Effect of Boundary Conditions on Impact Resistance of Concrete Slabs

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ABSRTUCT

A theoretical analysis based on the numerical solution of the slab impact integral equation is carried out to determine the impact force and deflection time histories, the strain energy absorbed by the slabs and the maximum bending moment.

Effect of slab boundary conditions on impact response of slab is also discussed. The theoretical results obtained in the present analysis are compared with experimental and theoretical works previously done. A good agreement is found between theoretical and experimental results. This indicates that the impact resistance of relatively large slabs may be predicted by using the theoretical approach based on equation of undamped slab vibration. All the derivations required to predict the effect of boundary conditions are performed for both forced and free vibrations. For the same falling mass and the same applied kinetic energy (height of drop) for all cases, the maximum central deflection and the maximum impact force are affected by the boundary conditions of the slabs

Keywords: Boundary Conditions, Deflection – Time Histories, Force – Time Histories, Free and Forced Vibration, Impact, Slab, Striker.

تـــأثير الشروط الحدية على مقاومة البلاطات الكونكريتية للأحمال الصدمية

الخلاصة

يتضمن البحث دراسة نظرية مستندة الى الحل العددي لمعادلة الصدم للبلاطات لايجاد التغير الزمني لقوة الصدم والهطول والطاقة الممتصه من قبل البلاطات وكذلك اقصى عزم تتعرض لـــه البلاطه. كذلك تضمن البحث دراسه تأثير الشروط الحديه على استجابه البلاطه لقوى الصدم.

النتائج النظرية المستحصله من الدراسة تمت مقارنتها مع نتائج عملية ونظريه سابقه وكانـــت النتائج متقاربه بشكل جيد. ان التقارب في النتائج اعلاه يؤكد امكانيه استخدام الاسلوب المعتمد في

هذه الدراسة، لايجاد مقاومه البلاطات الكبيره لقوى الصدم. جميع الاشتقاقات المطلوبه لمرحلتـــي الاهتزازالقسري والحر مقدمه في الدراسه ولكافة الشروط الحدية للبلاطات ولنفس مقدار الكتلـــه الساقطه وارتفاع السقوط، وجد ان التغير الزمني لقوة الصدم وكذلك الهطول يتأثر بنوع الشروط الحديه للبلاطه.

INTRODUCTION

Impact loads may be applied to many structures which have been designed only to resist their own dead loads in addition to the conventional static live loads. If the probability of impact loading is very small, it may be uneconomical to design against impact loads, but if the structure is subjected to impact the results could be very serious. Under these circumstances, it is useful to check the impact resistance of structures which have been designed to resist static loads. Some structures such as shelters and buildings of nuclear plant must be designed to resist impact loads. Missile impact, fragments impact, ship collision, vehicle impact with structures, and falling masses in industrial buildings are some examples of impact.

Local response and overall (structural) response are usually associated with impact. The structural responses are in the form of flexural and shear deformations, and the structure is to be dynamically analyzed under the applied force-time history. The effect of impact loading on concrete structures has received a considerable amount of attention of many researchers [1], [2], [3], [4], [5], [6], [7].

The objective of this study is to present a theoretical analysis based on the numerical solution of the slab impact integral equation. Based on equation of undamped slab vibration, the effect of boundary conditions on the slab response to impact force is also presented for both stages of vibration (forced and free vibrations).

The impact force and deflection-time histories, the strain energy absorbed by the slab and the bending moment are all determined. Concrete slab models of dimensions (500 x 500 x 20) mm are used in the present research. These slabs were tested experimentally by Hussain [(12)], with different falling masses (6, 9, 12 and 18) kg. dropped from different falling heights (300, 600 and 1000) mm.

IMPACT INTEGRAL EQUATIONS

The structural dynamic response of structures subjected to impact can be determined if the impact force time history is known. Therefore the main purpose of the impact analysis is to determine the impact force-time history F(t), deflection W (x, y, t). A slab is struck transversely by a mass (m_s) having a spherical surface at the point of contact and striking velocity (V_o), Fig. 1.

The formulation of this problem can be effected only under certain assumptions:-

a) All assumptions of the classical theory of beams and plates are applicable.

b) Hertz law of impact is valid [(8), (9)], hence

$$F(t)= K.\left(a(t)\right)^{3/2} \qquad \text{... (1)}$$

Where:

F (t): the impact force at any time (t) within the duration of impact.
a (t): the relative approach of striking bodies, Figure (1).
K : the Hertz (deformation) constant which depends on the elastic mechanical properties and the shapes of the two bodies at the contact zone.

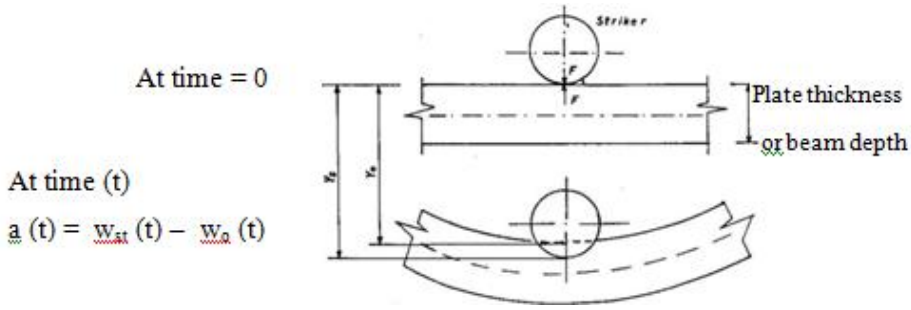


Figure (1) Displacement at Impact Zone

Timoshenko and Young [8] formulated a nonlinear integral equation for central impact of a sphere on a simply supported beam considering the striker and the beam displacement as shown in Figure (1) and using Hertz law at the point of contact. The deformation equation is:-

$$a(t) = Y_{st}(t) - Y_s(t) \qquad \text{... (2-a)}$$

Where

$Y_{st}(t)$ **: The displacement of the striker under the action of (F(t))**

$Y_s(t)$ **: The deflection of the beam at the point of contact.**

$$Y_{st}(t) = V_o.t - \frac{1}{m_{st}} \int_0^t \int_0^T F(t)\,dt\,dT \qquad \text{... (3-a)}$$

