UNIVERSITY OF TECHNOLOGY
BUILDING & CONSTRUCTION ENG.
SECOND CLASS EXPERIMENTAL TRAINING

(مختبر السوائل)

FLUID LABORATORY

Center of Pressure

Experiment # (1)

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- ***Objective:***

We want from this experiment to:

- Determine the position of the center of pressure on the rectangular face of the partially or entirely submerged object in the water.

- ***Equipments:***

Hydrostatic pressure apparatus is shown in figure 1 below.

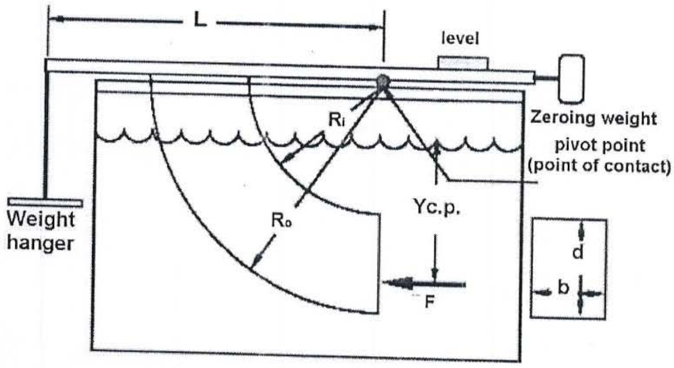


Figure 1: Experimental setup

- ***Theory:***

- The magnitude of the ***resultant hydrostatic force***:

$$\boxed{F = \overline{P} \cdot A = \gamma \cdot \overline{y} \cdot \sin \alpha \cdot A}$$

Where:

γ : is specific weight of the liquid.

$\overline{y}$: is slanted distance from the liquid surface to the centroid of the immerged surface.

α : inclination of the plane surface from the horizontal.

A : is the area on the immerged surface.

- The vertical location of line of action of resultant hydrostatic force is called ***center of pressure*** y_{cp} where :

$$\boxed{y_{cp} = \bar{y} + \frac{\bar{I}}{\bar{y} \cdot A}}$$ (Theoretically)

Where I is the area moment of inertia about the centroidal axis.

$$\boxed{I = b\,y^3 / 12}$$

- We have in this experiment two cases:

1. **Partially immerged (y < d)**

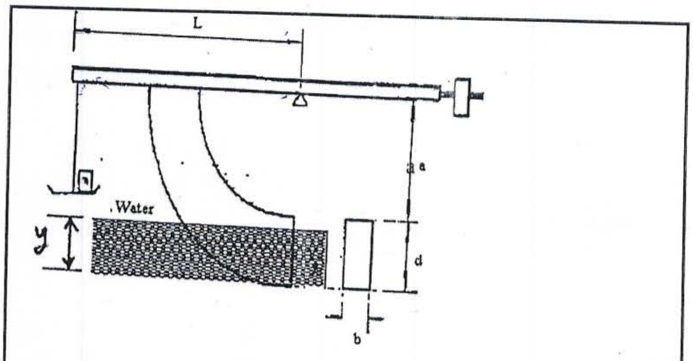


Figure 2: partially immerged case

The location of the center of pressure from experiment:

$$y_{c.p.Exp.} = \left(\frac{m.g.L}{F}\right) - d - a + y$$

Where : m is the mass added to balance F and L is the distance from the pivot to m

2. **Totally immerged (y > d)**

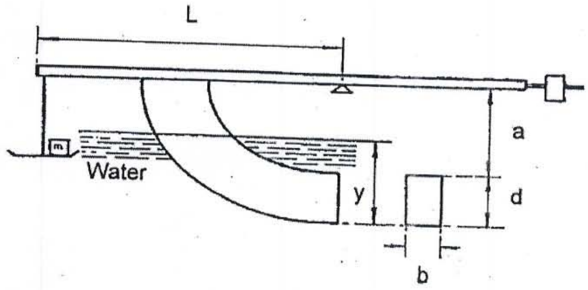


Figure 3: totally immerged case

The location of the center of pressure from experiment:

$$y_{c.p.Exp.} = \frac{m.g.L}{F} - a + \bar{y} - \frac{d}{2}$$

- ***Procedure:***

- The dimensions indicated on the figure are:

a = 9.5 cm, d = 10 cm, and b = 7 cm, L =28.5 cm.

- With the receiver placed on the bench, place the balance arm on the support (Sharp profile). Hang the pan at the end of the arm.

- Connect a length of flexible hose to receiver draining cock and connect the other end to drain. Level the receiver by properly acting on the support feet, which is adjustable, while the "bubble level" is observed. Displace the counterweight of the arm until getting the arm to be horizontal. Close the drain cock in the bottom of the receiver.

- Introduce water in the receiver until its free surface is tangent to the lower edge of the quadrant. The fine adjustment of that level can be achieved by slightly overreaching the established filling and then slowly draining through the cock.

- Place a calibrated weight on the balance pan and slowly add water until the balance arm recovers the horizontal position. Record the water level, indicated in the quadrant, and the value of the weight placed on the pan.

- Repeat the operation above several times, increasing progressively the weight in the pan until, the balance arm is at level, the level of the free water surface becomes flush with the upper edge of the flat rectangular surface that the end of the quadrant presents.
- From this point on , and in the order inverse to the operation above of placing the weights on the pan , the weight increments given in each step are removed , the arm is leveled (after every removal) by using the drain cock and the weight in the pan and the water level values are recorded.

- ***Data Results & Analysis:***

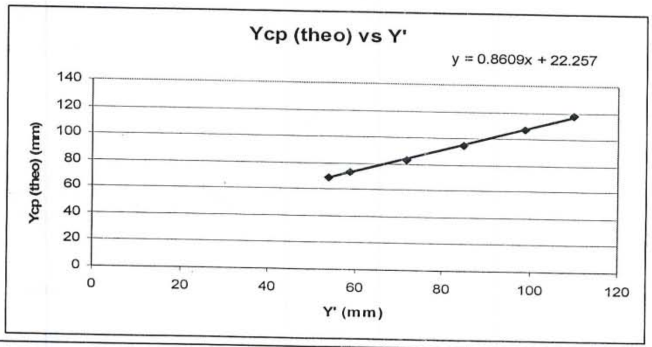
Totally:

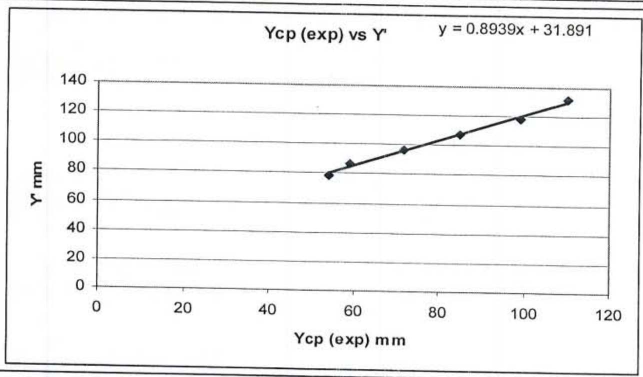
m	y	y'	F	y (Theo)	y (exp)
450	160	110	7.5537	117.5757	131.5584
400	149	99	6.79833	107.4175	118.5022
350	135	85	5.83695	94.80388	107.6471
300	122	72	4.94424	83.57403	96.64286
250	109	59	4.05153	73.12424	86.51816
225	104	54	3.70818	69.43204	78.64286

Partially:

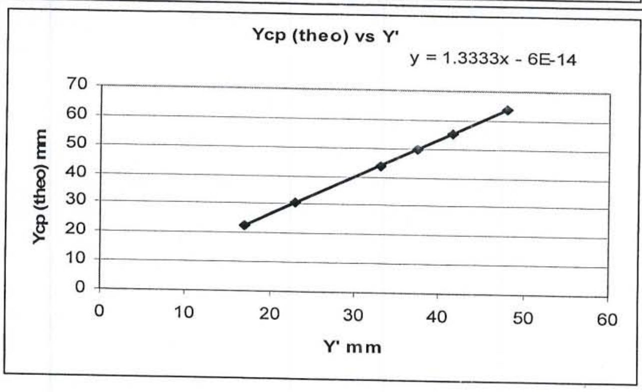
m	y	y'	F	y (Theo)	y (exp)
200	96	48	3.164314	64	77.71131
150	83	41.5	2.365338	55.33333	65.30129
125	75	37.5	1.931344	50	60.95238
100	66	33	1.495633	44	57.93428
50	46	23	0.726529	30.66667	43.41156
25	34	17	0.396913	22.66667	15.09985

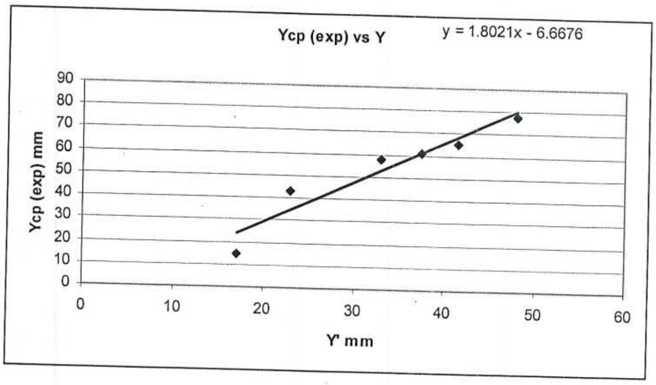
Totally





Partially





- **<u>Sample of calculations:</u>**

  - Totally: (FOR FIRST RAW)
    1. y' = y – d/2 = 16 – 10/2 = 11 cm.
    2. F = ρ g y' A = 1000 * 9.81 * 0.11 * 0.07 * 0.1 = 7.5537 N
    3. Y (theo) = 110 + (70 * 1003 / 12) / (110 * 70 * 100) = 117.5757 mm
    4. y (exp) = ((0.45 * 9.81 * 0.285 / 7.5537) – 0.095 + 0.11 – 0.050) * 1000

       = 131.5584 mm

  - Partially: (FOR FIRST RAW)
    1. y' = y /2 = 96/2 = 4.8 cm.
    2. F = ρ g y' A = 1000 * 9.81 * 0.048 * 0.07 * 0.096 = 3.164314 N
    3. Y (theo) = 2/3 y = 2/3 * 96 = 64 mm
    4. y (exp) = ((0.2 * 9.81 * 0.285 / 3.164314) – 0.095 – 0.1 + 0.096) *1000

       = 77.71131 mm

- ***Discussion & Conclusion:***

1- We can see from our results that the center of pressure is always deeper than the center of area.
2- **Also,** the center of pressure is affected by the depth of water, by increasing water level the center of pressure increase (i.e. direct downward), but the center of area is constant, so the difference between the two centers is increasing with the increase of water level.
3- We get some errors, and I think that its related to the following reasons:

a) The error in the measuring and reading.
b) The error in the level of the water.
c) The error due to unbalanced weights.

