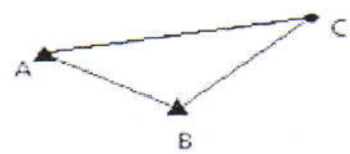
University Of Technology
Building and Construction Eng. Dept.
Final Exam – 2016/2015
Subject :Adjustment computation   Class: 4^{th}
Branch : Geomatics Engineering   Time : 3 hr.
Examiner : Dr. Abbas Z. Khalaf   Date :16/6 / 2016

Answer three Questions Only (Equal Marks)

Q1) The following table represent the field data of the triangulation problem for point C shown in the figure below.

Measured horizontal Angle to the right
ABC= 98° 07′ 48″
CAB= 61° 02′ 00″
BCA= 20° 51′ 35″



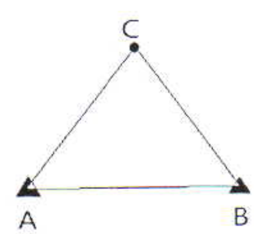
Knowing that the horizontal (X,Y) coordinates of the control points A,B

Points	X (m)	Y (m)
A	50	100
B	100	50

1- Write the observation equation for all observations.
2- Compute the approximate value of the horizontal coordinates (X, Y) for point (**C**).
3- Write the General form of [A,X,L,V] matrices of the least squares observation equation for this problem.
4- Compute the [A, L] matrices elements of the angle to the right **ABC** for the first iteration only.

Q2) The following table represent the field data of the intersection problem for point C shown in the figure below.

Horizontal distance (m)	Azimuth
AC= 113 ± 0.01	AZ_{AC}= 25° 00′ 00″ ± 15″
BC=183.5 ± 0.015	AZ_{BC}=303° 56′ 00″ ± 24″



Knowing that the horizontal (X,Y) coordinates of the control points A,B

Points	X (m)	Y (m)
A	0	0
B	200	0

1- Compute the elements of the weight matrix for this problem
2- Discuss in detail the procedure for computing the adjusted value of the horizontal coordinates (X, Y) for point **C** and their standard error using the least squares observation method.
3- Compute the elements of A,L matrices for the Horizontal Distance **AC**

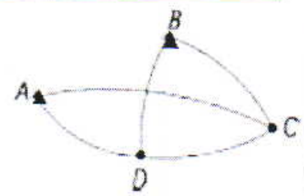
Q3) The following table represent the field data for the differential leveling network shown in the figure below.

Route	From	To	Difference in elevation (m)	Number of level setups	Route	From	To	Difference in elevation (m)	Number of level setups
R1	B	C	5.360	2	R3	D	A	-7.348	6
R2	C	D	-8.523	4	R4	B	D	-3.167	3
					R5	A	C	15.881	12

Knowing that the adjusted elevation of bench marks (A,B) are :-

ZA = 437.596 m

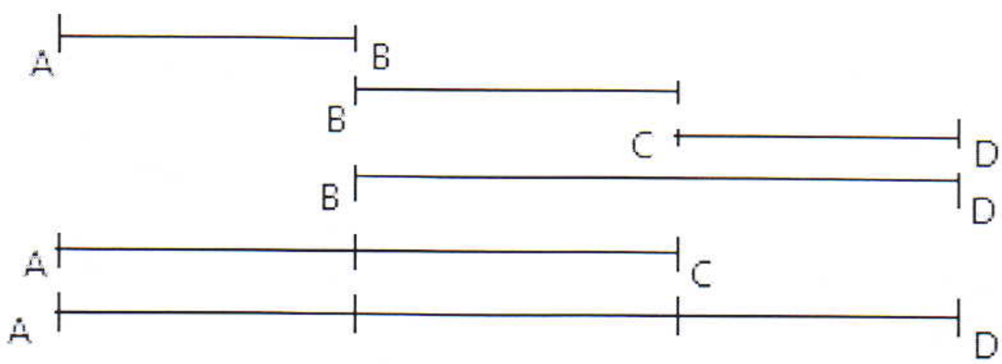
ZB = 448.116 m



Compute the adjusted elevation for points C, D and their standard errors.

