University of Technology
Building and Construction Eng. Dept.
Final Exam- First Attempt – 2014/2015

Branch : All Branches — **Class: 2nd year**
Subject : Mathematics II — **Time: 3 Hours**
Examiner :Mathematics Committee — **Date : 14/6/ 2015**

Note: Answer Eight questions only

Q1. If $f(x,y)=e^x y^2 - x^3 \ln y$, prove that (f_{xxy}, f_{xyx} , *and* f_{yxx}) all are equal. **(12.5 mark)**

Q2. Find all relative maxima and minima of $f(x,y)=2x^2+y^2+6xy+10x-6y+5$ **(12.5 mark)**

Q3. Solve the following differential equation: $y'+\frac{4}{x}y=x^4$ **(12.5 mark)**

Q4. Using undetermined coefficient method to find the general solution of **(12.5 mark)**

$$y''+y'=5e^x-\sin 2x$$

Q5. Find an equation of the plane through $P_0(2,-1,-1)$ and $P_1(1,2,3)$ and perpendicular to the plane $2x+3y-5z-6=0$. **(12.5 mark)**

Q6. Find the volume of the solid within the cylinder $x^2+y^2=16$ and between the planes $y-z=3$ and $z=2$. **(12.5 mark)**

Q7. Evaluate the integral $\int\int(x+y).dy.dx$, if region of integral (R) enclosed by $0\le y\le 3$ and $1\le x\le\sqrt{4-y}$ **(12.5 mark)**

Q8. Determine the eigin values only for for the following equations: **(12.5 mark)**

$$2x+3y=\lambda x$$
$$4x+y=\lambda y$$

Q9. Find the nature of the series $\sum_{n=1}^{\infty}\frac{1(n!)^2}{2(2n!)^2}$ **(12.5 mark)**

Q10. If $\cos z=\frac{e^{iz}+e^{-iz}}{2}$ and $\sin z=\frac{e^{iz}-e^{-iz}}{2i}$ prove that $1+\tan^2 z=\sec^2 z$

(12.5 mark)

Good Luck

Q1 $f(x,y) = e^x \cdot y^2 - x^3 \cdot \ln y$

$f_x = e^x \cdot y^2 - 3x^2 \cdot \ln y$

$f_{xx} = e^x \cdot y^2 - 6x \cdot \ln y$

$f_{xxy} = 2e^x \cdot y - 6\frac{x}{y}$

$f_{xy} = 2 \cdot e^x \cdot y - 3\frac{x^2}{y}$

$f_{xyx} = 2e^x \cdot y - 6\frac{x}{y}$

$f_y = 2e^x \cdot y - \frac{x^3}{y}$

$f_{yx} = 2e^x \cdot y - 3\frac{x^2}{y}$

$f_{yxx} = 2e^x \cdot y - 6\frac{x}{y}$

$\therefore f_{xxy} = f_{xyx} = f_{yxx}$

Q2 $f(x,y) = 2x^2 + y^2 + 6xy + 10x - 6y + 5$

Sol.

$f_x = 4x + 6y + 10 = 0$

$\therefore\ y = -\frac{2}{3}x - \frac{5}{3}$ ...(1)

$f_y = 2y + 6y - 6 = 0$

sub eq.(1) in f_y eq.

$-\frac{4}{3}x + 6x - 6 - \frac{10}{3} = 0$

$\therefore\ x = +2$

$y = -3$ $(2, -3)$

$f_{xx} = 4$

$f_{yy} = 2$

$f_{xy} = 6$

$f_{xx} \cdot f_{yy} - (f_{xy})^2 = 4 * 2 - 36 = -28 < 0$

$f_{xx} > 0$ @ $(2, -3)$

$\therefore$ No maxima & minima

Q3 $\quad y' + \frac{4}{x} y = x^4$

$$\frac{dy}{dx} + \frac{4}{x} y = x^4$$

linear eq.

