University of Technology

Building and Construction Eng. Dept.

Final Exam – First Attempt – 2015/2016

Subject: Engineering Analysis & Numerical Methods

Class: 3^{rd} year

Division: All Divisions

Time: 3 Hours

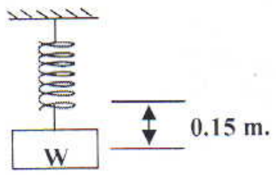
Date :24/ 05 / 2016

Note: All questions have the same marks

Part One: Engineering Analysis– Q1, Q2, Q3 and Q4

Answer three questions only *(50 marks)*

Q1: A mass of 9 kg is suspended from the end of a vertical spring stretches 0.150 m. Assuming no external force, find the position of the weight at any time, if the weight is initially pulled down 0.075m and given an initial velocity of 0.60 m/sec downward. Find the frequency ($\omega_\circ$) and time period ($T_\circ$) .

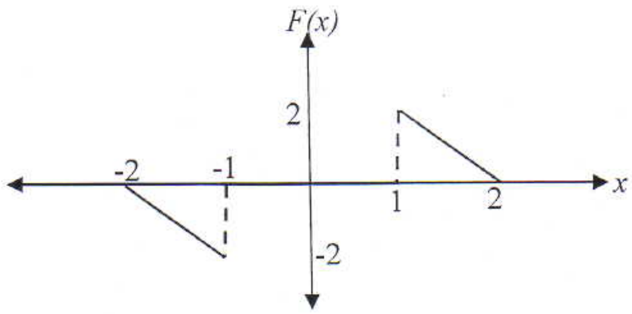


Q2: Solve the following simultaneous differential equations to find *x (t)* and *y (t)*. Given the initial condition y (0) =2 , x (0) =1.

$$\frac{dy}{dt} = -y + 3x$$

$$\frac{dx}{dt} = 4x - 2y$$

Q3: For the periodic function shown below, determine the Fourier coefficients and Fourier series.



Q4: Find the complete solution for the following partial differential equation:

$$x \,.\, U_x + y.\, U_y = 0$$

$$If \qquad U\,(1,1) = 1 \;\; and \;\; U(4,2) = 8$$

Part Two: Numerical Methods –Q5, Q6, Q7, and Q8
*Answer **three** questions **only** *(50 marks)*
Solve for 4 digits after decimal point.

Q5: Solve the following simultaneous linear equations system by using L and U decomposition method.

$$2x - 5y + z = 12$$
$$-x + 3y - z = 8$$
$$3x - 4y + 2z = 16$$

Q6: (a) Find the solution for the following non-linear equation by using Newton-Raphson method. Start with $X_0 = 1.4$:

$$(x-2)^2 - \ln x = 0$$

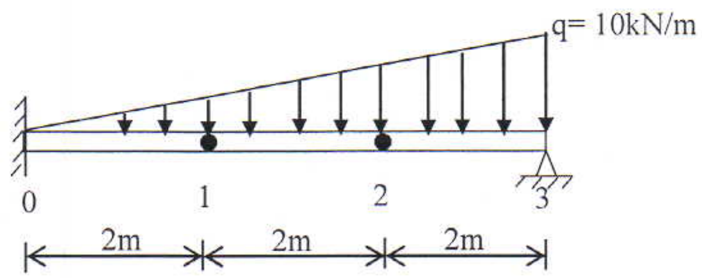
(b) Use Simpson's (3h/8) rule to find the value of the following integration, use n=3.

$$\int_{0.2}^{0.8} (x^3 + x\, e^{x^2}).\, dx$$

Q7: Solve the following initial value problem using Runge–Kutta method to find y (0.2) if y (0) =1 , h= 0.1

$$\frac{dy}{dx} = -2y + x + 4$$

Q8: For the beam shown below, determine the deflection at pivotal points. The stiffness (EI) of the beam is constant.