Q4) A) Suppose that the following values (in meter) were obtained in 15 independent distance observations, *Di* : 212.22, 212.25, 212.23, 212.15, 212.23,212.11, 212.29, 212.34, 212.22, 212.24, 212.19, 212.25, 212.27, 212.20, and 212.25., Calculate the mean, The range of the data, The median, The mode , *E*50 and *E*95.

B) Calculate the adjusted lengths *AB, BC, CD* and their standard errors *given* in the figure below for the following observation data (assume equal weights) using least square observation method. length observations (m): *AB* = 100.01, *BC* = 99.94 ,*CD* = 100.0, *BD* = 200.02, AC =200.0, *AD* = 299.98.



Good Luck

Adjustment computation - Final exam - 2016/2015 - class: 4th

Q1.) 1.

$$A\hat{B}C = \tan^{-1}\frac{X_C - X_B}{Y_C - Y_B} - \tan^{-1}\frac{X_A - X_B}{Y_A - Y_B} \pm 180$$

$$A\hat{B}C = \tan^{-1}\frac{X_C - 100}{Y_C - 50} - 315^\circ \quad ---- (1)$$

$$CAB = \tan^{-1}\frac{X_B - X_A}{Y_B - Y_A} - \tan^{-1}\frac{X_C - X_A}{Y_C - Y_A} \quad --- (2)$$

$$CAB = 135^\circ - \tan^{-1}\frac{X_C - 50}{Y_C - 100} \quad --- (2)$$

$$BCA = AZ_{CA} - AZ_{CB} \pm 180$$

$$BCA = \tan^{-1}\frac{50 - X_C}{100 - Y_C} - \tan^{-1}\frac{100 - X_C}{50 - Y_C} \quad -- (3)$$

2- approximate value for point (C)

Case 1. from eq. 1

$$98^\circ\ 07'\ 48'' = \tan^{-1}\frac{X_C - 100}{Y_C - 50} - 315$$

$$1.333 = \frac{X_C - 100}{Y_C - 50}$$

$$1.33\,Y_C + 33.33 = X_C \quad ---- (A)$$

from eq. 2

$$\tan 73^\circ\ 58'\ 0'' = \frac{X_C - 50}{Y_C - 100} \longrightarrow 3.479 = \frac{X_C - 50}{Y_C - 100} \quad -(B)$$

sub. eq. A in eq. B

$$3.479 = \frac{1.33y_c + 3.33 - 50}{y_c - 100}$$

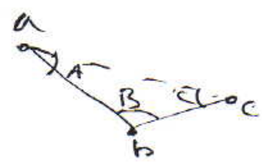
$$3.479y_c - 347.9 = 1.33y_c - 16.67$$

$$\boxed{\begin{aligned} y_c &= 154.132 \\ x_c &= 238.325 \end{aligned}}$$ in eq. (A)

Cas 2. to find the approximate value

from sin law

$$\frac{D_{AB}}{\sin C} = \frac{D_{BC}}{\sin A}$$



$$\frac{70.710}{\sin 20°51'35''} = \frac{D_{BC}}{\sin 61°02'00''} \longrightarrow D_{BC} = 173.738\ m$$

$$X_C = X_B + D_{BC} \sin Az_{BC}$$

$$x_c = 100 + 173.738 \sin(53.1) \longrightarrow \boxed{x_c = 238.99}$$

$$y_c = y_B + D_{BC} \cos Az_{BC} \longrightarrow \boxed{y_c = 154.243}$$

3. Three Horizontal angle to the right

3 eq. . 3 known, 1 unknown

$n > u$ using L.S.O.M.

nonlinear eq.