$$P = \frac{4}{x} \quad \& \quad Q = x^4$$

$$\rho = e^{\int P \cdot dx}$$

$$= e^{\int \frac{4}{x} \cdot dx}$$

$$= e^{4 \ln x} = e^{\ln x^4} = x^4$$

$$\rho \cdot y = \int \rho \cdot Q \cdot dx + c$$

$$x^4 \cdot y = \int x^4 \cdot x^4 \cdot dx + c$$

$$x^4 \cdot y = \frac{x^9}{9} + c$$

$$\therefore y = \frac{1}{9} x^5 + c \frac{1}{x^4}$$

Q4 $\quad y'' - y' = 5e^{x} - \sin 2x \qquad \text{--}(1)$

$$r^2 - r = 0 \Rightarrow r(r-1) = 0$$

$$\therefore r_1 = 0 \quad \& \quad r_2 = 1$$

$$y_h = c_1 + c_2 e^{x} \qquad \text{...}(2)$$

$$y_p = y_{p_1} + y_{p_2}$$

$$y_p = Ax.e^{x} + B\cos 2x + C\sin 2x$$

$$y_p' = A.e^{x} + Axe^{x} - 2B\sin 2x + 2C\cos 2x$$

$$y_p'' = 2Ae^{x} + Axe^{x} + Ae^{x} - 4B\cos 2x - 2C\sin 2x$$

$$= 2Ae^{x} + Axe^{x} - 4B\cos 2x - 4C\sin 2x$$

Sub in eq. (1)

$$Ae^{x} - 4B\cos 2x - 2C\cos 2x - 4B\sin 2x + 2B\sin 2x = 5e^{x} - \sin 2x$$

$$Ae^{x} = 5e^{x} \Rightarrow \boxed{A = 5}$$

$$-4B - 2C = 0 \Rightarrow 2B + C = 0 \Rightarrow \boxed{C = -2B}$$

$$-4C + 2B = -1 \Rightarrow B[-4 \times -2 + 2] = -1$$

$$\therefore \boxed{B = -\frac{1}{10}} \quad \& \quad \boxed{C = \frac{2}{10}}$$

$$\therefore y_p = 5xe^{x} - \frac{1}{10}\cos 2x + \frac{2}{10}\sin 2x$$

$$y = y_h + y_p$$

$$= C_2e^{x} + 5xe^{x} - \frac{1}{10}\cos 2x + \frac{2}{10}\sin 2x + C_1$$

Q5 $P_0(2,-1,-1)$, $P_1(1,2,3)$

$2x + 3y - 5z = 6$

$\overrightarrow{P_0P_1} = (1-2)i + (2-(-1))j + (3-(-1))k$

$= -i + 3j + 4k$

from eq of plane

$\vec{n} = 2i + 3j - 5k$

$\vec{N} = \vec{n} \times \overrightarrow{P_0P_1}$

$$= \begin{vmatrix} -i & +j & -k \\ -1 & 3 & 4 \\ 2 & 3 & -5 \end{vmatrix}$$

$\vec{N} = -27i + 3j - 9k$

$\vec{N} = Ai + Bj + Ck$

$\therefore$ $A = -27$

$B = 3$

$C = -9$

for $P_0(2,-1,-1)$ or $P_1(1,2,3)$

$Ax_1 + By_1 + Cz_1 = D$

$-27 * 2 + 3(-1) + (-9)(-1) = D = -48$

OR

$-27 * 1 + 3 * 2 - 9 * 3 = D = -48$

$\therefore$ The eq Required is

$-27x + 3y - 9 = -48$

$9x - y + 3z = 16$

Q6 $x^2+y^2=16$

$y-z=3$ & $z=2$

$$V=\iiint dz \cdot r \cdot dr \cdot d\theta$$

$x^2+y^2=16$

$r^2=16$

$r=4$

$y=r\sin\theta$

$z_1=r\sin\theta-3$

$z_2=2$

$$=\int_0^{2\pi}\int_0^4\int_{r\sin\theta-3}^{2} dz \cdot r \cdot dr \cdot d\theta$$

$$=\int_0^{2\pi}\int_0^4 (2-r\sin\theta+3)\, r dr \cdot d\theta$$

$$=\int_0^{2\pi}\int_0^4 (5r-r^2\sin\theta)\, dr \cdot d\theta$$

$$=\int_0^{2\pi}\left[\frac{5}{2}r^2-\frac{r^3}{3}\sin\theta\right]_0^4 d\theta$$

