



University of Technology  
Building and Construction Eng. Dept.  
Final Exam- First Attempt – 2014/2015

Branch : All Branches  
Subject : Mathematics II  
Examiner : Mathematics Committee

Class: 2<sup>nd</sup> year  
Time: 3 Hours  
Date : 14/6/ 2015



*Note: Answer Eight questions only*

- Q1. If  $f(x, y) = e^x y^2 - x^3 \ln y$ , prove that  $(f_{xxy}, f_{xyx}, \text{ and } f_{yxx})$  all are equal. (12.5 mark)
- Q2. Find all relative maxima and minima of  $f(x, y) = 2x^2 + y^2 + 6xy + 10x - 6y + 5$  (12.5 mark)
- Q3. Solve the following differential equation:  $y' + \frac{4}{x}y = x^4$  (12.5 mark)
- Q4. Using undetermined coefficient method to find the general solution of  $y'' + y' = 5e^x - \sin 2x$  (12.5 mark)
- Q5. Find an equation of the plane through  $P_0(2, -1, -1)$  and  $P_1(1, 2, 3)$  and perpendicular to the plane  $2x + 3y - 5z - 6 = 0$ . (12.5 mark)
- Q6. Find the volume of the solid within the cylinder  $x^2 + y^2 = 16$  and between the planes  $y - z = 3$  and  $z = 2$ . (12.5 mark)
- Q7. Evaluate the integral  $\int \int (x + y).dy.dx$ , if region of integral (R) enclosed by  $0 \leq y \leq 3$  and  $1 \leq x \leq \sqrt{4 - y}$  (12.5 mark)
- Q8. Determine the eigen values only for the following equations: (12.5 mark)  
 $2x + 3y = \lambda x$   
 $4x + y = \lambda y$
- Q9. Find the nature of the series  $\sum_{n=1}^{\infty} \frac{1(n!)^2}{2(2n!)^2}$  (12.5 mark)
- Q10. If  $\cos z = \frac{e^{iz} + e^{-iz}}{2}$  and  $\sin z = \frac{e^{iz} - e^{-iz}}{2i}$  prove that  $1 + \tan^2 z = \sec^2 z$  (12.5 mark)

*Good Luck*

Q.

$$f(x, y) = e^x \cdot y^2 - x^3 \cdot \ln y$$

$$f_x = e^x \cdot y^2 - 3x^2 \cdot \ln y$$

$$f_{xx} = e^x \cdot y^2 - 6x \cdot \ln y$$

$$f_{xxy} = 2e^x \cdot y - 6 \frac{x}{y}$$

$$f_{xy} = 2e^x \cdot y - 3 \frac{x^2}{y}$$

$$f_{xyx} = 2e^x \cdot y - 6 \frac{x}{y}$$

$$f_y = 2e^x \cdot y - \frac{x^3}{y}$$

$$f_{yx} = 2e^x \cdot y - 3 \frac{x^2}{y}$$

$$f_{yxx} = 2e^x \cdot y - 6 \frac{x}{y}$$

$$\therefore f_{xxy} = f_{xyx} = f_{yxx}$$

Q2

$$f(x, y) = 2x^2 + y^2 + 6xy + 10x - 6y + 5$$

Sol.

$$f_x = 4x + 6y + 10 = 0$$

$$\therefore y = -\frac{2}{3}x - \frac{5}{3} \quad \text{---(1)}$$

$$f_y = 2y + 6x - 6 = 0$$

Sub eq.(1) in  $f_y$  eq.

$$-\frac{4}{3}x + 6x - 6 - \frac{10}{3} = 0$$

$$\therefore x = +2 \quad (2, -3)$$

$$y = -3$$

$$f_{xx} = 4$$

$$f_{yy} = 2$$

$$f_{xy} = 6$$

$$f_{xx} \cdot f_{yy} - (f_{xy})^2 = 4 \times 2 - 36 = -28 < 0$$

$$f_{xx} > 0 \Rightarrow (2, -3)$$

$\therefore$  No maxima & minima

Q3

$$y' + \frac{A}{x}y = x^A$$

$$\frac{dy}{dx} + \frac{A}{x}y = x^A$$

linear eq.