$$\begin{bmatrix} \frac{\partial f_1}{\partial x_c} & \frac{\partial f_1}{\partial y_c} \\ \frac{\partial f_2}{\partial x_c} & \frac{\partial f_2}{\partial y_c} \\ \frac{\partial f_3}{\partial x_c} & \frac{\partial f_3}{\partial y_c} \end{bmatrix} \begin{bmatrix} \delta x_c \\ \delta y_c \end{bmatrix} - \begin{bmatrix} ABC_{measured} - ABC_{app.} \\ CAB_{meas.} - CAB_{app.} \\ BCA_{measured} - BCA_{app.} \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

3X2 2X1 3X1 3X1

4. $\frac{\partial f_1}{\partial x_c}$, $\frac{\partial f_1}{\partial y_c}$, angle $ABC_{measured}$ - angle $ABC_{appro.}$

$\frac{\partial f_1}{\partial x_c}$ from eq. 1

$$ABC = \tan^{-1} \frac{x_c - 100}{y_c - 50} - 315^\circ \quad \text{---} (f_1)$$

let: $\frac{x_c - 100}{y_c - 50} = AA$

$$AAA = \frac{1}{AA^2 + 1}$$

$$\frac{\partial f_1}{\partial x_c} = \frac{(1/y_c)}{1 + \left(\frac{x_c - 100}{y_c - 50}\right)^2} = AAA * \frac{1}{y_c}$$

$$\frac{\partial f_1}{\partial y_c} = \frac{\left(\frac{y_c - x_c}{y_c^2}\right)}{1 + \left(\frac{x_c - 100}{y - 50}\right)^2} = AAA \left(\frac{y_c - x_c}{y_c^2}\right)$$

using The approximate value $x_c = 238.325$
$y_c = 154.132$

$$\frac{\partial f_1}{\partial x_c} = 1.5177 * 10^{-3}$$

$$\frac{\partial f_1}{\partial y_c} = -1.2819 * 10^{-3}$$

angle ABC app. $= AZ_{BC} - AZ_{BA} = \tan^{-1}\frac{x_c - 100}{y_c - 50} - 315^\circ$

$= 98^\circ\ 1'\ 37.2''$

Q2.

1-

$$P = \begin{bmatrix} \frac{1}{\delta^2} & 0 & 0 & 0 \\ 0 & \frac{1}{\delta^2} & 0 & 0 \\ 0 & 0 & \frac{1}{\delta^2} & 0 \\ 0 & 0 & 0 & \frac{1}{\delta^2} \end{bmatrix} = \begin{bmatrix} \frac{1}{0.01^2} & 0 & 0 & 0 \\ 0 & \frac{1}{0.015^2} & 0 & 0 \\ 0 & 0 & \frac{1}{15''^2} & 0 \\ 0 & 0 & 0 & \frac{1}{20''^2} \end{bmatrix}$$

2. (A) Find the approximated value for (X, Y) point c by using

$$X_c = X_B + D_{BC} \sin AZ_{BC}$$
$$Y_c = Y_B + D_{BC} \cos AZ_{BC}$$

(B) write The observation eq. for all observations.

- in This problem 2 dist., 2. ~~Angles~~

$$D_{AC} = \sqrt{(X_c - X_A)^2 + (Y_c - Y_A)^2} \quad \text{---} \quad f_1$$

$$D_{BC} = \sqrt{(X_c - X_B)^2 + (Y_c - Y_B)^2} \quad \text{---} \quad f_2$$

$$AZ_{AC} = \tan^{-1} \frac{X_c - X_A}{Y_c - Y_A} \quad \text{------} \quad f_3$$

$$AZ_{BC} = \tan^{-1} \frac{X_c - X_B}{Y_c - Y_B} \quad \text{------} \quad f_4$$

(c) write The general form [AX − L = V] matrices .

