



University of Technology

Building and Construction Eng. Dept.
Final Exam – First Attempt – 2015/2016
Subject: Engineering Analysis & Numerical Methods

Class: 3rd year

Time: 3 Hours

Division: All Divisions

Date : 24/ 05 / 2016

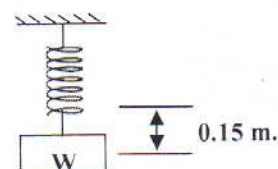
Note: All questions have the same marks



Part One: Engineering Analysis– Q1, Q2, Q3 and Q4

Answer three questions only (50 marks)

Q1: A mass of 9 kg is suspended from the end of a vertical spring stretches 0.150 m. Assuming no external force, find the position of the weight at any time, if the weight is initially pulled down 0.075m and given an initial velocity of 0.60 m/sec downward. Find the frequency (ω) and time period (T) .

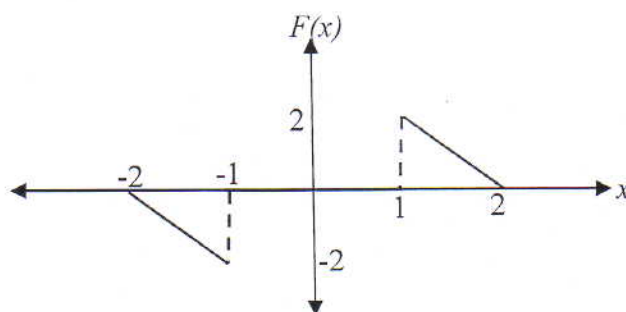


Q2: Solve the following simultaneous differential equations to find $x(t)$ and $y(t)$. Given the initial condition $y(0) = 2$, $x(0) = 1$.

$$\frac{dy}{dt} = -y + 3x$$

$$\frac{dx}{dt} = 4x - 2y$$

Q3: For the periodic function shown below, determine the Fourier coefficients and Fourier series.



Q4: Find the complete solution for the following partial differential equation:

$$x \cdot U_x + y \cdot U_y = 0$$

$$\text{If } U(1,1) = 1 \text{ and } U(4,2) = 8$$

Part Two: Numerical Methods –Q5, Q6, Q7, and Q8

***Answer three questions only (50 marks)**

***Solve for 4 digits after decimal point.**

Q5: Solve the following simultaneous linear equations system by using L and U decomposition method.

$$2x - 5y + z = 12$$

$$-x + 3y - z = 8$$

$$3x - 4y + 2z = 16$$

Q6: (a) Find the solution for the following non-linear equation by using Newton-Raphson method. Start with $X_0 = 1.4$:

$$(x - 2)^2 - \ln x = 0$$

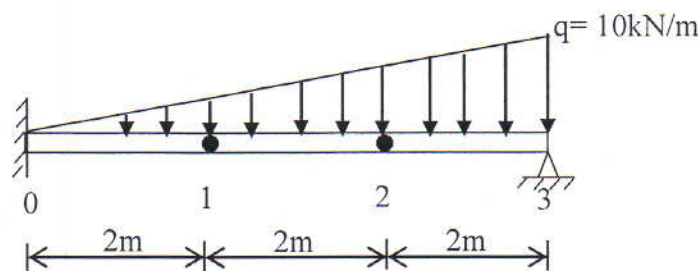
(b) Use Simpson's (3h/8) rule to find the value of the following integration, use $n=3$.

$$\int_{0.2}^{0.8} (x^3 + x e^{x^2}) \cdot dx$$

Q7: Solve the following initial value problem using Runge-Kutta method to find $y(0.2)$ if $y(0) = 1$, $h = 0.1$

$$\frac{dy}{dx} = -2y + x + 4$$

Q8: For the beam shown below, determine the deflection at pivotal points. The stiffness (EI) of the beam is constant.



