85\, f_{\bar{c}}\, b} = a\left(d - \frac{a}{2}\right)$$

$$\frac{M_n}{0.85\, f_{\bar{c}}\, bd^2} = \frac{a}{d}\left(1 - \frac{1}{2}\frac{a}{d}\right)$$ بقسمة الطرفين على d^2

Let

$$\frac{M_n}{0.85\, f_{\bar{c}}\, bd^2} = R$$

$$R = \frac{a}{d} - \frac{1}{2}\left(\frac{a}{d}\right)^2$$

$$\frac{1}{2}\left(\frac{a}{d}\right)^2 - \frac{a}{d} + R = 0$$

$$\left(\frac{a}{d}\right)^2 - 2\frac{a}{d} + 2R = 0$$

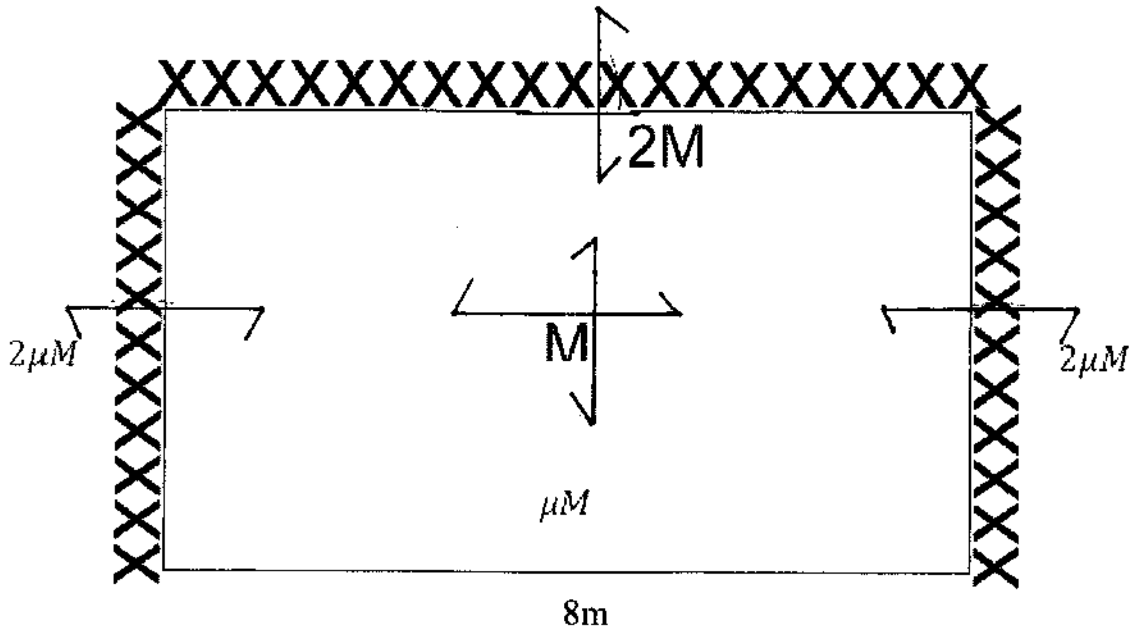
$$\frac{a}{d} = \frac{2 \pm \sqrt{4 - 4 * 2R * 1}}{2}$$