Where (Vo), (m_{st}) and (T) denote the velocity, the mass of the striker and the impact duration, respectively.
Substituting of equation (3-a) into equation (2-a) and making use of equation (1) leads to:

$$\left(\frac{F(t)}{K}\right)^{2/3} = V_o.t - \frac{1}{m_{st}} \int_0^t \int_0^T F(t)\,dt\,dT - Y_s(t) \qquad \text{... (4-a)}$$

Three different cases were discussed to determine the displacement function Y_s (t) of the beam [1], [9], [11], [23]. These cases are
(a): Impact between striker and a massive beam.
(b): Impact between striker and beam of effective mass.

(c): Impact between striker and a beam of distributed mass.

For the slab impact, equations (2-a) , (3-a) ,and (4-a) can be rewritten as

$$a(t) = W_{st}(t) - W_s(t) \qquad \text{... (2-b)}$$

$$W_{st}(t) = V_o . t - \frac{1}{m_{st}} \int_0^t \int_0^T F(t)\, dt\, dT \qquad \text{... (3-b)}$$

$$\left(\frac{F(t)}{K}\right)^{2/3} = V_o . t - \frac{1}{m_{st}} \int_0^t \int_0^T F(t)\, dt\, dT - W_s(t) \qquad \text{... (4-b)}$$

Where

W_{st} (t): The displacement of the striker under the action of (F (t)).

W_s (t): The displacement of the slab at the point of contact.

The case of impact between striker and a slab of distributed mass is discussed in the present research. The other two cases were discussed elsewhere [22].

Impact Between Striker and a Slab of Distributed Mass.

To get more accurate solution of the slab impact problem, a more accurate description of the slab vibration than that given by the effective or rigid mass models should be considered. This means that the free and forced vibration of slabs should be considered.

Partial Differential Equation of Rectangular Slab Vibration.

The well – known equation of rectangular slab due to impact is [13], [14], [15]

$$D\left[\frac{\partial^4 W}{\partial x^4} + 2\frac{\partial^4 W}{\partial x^2 \partial y^2} + \frac{\partial^4 W}{\partial y^4}\right] + \overline{m}\frac{\partial^2 W}{\partial t^2} = F(x,y,t) \qquad \text{... (5)}$$

where

$$D = \frac{EI}{1-\mu^2} = \frac{Eh^3}{12(1-\mu^2)} \qquad \text{... (6)}$$

which denotes the flexural rigidity of unit width of the slab.

and:

I : The moment of inertia of the slab per unit width.

h : Thickness of the slab.

E : Modulus of elasticity.

μ : Poisson's ratio.

$\overline{m}$: The mass of the slab per unit area.

F(x,y,t) : The external load intensity.

Equation (5) can be solved for W(x,y,t) ,the slab deflection as a function of both time and position.

Free Vibration.

For free vibration, the external dynamic load (F(x,y,t) = 0) in equation (5). Four cases will be given below, in all cases the deflection is represented by (W(x,y,t)) and the natural circular frequency is represented by (ω).

Square Slab Simply Supported at All Edges.

The deflection along each edge must be zero and there is no bending moments along each edge.

The slab displacement W(x,y,t) can be represented by [16]

$$W(x,y,t) = \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} W_{ij} = \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} w_{ij}(x,y).T_{ij}(t) \qquad \text{… (7)}$$

Then two differential equations can be yielded:

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} - \beta^2 w(x, y) = 0 \qquad \text{… (8)}$$

$$\ddot{T}(t) + \omega^2 . T(t) = 0 \qquad \text{… (9)}$$

$$\omega = \sqrt{\frac{D}{m}}\ \beta \qquad \text{… (10)}$$

The solution of equation (8) may have the form of [8], [15], [16]

$$w_{ij}(x,y) = \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} a_{ij} \sin\frac{i\pi x}{L} \sin\frac{j\pi y}{L} \qquad \text{… (11)}$$

Which satisfies the boundary conditions of the slab, substituting equation (11) and the corresponding derivatives into equation (8), gives:

$$\beta_{ij} = \frac{1}{L^2}\left[(i\pi)^2 + (j\pi)^2\right] \qquad \text{… (12)}$$

For each mode of vibration and equation (10) becomes:

$$\omega_{ij} = \frac{1}{L^2}\left[(i\pi)^2 + (j\pi)^2\right]\sqrt{\frac{D}{m}} \qquad \text{… (13)}$$

If $f_1(x,y)$ and $f_2(x,y)$ are the initial function of displacement and velocity for the slab, then [16], [22]

$$W(x,y,t) = \frac{4}{L^2}\sum_{i=1}^{\infty}\sum_{j=1}^{\infty} \sin\frac{i\pi x}{L}\sin\frac{j\pi y}{L}\left[\cos\omega_{ij}t\int_0^L\int_0^L f_1(x,y)\sin\frac{i\pi x}{L}\sin\frac{j\pi y}{L}\,dx\,dy\right.$$
$$\left. + \frac{1}{\omega_{ij}}\sin\omega_{ij}t\int_0^L\int_0^L f_2(x,y)\sin\frac{i\pi x}{L}\sin\frac{j\pi y}{L}\,dx\,dy\right] \qquad \text{… (14)}$$

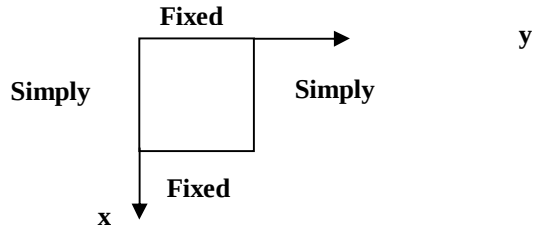
Timoshenko [8] applied equation (5) on square plate simply supported at all edges, and found:

$$W(x,y,t) = 4P_o L^2 \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} \frac{\sin^2 \frac{i\pi x}{L} \sin^2 \frac{j\pi y}{L}}{\pi^4 D\left(i^2 + j^2\right)^2} \cos(\omega_{ij} t) \qquad (15)$$

Where (ω_{ij}) can be obtained from equation (13).