Notes:

- 1) $\left(\frac{df}{dx}\right)_i = \frac{1}{2h} \cdot (y_{i+1} - y_{i-1})$
- 2) $\left(\frac{d^2f}{dx^2}\right)_i = \frac{1}{h^2} \cdot (y_{i+1} - 2y_i + y_{i-1})$
- 3) $\left(\frac{d^3f}{dx^3}\right)_i = \frac{1}{2h^3} \cdot (y_{i+2} - 2y_{i+1} + 2y_{i-1} - y_{i-2})$
- 4) $\left(\frac{d^4f}{dx^4}\right)_i = \frac{1}{h^4} \cdot (y_{i+2} - 4y_{i+1} + 6y_i - 4y_{i-1} + y_{i-2})$

Good luck

Q11

$$m = 9 \text{ kg}$$

stretches 0.15 m

Req: The position of the wt. at any time, if initially the wt. pulled down 0.075 m and given an initial velocity of 0.6 m/sec downward

$$F = k \times \delta \Rightarrow k = \frac{F(N)}{\delta} = \frac{9 \times 10}{0.15}$$

$$k = 600 \text{ N/m}$$

$$m y'' + \cancel{c y'} + k y = 0$$

$$9 D^2 + 600 = 0$$

$$D^2 + 66.667 = 0 \Rightarrow D^2 = -66.667$$

$$D = \pm 8.165 i$$

$$y_t = C_1 \cos 8.165 t + C_2 \sin 8.165 t$$

$$\text{at } t=0 \quad y = 0.075 \text{ m}$$

$$\therefore C_1 = 0.075$$

then $y_t = 0.075 \cos 8.165 t + C_2 \sin 8.165 t$

$$y'_t = -0.612 \sin 8.165 t + 8.165 C_2 \cos 8.165 t$$

$$\text{at } t=0 \quad y' = v = 0.6$$

$$\Rightarrow 0.6 = 8.165 C_2 \Rightarrow C_2 = 0.073$$

$$\Rightarrow y = 0.075 \cos 8.165t + 0.073 \sin 8.165t$$

$$\omega_0^* = 8.165 \quad \text{natural frequency}$$

$$\omega_0 = \frac{\omega_0^*}{2\pi} = \frac{8.165}{2\pi} = 1.299 \text{ 1/sec (Hz)}$$

$$T_0 = \frac{1}{\omega_0} = 0.769 \text{ sec time cycle}$$

Q2 $\frac{dy}{dt} = -y + 3x \quad \text{--- (A)}$

$$\frac{dx}{dt} = 4x - 2y$$

$$D = \frac{d}{dt} \Rightarrow Dy + y - 3x = 0 \Rightarrow -3x + y(D+1) = 0 \quad \text{--- (1)}$$

$$DX - 4X + 2y = 0 \Rightarrow X(D-4) + 2y = 0 \quad \text{--- (2)}$$

$$\text{det.} = \begin{vmatrix} -3 & D+1 \\ D-4 & 2 \end{vmatrix} = -D^2 + 3D - 2$$

$$DX = \begin{vmatrix} 0 & D+1 \\ 0 & 2 \end{vmatrix} = 0$$

$$Dy = \begin{vmatrix} -3 & 0 \\ D-4 & 0 \end{vmatrix} = 0$$

$$X = \frac{DX}{\text{det.}} = \frac{0}{-D^2 + 3D - 2} \Rightarrow (-D^2 + 3D - 2)X = 0$$

$$y = \frac{Dy}{\text{det.}} = \frac{0}{-D^2 + 3D - 2} \Rightarrow (-D^2 + 3D - 2)y = 0$$

To find $x(t)$ & $y(t)$

$$\text{let } -D^2 + 3D - 2 = 0 \Rightarrow D = 2$$

$$D = 1$$

$$x(t) = C_1 e^{2t} + C_2 e^t$$

$$y(t) = C_3 e^{2t} + C_4 e^t$$

now sub. $x(t)$, $y(t)$ & $\frac{dy}{dt}$ in eq. (A)

$$\frac{dy}{dt} = C_3 e^t + 2C_4 e^{2t}$$

$$\therefore C_3 e^t + 2C_4 e^{2t} = -C_3 e^t - C_4 e^{2t} + 3C_1 e^t + 3C_2 e^{2t}$$