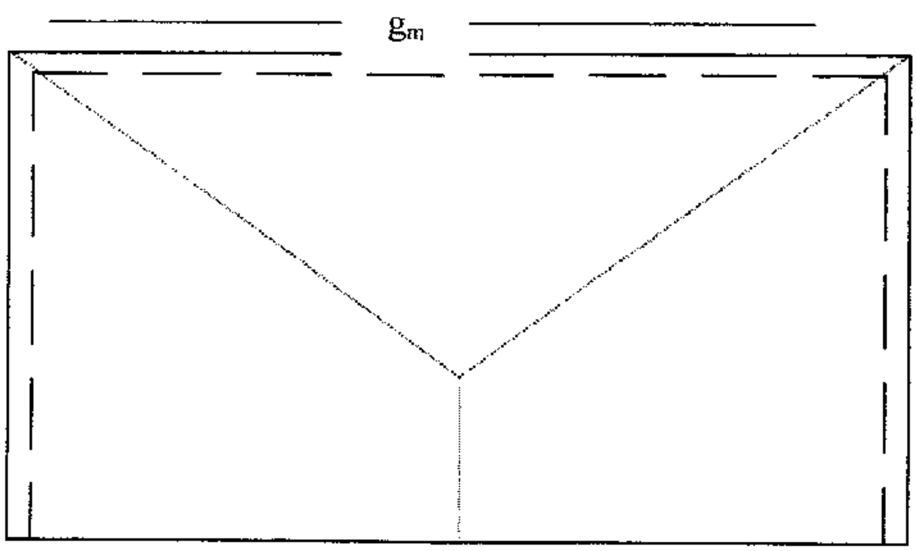
$$= 1 - \sqrt{1 - 2R}$$

$$\frac{a}{d} = 1 - \sqrt{1 - 2 * \frac{M_n}{0.85 f_{\bar{c}}\, bd^2}}$$

$$\frac{a}{d} = 1 - \sqrt{1 - 2 * \frac{M_n}{0.85 f_{\bar{c}}\, b d^2}}$$

Mode I

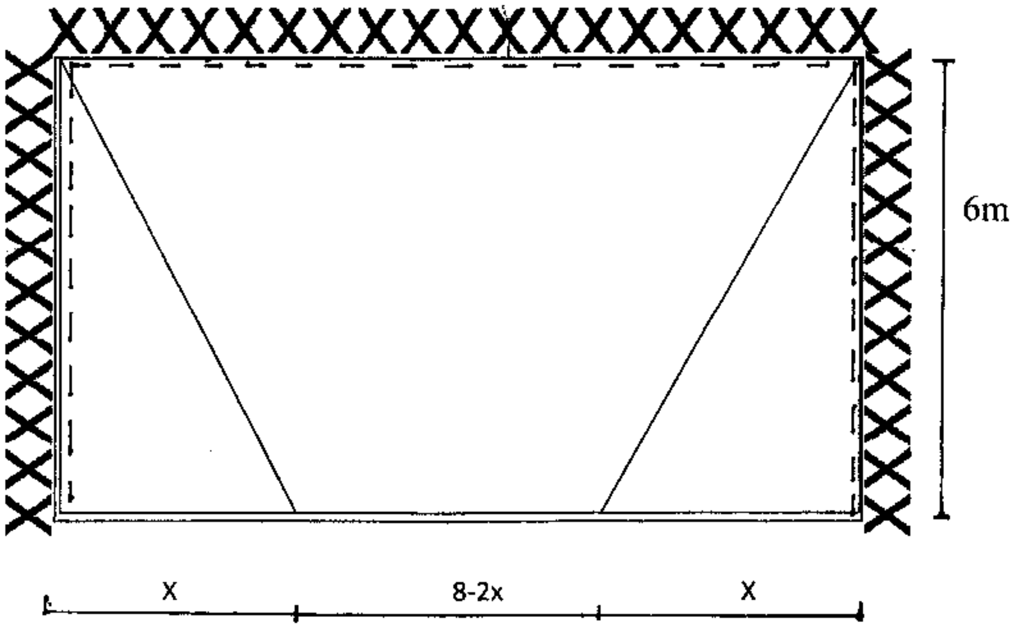




$$\frac{W_u L_x^2}{M} = 18\,\frac{\left[\frac{1}{k^2}+4\mu y\right]}{3y-y^2}$$

$$y = \frac{1}{4\mu}\cdot\frac{1}{k^2}\left[\sqrt{1+12\mu k^2}-1\right]$$

Mode 2



$$\frac{W_u L_x^2}{M} = 12 \cdot \frac{3\mu + \frac{x + x^2}{k^2}}{(3x - 2x^2)}$$