Square Slab Simply Supported at Two Parallel Edges and Fixed at the Other Edges

The solution of equation (8) may have the form of [15]:



$$w_{ij}(x,y) = \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} \cosh\alpha y - \cos f y - \beta(\sin f y - \frac{f}{\alpha}\sinh\alpha y) \qquad (16)$$

$$\beta = \frac{\cosh \alpha L - \cos f L}{\sin f L - \frac{f}{\alpha}\sinh \alpha L} \qquad (17)$$

$$\alpha = \frac{1}{L}(\beta_{ij} L^2 + i^2\pi^2)^{\frac{1}{2}} \qquad (18)$$

$$f = \frac{1}{L}(\beta_{ij} L^2 - i^2\pi^2)^{\frac{1}{2}} \qquad (19)$$

a **and** f **can be found from the equation [15] :**

$$(\cos f L - \cosh\alpha L)^2 + (\sin f L - \frac{f}{\alpha}\sinh\alpha L)\left(\sin f L + \frac{a}{f}\sinh a L\right) = 0 \qquad (20)$$

Then use can be made of equation (18) or (19) to obtain (β_{ij}) and the frequency (ω_{ij}) using equation (10).

Square Slabs With Combination of Fixed and Free EdgesSolved By Ritz Method.

An exact solution of the differential equation of a vibrating slab is known for the case of a rectangular slab which is simply supported at all four edges and for a rectangular slab which is simply supported at two parallel edges and fixed at the other edges. For other combination of edge conditions the solution is more complicated, and it has been necessary to use an approximate method.

Ritz method has been found to be very useful [8], [17]. The analysis herein is for a homogeneous slab of uniform thickness and is based upon the ordinary theory of thin plates.

For a uniform slab, the maximum potential energy is given by [8], (15) , (17) , (22).

$$U_{max} = \frac{D}{2}\iint\left[\left(\frac{\partial^2 w}{\partial x^2}\right)^2 + \left(\frac{\partial^2 w}{\partial y^2}\right)^2 + 2\mu\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} + 2(1-\mu)\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2\right]dxdy. \quad (21)$$

And the maximum kinetic energy is:

$$(K.E)_{max} = \frac{1}{2}\,\overline{m}\,\omega^2 \iint w^2\,dxdy \qquad \ldots(22)$$

The integrations are to be taken over the domain of the slab surface. Equating these two expressions yields:

$$\omega^2 = \frac{2}{\overline{m}}\,\frac{U_{max}}{\iint w^2 dxdy} \qquad \ldots (23)$$

For a rectangular slab, with edges parallel to the x and y axes, w(x,y) can have the form [8], [17]:

$$w\ (x,y) = \sum_{i=1}^{p}\sum_{j=1}^{q} A_{ij}.X_i(x).Y_j(y) \qquad \ldots (24)$$

where $X_i(x)$ and $Y_j(y)$ are the normal functions of vibration for beams. When w(x,y) as given by equation (24) is substituted in equation (23), the right-hand side becomes a function of the coefficients A_{ij} . This minimized by taking the partial derivative with respect to each coefficient and equating to zero. Thus a set of equations each of which has the form[17],[22]:

$$\frac{\partial U_{max}}{\partial A_{mn}} - \frac{\overline{m}\,\omega^2}{2}\,\frac{\partial}{\partial A_{mn}}\iint w^2\ dx\,dy = 0 \qquad \ldots (25)$$

Where A_{mm} is any one of the coefficients A_{ij} . Equation (25) represents a system of linear homogeneous equations in the unknowns A_{mm}. The natural frequencies $\omega 1, \omega 2, \ldots$.are determined from the condition that the determinant of the system must vanish.

As discussed before, the appropriate characteristic functions for vibrating beams will be used for X_i and Y_j .The different types of beams will be identified by a compound adjective which describes the end conditions. Thus a (fixed – fixed) beam is one which is rigidly fixed at both ends; a (fixed – free) beam is fixed at the end x = 0 and free at the end x = L; a (free – free) beam is free at both ends.

For each type of beam, there is an infinite number of normal modes in which the beam can vibrate laterally. The characteristic functions for the three types of beams are as follows[13],[8], [17].

1. **Fixed-fixed beam**

$$\varphi_r = cosh\frac{a_r x}{L} - cos\frac{a_r x}{L} - \psi_r\left(sinsh\ \frac{a_r\ x}{L} - sin\frac{a_r\ x}{L}\right) \ldots (26)$$

$$a_r = \frac{r + 0.5\pi}{L}$$

$$\psi_r = (\cosh a_r L - cos\ a_r L)/(sinsh\ a_r L - \sin a_r L)$$

The boundary conditions for this case are:

$$\varphi_r = \frac{d\varphi_r}{dx} = 0)\ at\ x = 0\ and\ x = L$$

2. Fixed-Free Beam

φ_r **is as given in equation** **…(26)**

$$a_r = \frac{r - 0.5\pi}{L}$$

$$\psi_r = (\cosh a_r L + cos\ a_r L)/(sinsh\ a_r L + \sin a_r L)$$

The boundary conditions for this case are:

$$\varphi_r = (\frac{d\varphi_r}{dx} = 0)\ at\ x = 0\ and(\frac{d^2\varphi_r}{dx^2} = \frac{d^3\varphi_r}{dx^3} = 0)x = L$$