$$=\int_0^{2\pi}\left(40-\frac{64}{3}\sin\theta\right)d\theta$$

$$=\left[40\theta+\frac{64}{3}\cos\theta\right]_0^{2\pi}$$

$$=\left[80\pi+\frac{64}{3}\cos 2\pi\right]-\left[0+\frac{64}{3}\cos 0\right]$$

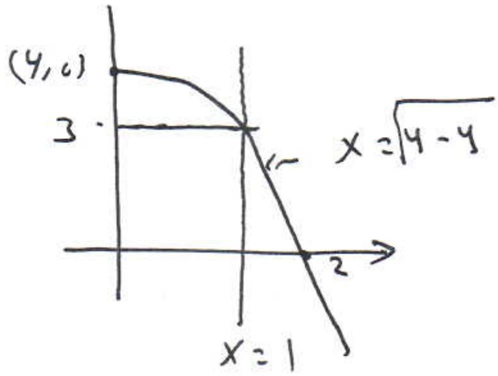
$$=80\pi+\cancel{\frac{64}{3}}-\cancel{\frac{64}{3}}$$

$$V=80\pi$$

Q7 $\iint (x+y)\, dy \cdot dx$

$0 \leq y \leq 3$

$1 \leq x \leq \sqrt{4-y}$



$x = \sqrt{4-y}$

$x^2 = 4-y$

if $y = 0 \Rightarrow x = 2$

$y = 3 \Rightarrow x = 1$

limits of integral

$1 \leq x \leq 2$

$0 \leq y \leq 4-x^2$

$$= \int_1^2 \int_0^{4-x^2} (x+y)\, dy \cdot dx$$

$$= \int_1^2 \left[xy + \frac{y^2}{2}\right]_0^{4-x^2} \cdot dx$$

$$= \int_1^2 \left[(4-x^2)x + \frac{1}{2}(4-x^2)^2\right] dx$$

$$= \int_1^2 \left[4x - x^3 + \frac{1}{2}(16 - 8x^2 + x^4\right] \cdot dx$$

$$= \left[\frac{4x^2}{2} - \frac{x^4}{4} + 8x - \frac{4}{3}x^3 + \frac{1}{10}x^5\right]_1^2$$

$$= \frac{241}{60}$$

Q8

$$2x + 3y = \lambda x$$
$$4x + y = \lambda y$$

$$\begin{vmatrix} 2-\lambda & 3 \\ 4 & 1-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)(1-\lambda) - 12 = 0$$

$$2 - 2\lambda - \lambda + \lambda^2 - 12 = 0$$

$$\lambda^2 - 3\lambda - 10 = 0$$

$$(\lambda - 5)(\lambda + 3) = 0$$

$$\lambda_1 = 5$$
$$\lambda_2 = -3$$

Q9 $\sum_{n=1}^{\infty} \frac{1}{2} \frac{(n!)^2}{(2n!)^2}$

$$\rho = \lim_{n\to\infty} \frac{a_{n+1}}{a_n}$$

$$a_n = \frac{1}{2} \frac{(n!)^2}{(2n!)^2}$$

$$a_{n+1} = \frac{1}{2} \frac{[(n+1)!]^2}{[2(n+1)!]^2}$$

$$= \frac{1}{2} \frac{((n+1)n!)^2}{((2n+2)(2n+1)2n!)^2}$$

$$= \frac{1}{2} \left[\frac{\cancel{(n+1)}\cancel{(n+1)}\, n!\, n!}{2\cancel{(n+1)}\, 2\cancel{(n+1)}\, (2n+1)(2n+1)\, 2n!\, 2n!} \right]$$

$$= \frac{1}{2} \left[\frac{(n!)^2}{4(2n+1)^2(2n!)^2} \right] = \frac{1}{8} \left[\frac{(n!)^2}{(2n+1)^2(2n!)^2} \right]$$

$$\frac{a_{n+1}}{a_n} = \frac{1}{8} \frac{\cancel{(n!)^2}}{(2n+1)^2 \cancel{(2n!)^2}} \cdot \frac{2\cancel{(2n!)^2}}{\cancel{(n!)^2}}$$

$$\rho = \lim_{n\to\infty} \frac{1}{4} \left(\frac{1}{(2n+1)^2} \right) = \frac{1}{\infty} \simeq 0$$

$$\rho = 0 < 1$$

The series is Convergent