7. Discharge over Weirs

Introduction

In hydraulic engineering, weirs are commonly used to regulate flow in rivers and other open channels. In some cases the relationship between the water level upstream of the weir and the discharge over it is known, so that the discharge at any time may be found by observing the upstream water level. Fig. 7.1 shows two different shapes of weir, one formed by cutting a rectangular notch, the other by cutting a V-shaped notch, in vertical metal plates. Such notches usually have sharp edges so that the water springs clear of the plate as it passes through the notch.

It is the purpose of this experiment to derive relationships between head on the weir and discharge for both a rectangular and a V-shaped sharp-edged notch.

Description of Apparatus

Fig. 7.2 shows the arrangement in which water from the bench supply valve is led through a flexible hose to a pipe which serves to distribute the water fairly evenly in the enlarged end of the tank. A contraction section leads the water to a short channel, in the end of which there is a sealed groove, into which either the rectangular, or V notched weir plates may be slotted.

Water which flows over the notch is collected in the exit tank, the outlet of which leads via a plastic bend, to the weigh tank.

The water level in the short approach channel may be observed in the still tube which is connected to the side of the channel. In this tube there is a hook gauge, which consists of a sharply-pointed hook secured to a vertical screwed rod. By turning a nut at the top of the tube the hook may be raised or lowered, its elevation at any time being read off a scale and markings on the nut. The screwed rod has 1 thread per mm and the nut is marked off into equal divisions, so that each division represents a tenth of a millimeter. The hook can be slid inside the hollow screwed rod so that for zero setting it coincides with the crest of the notch and hence the datum free water surface.

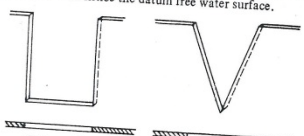


Fig. 7.1. Rectangular and V notches.

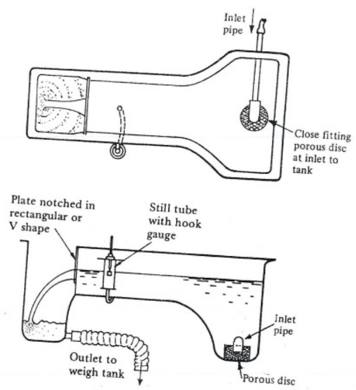


Fig. 7.2. Arrangement of apparatus for measuring flow over weirs.

The hook is best set in the surface by viewing it from below so that its image may be seen, internally reflected in the water surface. The hook is then slowly raised until the hook and its image just coincide as the hook enters the water surface. With a little practice, successive readings of a steady level may be repeated to an accuracy of ± 0.05mm.

Theory of Flow over Sharp-edged Weirs

Fig. 7.3 indicates the essential features of flow over rectangular or V notches. Consider the motion of a particle of fluid from a position M some distance upstream of the weir to its subsequent position N in the plane of the vertical weir plate. If there is no energy loss, according to Bernoulli's equation (1.6)

$$\frac{u_M^2}{2g} + \frac{p_M}{w} + z_M = \frac{u_N^2}{2g} + \frac{p_M}{w} + z_N \tag{7.1}$$

Now provided that the approach channel has a much larger cross-sectional area than the notch, the fluid in the vertical plane containing M will be comparatively

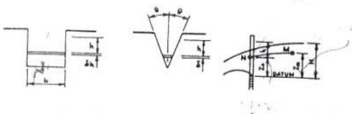


Fig. 7.3. Diagrammatic sketch of flow over weirs.

at rest, so that it is in almost a hydrostatic condition for which the total head of all points has the same value H relative to the datum shown. Making the further (and less justifiable) assumption that $p_n = 0$, i.e. the static pressure is atmospheric at N, equation (7.1) simplifies to

$$\frac{u_N^2}{2g} + z_N = H \tag{7.2}$$

Now

$$H - z_N = h \tag{7.3}$$

as may be seen from the figure, so

$$\frac{u_N^2}{2g} = h \tag{7.4}$$

This velocity is the same as that which would be attained by a particle falling freely from the level of the upstream surface to the position of N.

The discharge over each weir may now be found by integration. For the rectangular weir of width b, the area of an element having height δh is $b\delta h$, so that the flow rate δQ through it is

$$\delta Q = u_N b\delta h = \sqrt{2gh}\, b\, \delta h \tag{7.5}$$

The total flow rate Q obtained by integrating between zero and H gives a result which neglects the lowering of the surface level in the plane of the weir, and is

$$Q = \int_0^H \sqrt{2gh}\, b\, dh$$

$$\text{or}\quad Q = \frac{2}{3}\sqrt{2g}\, b\, H^{3/2} \tag{7.6}$$

for the rectangular weir.

For the V notch of angle 2θ, the width of an element is $2(H - h)\tan\theta$, so that the area of the element having height δh is $2(H - h)\tan\theta\, \delta h$. The flow rate through it is

$$\delta Q = u_N\, 2(H - h)\tan\theta\delta h = \sqrt{2gh}\,2(H - h)\tan\theta\delta h \tag{7.7}$$

so that, integrating as above,

$$Q = \int_0^H \sqrt{2gh}\,2(H - h)\tan\theta\, dh$$

or

$$Q = \frac{8}{15}\sqrt{2g}\tan\theta\, H^{5/2} \tag{7.8}$$

for the V notch.

Now there is in fact a considerable contraction of the stream as it passes through the notch. This can be seen clearly on the apparatus to take place both in the vertical plane, where the upper surface slopes downwards over the notch and the lower surface springs from the crest of the notch in an upward direction, and in the horizontal plane, where the water leaves the edges of the weir in a curve which reduces the width of the stream. This contraction is similar to that previously observed at a sharp-edged orifice and has the same effect of reducing the discharge. It is therefore customary to rewrite the equations in the form

$$Q = C_d \frac{2}{3}\sqrt{2g}\, b\, H^{3/2} \tag{7.9}$$

for the rectangular notch, and

$$Q = C_d \frac{8}{15}\sqrt{2g}\tan\theta\, H^{5/2} \tag{7.10}$$

for the V notch, in which C_d is a coefficient of discharge of the notch which is not necessarily independent of H and may be determined by experiment.

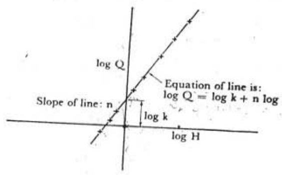


Fig. 7.4. Use of logarithmic graph to find k *and* n *in the equation* $Q = kH^n$ *from experimental results.*

A convenient way of finding C_d, and the exponent of H, in either of these expressions is as follows. Either of equations (7.9) or (7.10) may be written in the form

$$Q = kH^n \tag{7.11}$$

$$\text{or } \log Q = \log k + n \log H \tag{7.12}$$

If experimental results are plotted on a graph having log H as abcissae and log Q as ordinate, then provided that k and n are constant over the range of the results, they will lie on a straight line having slope n and intercept log k on the axis of log Q, as indicated on Fig. 7.4.

Experimental Procedure

The apparatus is first levelled and the zero of the hook gauge is established, i.e. the hook gauge reading corresponding to the level of the crest of the notch.

To do this water is admitted from the bench supply to the apparatus until the level is approximately correct, and then carefully baled out or in, using a small beaker, until the crest of the weir lies just in the surface. For the rectangular notch, this can be checked as illustrated in Fig. 7.5 (a) by placing a steel rule on the crest. For the V notch, the reflection of the V in the surface serves to indicate whether the level is correct or not as illustrated in Fig. 7.5 (b). When, the correct level has been obtained the hook should be set to coincide with the free water surface and the calibrated scale set to read zero.

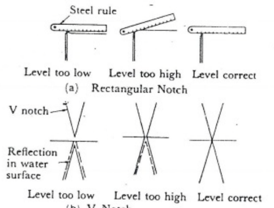


Fig. 7.5. Method of setting water level to height of crest of notch.

A series of measurements of discharge and head on the weir are then taken, the flow being regulated at the bench supply valve. It is recommended that the first reading is taken at maximum discharge, and subsequent readings with roughly equal decrements in head. Readings should be discontinued when the level has fallen to a point at which the stream ceases to spring clear of the notch plate; this is likely to occur when the head is reduced to about 10mm. for a rectangular notch and about 20mm for a V notch. About 8 different discharges for each notch should be sufficient.

The width of the rectangular notch and the angle of the V notch (best found

by measuring the depth and width of the V) should be recorded.

Results and Calculations

Rectangular Notch

Width of notch b . . . . 30mm.

Table 7.1 gives measurements of head H and discharge Q, together with log H and log Q. On Fig. 7.6 the results are plotted to natural scales and on Fig. 7.7 they are plotted logarithmically. The results on Fig. 7.7 lie on a straight line, having:

$$\text{Slope } n = 1.50$$

Intercept on log Q axis (i.e. when log H = 0) = $\bar{2}.72$ (By extrapolation).
The relationship between log Q and log H is thus

$$\log Q = \bar{2}.72 - 1.50 \log H$$

so that the relationship between Q and H is

$$Q = 0.0525\, H^{1.50} \tag{7.13}$$

Comparing this with the expression derived previously,

$$Q = C_d \frac{2}{3} \sqrt{2g}\, b\, H^{3/2} \tag{7.9}$$

we note that the exponent of H is the same in both, and that C_d is given by

$$C_d = \frac{0.0525}{{}^2/_3 \sqrt{2g}\, b} = \frac{0.0525}{{}^2/_3 \times \sqrt{2 \times 9.81} \times 0.030}$$

$$\underline{C_d = 0.59}$$

Gauge Reading mm	H mm	Qty kg	t s	$10^4 \times Q$ m^3/s	log Q	log H
3.93	0	0	–	–	–	–
62.61	58.68	30	39.4	7.62	– 3.1180	– 1.2315
57.12	53.19	30	44.7	6.70	– 3.1739	– 1.2742
49.92	45.99	30	55.3	5.43	– 3.2652	– 1.3373
43.07	39.14	30	70.1	4.28	– 3.3686	– 1.4074
39.34	35.41	30	81.6	3.68	– 3.4342	– 1.4509
31.98	28.05	30	116.0	2.58	– 3.5884	– 1.5520
28.01	25.08	15	72.6	2.06	– 3.6861	– 1.6007
22.38	18.45	7.5	51.9	1.445	– 3.8402	– 1.7350
17.06	13.13	7.5	95.5	0.785	– 4.1051	– 1.8817

Table 7.1. Results with Rectangular Notch.

Gauge Reading mm	H mm	Qty kg	t s	$10^4 \times Q$ m^3/s	log Q	log H
1.94	0	0	–	–	–	–
85.88	83.94	30	40.20	7.46	– 3.1273	– 1.0760
75.28	73.34	30	52.00	5.770	– 3.2388	– 1.1347
65.35	63.41	30	75.91	3.960	– 3.4023	– 1.1978
56.37	54.43	30	102.00	2.780	– 3.5560	– 1.2648
45.98	44.04	15	88.71	1.690	– 3.7721	– 1.3561
42.79	40.35	7.5	55.80	1.340	– 3.8729	– 1.3888
35.60	33.66	7.5	90.11	0.832	– 4.0799	– 1.4729

Table 7.2. Results with V Notch.