$$x = 1.2\mu\, k^2 \left[\sqrt{1 + \frac{1.25}{\mu\, k^2}} - 1\right]$$

$$k = \frac{6}{8} = 0.75$$

let $\mu = 0.8$ **Mode I**

$$y = \frac{1}{4\mu} \cdot \frac{1}{k^2}\left[\sqrt{1 + 12\mu k^2} - 1\right]$$

$$y = \frac{1}{4 * 0.8} \cdot \frac{1}{0.75^2}\left[\sqrt{1 + 12 * 0.8 * 0.75^2} - 1\right]$$

$$y = 0.85$$

$$\frac{W_u L_x^2}{M} = 18\, \frac{\left[\frac{1}{k^2} + 4\mu y\right]}{3y - y^2}$$

$$= 18\left[\frac{\frac{1}{0.75^2} + 4 * 0.8 * 0.85}{3 * 0.85 - 0.85^2}\right]$$

$$= 44.303\ \text{Kn/m}^2$$

Mode II

$$x = 1.2\mu\, k^2 \left[\sqrt{1 + \frac{1.25}{\mu\, k^2}} - 1\right]$$

$$x = 1.2 * 0.8 * 0.75^2 \left[\sqrt{1 + \frac{1.25}{0.8 * (0.75^2)}} - 1\right]$$

$$= 0.51$$

$$\frac{W_u L_x^2}{M} = 12 \cdot \frac{3\mu + \frac{x + x^2}{k^2}}{(3x - 2x^2)}$$

$$= 12 \cdot \frac{3 * 0.8 + \frac{0.51 + 0.51^2}{0.75^2}}{\{3(0.51) - 2(0.51)^2\}}$$

$$= 44.780 \ Kn/m^2$$

Mode I	Mode II
$y = \frac{1}{4\mu} \cdot \frac{1}{k^2} \left[\sqrt{1 + 12\mu k^2} - 1\right]$	$x = 1.2\mu k^2 \left[\sqrt{1 + \frac{1.25}{\mu k^2}} - 1\right]$
$\frac{W_u L_x^2}{M} = 18 \left[\frac{\left[\frac{1}{k^2} + 4\mu y\right]}{3y - y^2}\right]$	$\frac{W_u L_x^2}{M} = 12 \left[\cdot \frac{3\mu + \frac{x + x^2}{k^2}}{(3x - 2x^2)}\right]$

μ	$k = \frac{L_y}{L_x}$	y	$\frac{W_u L_x^2}{M}$	x	$\frac{W_u L_x^2}{M}$	True mode	True $\frac{W_u L_x^2}{M}$
0.8	0.6	0.85	44.303	0.51	44.780	I	44.303

$$\text{slab thickness} = \frac{\text{permitere slab}}{180}$$

$$= \frac{2(6+8) * 10^3}{180}$$

$$= 156 \text{ mm}$$

say use = 160 mm

<u>Dead Load =</u>

$$\text{self wt. of slab} = 0.16 * 24$$

$$= 3.84 \frac{Kn}{m^2}$$

$$\text{Tiles \& } Morter = 0.04 * 24$$

$$= 0.96 \frac{Kn}{m^2}$$

$$\therefore W_d = 3.84 + 0.96$$

$$= 4.08 \frac{Kn}{m^2}$$

$$\text{use live load} = 4 \frac{Kn}{m^2}$$

$$\therefore W_u = 1.2\, W_d + 1.6\, W_l$$

$$= 1.2 * 4.08 + 1.6 * 4$$

$$= 12.16 \frac{Kn}{m^2}$$

$$\frac{W_u\, L_x^2}{M} = 44.303 \frac{Kn}{m^2}$$

$$\therefore \frac{12.16\,(8)^2}{M} = 44.303$$

$$M = 17.566\ Kn.\frac{m}{m}$$

$$\therefore M_{u^+x} = 17.566\ Kn.\frac{m}{m}$$

$$M_{u^+y} = \mu\, M_{u^+x}$$

$$= 0.8 * 17.566$$

$$= 14.053\ Kn.\frac{m}{m}$$

$$M_{u^-x} = 2\,M$$

$$= 2 * 17.566$$

$$= 35.132\ Kn.\frac{m}{m}$$

$$M_{u^-y} = 2\,\mu\, M_{u^+x}$$

$$= 2 * 0.8 * 17.566$$

$$= 28.106\ Kn.\frac{m}{m}$$

1. for $M_{u^+x} = 17.566\ Kn.\frac{m}{m}$

$$\frac{a}{d} = 1 - \sqrt{1 - \frac{2\,M_n}{f_{\bar{c}}\,bd^2}}$$

$$M_n = \frac{17.566}{0.9} = 19.518\ Kn.\frac{m}{m}$$

$$\frac{a}{128} = 1 - \sqrt{1 - \frac{2 * 19.518 * 10^6}{0.85 * 20 * 1000 * 128^2}}$$

