University of Technology
Building and Construction Eng. Dept.
Final Exam- First Attempt – 2015/2016

Branch : All Branches — Class: 2^{nd} year
Subject : Mathematics IV — Time: 3 Hours
Examiner :Mathematics Committee — Date : 5/6/ 2016

Note: Answer five questions only

Q1. Convert the following integral to the cylindrical coordinates and then evaluate the new integral

$$\int_{0}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{-(x^2+y^2)}^{(x^2+y^2)} xy^2.dz.dy.dx$$ (20 mark)

Q2. A. Find the inverse of the matrix $A=\begin{bmatrix}6 & -4\\ -4 & -2\end{bmatrix}$ (10 mark)

Q2. B. Find eigen values of the matrix $B=\begin{bmatrix}4 & -1\\ 2 & 1\end{bmatrix}$ (10 mark)

Q3.A. Check the analyticity for the following function then find $\frac{dw}{dz}$

$$w = x^2 - y^2 + (2\,y\,x + 5)\,i$$ (10 mark)

Q3. B. Evaluate :

$$w = [2 + 2\sqrt{3}i]^{100}$$ (10 mark)

Q4. Find the nature of the series $\sum_{n=1}^{\infty}\frac{(3n+1)!}{4n!}$ (20 mark)

Q5. Find the Maclaurin series for the function $f_x = 6\sin 2x$ *at (x= 0)*. Write at least three terms. (20 mark)

Q6. Find the real and imaginary parts for the ($\frac{Z_1.Z_2}{Z_1+Z_2}$) If $Z_1 = 1-3i$ and $Z_2 = -2+5i$

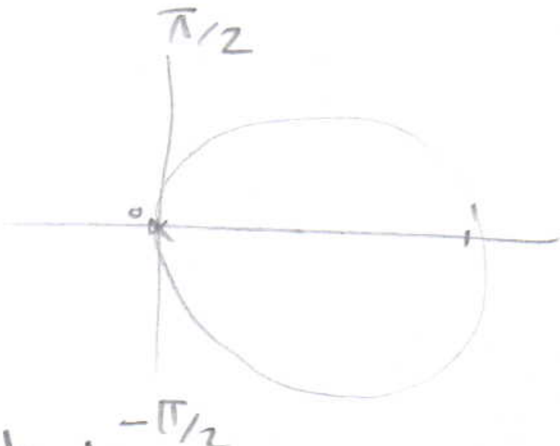
(20 mark)

Q7. Change to polar form and evaluate $\int\int e^{x^2+y^2}.dy.dx$ for first and fourth quadrants, where R is the region bounded by the line x = 0 and curve $x = \sqrt{1-y^2}$ (20 mark)

Good Luck

Q/ Convert to cylindrical coordinates, then evaluate the new integral.

$$\int_{0}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{-(x^2+y^2)}^{(x^2+y^2)} x\,y^2 \cdot dz \cdot dy \cdot dx$$



$$z_1 = -r^2 \qquad z_2 = r^2$$

$$y = -\sqrt{1-x^2} \Rightarrow r = 1$$

$$\int_{-\pi/2}^{\pi/2}\int_{0}^{1}\int_{-r^2}^{r^2} (r\cos\theta)\cdot(r^2\sin^2\theta)\cdot dz\cdot r\cdot dr\cdot d\theta$$

$$= \int_{-\pi/2}^{\pi/2}\int_{0}^{1}\int_{-r^2}^{r^2} r^4 \cdot \sin^2\theta \cdot \cos\theta \cdot dz \cdot dr \cdot d\theta$$

$$= \int_{-\pi/2}^{\pi/2}\int_{0}^{1} (z)\Big|_{-r^2}^{r^2} \cdot r^4 \cdot \sin^2\theta \cdot \cos\theta \cdot dr \cdot d\theta$$

$$= \int_{-\pi/2}^{\pi/2}\int_{0}^{1} (r^2+r^2)\cdot r^4 \cdot \sin^2\theta \cdot \cos\theta \cdot dr \cdot d\theta = \int_{-\pi/2}^{\pi/2}\int_{0}^{1} 2r^6 \cdot \sin^2\theta \cdot \cos\theta \cdot dr \cdot d\theta$$

$$= 2\int_{-\pi/2}^{\pi/2} \left(\frac{r^7}{7}\right)\Big|_{0}^{1} \cdot \sin^2\theta \cdot \cos\theta \cdot d\theta = \frac{2}{7}\cdot\left[\frac{\sin^3\theta}{3}\right]_{-\pi/2}^{\pi/2}$$

$$= \frac{2}{7}\left\{\frac{\sin^3 \pi/2}{3} - \frac{\sin^3(-\pi/2)}{3}\right\} = \underline{0.19}$$

Q2.A

$$A = \begin{bmatrix} 6 & -4 \\ -4 & -2 \end{bmatrix}$$

$$|A| = (6 \times -2) - (-4 \times -4) = -28$$

$$\operatorname{Cof} A = \begin{array}{c} \\ - \end{array} \begin{array}{cc} + & - \\ \begin{bmatrix} -2 & 4 \\ +4 & 6 \end{bmatrix} \end{array} \begin{array}{c} \\ + \end{array}$$

$$\operatorname{adj} A = \operatorname{Cof} A^T = \begin{bmatrix} -2 & 4 \\ 4 & 6 \end{bmatrix}$$

$$A^{-1} = \frac{\operatorname{Cof} A^T}{|A|} = -\frac{1}{28} \begin{bmatrix} -2 & 4 \\ 4 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{14} & -\frac{1}{7} \\ -\frac{1}{7} & -\frac{3}{14} \end{bmatrix}$$