Notes: 1) $\left(\frac{df}{dx}\right)_i = \frac{1}{2h}.(y_{i+1} - y_{i-1})$ 2) $\left(\frac{d^2f}{dx^2}\right)_i = \frac{1}{h^2}.(y_{i+1} - 2y_i + y_{i-1})$

3) $\left(\frac{d^3f}{dx^3}\right)_i = \frac{1}{2h^3}.(y_{i+2} - 2y_{i+1} + 2y_{i-1} - y_{i-2})$

4) $\left(\frac{d^4f}{dx^4}\right)_i = \frac{1}{h^4}.(y_{i+2} - 4y_{i+1} + 6y_i - 4y_{i-1} + y_{i-2})$

Good luck

Q1|

$m = 9$ ~~kg~~ kg   streches 0.15 m

Req: The position of the wt. at any time, if initially the wt. pulled down 0.075m and given an initial velocity of 0.6 m/sec downward

$$F = K * \delta \Rightarrow K = \frac{F(N)}{\delta} = \frac{9 \times 10}{0.15}$$

$$K = 600 \ N/m$$

$$my'' + \cancel{cy'}^{0} + Ky = 0$$

$$9D^2 + 600 = 0$$

$$D^2 + 66.667 = 0 \Rightarrow D^2 = -66.667$$

$$D = \mp 8.165 i$$

$$y_t = C_1 \cos 8.165t + C_2 \sin 8.165t$$

at $t = 0$ $y = 0.075$ m

$$\therefore C_1 = 0.075$$

then $y_t = 0.075 \cos 8.165t + C_2 \sin 8.165t$

$$y'_t = -.612 \sin 8.165t + 8.165 C_2 \cos 8.165t$$

at $t = 0$ $y' = v = 0.6$

$\therefore 0.6 = 8.165\, C_2 \Rightarrow C_2 = 0.073$

$\therefore y = 0.075 \cos 8.165t + 0.073 \sin 8.165t$

$\omega_0^* = 8.165$ natural frequency

$$\omega_0 = \frac{\omega_0^*}{2\pi} = \frac{8.165}{2\pi} = 1.299 \ {}^1/\text{sec}\ (Hz)$$

$$T_0 = \frac{1}{\omega_0} = 0.769 \text{ sec} \quad \text{time cycle}$$

Q2 $\quad \frac{dy}{dt} = -y + 3x \quad$ —— (A)

$$\frac{dx}{dt} = 4x - 2y$$

$D = \frac{d}{dt} \Rightarrow Dy + y - 3x = 0 \Rightarrow -3x + y(D+1) = 0$ —— (1)

$Dx - 4x + 2y = 0 \Rightarrow x(D-4) + 2y = 0$ —— (2)

$$det. = \begin{vmatrix} -3 & D+1 \\ D-4 & 2 \end{vmatrix} = -D^2 + 3D - 2$$

$$Dx = \begin{vmatrix} 0 & D+1 \\ 0 & 2 \end{vmatrix} = 0$$

$$Dy = \begin{vmatrix} -3 & 0 \\ D-4 & 0 \end{vmatrix} = 0$$

$$x = \frac{Dx}{det.} = \frac{0}{-D^2+3D-2} \Rightarrow (-D^2+3D-2)\,x = 0$$

$$y = \frac{Dy}{det.} = \frac{0}{-D^2+3D-2} \Rightarrow (-D^2+3D-2)\,y = 0$$

To find $x(t)$ & $y(t)$

let $-D^2 + 3D - 2 = 0 \Rightarrow D = 2$

$D = 1$

$$x(t) = c_1 e^{2t} + c_2 e^{t}$$

$$y(t) = c_3 e^{2t} + c_4 e^{t}$$

now sub. $x(t)$, $y(t)$ & $\frac{dy}{dt}$ in eq. (A)

$$\frac{dy}{dt} = c_3 e^{t} + 2c_4 e^{2t}$$

$$\therefore c_3 e^{t} + 2c_4 e^{2t} = -c_3 e^{t} - c_4 e^{2t} + 3c_1 e^{t} + 3c_2 e^{2t}$$

$$= (-c_3 + 3c_1) e^{t} + (-c_4 + 3c_2) e^{2t}$$

$$\therefore c_3 = -c_3 + 3c_1 \Rightarrow c_3 = \frac{3}{2}c_1$$

$$2c_4 = -c_4 + 3c_2 \Rightarrow c_4 = c_2$$

$$\therefore x(t) = c_1 e^{t} + c_2 e^{2t}$$

$$y(t) = \frac{3}{2}c_1 e^{t} + c_2 e^{2t}$$

when $t=0 \Rightarrow x=1,\ y=2 \Rightarrow c_1 + c_2 = 1$ — (3)