V Notch

Width across top of V . . . . 55mm.
Depth of V . . . . 102mm.

$$\tan\theta = \frac{55}{2 \times 102} \qquad \theta = 15.1^{\circ}$$
$$= 0.270$$

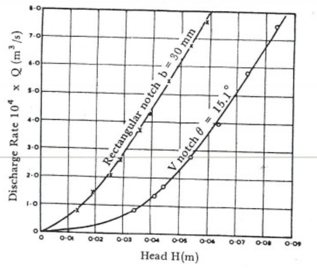


Fig. 7.6. Variation of discharge with head for rectangular and V notches.

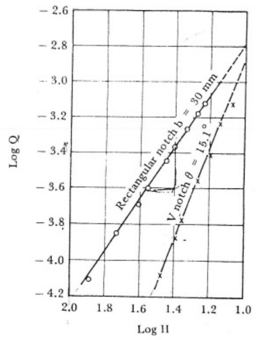


Fig. 7.7. Variation of log Q with log H for rectangular and V notches.

Table 7.2. gives measurements of head H and discharge Q, together with log H and log Q. On Fig. 7.6 the results are plotted to natural scales and on Fig. 7.7 they are plotted logarithmically. The results on Fig. 7.7 lie on a straight line having

$$\text{Slope, } n = 2.50$$

Intercept on log Q axis = $\bar{1}.60$ (By extrapolation)

The relationship between log Q and log H is thus

$$\log Q = \bar{1}.60 + 2.50 \log H$$

so that the relationship between Q and H is

$$Q = 0.398\, H^{2.50} \tag{7.14}$$

Comparing this with the expression derived previously,

$$Q = C_d \frac{8}{15} \sqrt{2g} \tan\theta \; H^{5/2} \qquad (7.10)$$

we note that the exponent of H is the same in both, and that C_d is given by

$$C_d = \frac{0.398}{8/15 \sqrt{2g} \tan\theta} = \frac{0.398}{8/15 \times \sqrt{2 \times 9.81} \times 0.270}$$

$$C_d = 0.62$$

Discussion of Results

The results show that discharge over the rectangular weir may be represented by the equation

$$Q = C_d \frac{2}{3} \sqrt{2g} \, b \, H^{3/2} \qquad (7.9)$$

in which

$$C_d = 0.59$$

over the range of the experiment. The expression $\frac{2}{3}\sqrt{2g}\, b\, H^{3/2}$ which appeared in the equation is the calculated discharge, neglecting losses and the contraction of the jet as it passes through the notch, and C_d is the empirical factor to take account of these effects.

The discharge over the V notch has been found to be represented by the equation

$$Q = C_d \frac{8}{15} \sqrt{2g} \tan\theta \; H^{5/2} \qquad (7.10)$$

in which

$$C_d = 0.62$$

over the range of the experiment. In this case, $\frac{8}{15}\sqrt{2g} \tan\theta \; H^{5/2}$ represents the calculated discharge, and C_d again takes account of losses and of contraction of the jet.

It should be noted that, over the range of the experiments, slightly different exponents of H, with corresponding different values of C_d, could be fitted to the results. For example, the equation

$$Q = C_d \frac{2}{3} \sqrt{2g} \, b \, H^{1.47} \qquad (7.15)$$

in which

$$C_d = 0.54$$

fits the results for the rectangular weir almost as well as equation (7.9) with $C_d = 0.59$. A wider range of H would be required to differentiate between the various alternatives, but, without any evidence to suggest that the exponents 1.50

and 2.50 for the rectangular and V notches do not apply exactly in practice, it is reasonable and convenient to take these values. Moreover, the values of C_d associated with these exponents are close to the value obtained for flow through an orifice.

Questions for Further Discussion

1. What suggestions have you for improving the apparatus?
2. How would you interpret results, which, when plotted logarithmically, as on Fig. 7.7, fall on a line which is not straight, but slightly curved?
3. To what extent does the experiment confirm the theoretical treatment? Has the dependence on b (for the rectangular notch) or θ (for the V notch) been established? A suggested project is to plan a series of tests to explore the dependence on b, using a set of notches or by partially covering the width of the one supplied with a sharp-edged metal strip. What range of b should be chosen? What is the best way to present the results? Is there a recognised modification to the form of equation (7.9) which allows for the effect of the contractions at the sides?

6. Flow Through a Venturi Meter

Introduction

The Venturi is a device which has been used over many years for measuring the discharge along a pipe. The fluid flowing in the pipe is led through a contraction section to a throat, which has a smaller cross-sectional area than the pipe, so that the velocity of the fluid through the throat is higher than that in the pipe. This increase of velocity is accompanied by a fall in pressure, the magnitude of which depends on the rate of flow, so that by measuring the pressure drop, the discharge may be calculated. Beyond the throat the fluid is decelerated in a pipe of slowly diverging section (sometimes referred to as a diffuser), the pressure increasing as the velocity falls.

Description of Apparatus

Fig. 6.1 shows the arrangement of the meter, which is manufactured in clear plastic material. Water is admitted from the bench supply valve and passes through a flexible hose into the meter. Beyond the control valve, which is just downstream of the meter, a further flexible hose leads to the measuring tank. At a number of points along the length of the convergent-divergent passage of the

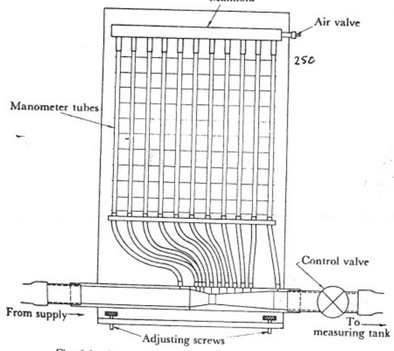


Fig. 6.1. Arrangement of Venturi meter apparatus.

Venturi, piezometer tubes are drilled into the wall and connections are made from each of these to vertical manometer tubes which are mounted in front of a scale marked in millimetres. The manometer tubes are connected at their top ends to a common manifold in which the amount of air may be controlled by a small air valve at one end. The whole assembly of Venturi meter, manometer tubes, scale and manifold is supported on a base mounted on screwed feet which may be adjusted to level the equipment.

It may be noted that in the usual form of Venturi meter intended for flow measurement, pressure tappings are made only at the entrance and at the throat, as these two readings suffice to measure the discharge. The larger number of tappings on this experimental Venturi tube are intended to show the distribution of pressure along the length of the convergent-divergent passage.

Theory of the Venturi Meter

Consider the flow of an incompressible fluid through the convergent-divergent pipe shown in Fig. 6.2. The cross-sectional area at the upstream section 1 is a_1, at the throat section 2 is a_2, and at any other arbitrary section n is a_n. Piezometer tubes at these sections register h_1, h_2 and h_n as shown.

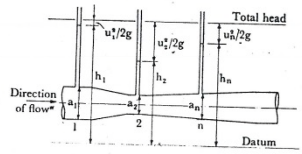


Fig. 6.2. Ideal conditions in a Venturi meter.

Assuming that there is no loss of energy along the pipe, and that the velocity and piezometric heads h are constant across each of the sections considered, then Bernoulli's theorem states that

$$\frac{u_1^2}{2g} + h_1 = \frac{u_2^2}{2g} + h_2 = \frac{u_n^2}{2g} + h_n \qquad (6.1)$$

in which u_1, u_2 and u_n are the velocities of flow through sections 1, 2 and n. The equation of continuity is

$$u_1 a_1 = u_2 a_2 = u_n a_n = Q \qquad (6.2)$$

in which Q denotes the volume-flow or discharge rate.

Substituting in equation (6.1) for u_1 from equation (6.2)

$$\frac{u_2^2}{2g}\left(\frac{a_2}{a_1}\right)^2 + h_1 = \frac{u_2^2}{2g} + h_2$$

and solving this equation for u_2 leads to

$$u_2 = \sqrt{\frac{2g(h_1 - h_2)}{1 - \left(\frac{a_2}{a_1}\right)^2}}$$

so that the discharge rate, from equation (6.2) becomes

$$Q = a_2\sqrt{\frac{2g(h_1 - h_2)}{1 - \left(\frac{a_2}{a_1}\right)^2}} \tag{6.3}$$

In practice, there is some loss of energy between sections 1 and 2, and the velocity is not absolutely constant across either of these sections. As a result, measured values of Q usually fall a little short of those calculated from equation (6.3) and it is customary to allow for this discrepancy by writing

$$Q = Ca_2\sqrt{\frac{2g(h_1 - h_2)}{1 - \left(\frac{a_2}{a_1}\right)^2}} \tag{6.4}$$

in which C is known as the *coefficient* of the meter, which may be established by experiment. Its value varies slightly from one meter to another, and, even for a given meter it may vary slightly with the discharge, but usually lies within the range 0.92 to 0.99.

The ideal pressure distribution along the convergent-divergent pipe may be seen from Bernoulli's equation (6.1) to be given by

$$h_n - h_1 = \frac{u_1^2 - u_n^2}{2g}$$

For the purpose of calculation and of comparison of experimental results with calculation, it is convenient to express $(h_n - h_1)$ as a fraction of the velocity head at the throat of the meter, i.e.

$$\frac{h_n - h_1}{\frac{u_2^2}{2g}} = \frac{u_1^2 - u_n^2}{u_2^2}$$

Substituting on the right hand side area ratios in place of velocity ratios from the equation of continuity (6.2), the ideal pressure distribution becomes

$$\frac{h_n - h_1}{\frac{u_2^2}{2g}} = \left(\frac{a_2}{a_1}\right)^2 - \left(\frac{a_2}{a_n}\right)^2 \tag{6.5}$$

Experimental Procedure

To establish the coefficient of the meter it is necessary to obtain the variation of $(h_1 - h_2)$ with discharge rate Q. As preliminary, the manometer scale must be levelled. This is conveniently done by first opening both the control valve downstream of the meter and the bench supply valve, so as to allow water to flow for a few seconds to clear air pockets from the supply system. The control valve is then gradually closed, so that the meter is subjected to a gradually increasing pressure which will cause water to pass up the piezometer tubes, thereby compressing the air contained in the manifold. When the water levels have risen to a convenient height, the bench valve is also gradually closed, so that, as both valves are finally shut off, the meter is left containing static water under moderate pressure. The adjusting screws at the base are then set so that the piezometers each read the same value when the scale is viewed from the front, and the tubes are reasonably vertical when viewed from the end.

Having levelled the scale, measurements of a series of values of $(h_1 - h_2)$ and Q may be made. The first reading may be taken at the maximum available value of $(h_1 - h_2)$, i.e. with h_1 close to the top of the scale and h_2 close to the bottom. This condition may be achieved by gradually opening both the bench valve and the control valve. Successive opening of either valve will increase the flow and the difference between h_1 and h_2. Opening of the bench valve is accompanied by a general rise in levels in the manometer, while opening of the control valve is accompanied by a general fall in the manometer, so that by judiciously balancing the two valve settings the required condition may be obtained. If difficulty is experienced in reaching the desired condition, air may be released from or admitted to the manifold through the small air valve at its end.