$$P = \frac{A}{x} \quad \& \quad Q = x^A$$

$$\rho = \frac{\int P \cdot dx}{e}$$

$$= \frac{\int \frac{A}{x} \cdot dx}{e}$$

$$= \frac{A \ln x}{e} = \frac{\ln x^A}{e} = x^A$$

$$\rho \cdot y = \int \rho \cdot Q \cdot dx + C$$

$$x^A \cdot y = \int x^A \cdot x^A \cdot dx + C$$

$$x^A \cdot y = \frac{x^9}{9} + C$$

$$\therefore y = \frac{1}{9} x^5 + C \frac{1}{x^4}$$

Q4

$$y'' - y' = 5e^x - \sin 2x \quad \dots (1)$$

$$r^2 - r = 0 \Rightarrow r(r-1) = 0$$

$$\therefore r_1 = 0 \quad \& \quad r_2 = 1$$

$$y_h = c_1 + c_2 e^x \quad \dots (2)$$

$$y_p = y_{p1} + y_{p2}$$

$$y_p = Ax \cdot e^x + B \cos 2x + C \sin 2x$$

$$y_p' = A \cdot e^x + Ax e^x - 2B \sin 2x + 2C \cos 2x$$

$$y_p'' = 2A e^x + Ax e^x + A e^x - 4B \cos 2x - 2C \sin 2x$$

$$= 2A e^x + Ax e^x - 4B \cos 2x - 4C \sin 2x$$

Sub in eq. (1)

$$A e^x - 4B \cos 2x - 2C \cos 2x - 4B \sin 2x + 2B \sin 2x = 5e^x - \sin 2x$$

$$A e^x = 5e^x \Rightarrow \boxed{A = 5}$$

$$-4B - 2C = 0 \Rightarrow 2B + C = 0 \Rightarrow \boxed{C = -2B}$$

$$-4C + 2B = -1 \Rightarrow B[-4 \times -2 + 2] = -1$$

$$\therefore \boxed{B = -\frac{1}{10}} \quad \& \quad \boxed{C = \frac{2}{10}}$$

$$\therefore y_p = 5x e^x - \frac{1}{10} \cos 2x + \frac{2}{10} \sin 2x$$

$$y = y_h + y_p$$

$$= c_2 e^x + 5x e^x - \frac{1}{10} \cos 2x + \frac{2}{10} \sin 2x + c_1$$

Q5

$$R_0(2, -1, -1), R_1(1, 2, 3)$$

$$2x + 3y - 5z = 6$$

$$\begin{aligned}\vec{R_0 R_1} &= (1-2)i + (2-(-1))j + (3-(-1))k \\ &= -i + 3j + 4k\end{aligned}$$

normal of plane

$$\vec{n} = 2i + 3j - 5k$$

$$\vec{n} = \vec{r} \times \vec{R_0 R_1}$$

$$= \begin{vmatrix} -i & +j & -k \\ -1 & 3 & 4 \\ 2 & 3 & -5 \end{vmatrix}$$

$$\vec{n} = -27i + 3j - 9k$$

$$\vec{n} = Ai + Bj + Ck$$

$$\therefore A = -27$$

$$B = 3$$

$$C = -9$$

$$\text{for } R_0(2, -1, -1) \text{ or } R_1(1, 2, 3)$$

$$Ax_1 + By_1 + Cz_1 = D$$

$$-27 \times 2 + 3(-1) + (-9)(-1) = D = -48$$

or

$$-27 \times 1 + 3 \times 2 - 9 \times 3 = D = -48$$

$\therefore$  The eq. Required is

$$-27x + 3y - 9z = -48$$

$$9x - y + 3z = 16$$



Q6

$$x^2 + y^2 = 16$$

$$y - z = 3 \quad \& \quad z = 2$$

$$V = \iiint dz \cdot r \cdot dr \cdot d\alpha$$

$$= \int_0^{2\pi} \int_0^4 \int_{r \sin \alpha - 3}^2 dz \cdot r \cdot dr \cdot d\alpha$$

$$= \int_0^{2\pi} \int_0^4 (2 - r \sin \alpha + 3) r dr \cdot d\alpha$$

$$= \int_0^{2\pi} \int_0^4 (5r - r^2 \sin \alpha) dr \cdot d\alpha$$

$$= \int_0^{2\pi} \left[ \frac{5}{2} r^2 - \frac{r^3}{3} \sin \alpha \right]_0^4 d\alpha$$

$$= \int_0^{2\pi} \left( 40 - \frac{64}{3} \sin \alpha \right) d\alpha$$

$$= \left[ 40\alpha + \frac{64}{3} \cos \alpha \right]_0^{2\pi}$$

$$= \left[ 80\pi + \frac{64}{3} \cos 2\pi \right] - \left[ 0 + \frac{64}{3} \cos 0 \right]$$

