الجامعة التكنولوجية

قسم هندسة البناء والإنشاءات

المرحلة الأولى

العـدد : ـــــــ

التاريخ : 2015/6/08

الى / السيد معاون رئيس القسم

م/ الاجابة النموذجية لمادة (الرياضيات 2)

تحية طيبة ....

نرفق لكم طياً نسخة من الأسئلة الخاصة بمادة الرياضيات 2 و للإمتحان النهائي للفصل الدراسي الثاني – الدور الأول و للعام الدراسي 2014 – 2015 و الذي تم اجراءه بتاريخ 2015/06/08 مع الاجابة النموذجية الخاصة بها.

مع التقدير

أ.م.د. قيس جواد فريح

مسؤول المرحلة الأولى

2015/6/08

<u>نسخة منه الى/</u>

- ملف اللجنة الامتحانية

University of Technology

Building and Construction Eng. Dept.

Final Exam 2nd course / 2014-2015

Subject : Mathematics Class: 1st Year

Time: 3 hrs.

Note: Answer Five Questions only

Q. No. 1: (20 marks)

A) Evaluate the following integral $\int \frac{\tan^{-1}(x)}{x^2}\, dx$.

B) Using vectors show that the triangle ABC is right at A if A(1,2,0), B(3,0,1) and C(2,2,-2).

Q. No. 2: (20 marks)

A) Evaluate the following integral $\int \frac{x^4 dx}{x^2+1}$

B) Find the area bounded by y=ln x , x=1 , x=3 and the x-axis.

Q. No. 3: (20 marks)

A) Evaluate the following integral $\int \frac{dx}{4x^2+4x+2}$

B) Find dy/dx if : $y = \pi^{sinx} + 2^{lnx} + x^2 tan^{-1}(e^x)$.

Q. No. 4: (20 marks)

A) Evaluate the following integral $\int \frac{x^2\, dx}{\sqrt{9-x^2}}$.

B) If v_1= 2i+j-k & v_2= i+2j+2k, find the scalar (dot) and vector (Cross) product.

Q. No. 5: (20 marks)

A) Evaluate the following integral $\int \frac{6x+7}{(x+2)^2}\, dx$.

B) Find the volume of the solid generated by rotating the region bounded by $y = x^2, y = \sqrt{x}$ about the line x=1.

Q. No. 6: (20 marks)

A) Evaluate the following integral $\int \frac{x dx}{x+\sqrt{x}+1}$.

B) Find dy/dx if : $y = \sqrt[x]{tanx}$.

رياضيات / كورس 2 / 2015

Q.1 a) $\int \underbrace{\tan^{-1}x}_{u} * \underbrace{x^{-2}\,dx}_{dv}$

$I = \tan^{-1}x * \frac{x^{-1}}{-1} + \int \frac{1}{x} * \frac{1}{1+x^2}\,dx$

$= \frac{A}{x} + \frac{Bx+C}{x^2+1} = \frac{A(x^2+1)+(Bx+C)x}{x(x^2+1)}$

$1 = Ax^2 + A + Bx^2 + Cx$

$A+B=0$

$C=0$

$A=1 \Rightarrow B=-1$

$\int \frac{1}{x} + \int \frac{-x}{x^2+1}$

$= \ln|x| - \frac{1}{2}\int \frac{2x}{x^2+1}$

$I = \tan^{-1}x * \frac{x^{-1}}{-1} + \ln|x| - \frac{1}{2}\ln|x^2+1| + c$

Q.1 b) $\overrightarrow{AB} = 2i - 2j + k,\ \overrightarrow{AC} = i + 0j - 2k$

$AB \cdot AC = 2 - 2 = 0 \quad \therefore AB \perp AC$

Q.2 a) $I = \int \left(x^2 - 1 + \frac{1}{x^2+1}\right)dx$

$= \frac{x^3}{3} - x + \tan^{-1}x + c$

$$\begin{array}{r|l} & x^2-1 \\ \hline x^2+1 & x^4 \\ & x^4+x^2 \\ \hline & -x^2 \\ & -x^2-1 \\ \hline & +1 \end{array}$$

Q.2 b) $A = \int y\,dx$

$A = \int_1^3 \ln x\,dx$

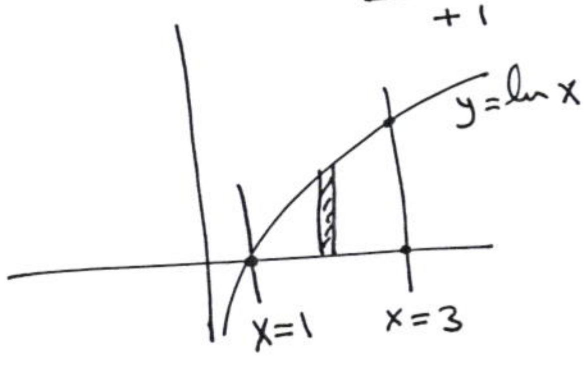
$= \ln x * x - \int x \cdot \frac{1}{x}\,dx$

$= x\ln x - x \Big|_1^3$

$= 3\ln 3 - 3 - (\ln 1 - 1)$

$= 3\ln 3 - 3 + 1 = 3\ln 3 - 2 = 1.295$

unit Area



Q.3 A) $\frac{1}{4}\int \frac{dx}{x^2+x+\frac{1}{2}} = \frac{1}{4}\int \frac{dx}{\underline{x^2+x+\frac{1}{4}}-\frac{1}{4}+\frac{1}{2}}$