The rate of flow is now measured by collection in the weighing tank, and while this is in progress, values of h_1 and h_2 are read from the scale. Similar readings may be taken at a series of reducing values of $(h_1 - h_2)$, about 10 readings, roughly equally spread, in the range of 250mm to zero being adequate for the purpose.

By reading off from all the piezometer tubes at any of the settings used above, the pressure distribution along the length of the Venturi meter may be recorded. If, however, this is done in every case, the reduction of all the results becomes lengthy, so it is suggested that only one or two such comprehensive readings be taken, preferably, for the sake of accuracy, near the maximum flow.

Results and Calculations

The dimensions of the meter and the position of the piezometer tappings are shown on Fig. 6.3. Table 6.1 shows the diameter of the cross-section at each of the piezometer stations, and the calculations of the ideal pressure distribution according to equation (6.5)

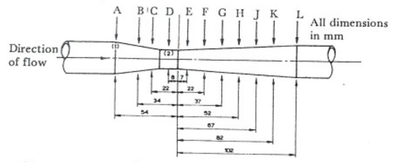


Fig. 6.3. Dimensions of Venturi meter and positions of piezometer tubes.

Piezometer Tube No. n	*Dia. of cross-section* d_n (mm)	$\frac{d_2}{d_n}$	$\left(\frac{a_2}{a_n}\right)^2$	$\left(\frac{a_2}{a_1}\right)^2 - \left(\frac{a_2}{a_n}\right)^2$
A (1)	26.00	0.615	0.144	0.000
B	23.20	0.690	0.226	− 0.082
C	18.40	0.869	0.575	− 0.431
D (2)	16.00	1.000	1.000	− 0.856
E	16.80	0.953	0.830	− 0.686
F	18.47	0.867	0.565	− 0.421
G	20.16	0.787	0.400	− 0.256
H	21.84	0.730	0.289	− 0.145
J	23.53	0.680	0.215	− 0.071
K	25.24	0.633	0.168	− 0.024
L	26.00	0.615	0.144	0.000

Table 6.1. Calculation of Ideal Pressure Distribution according to Equation (6.5)

In table 6.2 measurements of h_1 and h_2 at various discharges are recorded together with values of $(h_1 - h_2)^{1/2}$ calculated from these measurements. A typical calculation, referring to the values in the third line of the table, is set out below.

$$d_1 = 26.0\text{mm} \quad a_1 = 531\text{mm}^2 = 5.31 \times 10^{-4}\text{m}^2$$

$$d_2 = 16.0\text{mm} \quad a_2 = 201\text{mm}^2 = 2.01 \times 10^{-4}\text{m}^2$$

$$\left(\frac{a_2}{a_1}\right)^2 = 0.144 \quad 1 - \left(\frac{a_2}{a_1}\right)^2 = 0.856$$

Qty. (kg)	t (s)	h_1 (mm)	h_2 (mm)	$10^4 \times Q$ (m^3/s)	$(h_1 - h_2)$ (m)	$(h_1 - h_2)^{1/2}$ ($m^{1/2}$)
45	100.9	247.5	6.0	4.46	0.2415	0.492
45	103.6	246.0	13.0	4.34	0.2330	0.483
45	110.7	232.5	26.5	4.06	0.2060	0.454
30	80.2	215.5	41.0	3.74	0.1745	0.418
30	87.9	201.0	56.0	3.42	0.1450	0.381
30	101.5	181.0	69.5	2.96	0.1115	0.334
30	118.5	159.5	79.5	2.53	0.0800	0.283
15	84.4	136.5	97.0	1.78	0.0395	0.199
15	134.9	121.0	105.0	1.11	0.0160	0.126

Table 6.2. Measurement of $(h_1 - h_2)$ *and* Q

For measured values as follows:

$$\text{Quantity collected } 45\text{kg} = 45 \times 10^{-3}\text{m}^3$$
$$\text{Time of collection } = 110.7\text{s}$$
$$h_1 = 232.5\text{mm}$$
$$h_2 = 26.5\text{mm}$$

$$\text{Discharge rate } Q = \frac{45 \times 10^{-3}}{110.7} = 4.06 \times 10^{-4}\text{m}^3/\text{s}$$

$$\text{or } 10^4 \times Q = 4.06$$
$$h_1 - h_2 = 0.2060\text{m}$$
$$(h_1 - h_2)^{1/2} = 0.454\text{m}^{1/2}$$

Fig. 6.4 is a graphical representation of the variation of $(h_1 - h_2)^{1/2}$ with Q which, from equation (6.4), should be a straight line through the origin, if the value of C is constant over the range of measurement. Such a straight line has been drawn on the graph, and its equation is

$$(h_1 - h_2)^{1/2} = 1110\,Q$$

$$\text{or} \quad Q = 0.900 \times 10^{-3}\,(h_1 - h_2)^{1/2} \tag{6.6}$$

in which Q and $(h_1 - h_2)$ are measured in m-s units. Comparing this with equation (6.4), the value of C may be calculated from

$$Ca_2 \sqrt{\frac{2g}{1 - \left(\frac{a_2}{a_1}\right)^2}} = 0.900 \times 10^{-3}$$

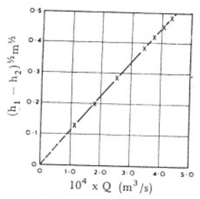


Fig. 6.4. Variation of $(h_1 - h_2)^{1/2}$ *with Q for Venturi meter.*

which, by substitution of numerical values, gives

$$C = \frac{0.900 \times 10^{-3}}{2.01 \times 10^{-4} \sqrt{2 \times 9.81/0.856}}$$

$$\underline{C = 0.935}$$

A closer inspection of Fig. 6.4 shows that the experimental points towards the top of the range lie very slightly below the straight line, and towards the bottom of the range above it. This suggests that the value of C is not quite constant as Q varies, but increases slightly with increasing Q. In view of this, table 6.3 has been drawn up, in which the value of C corresponding to each of the experimental readings in turn has been calculated, and these values are plotted

$10^4 \times Q$ (m^3/s)	$(h_1 - h_2)^{1/2}$ $(m)^{1/2}$	C
4.46	0.492	0.940
4.34	0.483	0.938
4.06	0.454	0.920
3.74	0.418	0.930
3.42	0.381	0.938
2.96	0.334	0.928
2.53	0.283	0.930
1.78	0.199	0.932
1.11	0.126	0.925

Table 6.3. Values of C calculated from individual experimental results.

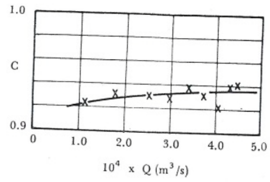


Fig. 6.5. Variation of C with Q for Venturi meter.

against Q in Fig. 6.5. Although there is a certain amount of scatter because of the large scale chosen for C, a trend of increasing C, from about 0.925 to about 0.940 with increasing Q is discernible.

The piezometer tappings along the length of the tube may be used for further

Piezometer Tube No.	$Q = 4.46 \times 10^{-4} m^3/s$, $\frac{u_2^2}{2g} = 0.251m$: h_n (mm)	$h_n - h_1$ (m)	$\frac{h_n - h_1}{u_2^2/2g}$	$Q = 2.96 \times 10^{-4} m^3/s$, $\frac{u_2^2}{2g} = 0.111m$: h_n (mm)	$h_n - h_1$ (m)	$\frac{h_n - h_1}{u_2^2/2g}$
A (1)	247.5	0	0	181.0	0	0
B	228.5	− 0.019	− 0.077	172.0	− 0.009	− 0.081
C	140.5	− 0.1075	− 0.429	131.0	− 0.050	− 0.450
D (2)	6.0	− 0.2415	− 0.961	69.5	− 0.1115	− 1.004
E	26.0	− 0.2215	− 0.880	77.0	− 0.1045	− 0.941
F	112.0	− 0.1355	− 0.540	115.5	− 0.0655	− 0.590
G	150.5	− 0.0970	− 0.388	134.0	− 0.0475	− 0.428
H	176.0	− 0.0715	− 0.297	145.0	− 0.0360	− 0.324
J	193.0	− 0.0545	− 0.216	152.0	− 0.0290	− 0.261
K	204.0	− 0.0435	− 0.173	157.5	− 0.0240	− 0.216
L	209.0	− 0.0385	− 0.153	159.0	− 0.0220	− 0.198

Table 6.4. Measurements of pressure distribution along Venturi meter.

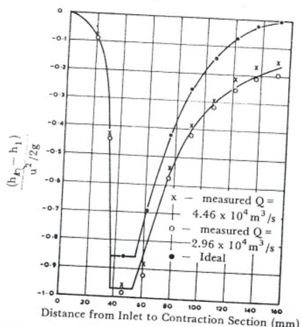


Fig. 6.6. Measured and ideal pressure distribution along Venturi meter.

experiments, possibly of the project type. Measurements made at two different discharge rates are recorded in Table 6.4, and are compared with the ideal pressure distribution previously calculated in Table 6.1 on Fig. 6.6. By expressing piezometric head changes $h_n - h_1$ as a fraction of the velocity head $\frac{u_2^2}{2g}$ at the throat, results at different discharges become directly comparable, and it is seen that the experimental values follow the ideal curve quite well up to the throat, after which a steadily increasing loss of energy becomes apparent.

Discussion of Results

It is seen from Fig. 6.4 that the linear relationship between Q and $(h_1 - h_2)^{1/2}$ which is predicted by the expression

$$Q = Ca_2\sqrt{\frac{2g(h_1 - h_2)}{1 - \left(\frac{a_2}{a_1}\right)^2}} \tag{6.4}$$

is reasonably well supported by the experimental results. The value of the constant C, which takes account of the loss of energy in the contraction section of the meter, and of the non-uniformity of velocity distribution across the two cross-sections, has been found to be 0.938. A close inspection of Fig. 6.4, how-

ever, indicates that C rises slightly with increasing Q, and when values of C are calculated separately for each set of experimental readings, an increase from about 0.925 to about 0.940 is found as Q increases from 1.1×10^{-4} to $4.4 \times 10^{-4} m^3/s$.

It is unlikely that this change is due to error in measurement of Q, which should have negligible systematic error and, with times of collection generally about 100 sec, should have random error due to inaccurate timing of less than ±½%. If the manometer were not levelled correctly in the plane of the manometer scale, however, a constant error in measurement of $(h_1 - h_2)$ would be produced. This error should certainly not exceed 1.5mm. Subtracting this amount from each of the values of $(h_1 - h_2)$ in Table 6.2 and recalculating the values of C given in Table 6.3, the variation C is found to be between about 0.935 and 0.945 as Q increases over the measured range. The variation of C is thus reduced to rather more than half the previous amount. It may be concluded that some increase in C is genuinely taking place, but the amount of the variation is rather uncertain, and could be reduced only by increasing the accuracy of the measurements. (When in fact, this is done in an accurate Venturi meter calibration, a slight increase in C with increasing Q is usually found.)