3. Free – Free Beam

$$j_r = cosh\frac{a_r x}{L} + cos\frac{a_r x}{L} - \psi_r(sinh\frac{a_r x}{L} + sin\frac{a_r x}{L}) \quad \ldots (27)$$

$$a_r = \frac{r + 0.5\pi}{L}$$

$$\psi_r = (cosha_r L - cos a_r L)/(sinha_r L - sin\ a_r L)$$

The boundary conditions for this case are:

$(d^2 j_r/dx^2 = d^3 j_r/dx^3 = 0)$ **at x = 0 and x = L**

Where (r) is the mode of vibration and (j_r) is the normal functions of vibration for beam. The boundary conditions satisfied by the functions in each set are the same as the end conditions of the corresponding beam. Each set of the functions is orthogonal in the interval ($0 \rightarrow L$), that is for any functions (j_r, j_s) in the same set, the following relations hold [17]:

$$\int_0^L j_r\ j_s\ dx = L \quad (for\ r = s)$$
$$= 0 \quad (for\ r \neq s) \quad \ldots (28)$$

The second derivatives of the function in each set are also orthogonal and satisfy the relations [17]:

$$\int_0^L \frac{d^2 j_r}{dx^2}\,\frac{d^2 j_s}{dx^2}\,dx = \frac{a_r^{\,4}}{L^3} \quad (for\ \ r = s) \qquad \ldots(29)$$
$$= 0 \quad (for\ \ r \neq s)$$

Numerical values of φ_r and a^4_r are given in Ref.[17], [22]
In addition to the integrals defined by equations (28) and (29), it is necessary to evaluate.

$$\int_0^L j_r\,\frac{d^2 j_s}{dx^2}\,dx \ \ ,\ and\ \int_0^L \frac{dj_r}{dx}\,\frac{dj_s}{dx}\,dx$$

Values of these integrals have been computed and given in Ref.[17], [22].

Consider a square slab bounded by the lines x = 0, x = L, y = 0, y = L. and assume, for example that the slab is fixed along the edge x = 0 and free along the other three edges. In this case the fixed – free functions, equation (26) should be used for X_i , and the free – free functions, equation (27) should be used for Y_j.

With the three sets of function given herein, solutions can be obtained for rectangular slabs having any combination of free and fixed edges [17]:

$$E_{mi} = L\int_0^L X_m\,\frac{d^2X_i}{dx^2}\,dx \ ,\ E_{im} = L\int_0^L X_i\,\frac{d^2X_m}{dx^2}\,dx \qquad \ldots (30)$$

$$F_{nj} = L\int_0^L Y_n\,\frac{d^2Y_j}{dy^2}\,dy \ ,\ F_{jn} = L\int_0^L Y_j\,\frac{d^2Y_n}{dy^2}\,dy \qquad \ldots (31)$$

$$H_{mi} = L\int_0^L \frac{dX_m}{dx}\,\frac{dX_i}{dx}\,dx \ ,\ K_{nj} = L\int_0^L \frac{dY_n}{dy}\,\frac{dY_j}{dy}\,dy \qquad \ldots (32)$$

Since the appropriate φ – functions are to be used for Xi and Y_j . The numerical values of integrals can be taken directly from Ref.[17], [22]. Using equation (24) and (21), and taking into account the orthogonality relations, equations (28) and (29), the set of equations (25) can be reduced to the form [17]:

$$\sum_{i=1}^{p}\ \sum_{j=1}^{q}\ [C_{ij}^{\ mn} - \lambda\,\delta_{ij}]\,A_{ij} = 0 \qquad \ldots (33)$$

Where:

$$\lambda_{ij} = \overline{m}.\omega_{ij}^{2}.L^{4}/D \qquad \text{… (34)}$$

$$\delta_{ij} = 1 \text{ for } ij = mn$$

$$= 0 \text{ for } ij \neq mn$$

And for ij ≠ mn

$$C_{ij}^{(mn)} = \mu[E_{im}F_{nj} + E_{mi}F_{jn}] + 2(1-\mu)H_{mi}K_{nj} \qquad \text{… (35)}$$

For ij = mn the coefficient is

$$C_{ij}^{(ij)} = a_i^{4} + a_j^{4} + 2\mu E_{ii}F_{jj} + 2(1-\mu)H_{ii}K_{jj} \qquad \text{… (36)}$$

In eq.(36) a_i is taken corresponding to the φ- function represent X_i while a_j is to be taken from data for the φ-function that represent Y_j
According to the Timoshenko's equation [18], the deflection – time history of square slab due to impact load is:

$$W(x, y, t) = \frac{P_o}{L^4} \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \frac{w_{ij}(x, y).w_{ij}(\overline{x}, \overline{y})}{\overline{m}.\omega_{ij}^{2}} \cos(\omega_{ij} t) \qquad \text{(37)}$$

Where the initial displacement is caused by the concentrated force P_o acted at the point ($\overline{x}, \overline{y}$).and P_o is the magnitude of impact force at the beginning of last time increment.
Two cases will be discussed here, using Ritz method [8], [17] to evaluate the frequency and the deflection.
Square Cantilever Slab

Consider the case of a square slab which is fixed along one edge and free along the other three edges. Therefore, let take x = 0 as the fixed edge. For X_i the fixed – free functions, equation (26), and for Y_j the free – free functions, equation (27) are used. The frequency for fifth modes and the coefficients A_{ij} for each frequency have been calculated and the results are given in (22). Making use of equation (24) and (37) can be determined the deflection – time history of square slab due to impact load.
Square Fixed Slab

Consider the case of a square slab which is fixed along all four edges. For this case the fixed – fixed functions equation (26) for both X_i and Y_j is used. The frequency for sixth modes and the coefficients A_{ij} for each frequency have been calculated and given in (22). Equation (24) and (37) can be used to determine the deflection–time history of square slab due to impact load.
Forced Vibration

For the case of forced vibration (the initial displacement and the initial velocity are equal to zero). The external dynamic load (F(x,y,t)) is included in equation (5). It should be noted that the forced vibration occurs before the free vibration.