$* \underset{n}{4} > \underset{u}{1}$ using L.s.o.m. for point [C]

$$A = \begin{bmatrix} \frac{\partial f_1}{\partial x_c} & \frac{\partial f_1}{\partial y_c} \\ \frac{\partial f_2}{\partial x_c} & \frac{\partial f_2}{\partial y_c} \\ \frac{\partial f_3}{\partial x_c} & \frac{\partial f_3}{\partial y_c} \\ \frac{\partial f_4}{\partial x_c} & \frac{\partial f_4}{\partial y_c} \end{bmatrix}_{4\times2} , \quad L = \begin{bmatrix} D_{meas.} - D_{app.} \\ D_{meas.} - D_{app.} \\ AZ_{measured} - AZ_{approx.} \\ AZ_{measured} - AZ_{approx.} \end{bmatrix}_{4\times1} , \quad V = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix}_{4\times1}$$

$$X = \begin{bmatrix} \delta x_c \\ \delta y_c \end{bmatrix}_{2\times1}$$

. normal eq. $NX = D$

$$N = A^T P A \quad , \quad D = A^T P L$$

using (P) as The weight matrix from point (1) in The question.

$$X = N^{-1} D$$

$$X_c = X_{c\,approx.} + \delta X_c \quad , \quad y_c = y_{c\,app.} + \delta y_c$$

$$\delta_o = \sqrt{\frac{v^T P v}{n-u}} \quad , \quad \delta_x = \delta_o \sqrt{Q_{xx}} \quad , \quad \delta_y = \delta_o \sqrt{Q_{yy}}$$

$$\delta = \sqrt{\delta_x^2 + \delta_y^2}$$

Q3.

$\Delta Z_{BC} = Z_C - Z_B$

$\Delta Z_{CD} = Z_D - Z_C$

$\Delta Z_{DA} = Z_A - Z_D$

$\Delta Z_{BD} = Z_D - Z_B$

$\Delta Z_{AC} = Z_C - Z_A$

$$\underset{5\times 2}{\begin{bmatrix} 1 & 0 \\ -1 & 1 \\ 0 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}} \underset{2\times 1}{\begin{bmatrix} Z_C \\ Z_D \end{bmatrix}} - \underset{5\times 1}{\begin{bmatrix} 453.476 \\ -8.523 \\ 444.944 \\ 444.949 \\ 453.477 \end{bmatrix}} = \underset{5\times 1}{\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix}}$$

$$P = \underset{5\times 5}{\begin{bmatrix} 1/2 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{8} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{12} \end{bmatrix}}$$

or

$$P = \underset{5\times 5}{\begin{bmatrix} 6 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}}$$

$NX = D$

$$N = \begin{bmatrix} 10 & -3 \\ -3 & 9 \end{bmatrix} \longrightarrow N^{-1} = \begin{bmatrix} 0.111 & 0.039 \\ 0.039 & 0.118 \end{bmatrix}$$

$$D = A_p^{T} L = \begin{bmatrix} 1 & -1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 6 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 435.476 \\ -8.523 \\ 444.944 \\ 444.949 \\ 453.477 \end{bmatrix}$$

$$\underset{2\times 1}{D} = \begin{bmatrix} 3199.903 \\ 2644.216 \end{bmatrix} \Rightarrow X = N^{-1} D \longrightarrow X \begin{bmatrix} 453.345 \\ 444.643 \end{bmatrix}$$

$AX - L = V$

$$\begin{bmatrix} 1 & 0 \\ -1 & 1 \\ 0 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 453.345 \\ 444.643 \end{bmatrix} - \begin{bmatrix} 453.476 \\ -8.25 \\ 444.924 \\ 444.949 \\ 453.477 \end{bmatrix}$$

$\delta_o = \pm \sqrt{\frac{V^T P V}{n - u}} = \pm 0.453$

$Q_{xa} = N^{-1} = 0.111$

$\delta x_1 = \pm \delta_o \sqrt{q_{11}} = \pm 0.453 \sqrt{0.111} = \pm 0.151$

$\delta x_2 = \pm \delta_o \sqrt{q_{22}} = 0.452 \sqrt{0.123} = \pm 0.159$

Q4) A)