Fig. 6.6 shows clearly how the piezometric head falls to a minimum at the throat and subsequently rises along the diffuser. The ideal rise brings the piezometric pressure back to its initial value, and if this were possible in practice, the flow measurement could be made without expenditure of energy. The experimental result shows that only about 16% of the differential head between the upstream piezometer and the piezometer at the throat is lost in passing through the meter, and that this is incurred mainly beyond the end of the contraction section.

Questions for Further Discussion

1. What suggestions have you for improving the apparatus?
2. What would be the effect on the results if the Venturi meter were not horizontal? Would you make any correction to the piezometer readings if the meter were mounted with its axis vertical?
3. The experimental results show that there is a pressure drop across the meter from inlet to exit, and that this pressure drop increases with discharge. Would there be any disadvantage in using this pressure drop, rather than that between inlet and the throat, for the purpose of calibrating the meter?
4. Using the value of C which you have obtained by experiment, determine the throat diameter of a Venturi meter which would measure a flow of $0.4 m^3/s$ in a pipe of 0.6m diameter with a differential head of 0.8m (0.37m approx).

8. Friction Loss along a Pipe

Introduction

The frictional resistance to which fluid is subjected as it flows along a pipe results in a continuous loss of energy or total head of the fluid. Fig. 8.1 illustrates this in a simple case; the difference in levels between piezometers A and B represents the total head loss h in the length of l of pipe. In hydraulic engineering it is customary to refer to the rate of loss of total head along the pipe, dh/dl, by the term *hydraulic gradient*, denoted by the symbol i, so that

$$\frac{dh}{dl} = i \qquad (8.1)$$

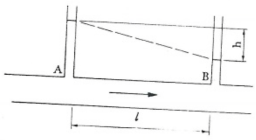


Fig. 8.1. Diagram illustrating the hydraulic gradient.

Osborne Reynolds, in 1883, recorded a number of experiments to determine the laws of resistance in pipes. By introducing a filament of dye into the flow of water along a glass pipe he showed the existence of two different types of motion. At low velocities the filament appeared as a straight line which passed down the whole length of the tube, indicating laminar flow. At higher velocities, the filament, after passing a little way along the tube, suddenly mixed with the surrounding water, indicating that the motion had now become turbulent. Experiments with pipes of different diameters and with water at different temperatures led Reynolds to conclude that the parameter which determines whether the flow shall be laminar or turbulent in any particular case is

$$R = \frac{\rho u D}{\mu} \qquad (8.2)$$

in which R denotes the *Reynolds number* of the motion
ρ denotes the density of the fluid
u denotes the velocity of flow
D denotes the diameter of the pipe
μ denotes the coefficient of viscosity of the fluid.

The motion is laminar or turbulent according as the value to R is less than or greater than a certain critical value. If experiments are made with increasing rates

of flow, this value of R depends on the degree of care which is taken to eliminate disturbances in the supply and along the pipe. On the other hand, if experiments are made with decreasing flow, transition from turbulent to laminar flow takes place at a value of R which is very much less dependent on initial disturbances. This value of R is about 2000, and below this, the flow becomes laminar sufficiently downstream of any disturbance, no matter how severe it is.

Different laws of resistance apply to laminar and to turbulent flow. For a given fluid flowing along a given pipe, experiments show that

$$i \propto u \qquad (8.3)$$

for laminar motion, and

$$i \propto u^n \qquad (8.4)$$

for turbulent motion, n being an index which lies between 1.7 and 2.0 (depending on the value of R and on the roughness of the wall of the pipe). Equation (8.3) is in accordance with Poiseuille's equation which can be written in the form

$$i = \frac{32\mu u}{\rho g D^2} \qquad (8.5)$$

There is no similar simple result for turbulent flow; in engineering practice it is customary to use Darcy's equation

$$i = \frac{4f}{D}\frac{u^2}{2g} \qquad (8.6)$$

in which f denotes an experimentally determined friction factor which varies with R and pipe roughness.

The object of the present experiment is to demonstrate the change in the law of resistance and to establish the critical value of R. Measurements of i in the laminar region may be used to find the coefficient of viscosity from equation (8.5) and measurements in the turbulent region may be used to find the friction factor f from equation (8.6).

Description of Apparatus

Fig. 8.2 shows the arrangement in which water from a supply tank is led through a flexible hose to the bell-mouthed entrance to a straight tube along which the frictional loss is measured. Piezometer tappings are made at an upstream section which lies approximately 45 tube diameters away from the pipe entrance, and at a downstream section which lies approximately 40 tube diameters away from the pipe exit. These clear lengths upstream and downstream of the test section are required to prevent the results from being affected by disturbances near the entrance and exit of the pipe. The piezometer tappings are connected to an inverted U-tube manometer, which reads the differential pressure directly in inches of water, or a U-tube which reads in inches of mercury.

The rate of flow along the pipe is controlled by a needle valve at the pipe exit, and is measured by timing the collection of water in a measuring cylinder (the

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discharge being so small as to make the use of the bench weighing tank impracticable).

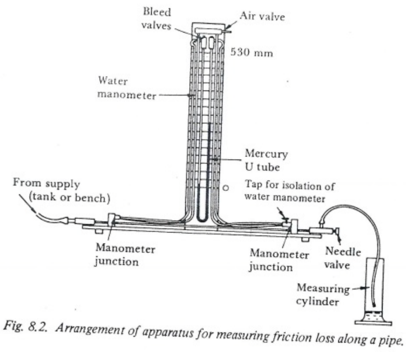

Fig. 8.2. Arrangement of apparatus for measuring friction loss along a pipe.

Derivation of Poiseuille's Equation

To derive Poiseuille's equation which applies to laminar flow along a tube, consider the motion indicated on Fig. 8.3. Over each cross-section of the tube the piezometric pressure is constant, and this pressure falls continuously along the tube. Suppose that between cross-sections A and B separated by length l of tube, the fall in pressure is p. Then the force exerted by this pressure difference on the

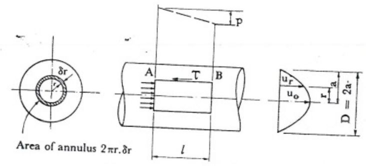

Fig. 8.3. Derivation of Poiseuille's equation

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ends of a cylinder having radius r, and its axis on the centre line of the tube, is $p.\pi r^2$. Over any cross-section of the tube, the velocity varies with radius, having a maximum value of u_0 at the centre and falling to zero at the wall; let the velocity at radius r in any cross-section be denoted by u_r. Then the shear stress τ, in the direction shown on Fig. 8.3, due to viscous action on the curved surface of the cylinder, is given by

$$\tau = \mu \frac{du_r}{dr} \tag{8.7}$$

(Note that $\frac{du_r}{dr}$ is negative so that the stress acts in the direction shown in the figure). The force on the cylinder due to this stress is $\frac{\mu du_r}{dr}.2\pi rl$. Since the fluid is in steady motion under the action of the sum of pressure and viscous forces,

$$p.\pi r^2 + \mu\frac{du_r}{dr}.2\pi rl = 0$$

$$\therefore \frac{du_r}{dr} = -\frac{pr}{2l\mu} \tag{8.8}$$

Integrating this and inserting a constant of integration such that

$$u_r = 0 \text{ when } r = a$$

$$u_r = \frac{p}{4l\mu}(a^2 - r^2) \tag{8.9}$$

This result shows that the velocity distribution across a section is parabolic, as indicated on Fig. 8.3, and that the velocity on the centre line, given by putting $r = 0$ in equation (8.9) is

$$u_0 = \frac{pa^2}{4l\mu} \tag{8.10}$$

The discharge rate Q may now be calculated. The flow rate δQ through an annulus of radius r and width δr is

$$\delta Q = u_r.2\pi r\delta r$$

Inserting u_r from equation (8.9) and integrating

$$Q = \frac{p}{4l\mu}2\pi\int_0^a (a^2 r - r^3)\,dr$$

$$\therefore Q = \frac{p\pi a^4}{8l\mu} \tag{8.11}$$

Now the mean velocity u over the cross-section is, by definition, given by

$$Q = u.\pi a^2 \tag{8.12}$$

and eliminating Q between equation (8.11) and (8.12) gives

$$u = \frac{pa^2}{8l\mu} = \frac{pD^2}{32l\mu}$$

which may be written in the form (8.13)

$$i = \frac{32\mu u}{\rho g D^2}$$

by use of the substitutions (8.5)

$$\rho g h = p$$

$$\text{and } \frac{h}{l} = i$$

Derivation of Darcy's Equation

If the flow is turbulent, the analysis given above is invalidated by the continuous mixing process which takes place. Across the curved surface of the cylinder having radius r in Fig. 8.3, this mixing is manifest as a continuous unsteady and random flow into and out of the cylinder, so that the apparent shear stress on this surface is greater than the value given in equation (8.7). Because of the mixing the distribution of velocity over a cross-section is more uniform than the parabolic shape deduced for laminar flow, as indicated on Fig. 8.4.

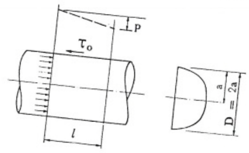

Fig. 8.4. Derivation of Darcy's equation

Although it is not possible to perform a complete analysis for turbulent flow, a useful result may be obtained by considering the whole cross-section as shown in Fig. 8.4. It is reasonable to suppose that the shear stress τ_o on the wall of the tube will depend on the mean velocity u; let us assume for the present that

$$\tau_o = f.\tfrac{1}{2}\rho u^2 \qquad (8.14)$$

in which $\frac{1}{2}\rho u^2$ denotes the dynamic pressure corresponding to the mean velocity

u and f is a friction factor (not necessarily constant). Since τ_o and $\frac{1}{2}\rho u^2$ each have dimensions of force per unit area, f is dimensionless. The force on a cylinder of length l due to this stress is $f.\frac{1}{2}\rho u^2.2\pi a l$, and the force due to the fall in pressure is $p.\pi a^2$, so that

$$p.\pi a^2 = f.\tfrac{1}{2}\rho u^2.2\pi a l$$

Substituting

$$\rho g h = p$$

$$h/l = i$$

$$\text{and } a = \frac{D}{2}$$

leads to the result

$$i = \frac{4f}{D}\cdot\frac{u^2}{2g}$$

which is a form of Darcy's equation. (8.6)

The friction factor f which occurs in this equation was defined by equation (8.14) and is not necessarily constant. The results of many experiments show that f does, in fact, depend on both R and on the roughness of the pipe wall. At a given value of R, f increases with increasing surface roughness. For a given surface roughness, f generally decreases slowly with increasing R. This means that if R is increased by increasing u along a given pipe, f will decrease slowly with increasing u, so that the product fu^2 on which i depends in equation (8.6) will increase somewhat less than u^2. In fact, over a fairly wide range, it is often possible to represent the variation of i with u by the approximation.

$$i = ku^n$$

where k and n are constants for a given fluid flowing along a given pipe, n having a value between 1.7 and 2.0.