Square Slab Simply Supported at All Edges.

The deflection at the point of contact is given by [15]

$$W(x,y,t) = \frac{2}{m_s} \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \frac{Cr_{ij}}{\omega_{ij}} \int_0^t F(T) \cdot \sin[\omega_{ij}(t-T)]\, dT \qquad \text{... (38)}$$

$$Cr_{ij} = w_{ij}^2(x,y) \cdot A / 2b_{ij} \qquad \text{... (39)}$$

$$w_{ij}(x,y) = \sin\frac{i\pi x}{L} \sin\frac{j\pi y}{L} \quad , \quad b_{ij} = L^2/4$$

where ω_{ij} is from equation (13), and A is the total area of the slab, and m_s is the total mass of the slab.

The solution of equation (5) for midspan deflection of this case resulting from central impact is given by [8], [9], [15]:

$$W_o(L/2, L/2, t) = \frac{4}{m_s} \sum_{i=1,3...}^{\infty} \sum_{j=1,3...}^{\infty} \frac{1}{\omega_{ij}} \int_0^t F(T) \cdot \sin[\omega_{ij}(t-T)]\, dT \qquad \text{... (40)}$$

Substituting equation (35) into equation (4):

$$\left(\frac{F(t)}{K}\right)^{2/3} = V_o t - \frac{1}{m_{st}} \int_0^t dT \int_0^T F(t)\, dt$$

$$- \frac{4}{m_s} \sum_{i=1,3...}^{\infty} \sum_{j=1,3...}^{\infty} \frac{1}{\omega_{ij}} \int_0^t F(T) \cdot \sin[\omega_{ij}(t-T)]\, dT \qquad \text{... (41)}$$

This equation can not be solved in a closed form, but it may be solved numerically to give the impact force and the deflection – time histories. A computer program was written for this purpose. After that, it was possible to determine the following quantities, the displacement, the central bending moment, the total kinetic energy and the total strain energy. All these are given in Ref.[22]

Square Slab Simply Supported at Two Parallel Edges and Fixed at the Other Edges.

Equations (38) and (39) are used to find the deflection at the point of contact, but b_{ij} for this case is [15] :

$$b_{ij} = \frac{1}{4}L^2\left(1+\frac{f^2}{\alpha^2}\right) + \frac{L}{4\alpha}\left(\sin fL - \frac{f}{\alpha}\sinh \alpha L\right)^{-1}$$

$$\left[\left(1-\frac{f^2}{\alpha^2}\right)\sinh \alpha L.\sin fL - \frac{2f}{\alpha}\cosh \alpha L.\cos fL\right]$$

$$\left[\left(\cosh \alpha L + \frac{\alpha^2}{f^2}\cos \alpha L\right) + \frac{2\alpha}{f}\left(\cosh \alpha L + \frac{f^2}{\alpha^2}\cos fL\right)\right]$$

To calculate midspan deflection for this case, it should be calculate C_{ij} for each mode of vibration at (x = y = L/2) and substituting the results in equation (38). Therefore the displacement at midspan of the slab can be determined using equation (38) and substituting in equation (4):

$$\left(\frac{F(t)}{K}\right)^{2/3} = V_{o.}t - \frac{1}{m_{st}}\int_0^t d\tau \int_0^\tau F(t)\,dt$$

$$-\frac{2}{m_s}\sum_{i=1,3..}\sum_{j=1,3...}\frac{Cr_{ij}}{\omega_{ij}}\int_0^t F(\tau).\sin[\omega_{ij}(t-\tau)]\,d\tau \quad \text{... (42)}$$

As mentioned before, the above equation may be solved numerically to give the impact force and deflection – time histories and the following may be computed; the displacement, the central bending moment, the total kinetic energy and the total strain energy and are given in Ref.[22]

Square Slabs Have Combination of Fixed and Free Edges Solved By Ritz Method.

As mentioned in section (1.1.2.3), for other combination of edge conditions the solution is more complicated, and it has been necessary to use an approximate method. For these cases, Ritz assumed:

$$W(x, y, t) = Z.\cos(\omega_{ij}t - \gamma) \quad \text{... (43)}$$

$$Z(x, y) = \frac{g}{L^2}\sum_{i=1}\sum_{j=1}\frac{w_{ij}(x,y).w_{ij}(\bar{x},\bar{y})}{\omega_{ij}^2} \quad \text{... (44)}$$

Where g is the acceleration due to gravity, w_{ij} can obtained from equation (24) and ω_{ij} from equation (23) and:

$$\gamma = \frac{\omega_{ij}}{\sqrt{D/\overline{m}\,L^4}} \qquad \text{... (45)}$$

Solutions are obtained for three specific slabs.

1. **Square cantilever slab.**
2. **Square fixed slab.**

For these two cases (γ) are given in Ref. [17], [22].

Dynamic Flexural rigidity

It is difficult to determine an effective flexural rigidity for the slabs, equation (6), because the amount of cracking varies along the span since it depends on the deflection caused by the applied load which means that the moment of inertia also varies during the period of impact. To estimate the moment of inertia, the following approaches will be discussed:

1- **A fully cracked transformed section is used for computing the moment of inertia. The flexural rigidity of the slab is assumed constant during the period of impact [9], [13], [22], [23].**
2- **The average moment of inertia of cracked and gross transformed sections is considered [13], [14] , [22], [23] .**
3- **A Constant moment of inertia within the duration of impact using the dynamic effective moment of inertia given [19], [20], [22], [23] :**

$$I_{deff} = \left(\frac{\lambda M_{cr}}{M_u}\right)(I_\alpha - I_{cr}) + I_{cr} \qquad \text{... (46)}$$

Where:

I_{deff} : Effective dynamic moment of inertia of the section.
M_{cr} : Static cracked moment capacity of the section.
M_u : Ultimate static moment capacity of the section.
I_α : Moment of inertia for the gross section.
I_{cr} : Moment of inertia for the transformed cracked section.
λ : A constant equals to (o.6 – 0.8) [19] , (1) [20].