212.11, 212.15, 212.19, 212.20
212.22, 212.22, 212.23, 212.23
212.24, 212.25, 212.23, 212.25
212.27, 212.27, 212.34

① The mean $\bar{x}$ = 212.229 m

② Range = 212.34 − 212.11 = 0.23

③ median = 212.23

④ mode = 212.25

⑤ $\delta x = \sqrt{\frac{0.051295}{14}} = \pm 0.055$ m

$E_{50} = 0.6745\delta = \pm 0.6745 (0.055) = \pm 0.04$ m

* nine observation lie between 212.22 ± 0.04 m and this corresponds to 9/15 * 100% or 60% of observ

$E_{95} = 1.960\delta = \pm 1.960 (0.055) = \pm 0.11$ m

14 of observation lies between 212.22 ± 0.11 or 93%

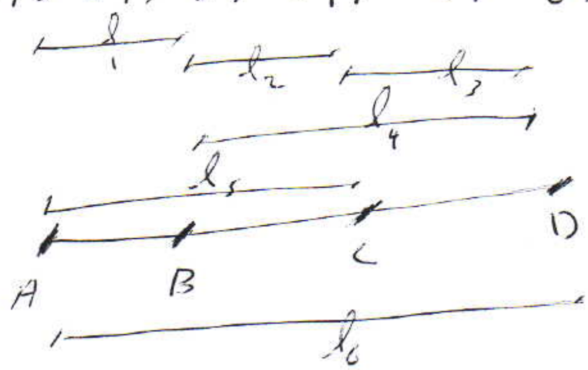
at 99.7% level the range ± 2.965δ almost 3δ corresponds of ± 0.16 m, where all values in the data are within this range

Q4) B. using least squares observation method.

build The eqn.

for The unknown (X_1, X_2, X_3)

The nKnown is ($l_1, l_2, l_3, l_4, l_5, l_6$)



$$X_1 = l_1 + v_1 \quad \text{--- (1)}$$

$$X_2 = l_2 + v_2 \quad \text{--- (2)}$$

$$X_3 = l_3 + v_3 \quad \text{--- (3)}$$

$$X_2 + X_3 = l_4 + v_4 \quad \text{--- (4)}$$

$$X_1 + X_2 = l_5 + v_5 \quad \text{--- (5)}$$

$$X_1 + X_2 + X_3 = l_6 + v_6 \quad \text{--- (6)}$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}, \quad X = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}, \quad L = \begin{bmatrix} l_1 \\ l_2 \\ l_3 \\ l_4 \\ l_5 \\ l_6 \end{bmatrix}, \quad V = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{bmatrix}$$

using least-squares observation form ($AX - L = V$)

$$NX = D$$

$$N = A^T P A, \quad D = A^T P L$$

* $P = 1$ (equal weight)

$$N = \underset{3\times6}{\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}} \underset{6\times3}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}} = \underset{3\times3}{\begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 3 \end{bmatrix}}$$

$$\underset{3\times1}{D} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 100.01 \\ 99.94 \\ 100 \\ 200.02 \\ 200 \\ 299.98 \end{bmatrix} = \begin{bmatrix} 599.99 \\ 799.94 \\ 600 \end{bmatrix}$$

3×6 $\quad 6 \times 1 \quad$ 3×1

$$X = N^{-1} D = \begin{bmatrix} 0.5 & -0.25 & 0 \\ -0.25 & 0.5 & -0.25 \\ 0 & -0.25 & 0.5 \end{bmatrix} \begin{bmatrix} 599.99 \\ 799.94 \\ 600 \end{bmatrix}$$

$$X = \begin{bmatrix} 100.01 \\ 99.972 \\ 100.01 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 100.01 \\ 99.972 \\ 100.01 \end{bmatrix} - \begin{bmatrix} 100.01 \\ 99.94 \\ 100 \\ 200.02 \\ 200 \\ 299.98 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -0.037 \\ -0.018 \\ 0.012 \end{bmatrix}$$