Experimental Procedure

The apparatus is set on the bench and levelled so that the manometers stand vertically. The water manometer is then introduced into the circuit by directing the lever on the tap towards the relevant connecting pipe. The bench supply valve is opened and adjusted until there is a steady flow down the supply tank overflow pipe. With the needle valve partly open to allow water to flow through the system, any trapped air is removed by manipulation of the flexible pipes. Particular care should be taken to clear the piezometer connectio needle valve is then closed whereupon the levels in the two limbs of the inverted U-tube should settle to the same value. If they do not, check that flow has been stopped absolutely, and that all air bubbles have been cleared from the piezometer connections. The height of the water level in the manometer may be raised to a suitable value by allowing air to escape through the air valve at the top, or by pumping air in through the valve.

The first reading of head loss and flow may now be taken. The needle valve is opened fully to obtain a differential head of at least 400mm, and the collection of a suitable quantity of water in the measuring cylinder timed. During this operation care should be taken a) to ensure that the flow pipe exit is never below the surface of the water in the measuring cylinder; and b) to stand the measuring cylinder below the apparatus. Failure to observe these conditions will result in inaccurate flow rate readings, especially at the lower flow rates. Further readings may be taken at various flows, the needle valve serving to reduce the discharge from each reading to the next. The water temperature should be measured as accurately as possible at frequent intervals.

These readings should comfortably cover the whole of the laminar region and the transition to turbulent flow; it is advisable to plot a graph of differential head against discharge as the experiment proceeds to ensure that sufficient readings have been taken to establish the slope of the straight line in the laminar region.

To obtain a range of results in the turbulent region it is necessary to work with much great differential-heads than can be measured by the inverted U-tube manometer which is now isolated using the downstream tap. The mercury manometer is now used, and the supply to the apparatus is taken directly from the bench supply valve instead of the elevated supply tank. Since the flexible hose between the bench supply valve and the apparatus will be subjected to the full pump pressure, it is advisable to secure the joints with hose clips.

With the needle valve partially open and the pump running, the bench supply valve is opened fully. Air which may be trapped in the flexible hose is removed by manipulation, and bubbles in the piezometer connections are induced to rise to the top of the U-tube, where they are expelled through bleed valves. There should then be continuous water connections from the piezometer tappings to the two surfaces of mercury in the U-tube, and when the needle valve is closed, the two surfaces should settle at the same level.

Readings of head loss and flow are now taken, starting at maximum discharge, and reducing in steps, the needle valve being used to set the desired flows. The water temperature should be recorded at frequent intervals.

It is desirable to take one or two readings, at the lower end of the range, which overlap the range already covered by the water manometer. Since a reading of 20mm on the mercury U-tube corresponds to 252mm on the water manometer, this requires one or two readings in the region of 20mm.

The diameter of the tube and the length between piezometer tappings should be noted.

Results and Calculations

Length of pipe between piezometer tappings, l 524mm
Nominal diameter of pipe, D 3mm
Cross-sectional area of pipe, A . . . . 7.06mm^2

Tables 8.1 and 8.2 give results with the water and mercury manometers in turn.

Qty. (ml)	t (s)	u (m/s)	h_1 (mm)	h_2 (mm)	$h_1 - h_2$ (m)	i	θ (°C)	log i	log u
400	50.8	1.110	521.0	56.0	0.465	0.887	15.3	– 0.0521	0.0453
400	54.0	1.049	500.0	85.0	0.415	0.794		– 0.1002	0.0208
400	58.8	0.961	476.0	114.0	0.362	0.692		– 0.1599	– 0.0173
400	61.8	0.915	452.0	145.0	0.307	0.586		– 0.2321	– 0.0586
400	67.2	0.843	427.5	174.0	0.2535	0.483		– 0.3161	– 0.0742
300	57.8	0.734	390.0	223.0	0.167	0.319	15.3	– 0.4962	– 0.1343
300	71.9	0.592	375.0	245.0	0.130	0.248		– 0.6055	– 0.2277
300	92.9	0.457	362.0	263.0	0.099	0.189		– 0.7235	– 0.3401
200	92.4	0.306	349.0	282.0	0.067	0.128		– 0.8928	– 0.5143
150	100.8	0.220	340.0	295.5	0.455	0.085		– 1.0771	– 0.6576
85	113.6	0.106	332.5	306.0	0.0265	0.050	15.3	– 1.2958	– 0.9747
50	129.4	0.055	325.0	316.0	0.009	0.017		– 1.7645	– 1.2596

Table 8.1. Results with Water Manometer

Qty. (ml)	t (s)	u (m/s)	h_1 (mm)	h_2 (mm)	$h_1 - h_2$ (m)	i	θ (°C)	log i	log u
900	39.0	3.270	431.0	195.0	0.236	5.770		0.7612	0.5145
900	42.9	2.980	414.0	214.0	0.200	4.810		0.6821	0.4742
900	46.6	2.740	402.0	226.0	0.176	4.230		0.6263	0.4378
900	51.7	2.470	390.0	240.0	0.150	3.600	15.5	0.5563	0.3927
900	58.0	2.200	377.0	254.5	0.1225	2.940		0.4683	0.3424
900	62.7	2.030	370.5	261.1	0.1095	2.520		0.4014	0.3075
900	68.5	1.860	362.0	270.5	0.0915	2.201		0.3426	0.2695
600	47.5	1.770	358.5	275.0	0.0875	2.010		0.3052	0.2480
600	54.6	1.550	351.5	283.5	0.0680	1.689	15.9	0.2146	0.1903
600	70.4	1.190	340.0	294.0	0.0460	1.108		0.0434	0.0755
300	48.0	0.885	331.5	305.5	0.0360	0.866		– 0.0625	– 0.0531

Table 8.2. Results with Mercury U-tube.

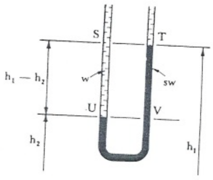

Fig. 8.5. Diagram of U-tube.

To obtain i in Table 8.1 where h_1 and h_2 are measured in mm of water merely entails dividing $(h_1 - h_2)$ by *l* in mm.

To obtain i in Table 8.2 where h_1 and h_2 are measured on the mercury U-tube, a conversion of $(h_1 - h_2)$ to the equivalent value in m of water is necessary. On Fig. 8.5 the differential pressure to be measured is the difference between pressures at S and T in the two limbs. Now the pressures at U and V are equal, since these points are at the same level and are connected hydrostatically round

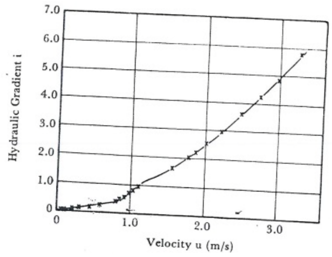

Fig. 8.6 (a). Variation of hydraulic gradient i with velocity u along pipe.

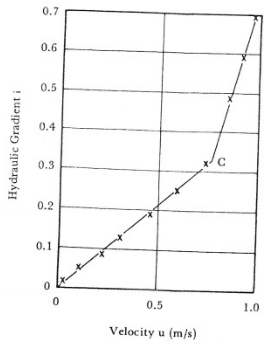

Fig. 8.6(b). Variation of Hydraulic Gradient i with velocity u along pipe, up to 1.0 m/s.

the bottom of the U-tube. But the pressure at S is $w(h_1 - h_2)$ less than the pressure at U, and expressed as a head of water this is $(h_1 - h_2)$ m. The pressure at T is $sw(h_1 - h_2)$ less than the pressure at V, and expressed in m of water this is $s(h_1 - h_2)$ m. So the difference between pressures at S and T is

$$s(h_1 - h_2) - (h_1 - h_2) \text{ m water}$$
$$\text{or } (s - 1)(h_1 - h_2) \text{ m water}$$

The specific gravity s of mercury is 13.6, so

$$s - 1 = 12.6$$

and to convert $(h_1 - h_2)$ to an equivalent amount of water head it is necessary to multiply by 12.6.

In the turbulent region where $i \propto u^{1.71}$, the value of f in Darcy's equation will fall with increasing u. Reading off Fig. 8.6 (a) values of i at a few values of u, corresponding values of f may be calculated from Darcy's equation.

Corresponding values of R may also be calculated, taking the mean value of water temperature to be 15.6°C at which $\mu = 11.4 \times 10^{-4}\ \mathrm{Ns/m^2}$ from the expression

$$R = \frac{\rho u D}{\mu}$$

$$\therefore \quad R = \frac{10^3 \times 0.203\ u}{11.4 \times 10^{-4}}$$

$$R = 2660\ u$$

The calculations are set out in Table 8.4.

θ (°C)	$10^4\ \mu$ (Ns / m²)
0	17.90
10	13.10
20	10.10
30	8.00
40	6.56
50	5.48
60	4.68
70	4.06
80	3.57
90	3.17
100	2.84

Table 8.3. Coefficient of viscosity of water at various temperatures

u (m/s)	*i*	$\frac{u^2}{2gD}$	f	R
1.6	1.75	43.7	0.0100	4260
2.2	3.00	82.2	0.0092	5860
2.8	4.45	132.8	0.0084	7450

Table 8.4. Calculation of the friction factor f in Darcy's equation.

Discussion of Results

Measurements of frictional loss along the pipe at different velocities have shown two well-defined regions to which different laws of resistance apply. As

the velocity i decreased from 3.3 to 1.5 m/s, frictional loss varied as $u^{1.71}$. Between 1.5 and 0.765, the loss decreased rather more steeply and as u decreased from 0.765 to zero the loss varied directly as u. The critical velocity of 0.765 corresponds to a Reynolds number of 1930, this value being close to the figure of about 2000 at which transition from turbulent to laminar flow is usually found to take place.

The value of μ calculated by Poiseuille's equation applied to the results in the laminar region is

$$\mu = 11.9 \times 10^{-4}\ \mathrm{Ns/m^2} \text{ at } 15.3^{\circ}\mathrm{C}.$$

The accepted value at this temperature is

$$\mu = 11.4 \times 10^{-4}\ \mathrm{Ns/m^2}$$

Since the accepted values are based on experiments with similar but more refined apparatus, the discrepancy reveals an error of about 5 per cent. in the apparatus used here.

The results in the turbulent region have been used to calculate the friction factor f in Darcy's equation, and are found to fall with increasing u as shown in Table 8.4.

Questions for Further Discussion

1. What suggestions have you for improving the apparatus?
2. What change in the calculated value of μ, expressed as a percentage, would you expect to be produced by errors of measurement as follows?
   (a) Error of 1.0mm in measurement of the length of the pipe between the piezometer tappings.
   (b) Error of 0.03mm in measurement of the diameter of the pipe.
3. What methods would you consider suitable for measurement of the diameter of the pipe?
4. Compare the values of f which you have measured with the equation
   $$f = 0.079R^{-0.25}$$
   which was proposed by Blasius for turbulent flow in smooth pipes.
5. A possible project is the adaptation of the apparatus to operate with air instead of water as the working fluid. Using a value of μ taken from physical tables, calculate the critical velocity and the corresponding pressure drop. Consider whether this could be measured using a water U-tube. Devise a simple method of producing a steady air flow at a known rate by displacement from a closed vessel.