4- **A variable moment of inertia within the duration of impact[1] is to be considered. This depends on the dynamic deflection at any time within the impact duration and assuming that the static and dynamic load resistance-deflection curves are similar.**

VALIDITY OF PROGRAMME

Steel ratio ($\rho = 8x10^{-3}$), thus amount of steel reinforcement is (As =136mm^2/m). Based on the effect of moment of inertia discussed in section (1.1.4), four approaches of moment of inertia are used to determine the maximum deflection – time history, and the maximum impact force – time history, by using the

computer program. For the present research the input data in the computer program are:

m_{st} : Striker mass, m_s : Slab mass, D : Flexural rigidity of slab, V_o : Impact velocity of striker, K : Deformation constant, L : Span of slab.

a) To check the accuracy of the developed program solution, Figure (2) show the force – time history of a simply supported square slab subjected to a central impact load which is compared with the solution given by Eringen [15] . The following parameters were used:

m_{st}: striker mass = 0.03334 kg.
m_s: slab mass = 2.55 kg.
Vo: impact velocity of striker = 1 m/sec
K: deformation constant = 1.4 E6 $N/m^{1..5}$
ω_1: 62.2 rad/sec

A comparison is made between the present research and Eringen [15], Figure (2), from which it can be concluded that the theoretical results based on a dynamic effective moment of inertia [21]' have a reasonable agreement with Eringen [15].

b) Also a comparison is made between the present approach and Hussain [12] (the dimensions of slabs used are (500 x 500 x 20) mm), Figure (3). A good agreement is found between results of present study and Hussain[12] results. Maximum experimental deflection (2.30mm) found by Hussain [12] is conformable with the present theoretical results using dynamic effective moment of inertia. The model dimensions and properties used by Hussain [12] are adopted in present work for other slabs.

CONCLUSIONS

1. **The theoretical maximum central impact force of the slabs obtained from the solution of the impact integral equation of the present study is found to have a good agreement with Eringen [15] and Hussain [12], Figures (2) and (3).**
2. **For the same falling mass and the same applied kinetic energy (height of drop), the slabs stiffened with steel pads ($K = 1.72 \times 10^{9}$ $N / m^{1.5}$) exhibit smaller central deflection by about (32%) in comparison with unstiffened slabs ($K = 24.7 \times 10^{6}$ $N / m^{1.5}$) for square slab simply supported at all edges, fig. (6).**
3. **For the same falling mass and the same applied kinetic energy (height of drop), the slabs stiffened with steel pads ($K = 1.72 \times 10^{9}$ $N / m^{1.5}$) exhibit higher impact force by about (73%) in comparison with unstiffened slabs($K= 24.7 \times 10^{6}$ $N / m^{1.5}$) for square slab simply supported at all edges, fig. (6).**
4. **The calculated impact duration is longer for heavier falling mass.**
5. **For the same falling mass the calculated peak impact forces, central deflection and the absorbed strain energy are all increased as the impact velocity (height of drop) is increased.**
6. **The calculated impact duration is found independent of the impact velocity (height of drop).**

7. **For the same applied kinetic energy (height of drop) the calculated peak impact forces, central deflection and the absorbed strain energy are all increased as the falling mass being heavier.**
8. **For the same falling mass and the same applied kinetic energy (height of drop) for all cases, the maximum central deflection and the maximum impact force are affected by the boundary conditions of the slabs. The maximum central impact force for square fixed slab is higher than other cases by about (61%) for simply supported square slab, (38%) for square slab simply supported at two parallel edges and fixed at the other edges and (2%) for square cantilever slab, the maximum central deflection for square cantilever slab is smaller than other cases by about (67%) for simply supported square slab, (60%) for square slab simply at two parallel edges and fixed at the other edges and (51%) for square fixed slab, Figure (7).**
9. **For the same falling mass and the same height of drop the free end deflection is greater than the central deflection by about (36%), for the case of the case of square cantilever slab, Figure (8).**

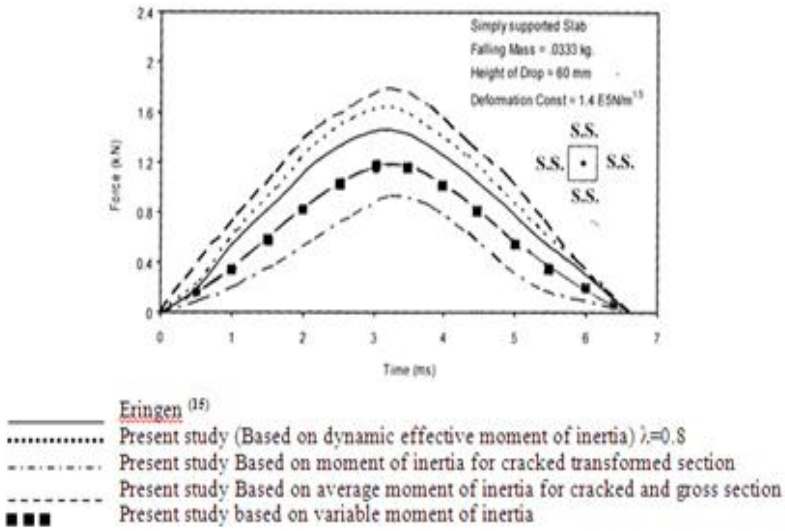


Figure (2) typical theoretical impact force – time history for simply supported slab.