$$= (-C_3 + 3C_1) e^t + (-C_4 + 3C_2) e^{2t}$$

$$\therefore C_3 = -C_3 + 3C_1 \Rightarrow C_3 = \frac{3}{2}C_1$$

$$2C_4 = -C_4 + 3C_2 \Rightarrow C_4 = C_2$$

$$\therefore x(t) = C_1 e^t + C_2 e^{2t}$$

$$y(t) = \frac{3}{2}C_1 e^t + C_2 e^{2t}$$

$$\text{When } t=0 \Rightarrow x=1, y=2 \Rightarrow \begin{aligned} C_1 + C_2 &= 1 & \text{--- (3)} \\ \frac{3}{2}C_1 + C_2 &= 2 & \text{--- (4)} \end{aligned}$$

$$\text{solving eqs. (3) \& (4) To get } \begin{aligned} C_1 &= 2 \\ C_2 &= -1 \end{aligned}$$

$$\therefore x(t) = 2e^{2t} - e^t$$

$$y(t) = 3e^t - e^{2t}$$

O.K

Q3. Two marks

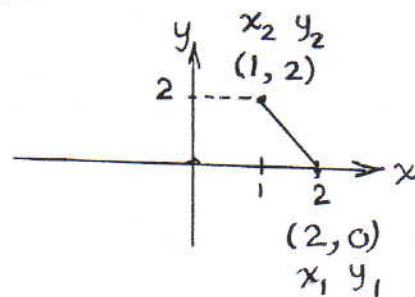
$$l = \frac{\text{Max.} - \text{Min.}}{2} = \frac{2 - (-2)}{2} = 2$$

$$F_1(x) = 0$$

To find $F_2(x)$:

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{y - y_1}{x - x_1}$$

$$\frac{2 - 0}{1 - 2} = \frac{y - 0}{x - 2} \Rightarrow y = -2x + 4$$



$$F(x) = \begin{cases} 0 & 0 \leq x \leq 1 \\ -2x + 4 & 1 \leq x \leq 2 \end{cases}$$

The function is odd.

$$\alpha_0 = 0 \text{ and } \alpha_n = 0$$

$$b_n = \frac{2}{l} \int_0^l F(x) \sin \frac{n\pi}{l} x \, dx$$

$$= \frac{2}{2} \int_0^2 F(x) \sin \frac{n\pi}{2} x \, dx$$

$$= \int_0^1 0 \sin \frac{n\pi}{2} x \, dx + \int_1^2 (-2x + 4) \sin \frac{n\pi}{2} x \, dx$$

$$= \left[(-2x + 4) \frac{-2}{n\pi} \cos \frac{n\pi}{2} x - \frac{8}{n^2 \pi^2} \sin \frac{n\pi}{2} x \right]_1^2$$

$$= \left[\frac{-2}{n\pi} (-4 + 4) \cos n\pi - \frac{8}{n^2 \pi^2} \sin n\pi \right] - \left[\frac{-4}{n\pi} \cos \frac{n\pi}{2} - \frac{8}{n^2 \pi^2} \sin \frac{n\pi}{2} \right]$$

$$= \frac{4}{n\pi} \cos \frac{n\pi}{2} + \frac{8}{n^2 \pi^2} \sin \frac{n\pi}{2} - \frac{8}{n^2 \pi^2} \sin \frac{n\pi}{2}$$

$$\therefore b_n = \frac{8}{n^2 \pi^2} \sin \frac{n\pi}{2}$$

$$b_1 = \frac{8}{\pi^2}, b_2 = 0, b_3 = -\frac{8}{9\pi^2}, b_4 = 0, b_5 = \frac{8}{25\pi^2}$$

$$f(x) = \frac{8}{\pi^2} \left[\sin \frac{\pi}{2} - \frac{1}{9} \sin \frac{3\pi}{2} + \frac{1}{25} \sin \frac{5\pi}{2} \right]$$