$$= 80\pi + \frac{64}{3} - \frac{64}{3}$$

$$V = 80\pi$$

$$x^2 + y^2 = 16$$

$$r^2 = 16$$

$$r = 4$$

$$y = r \sin \alpha$$

$$z = r \sin \alpha - 3$$

$$z = 2$$

$$Q7 \quad \iint (x+y) dy \cdot dx$$

$$0 \leq y \leq 3$$

$$1 \leq x \leq \sqrt{4-y}$$

$$x = \sqrt{4-y}$$

$$x^2 = 4-y$$

$$\text{if } y=0 \Rightarrow x=2$$

$$y=3 \Rightarrow x=1$$

limits of integral

$$1 \leq x \leq 2$$

$$0 \leq y \leq 4-x^2$$

$$= \int_1^2 \int_0^{4-x^2} (x+y) dy \cdot dx$$

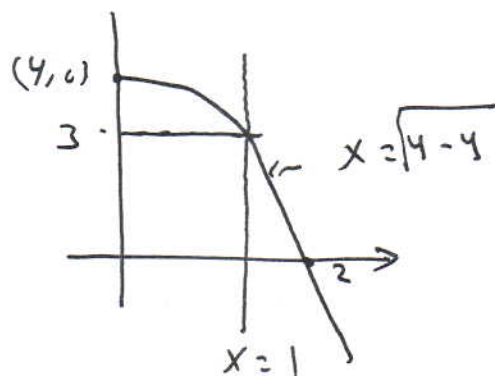
$$= \int_1^2 \left[ xy + \frac{y^2}{2} \right]_0^{4-x^2} \cdot dx$$

$$= \int_1^2 \left[ (4-x^2)x + \frac{1}{2}(4-x^2)^2 \right] dx$$

$$= \int_1^2 \left[ 4x - x^3 + \frac{1}{2}(16 - 8x^2 + x^4) \right] \cdot dx$$

$$= \left[ \frac{4x^2}{2} - \frac{x^4}{4} + 8x - \frac{4}{3}x^3 + \frac{1}{10}x^5 \right]_1^2$$

$$= \frac{241}{60}$$





Q8

$$2x + 3y = 1x$$
$$4x + y = 1y$$

$$\begin{vmatrix} 2-1 & 3 \\ 4 & 1-1 \end{vmatrix} = 0$$

$$(2-1)(1-1) - 12 = 0$$

$$2 - 2 - 1 - 1 + 1^2 - 12 = 0$$

$$1^2 - 3 - 10 = 0$$

$$(1-5)(1+3) = 0$$

$$1_1 = 5$$

$$1_2 = -3$$

Q9

$$\sum_{n=1}^{\infty} \frac{1}{2} \frac{(n!)^2}{(2n!)^2}$$

$$\rho = \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n}$$

$$a_n = \frac{1}{2} \frac{(n!)^2}{(2n!)^2}$$

$$a_{n+1} = \frac{1}{2} \frac{[(n+1)!]^2}{[2(n+1)!]^2}$$

$$= \frac{1}{2} \frac{((n+1)n!)^2}{((2n+2)(2n+1)2n!)^2}$$

$$= \frac{1}{2} \left[ \frac{\cancel{(n+1)} \cancel{(n+1)} n! n!}{2 \cancel{(n+1)} 2 \cancel{(n+1)} (2n+1) (2n+1) 2n! 2n!} \right]$$

$$= \frac{1}{2} \left[ \frac{(n!)^2}{4 (2n+1)^2 (2n!)^2} \right] = \frac{1}{8} \left[ \frac{(n!)^2}{(2n+1)^2 (2n!)^2} \right]$$

$$\frac{a_{n+1}}{a_n} = \frac{1}{8} \frac{\cancel{(n!)}^2}{(2n+1)^2 \cancel{(2n!)}^2} \cdot \frac{2 \cancel{(2n+1)}^2}{\cancel{(n!)}^2}$$

$$\rho = \lim_{n \rightarrow \infty} \frac{1}{4} \left( \frac{1}{(2n+1)^2} \right) = \frac{1}{\infty} = 0$$

$$\rho = 0 < 1$$

The series is Convergent

Q10

$$\cos z = \frac{e^{iz} + e^{-iz}}{2}$$

$$\sin z = \frac{e^{iz} - e^{-iz}}{2i}$$

$$1 + \tan^2 z = \sec^2 z$$

$$1 + \frac{\sin^2 z}{\cos^2 z} = 1 + \left( \frac{\frac{e^{iz} - e^{-iz}}{2}}{\frac{e^{iz} + e^{-iz}}{2}} \right)^2$$

$$1 + \frac{e^{iz} - 2 + e^{-iz}}{-4} = 1 - \left( \frac{e^{iz} - 2 + e^{-iz}}{e^{iz} + 2 + e^{-iz}} \right)^2$$

$$= \frac{\cancel{e^{iz}} + 2 + \cancel{e^{-iz}} - 2 + \cancel{e^{iz}}}{e^{iz} + 2 + e^{-iz}}$$

$$= \frac{4}{e^{iz} + 2 + e^{-iz}} = \left[ \frac{2}{e^{iz} + e^{-iz}} \right]^2$$

$$= \left( \frac{1}{\cos z} \right)^2 = \frac{1}{\cos^2 z} = \sec^2 z$$