$\frac{3}{2}c_1 + c_2 = 2$ — (4)

solving eqs. (3) & (4) To get $c_1 = 2$

$c_2 = -1$

$$\therefore x(t) = 2e^{2t} - e^{t}$$

$$y(t) = 3e^{t} - e^{2t}$$

o.k

Q3: تحليلات

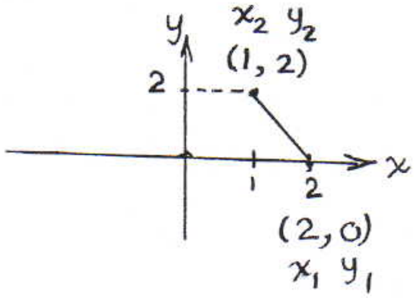
$$l = \frac{Max. - Min.}{2} = \frac{2-(-2)}{2} = 2$$

$F_1(x) = 0$

To find $F_2(x)$:

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{y - y_1}{x - x_1}$$

$$\frac{2-0}{1-2} \quad \frac{y-0}{x-2} \Rightarrow y = -2x+4$$



$$\therefore F(x) = \begin{cases} 0 & 0 \leqslant x \leqslant 1 \\ -2x+4 & 1 \leqslant x \leqslant 2 \end{cases}$$

The function is odd.

$a_0 = 0$ and $a_n = 0$

$$b_n = \frac{2}{l}\int_0^l F(x)\ \sin\frac{n\pi}{l}x\ dx$$

$$= \frac{2}{2}\int_0^2 F(x)\ \sin\frac{n\pi}{l}x\ dx$$

$$= \int_0^1 \cancel{0\ \sin\frac{n\pi}{2}x\ dx} + \int_1^2 (-2x+4)\ \sin\frac{n\pi}{2}x\ dx$$

$$= \left[(-2x+4)\ \frac{-2}{n\pi}\cos\frac{n\pi}{2}x - \frac{8}{n^2\pi^2}\sin\frac{n\pi}{2}x\right]_1^2$$

$$= \left[\frac{-2}{n\pi}\ \cancel{(-4+4)}^{\,0}\cos n\pi - \frac{8}{n^2\pi^2}\ \cancel{\sin n\pi}^{\,0}\right] - \left[\frac{-4}{n\pi}\cos\frac{n\pi}{2} - \frac{8}{n^2\pi^2}\sin\frac{n\pi}{2}\right]$$

$$= \frac{4}{n\pi}\ \cancel{\cos\frac{n\pi}{2}}^{\,0} + \frac{8}{n^2\pi^2}\sin\frac{n\pi}{2}$$

$$\therefore b_n = \frac{8}{n^2\pi^2}\sin\frac{n\pi}{2}$$

$$b_1 = \frac{8}{\pi^2},\ b_2 = 0,\ b_3 = -\frac{8}{9\pi^2},\ b_4 = 0,\ b_5 = \frac{8}{25\pi^2}$$

$$F(x) = \frac{8}{\pi^2}\left[\sin\frac{\pi}{2} - \frac{1}{9}\sin\frac{3\pi}{2} + \frac{1}{25}\sin\frac{5\pi}{2}\right]$$

Eng 2016

Q4

$$u(x,y) = G(x).F(y) = G.F$$
$$u_x = G'F \quad , \quad u_y = GF'$$

$$[xG'F + yGF' = 0] \div GF$$

$$x\frac{G'}{G} + y\frac{F'}{F} = 0$$

$$\therefore x\frac{G'}{G} = \lambda \quad — (1)$$

$$-y\frac{F'}{F} = \lambda \quad — (2)$$

from (1) $\Rightarrow x\dfrac{dG/dx}{G} = \lambda \Rightarrow \dfrac{dG}{G} = \dfrac{\lambda}{x}dx$

$$\ln G = \lambda \ln x + C_1 \Rightarrow G(x) = Ax^{\lambda}$$

From (2) $\Rightarrow \dfrac{dF/dy}{F} = \dfrac{-\lambda}{y} \Rightarrow \dfrac{dF}{F} = \dfrac{-\lambda}{y}dy$

$$\ln F = -\lambda \ln y + C_2 \Rightarrow F(y) = \frac{B}{y^{\lambda}}$$

$$\therefore u(x,y) = Ax^{\lambda} \cdot \frac{B}{y^{\lambda}} = C\left(\frac{x}{y}\right)^{\lambda}$$

now at $x=1$, $y=1$, $u=1 \Rightarrow C=1$

$$x=4,\ y=2,\ u=8 \Rightarrow 8=\left(\frac{4}{2}\right)^{\lambda} \Rightarrow \lambda=3$$

$$\therefore u(x,y) = \left(\frac{x}{y}\right)^3 = \frac{x^3}{y^3}$$

ans.