Q2.B

$$B = \begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix}$$

$$\begin{vmatrix} 4-\lambda & -1 \\ 2 & 1-\lambda \end{vmatrix} = 0$$

$$(4-\lambda)(1-\lambda) + 2 = 0$$

$$\lambda^2 - 5\lambda + 4 + 2 = 0$$

$$\lambda^2 - 5\lambda + 6 = 0$$

$$(\lambda - 3)(\lambda - 2) = 0$$

$$\lambda_1 = 3$$

$$\lambda_2 = 2$$

Q 3. A

$$w = x^2 - y^2 + (2yx + 5)i$$

$$u = x^2 - y^2 \qquad v = 2xy + 5$$

$$\frac{\partial u}{\partial x} = 2x \qquad \frac{\partial v}{\partial y} = 2x$$

$$\frac{\partial u}{\partial y} = -2y \qquad -\frac{\partial v}{\partial x} = -2y$$

$\therefore$ ~~The~~ function is analytical

$$\frac{dw}{dz} = \frac{du}{dx} + i\frac{dv}{dx}$$

$$= 2x + 2yi$$

Q 3. B $\quad w = [2 + 2\sqrt{3}\, i]^{100} = z^a$

$$a = 100 \quad \& \quad z = (2 + 2\sqrt{3}\, i)$$

$$|z| = \sqrt{4 + 4(3)} = \sqrt{16} = 4$$

$$\cos\theta = \frac{2}{4} = \frac{1}{2} \qquad \sin \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$$

$$\therefore \theta = \frac{\pi}{6}$$

$$w = 4^{100} \exp\left[i(100)\left(\frac{\pi}{6} + 2k\pi\right)\right]$$

$$= 4^{100} \exp\left[i\left(\frac{50}{2}\pi + 200k\pi\right)\right]$$

Q4

$$\sum_{n=1}^{\infty} \frac{(3n+1)!}{4!}$$

$$a_n = \frac{(3n+1)!}{4n!}$$

$$a_{n+1} = \frac{(3n+4)!}{4(n+1)!}$$

$$\rho = \lim_{n\to\infty} \frac{a_{n+1}}{a_n}$$

$$= \lim_{n\to\infty} \frac{\dfrac{(3n+4)!}{4(n+1)!}}{\dfrac{(3n+1)!}{4n!}}$$

$$= \lim_{n\to\infty} \frac{\dfrac{(3n+4)(3n+3)(3n+2)\cancel{(3n+1)}\cancel{3n!}}{\cancel{4}(n+1)\cancel{(4n!)}}}{\dfrac{\cancel{(3n+1)}\cancel{(3n!)}}{\cancel{(4n!)}}}$$

$$= \lim_{n\to\infty} \frac{(3n+4)(3n+3)(3n+2)}{n+1}$$

$$= \infty$$

$\rho > 1$ — The series is divergent

Q5 $6\sin 2x$

$f_x = 6\sin 2x$ $\quad f(0) = 0$

$f'(x) = 12\cos 2x$ $\quad f(0) = 12$

$f'' = -24\sin 2x$ $\quad f(0) = 0$

$f''' = -48\cos 2x$ $\quad f(0) = -48$

$f^{IV} = +96\sin 2x$ $\quad f(0) = 0$

$f^{V} = 192\cos 2x$ $\quad f(0) = 192$

$f(x) = f(0) + x \cdot f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \dots$

$6\sin 2x = 12x + (-48)\frac{x^3}{3!} + 192\frac{x^5}{5!}$

$6\sin 2x = 12x - 8x^3 + 1.6x^5$

Q6

$$\frac{z_1 \cdot z_2}{z_1 + z_2} = \frac{(1-3i)(-2+5i)}{(1-3i)+(-2+5i)}$$

$$= \frac{-2+6i+5i+15}{-1+2i}$$

$$= \frac{13+11i}{-1+2i}$$

$$= \frac{13+11i}{-1+2i} * \frac{-1-2i}{-1-2i}$$

$$= \frac{-13-11i-26i+22}{1+4}$$

$$= \frac{9-37i}{5}$$

$$= \frac{5}{9} - \frac{37}{5}i$$

$$\therefore \; x = \frac{5}{9}$$

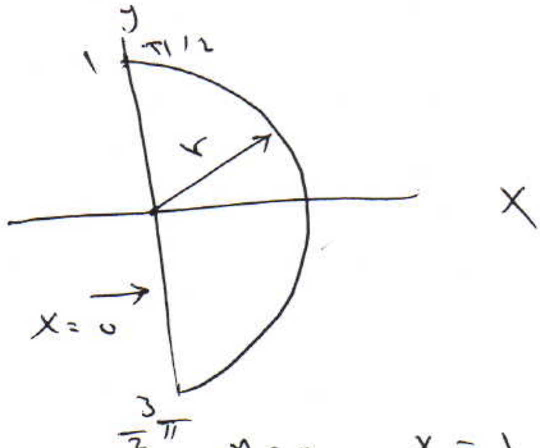
$$y = -\frac{37}{5}$$

Q7 $\iint e^{x^2+y^2} \cdot dx \cdot dy$

$x=0 \quad , \quad x=\sqrt{1-y^2}$

$x^2+y^2=r^2$

$= 2\int_0^{\pi/2}\int_0^1 e^{r^2} \cdot r \cdot dr \cdot d\theta$



$y=0 \quad x=1$

$y=1 \quad x=0$

$y=-1 \quad x=0$

$= 2\int_0^{\pi/2} \left[\frac{1}{2} e^{r^2}\right]_0^1 d\theta$

$= \int_0^{\pi/2} (e^1-1) \cdot d\theta$

$= (e^1-1)\left[\theta\right]_0^{\pi/2}$

$= \frac{\pi}{2}(e^1-1) = 2\ 718$