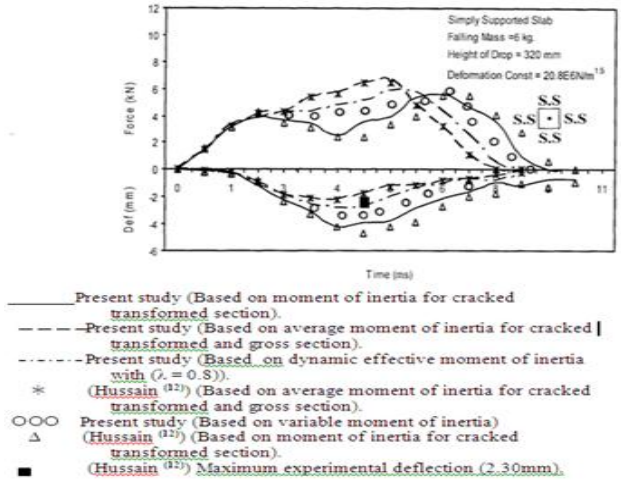


Figure (3) typical theoretical central impact force and deflection – timehistories for simply supported slab.

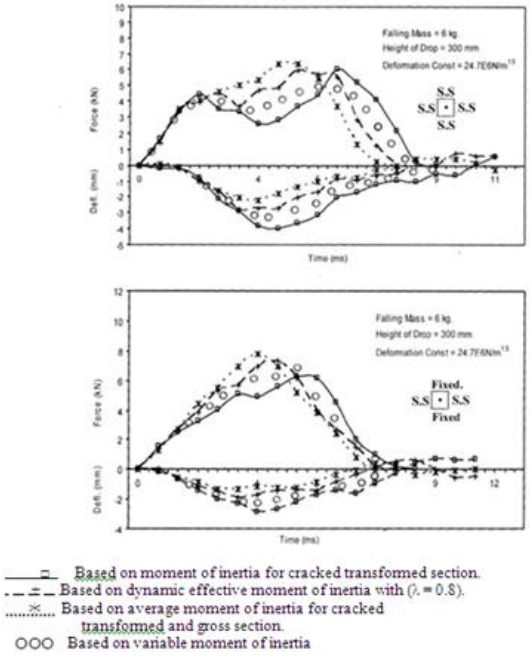


Figure (4) Typical theoretical central impact force and deflection –time histories.

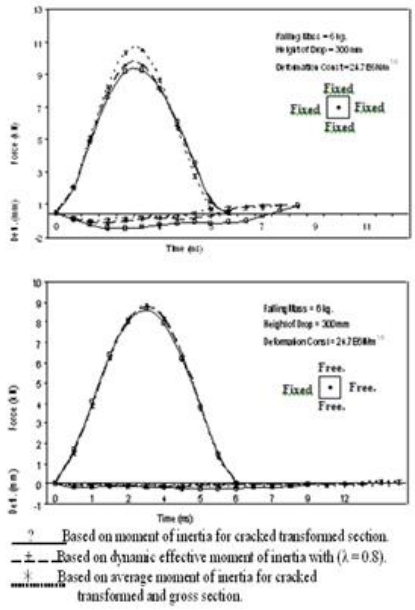


Figure (5) Typical theoretical central impact force and deflection – time histories.

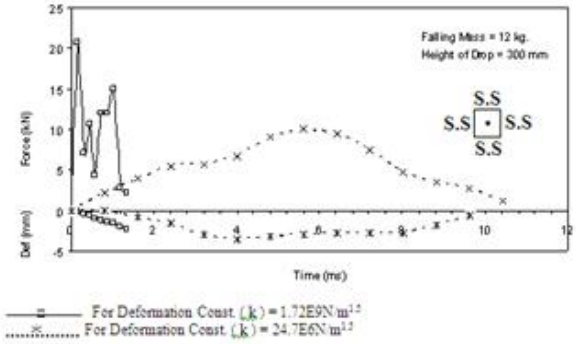


Figure (6) Typical theoretical impact fo deflection – time histories. (based on dynamic effective

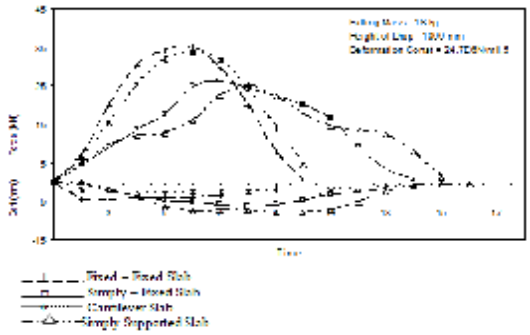
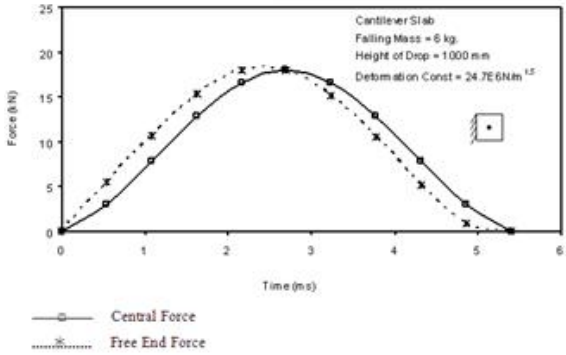


Figure (7) Typical theoretical central central impact force and deflection – time histories(based on dynamic effective moment of inertia,λ=0.8).



Results based on a dynamic effective moment of inertia, (λ=0.8).

Figure (8) Typical theoretical impact force – time history.

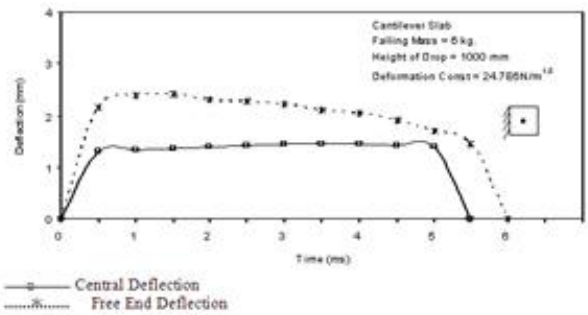


Figure (9) Typical theoretical maximum deflection – time history. (based on dynamic effective moment of inertia, λ=0.8).