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Q4

$$u(x, y) = G(x) \cdot F(y) = G \cdot F$$

$$u_x = G' \cdot F, \quad u_y = G \cdot F'$$

$$[xG'F + yGF' = 0] \div GF$$

$$x \frac{G'}{G} + y \frac{F'}{F} = 0$$

$$\therefore x \frac{G'}{G} = -\lambda \quad \text{--- (1)}$$

$$-y \frac{F'}{F} = -\lambda \quad \text{--- (2)}$$

$$\text{from (1)} \Rightarrow x \frac{dG/dx}{G} = -\lambda \Rightarrow \frac{dG}{G} = -\frac{\lambda}{x} dx$$

$$\ln G = -\lambda \ln x + C_1 \Rightarrow G(x) = A x^{-\lambda}$$

$$\text{from (2)} \Rightarrow \frac{dF/dy}{F} = \frac{-\lambda}{y} \Rightarrow \frac{dF}{F} = -\frac{\lambda}{y} dy$$

$$\ln F = -\lambda \ln y + C_2 \Rightarrow F(y) = \frac{B}{y^{\lambda}}$$

$$\therefore u(x, y) = A x^{-\lambda} \cdot \frac{B}{y^{\lambda}} = C \left(\frac{x}{y} \right)^{-\lambda}$$

$$\text{now at } x=1, y=1, u=1 \Rightarrow C=1$$

$$x=4, y=2, u=8 \Rightarrow 8 = \left(\frac{4}{2} \right)^{-\lambda} \Rightarrow \lambda = -3$$

$$\therefore u(x, y) = \left(\frac{x}{y} \right)^3 = \frac{x^3}{y^3}$$

ans.

Q5

$$\begin{bmatrix} 2 & -5 & 1 \\ -1 & 3 & -1 \\ 3 & -4 & 2 \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

$$l_{11} = 2, \quad l_{21} = -1, \quad l_{31} = 3$$

$$l_{11} u_{12} = -5 \Rightarrow u_{12} = -\frac{5}{2}$$

$$l_{21} u_{12} + l_{22} = 3 \Rightarrow l_{22} = \frac{1}{2}$$

$$l_{31} u_{12} + l_{32} = -4 \Rightarrow l_{32} = \frac{7}{2}$$

$$l_{11} u_{13} = 1 \Rightarrow u_{13} = \frac{1}{2}$$

$$l_{21} u_{13} + l_{22} u_{23} = -1 \Rightarrow u_{23} = -1$$

$$l_{31} u_{13} + l_{32} u_{23} + l_{33} = 2 \Rightarrow l_{33} = 4$$

$$\therefore L = \begin{bmatrix} 2 & 0 & 0 \\ -1 & 1/2 & 0 \\ 3 & 7/2 & 4 \end{bmatrix}, \quad U = \begin{bmatrix} 1 & -5/2 & 1/2 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$Lb' = b$$

$$\begin{bmatrix} 2 & 0 & 0 \\ -1 & 1/2 & 0 \\ 3 & 7/2 & 4 \end{bmatrix} \begin{bmatrix} b'_1 \\ b'_2 \\ b'_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 8 \\ 16 \end{bmatrix}$$

$$\therefore b'_1 = 6$$

$$b'_2 = 28$$

$$b'_3 = -25$$



$$Ux = b'$$

$$\begin{bmatrix} 1 & -5/2 & 1/2 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 28 \\ -25 \end{bmatrix}$$

$$\therefore z = -25$$

$$y - z = 28 \Rightarrow y = 3$$

$$x - \frac{5}{2}y + \frac{1}{2}z = 6 \Rightarrow x = 26$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 26 \\ 3 \\ -25 \end{bmatrix}$$

Q6: a $x_0 = 2$

$$F(x) = (x-2)^2 - \ln x = 0$$

$$F'(x) = 2(x-2) - \frac{1}{x}$$

The Newton's method is:

$$x_{n+1} = x_n - \frac{(x_n-2)^2 - \ln x_n}{2(x_n-2) - \frac{1}{x_n}}$$

x_n	$F(x_n)$	$F'(x_n)$	x_{n+1}
1.4	0.0235	-1.9143	1.4123
1.4123	1.7171×10^{-4}	-1.8835	1.4123

$$\therefore x = 1.4123$$

Q6/b $I = \int_{0.2}^{0.8} (x^3 + x e^{x^2}) dx$

$$h = \frac{b-a}{n} = \frac{0.8-0.2}{3} = 0.2$$

$$f(x) = (x^3 + x e^{x^2})$$

x	f(x)	
0.2	0.2162	f_0
0.4	0.5334	f_1
0.6	1.0759	f_2
0.8	2.0292	f_3

$$I = \frac{3h}{8} [f_0 + 3f_1 + 3f_2 + f_3]$$

$$= \frac{3 \times 0.2}{8} [0.2162 + 3 \times 0.5334 + 3 \times 1.0759 + 2.0292]$$