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Orifice and Jet FlOW

Experiment # (2)

Asst. Prof Dr.Jaafar S. Maatooq

- ***Objective:***

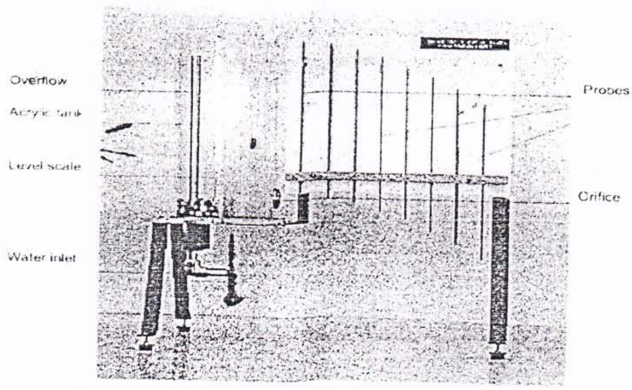
We want from this experiment to:

Study the flow through an orifice and determine the discharge coefficient, velocity coefficient and the actual jet profile.

- ***Equipments:***

We use the ***Hydraulic bench*** (to measure the flow rate) and the setup consists of:

1. Clear acrylic tank 20 cm diameter and 48 cm height. Water inlet is at the bottom of the tank and level scale is attached to the side of the tank.
2. An overflow pipe can adjust water level in the tank to as high as 42 cm from the center of the orifice. Water from overflow should be directed to the storage tank.
3. The sharp edged orifice is attached next and vertically parallel to the tank wall and perpendicular to the jet. The probes are at a distance of 5, 10, 15, 20, 25, 35 and 40 cm from the orifice. All probe lengths are equal then the tips of probes at top or bottom provide the same projectile.



- ***Theory:***

  ✓ For water at a level of H above the orifice, apply the Bernoulli's equation from the top surface to the orifice then the velocity of water discharge through the orifice can be written as:

$$V = \sqrt{2gH} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(1)$$

where **g** is acceleration due to gravity **(m/s^2)** and **H** is the water Height **(m)**

  ✓ The jet velocity trajectory consists of two components. At the exit of the orifice the vertical component is zero, thus the velocity leaves the orifice horizontally. Neglecting air resistance, the horizontal velocity $\dot{X}$ can be considered as a constant and equal to V.

The jet vertical velocity increases from zero at the orifice exit due to the gravitational acceleration such that: $\dot{Y}$ = **gt** —(1)

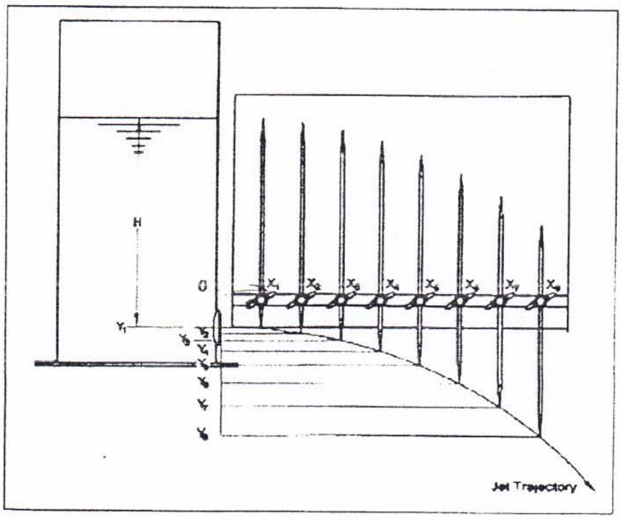
Where $\dot{Y}$ is the vertical component of the velocity in m/s, g is the gravitational acceleration and t is the time in second.

$Y = (½)\ g\ t^2$

Y: vertical distance in m

Y = 0 when the bottom end of the needle is at the same level as the center of the orifice.

The Y =0 mark is made on the panel behind the first needle.



✓ We assume the jet path touches the probes at point 1,2, 3 up to point 8.
Horizontal distance from 0 to 1 = X1 → middle(1) = 0
Horizontal distance from 0 to 2 = X2 etc. up to X8

قيم ال X

Vertical distance from 0 to 1 = Y1 ([illegible]) $X_1 = 0$

$X_2 = 0.07$

Vertical distance from 0 to 2 = Y2 etc. up to Y8 $X_3 = 0.14$

$X_4 = 0.21$

since $Y = (\frac{1}{2})\, g\, t^2$ yields $t = \sqrt{\frac{2Y}{g}}$

$X_5 = 0.28$

Therefore $t_1 = \sqrt{\frac{2Y_1}{g}}$

$X_6 = 0.35$

$t_2 = \sqrt{\frac{2Y_2}{g}}$ etc. up to t_8

$X_7 = 0.42$

$X_8 = 0.49$

at the same time $X_1 = \dot{X}_1 t_1$
and $X_2 = \dot{X}_2 t_2$ etc.

Then $\frac{X_1}{\dot{X}_1} = \sqrt{\frac{2Y_1}{g}}$ *then* $\dot{X}_1 = \frac{X_1}{\sqrt{\frac{2Y_1}{g}}}$ ……………………(2)

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Impact water jet

Experiment # ( )

Asst. Prof Dr.Jaafar S. Maatooq

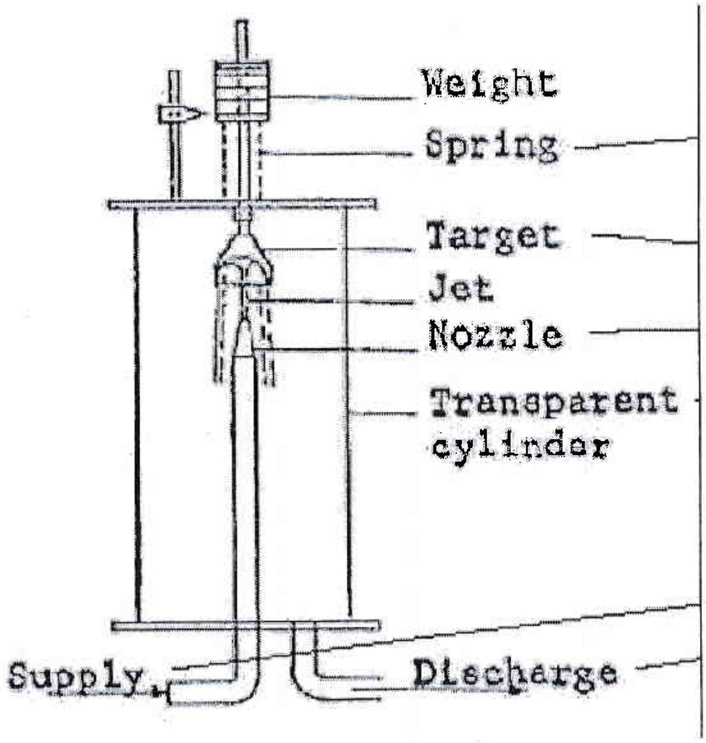
☒ *Objective:*

We want from this experiment to:

- To *produce and measure force* resulted by a water jet when it strikes a target.
- To *compare the results* with the theoretical values that calculated from the momentum equation.

☒ *Equipments:*

The equipment to be *used with the Hydraulic bench* for measurement of flow rate and continuous supply facility.

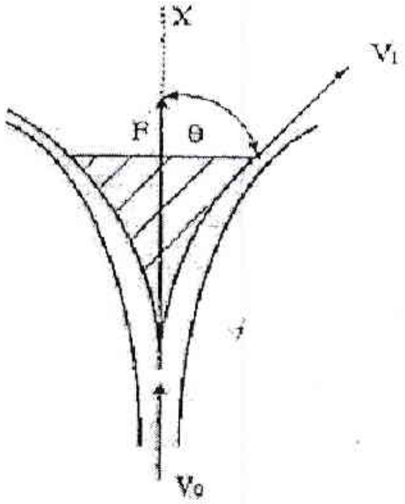


The equipment consists of:

1. Water supply from the Hydraulic bench which is connected to the inlet pipe at the bottom of the clear acrylic tank diameter 200 mm such that water jet and its target plate can be easily seen.
2. Water is discharged vertically through a nozzle of 7 mm diameter.
3. Three target plates each diameter 36 mm are to be considered: Flat plate, 120_{0} cone and hemisphere.
4. The target plate is connected to a vertical stem with spring near which a stem level indicator is provided. This indicator is set when there is no jet and also no weight on the stem.
5. Selected weights are then placed on the stem and at the same time flow rate of water jet is adjusted to hit the target plate raising it to its original level. Use weights of 100, 200, 300, 400, 500, and 600 gram.
6. Flow rate of the water jet can be obtained from the Hydraulic bench measuring tank.

☒ Theory:

✓ When a water jet of velocity V_0 hits a target plate, its velocity will change direction to V_1 as shown in the figure.



Note: The velocity that hits the target plate is not the same as the velocity at the nozzle tip. You can calculate the velocity that hits the target(s) from Bernoulli's equation

$$\boxed{(V_0^2 = V_{nozzle}^2 - 2gS).}$$

Where

$\underline{V_o}$ is the velocity when hitting the target plate

$\underline{S}$ is the distance from the nozzle tip to the target plate which is designed to be 30 mm for all target plates.

$\underline{V_{nozzle}}$ is the velocity at the nozzle tip (the nozzle tip is 7 mm Diameter)

✓ Assuming that the jet mass flow rate is $\dot{m}$ (kg/s) with velocity V_0 (m/s). After striking the target the jet velocity becomes V_1 (m/s) and its direction deviates from the original direction by angle θ.