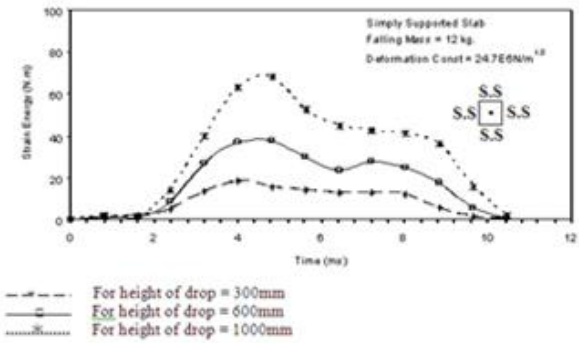
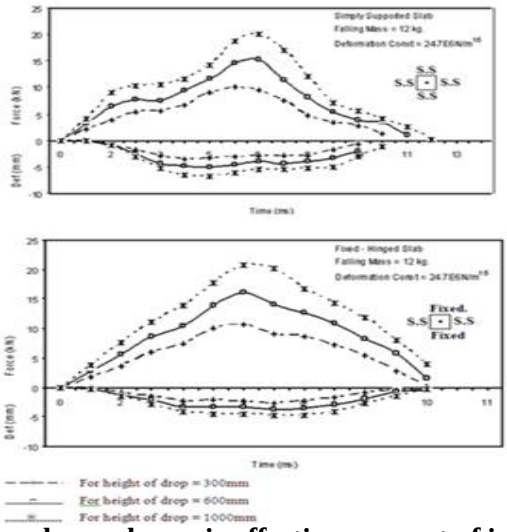
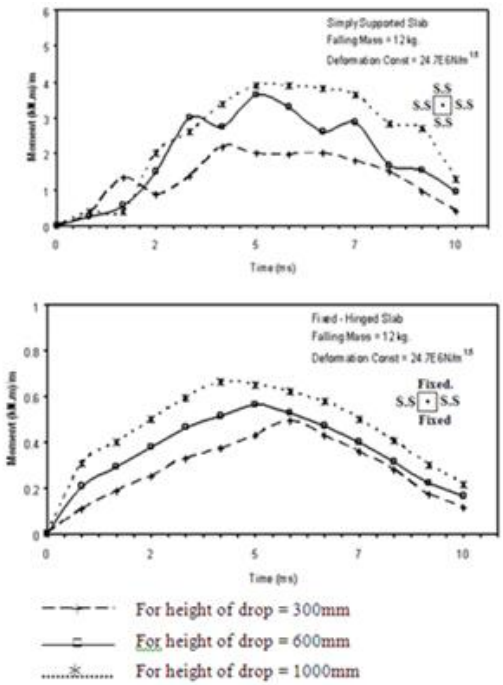


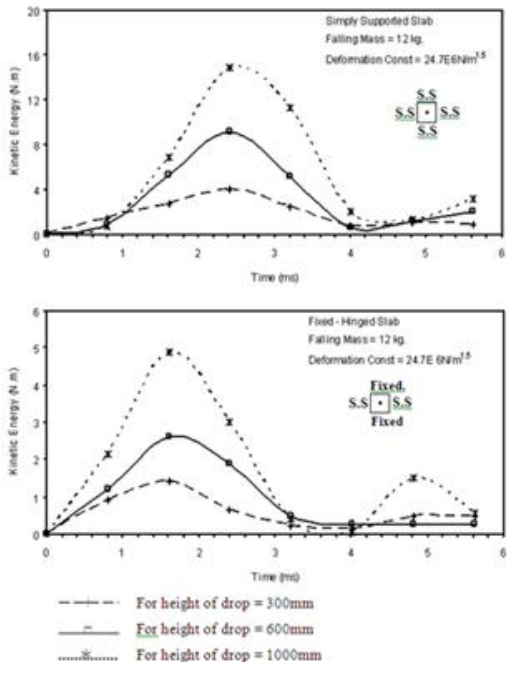
Figure (10) Typical theoretical impact strain energy – time history. (based on dynamic effective moment of inertia, λ=0.8).



Based on a dynamic effective moment of inertia, (λ=0.8).
Figure (11) typical theoretical central impact force and dflection – time histories



Based on a dynamic effective moment of inertia, (λ=0.8).
Figure (12) typical theoretical Central impact moment–time history.



Based on a dynamic effective moment of inertia, (λ=0.8).
Figure (13) Typical theoretical Impact kinetic energy–time history.

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LIST OF SYMBOLS

a	**Relative approach of striking bodies**
D	**Flexural rigidity of the slab**
D_{cr}	**Cracked flexural rigidity of the slab**
D_{deff}	**Dynamic effective flexural rigidity of the slab**
F	**Impact force.**
h	**Slab thickness**
I	**Moment of inertia**
K	**Hertz (deformation) constant.**
K E	**Kinetic energy**
L	**Length**
m_b	**Total mass of the beam**
m_s	**Total mass of the slab**
m_{st}	**Mass of the striker**
T	**Impact duration**
t	**Time.**
U	**Strain energy**
W	**Deflection**
W_o	**Central deflection of the slab**
W_s	**Displacement of the slab**
W_{st}	**Displacement of the striker**
$\overline{m}$	**Mass per unit length of the beam or mass per unit area of the slab**
ω_{ij}	**Angular frequency of free vibration of the slab**
α_e	**Mass ratio (m_s/m_{st})**
μ	**Poisson's ratio**
ρ:	**Steel ratio**
π	**Independent dimensionless Buckingham's pi – factor**
S.S.	**Simply Supported edge**
F.	**Fixed Supported Edge**
Fr	**Free edge**