$$= 0.5305 \text{ ans.}$$

Q7 | $\frac{dy}{dx} = -2y + x + 4$, $x_0 = 0$, $y_0 = 1$, $h = 0.1$

$$k_1 = f(x_0, y_0) h = f(0, 1) \times 0.1 = (-2 \times 1 + 0 + 4) \times 0.1$$

$\therefore k_1 = 0.2$

$$k_2 = f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right) \times h = f\left(0 + \frac{0.1}{2}, 1 + \frac{0.2}{2}\right) \times 0.1$$

$$k_2 = 0.185$$

$$k_3 = f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right) h = f\left(0.05, 1 + \frac{0.185}{2}\right) \times 0.1$$

$$k_3 = 0.1865$$

$$k_4 = f(x_0 + h, y_0 + k_3) h = f(0.1, 1 + 0.1865) \times 0.1$$

$$= f(0.1, 1.1865) \times 0.1 = 0.1727$$

$$\Delta = \frac{1}{6} (k_1 + 2k_2 + 2k_3 + k_4) = 0.1859$$

$$y_{0.1} = y_0 + \Delta = 1.1859$$

now $x_0 = 0.1$, $y_0 = 1.1859$, $h = 0.1$

$$k_1 = f(x_0, y_0) h = 0.1728$$

$$k_2 = f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right) h = f(0.15, 1.2723) \times 0.1$$

$$= 0.1605$$



$$k_3 = f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right) h$$

$$= f(0.15, 1.26615) \times 0.1 = 0.16177$$

$$k_4 = f(x_0 + h, y_0 + k_3) \times h$$

$$= f(0.2, 1.3477) \times 0.1 = 0.15046$$

$$\Delta = \frac{1}{6} (k_1 + 2k_2 + 2k_3 + k_4)$$

$$= 0.1613$$

$$y_{0.2} = y_{0.1} + \Delta = 1.1859 + 0.1613$$

$$\therefore y_{0.2} = 1.3472 \quad \underline{\underline{\text{ans.}}}$$

Q.3: عرصة

$$\frac{d^4 y}{dx^4} = \frac{w}{EI}$$

$$\frac{w_i}{EI} = \frac{y_{i+2} - 4y_{i+1} + 6y_i - 4y_{i-1} + y_{i-2}}{h^4}$$

B.C(1):

At $i=0$: $y_0=0$ and $\theta=0$

$$\theta = \frac{dy}{dx} = \frac{y_{i+1} - y_{i-1}}{2h}$$

$$0 = \frac{y_1 - y_{-1}}{2(2)} \Rightarrow y_1 = y_{-1}$$

B.C(2):

At $i=3$: $y_3=0$ and $M=0$

$$\frac{d^2 y}{dx^2} = \frac{M_i}{EI} = \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}$$

$$0 = \frac{y_4 - 2y_3 + y_2}{(2)^2} \Rightarrow y_4 = -y_2$$

At $i=1$:

$$\frac{w_1}{h} = \frac{10}{3h} \Rightarrow w_1 = \frac{10}{3} \text{ KN/m}$$

$$\frac{-10}{3EI} = \frac{y_3 - 4y_2 + 6y_1 - 4y_0 + y_{-1}}{(2)^4}$$

$$7y_1 - 4y_2 = \frac{160}{3EI}$$

$$\therefore y_1 = \frac{-160}{3EI} + \frac{4}{7} y_2 \quad \text{--- (1)}$$

At i = 2

$$\frac{w_2}{2h} = \frac{10}{3h} \Rightarrow w_2 = \frac{20}{3} \text{ KN/m}$$

$$\frac{-20}{3EI} = \frac{y_4 - 4y_3 + 6y_2 - 4y_1 + y_0}{(2)^4}$$

$$5y_2 - 4y_1 = \frac{-320}{3EI}$$

$$\therefore y_2 = \frac{-320}{3EI} + \frac{4}{5} y_1 \quad \text{--- (2)}$$

Solving Eqs. ① and ②:

$$y_1 = \frac{224.561}{EI}$$