✓ The momentum equation in the Cartesian coordinate for uniform flow across each flow section and steady state case can be written as:

$$\boxed{\sum F_{external} = \sum_{c.s} \dot{m}\vec{V}_0 - \sum_{c.s} \dot{m}\vec{V}_i}$$

or the impact force = change in momentum

Momentum of jet before hitting the target plate = $\dot{m}\,V_0$ (kg-m/s²)

Momentum of jet after hitting the target plate = $\dot{m}\,V_1$ (kg-m/s²)

Then the impact force = $\dot{m}\,V_1\cos\theta - \dot{m}\,V_0$

Then the reaction F by the target plate

$$F = \dot{m}\,(V_0 - V_1\cos\theta) \quad \text{N}$$

✓ For the three targets under consideration the reaction F will be as:

1. *Flat plate ($\theta = 90^0$)*
   Cos $\theta = 0$ then $\boxed{\mathbf{F = m\&\,V_0}}$ (N)

2. *120° cone (q = 120o)*
   Cos $\theta = -0.5$ then
   Assume very little energy loss between nozzle (jet) and target (cone) then we can assume $V_0 = V_1$
   Thus

$$\boxed{F = \dot{m}\,(V_0 - V_1\cos\theta) = \dot{m}\,V_0\,(1-(-0.5)) = 1.5\,\dot{m}\,V_0 \quad \text{or} \quad (\textit{1.5 times the flat plate})}$$

3. *Hemisphere ($\theta = 180^0$)*
   Cos θ = -1 then
   Assume very little energy loss between nozzle (jet) and target (Hemisphere) then we can assume $V_0 = V_1$
   Thus

$$\boxed{F = \dot{m}\,(V_0 - V_1 \cos\theta) = \dot{m}\,V_0\,(1-(-1)) = 2\,\dot{m}\,V_0 \quad \text{or} \quad (\textit{2 times the flat plate})}$$

☒ *Procedure:*

1. Adjust the equipment to an approximate level so that the jet is vertical.
2. Install the required target plate and adjust the stem level indicator to a fixed point on the stem.
3. Put 100 g weight on the stem, then the stem will go down.
4. Open the Hydraulic bench valve to allow the jet to hit the target and adjust the flow until the stem rises to its original position.
5. Record volume and time on measuring tank.
6. Repeat step 3 for higher weights (200, 300, 400, 500, and 600g)
7. Repeat for other types of targets.

☒ *Data Results & Analysis:*

☒ *Discussion & Conclusion:*

1. We can see from our results that

- We get some errors (not that big one), & I think that its related to the error in the measuring and reading.

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REYNOLDS NUMBER

EXPERIMENT # 4

Asst. Prof Dr.Jaafar S. Maatooq

EXPERIMENT 5

CRITICAL REYNOLDS NUMBER IN PIPE FLOW

The Reynolds number is a dimensionless ratio of inertia forces to viscous forces and is used in identifying certain characteristics of fluid flow. The Reynolds number is extremely important in modeling pipe flow. It can be used to determine the type of flow occurring: **laminar** or **turbulent**. Under laminar conditions the velocity distribution of the fluid within the pipe is essentially parabolic and can be derived from the equation of motion. When turbulent flow exists, the velocity profile is "flatter" than in the laminar case because the mixing effect which is characteristic of turbulent flow helps to more evenly distribute the kinetic energy of the fluid over most of the cross section.

In most engineering texts, a Reynolds number of 2 100 is usually accepted as the value at transition; that is, the value of the Reynolds number between laminar and turbulent flow regimes. This is done for the sake of convenience. In this experiment, however, we will see that transition exists over a range of Reynolds numbers and not at an individual point.

The Reynolds number that exists anywhere in the transition region is called the **critical Reynolds number**. Finding the critical Reynolds number for the transition range that exists in pipe flow is the subject of this experiment.

Critical Reynolds Number Measurement

Equipment

Critical Reynolds Number Determination Apparatus

Figure 5.1 is a schematic of the apparatus used in this experiment. The constant head tank provides a controllable, constant flow through the transparent tube. The flow valve in the tube itself is an on/off valve, not used to control the flow rate. Instead, the flow rate through the tube is varied with the rotameter valve at *A*. The head tank is filled with water and the overflow tube maintains a constant head of water. The liquid is then allowed to flow through one of the transparent tubes at a very low flow rate. The valve at *B* controls the flow of dye; it is opened and dye is then injected into the pipe with the water. The dye injector tube is not to be placed in the pipe entrance as it could affect the results. Establish laminar flow by starting with a very low flow rate of water and of dye. The injected dye will flow downstream in a threadlike pattern for very low flow rates. Once steady state is achieved, the rotameter valve is opened slightly to increase the water flow rate. The valve at *B* is opened further if necessary to allow more dye to enter the tube. This procedure of increasing flow rate of water and of dye (if necessary) is repeated throughout the experiment.

Establish laminar flow in one of the tubes. Then slowly increase the flow rate and observe what happens to the dye. Its pattern may change, yet the flow might still appear to be laminar. This is the beginning of transition. Continue increasing the flow rate and again observe the behavior of the dye. Eventually, the dye will mix with the water in a way that will be recognized as turbulent flow. This point is the end of transition. Transition thus will exist over a range of flow rates. Record the flow rates at key points in the experiment. Also record the temperature of the water.

The object of this procedure is to determine the range of Reynolds numbers over which transition occurs. Given the tube size, the Reynolds number can be calculated with:

$$\mathrm{Re} = \frac{VD}{\nu}$$

where $V (= Q/A)$ is the average velocity of liquid in the pipe, D is the hydraulic diameter of the pipe, and ν is the kinematic viscosity of the liquid.

The hydraulic diameter is calculated from its definition:

$$D = \frac{4 \text{ x Area}}{\text{Wetted Perimeter}}$$

For a circular pipe flowing full, the hydraulic diameter equals the inside diameter of the pipe. For a square section, the hydraulic diameter will equal the length of one side (show that this is the case). The experiment is to be performed for both round tubes and the square tube. With good technique and great care, it is possible for the transition Reynolds number to encompass the traditionally accepted value of 2 100.

Questions

1. Can a similar procedure be followed for gases?
2. Is the Reynolds number obtained at transition dependent on tube size or shape?
3. Can this method work for opaque liquids?

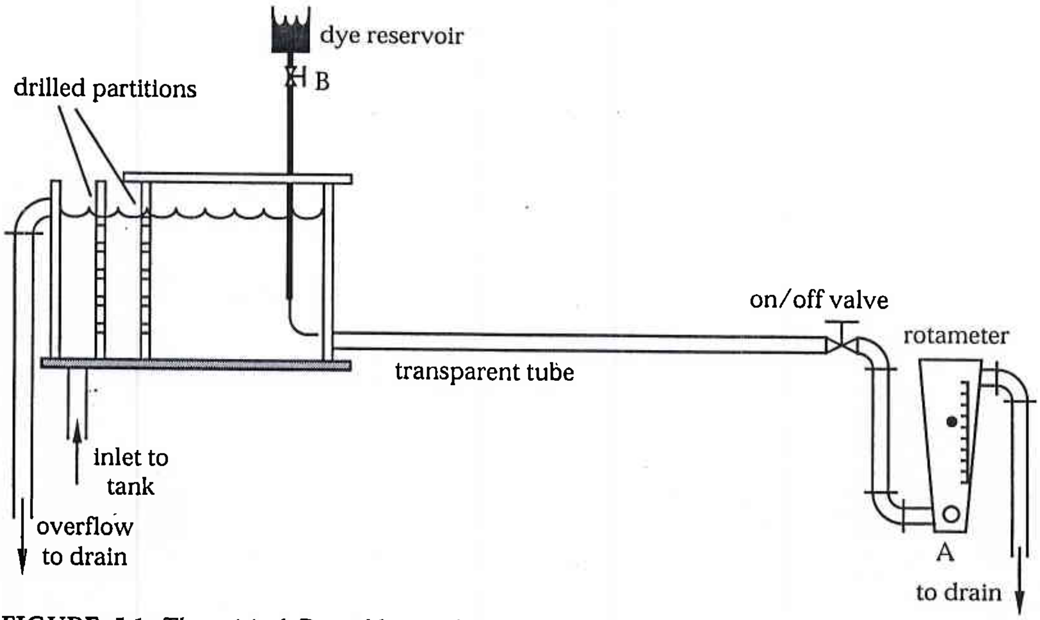


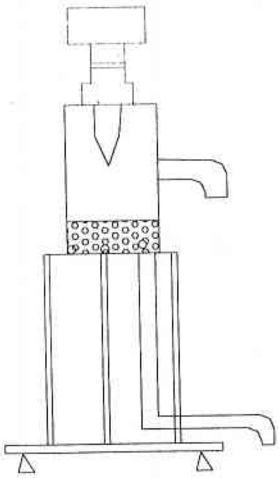
FIGURE 5.1. *The critical Reynolds number determination apparatus.*

Objective:

- To perform the classical experiment conducted by Reynolds concerning fluid flow (i.e.,laminar , transition,and turbulent flow).

Apparatus:

Water supply, Osborne Reynolds' Apparatus) , Measuring Cylinder , Stop Watch ,Vegetable Dye , and Thermometer .



Theory:

The nature of agiven flow of in compressible fluid is characterized by its renold number.

- The incompressible fluid is chracterized by its Reynolds number,which is defined as the ratio of internal forces to viscous force ,or mahtmatically as:

$$Re = \frac{vd}{\nu}$$

v = flow velocity (m/sec.) ,
d = flow visualization pipe diameter (mm) ,
ν= kinematic viscosity (m^2/sec.)

- the flow is considered :
  - -Laminar flow if $Re \leq 2000$,
  - -Turbulent flow if $Re \geq 4000$,
  - -Transition flow if $2000 < Re < 4000$.

Procedure:

1-Fill the small reservoir with dye , position the apparatus on the bench and connect the inlet pipe to the bench feed , ...
2-Open & close the control valve slightly to admit water to flow through the visualization pipe .

3-Measure the temperature of the water .
4-Open the inlet valve slightly until water trickles from the outlet pipe fractionally open the control valve and adjust the dye control valve until slow flow with dye indication is achieved. Measure the flow rate.
5-Repeat for increasing flow rates .
6-Repeat for decreasing flow rates .

Results and Calculations:

- Internal diameter of visualisation pipe = 10 mm .
- Temperature of water = 18° C .

Flow Condition	Laminar Flow			Transition Flow			Turbulent Flow		
Water volume (cm^3)	300	300	300	300	300	300	300	300	300
Time (sec)	224.78	46.18	18.42	12.4	10.65	9.74	8.14	7.24	6.57
Re Number	1204	1322	1452	3548	3879	4122	5108	5421	6894

Discussion:

it was noticed that the water has suffered from the follwing disturbances:

- The dye needle was in bad condition , and this affected the amount of dye released from the tank .

- The water level in the tank was not constant , that's because the rate of flow from the flow control valve and the inlet valve was not always the same , and this caused a diiference in the pressure head, which caused a decrease in the velocity of the flow in the visualisation pipe , causing the change in the flow rate .

In the laminar flow ,the dye in the visualisation pipe was a straight line , this is because the water moves in layers , one layer gliding smoothly over an adjacent layer. And , in the transition flow,the dye line became wavy -but we were able to see it- that's because the velocity increased, causing more disturbance to the flow,and it can be said that the flow is in the way of being turbulent .
In the turbulent flow, the dye has disappeared totally.

Conclusion:

- small value of Reynolds number means that the flow is laminar,and as Re increase ,then the flow is in the way of being turbulent , with a transition phase between the two types of flow.
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in the laminar flow , the fluid is moving smoothly,this could be due to the conditions surrounding the flow:
*The velocity of the flow , and as the velocity increase, the flow gets more and more disturbed ,and finally reaches the turbulent stage.
* The viscosity of the fluid , as the viscosity increase ,the flow will become smoother.

References:

-Fluid Mechanics .