$a = 9.308\ mm$

$T = C$

$f_y\ As = f_{\bar{\bar{c}}}\ Ac$

$400\ As = 20 * 0.85 * 1000 * 9.308$

$As = 395.59\ mm^2$

$As_{min} = 0.0018\ bh$

$= 0.0018 * 1000 * 160$

$= 288\ mm^2$

use Ø 12 mm

$$s = \frac{ab.b}{As}$$

$$s = \frac{113.04 * 1000}{395.59}$$

$s = 285.750 \Rightarrow s = 280\ mm$

$s_{max.} = 2h$

$= 2 * 160$

$= 320\ mm$

$s_{max.} = 450\ mm$

use Ø 12 @ 280 mm

$$\text{for } M_{u^-x} = 35.132 = 2M$$

$$M_n = \frac{35.132}{0.9}$$

$$= 39.036 \frac{Kn}{m}.m$$

$$\frac{a}{128} = 1 - \sqrt{1 - \frac{2 * 39.036 * 10^6}{0.85 * 20 * 1000 * 128^2}}$$

$$a = 19.411 \text{ mm}$$

$$400 \text{ As} = 20 * 0.85 * 1000 * 19.411$$

$$\text{As} = 824.96 \text{ mm}^2$$

$$s = \frac{113040}{824.968}$$

$$s = 137.024 \text{ mm}$$

$$\text{use } s = 130 \text{ mm}$$

$$s_{max.} = 2h$$

$$= 2 * 160$$

$$= 320 \text{ mm}$$

$$s_{max.} = 450 \text{ mm}$$

$$\therefore \text{use } \emptyset\, 12 \text{ @ } 130 \text{ mm}$$

$$\text{or } M_{u^+y} = \mu\, M_{u^+x}$$

$$= 0.8 * 17.566$$

$$= 14.053 \text{ Kn}.\frac{m}{m}$$

$$M_n = \frac{14.053}{0.9}$$

$$= 15.614\ \frac{Kn}{m}.m$$

$$\frac{a}{128} = 1 - \sqrt{1 - \frac{2 * 15.614 * 10^6}{0.85 * 20 * 1000 * 128^2}}$$

$$a = 7.389\ mm$$

$$400\ As = 20 * 0.85 * 1000 * 7.389$$

$$As = 314.033\ mm^2$$

$$s = \frac{113040}{314.033}$$

$$s = 359.962\ mm$$

$$\text{use } s = 350\ mm$$

$$s_{max.} = 2h$$

$$= 2 * 160$$

$$= 320\ mm$$

$$s_{max.} = 450\ mm$$

$$\therefore \text{use } \emptyset\ 12\ @\ 320\ mm$$

$$\text{for } M_{u^-y} = 2\ \mu\ M_{u^+x}$$

$$= 2 * 0.8 * 17.566$$

$$= 28.106\ Kn.\frac{m}{m}$$

$$M_n = \frac{28.106}{0.9}$$

$$= 31.229\ \frac{Kn}{m}.m$$

$$\frac{a}{128} = 1 - \sqrt{1 - \frac{2 * 31.229 * 10^6}{0.85 * 20 * 1000 * 128^2}}$$

$$a = 15.261\ mm$$

$$400\ As = 20 * 0.85 * 1000 * 15.261$$

$$As = 648.5925\ mm^2$$

$$s = \frac{113040}{648.5925}$$

$$s = 174.285\ mm$$

$$use\ s = 170\ mm$$

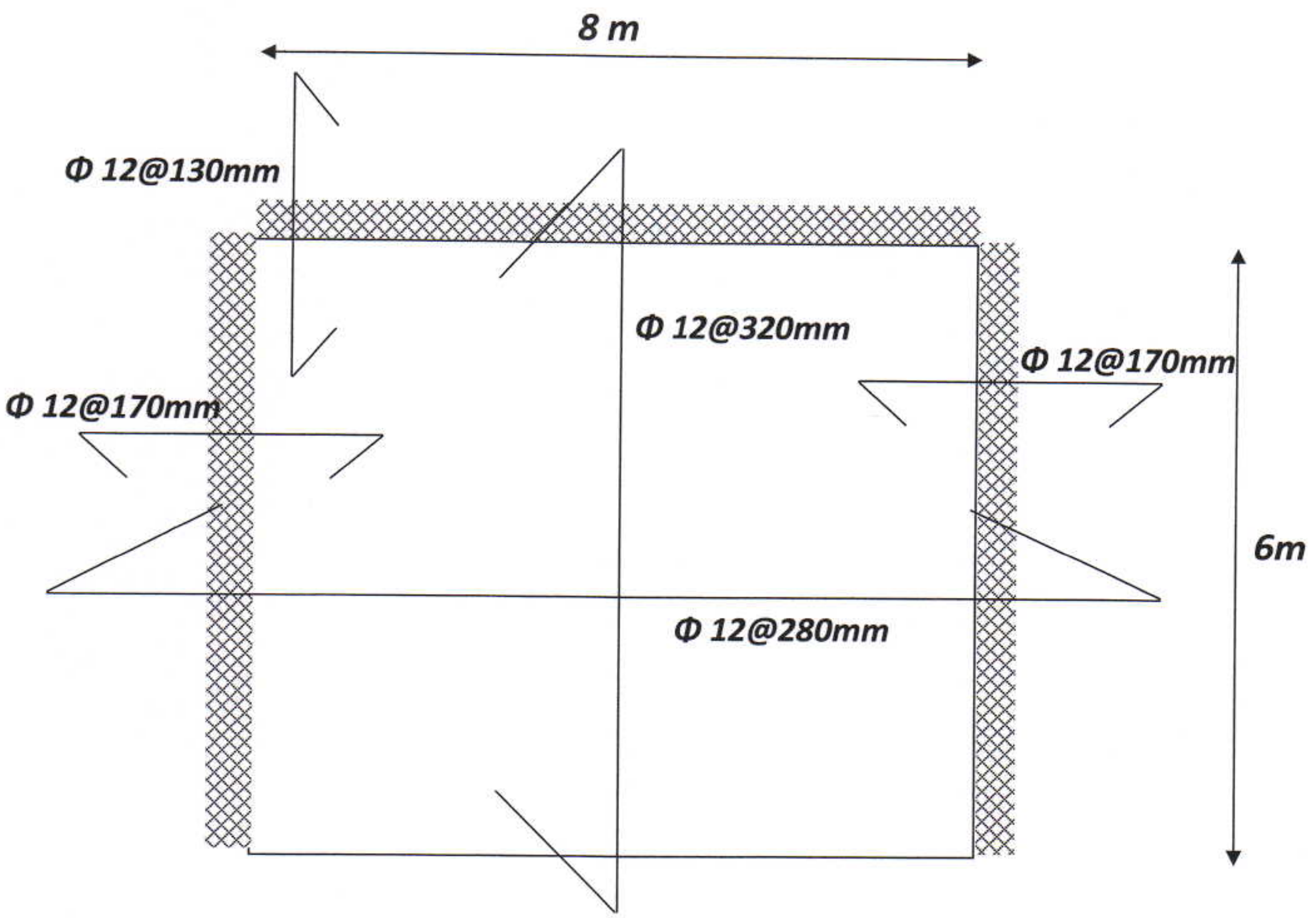
$$s_{max.} = 2h$$

$$= 2 * 160$$

$$= 320\ mm$$

$$s_{max.} = 450\ mm$$

$$\therefore use\ \emptyset\ 12\ @\ 170\ mm$$


8 m
Φ 12@130mm
Φ 12@320mm
Φ 12@170mm
Φ 12@170mm
6m
Φ 12@280mm