$= \frac{1}{4}\int \frac{dx}{(x+\frac{1}{2})^2+\frac{1}{4}} = \frac{1}{4} * \frac{1}{\frac{1}{2}} \tan^{-1} \frac{x+\frac{1}{2}}{\frac{1}{2}}$

$= \frac{1}{2} \tan^{-1}(2x+1) + c$

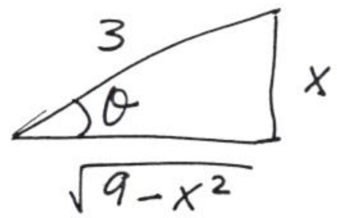
$\frac{1}{2}\int \frac{2\,dx}{(2x+1)^2+1} = \frac{1}{2}\tan^{-1}(2x+1)$ او مباشرة

Q.3 b) $y' = \pi^{\sin x} * \cos x * \ln\pi + 2^{\ln x} * \frac{1}{x} * \ln 2$
$+ x^2 * \frac{e^x}{1+(e^x)^2} + \tan^{-1}(e^x) * 2x$

Q.4A) let $x = 3\sin\theta$, $dx = 3\cos\theta\, d\theta$

$I = \int \frac{9\sin^2\theta * \cancel{3\cos\theta}\, d\theta}{\cancel{\sqrt{9-9\sin^2\theta}}} = 9\int \sin^2\theta\, d\theta$

نصف الزاوية



Q.4 b) $v_1 \cdot v_2 = (2*1)+(1*2)+(-1*2)$
$= 2+2-2 = 2$

$v_1 \times v_2 = \begin{vmatrix} i & j & k \\ 2 & 1 & -1 \\ 1 & 2 & 2 \end{vmatrix}$

$= i(2+2) - j(4+1) + k(4-1)$
$= 4i - 5j + 3k$

Q.5 A) $\frac{6x+7}{(x+2)^2} = \frac{A}{x+2} + \frac{B}{(x+2)^2} = \frac{A(x+2)+B}{(x+2)^2}$

$6x+7 = Ax + 2A + B \Rightarrow 6 = A$

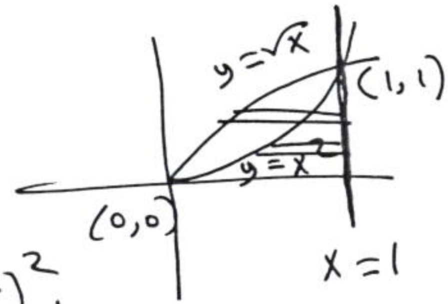
$7 = 2A + B$

$7 = 12 + B \Rightarrow B = -5$

$I = \int \frac{6}{x+2} dx - \int \frac{5}{(x+2)^2} = 6 \ln|x+2| - 5 \frac{(x+2)^{-1}}{-1} + c$

$I = 6 \ln|x+2| + \frac{5}{x+2} + c$

Q.5 B) $V = \int_0^1 \pi R^2 dy - \int_0^1 \pi R^2 dy$



$V = \int_0^1 \pi (1-x_1)^2 dy - \int_0^1 \pi (1-x_2)^2 dy$

$V = \int_0^1 \pi (1-y^2)^2 dy - \int_0^1 \pi (1-\sqrt{y})^2 dy$

$V = \pi \int (1 - 2y^2 + y^4) \Big|_0^1 - \pi \int (1 - 2\sqrt{y} + y) \Big|_0^1$

$= \pi \left(y - \frac{2y^3}{3} + \frac{y^5}{5}\right)\Big|_0^1 - \pi \left(y - 2\frac{y^{3/2}}{3/2} + \frac{y^2}{2}\right)\Big|_0^1$

$= \pi \left(1 - \frac{2}{3} + \frac{1}{5}\right) - \pi \left(1 - \frac{4}{3} + \frac{1}{2}\right)$

$= \pi \left(\frac{15-10+3}{15}\right) - \pi \left(\frac{6-8+3}{6}\right)$

$= \pi \left(\frac{8}{15} - \frac{1}{6}\right) = \pi \frac{16-5}{30} = \frac{11\pi}{30}$

Q6 A) let $x = y^2$

$dx = 2y\,dy$

$$\int \frac{y^2 * 2y\,dy}{y^2+y+1} = 2\int \frac{y^3}{y^2+y+1}\,dy$$

$$\begin{array}{r|l} & y-1 \\ \hline y^2+y+1 & y^3 \\ & y^3+y^2+y \\ \hline & -y^2-y \\ & -y^2-y-1 \\ \hline & +1 \end{array}$$

$$I = 2\int (y-1) + \frac{1}{y^2+y+1}\,dy$$

$$\int \frac{1}{y^2+y+1} = \int \frac{1}{y^2+y+\frac{1}{4}-\frac{1}{4}+1} = \int \frac{1}{\left(y+\frac{1}{2}\right)^2+\frac{3}{4}}$$

let $y+\frac{1}{2} = \frac{\sqrt{3}}{2}\tan\theta$

$dy = \frac{\sqrt{3}}{2}\sec^2\theta\,d\theta$

or

$$\frac{1}{\frac{\sqrt{3}}{2}}\tan^{-1}\frac{\left(y+\frac{1}{2}\right)}{\frac{\sqrt{3}}{2}}$$

$$= \frac{2}{\sqrt{3}}\tan^{-1}\frac{y+\frac{1}{2}}{\frac{\sqrt{3}}{2}}$$

Q.6 B) $y = (\tan x)^{1/x}$

$\ln y = \ln(\tan x)^{\frac{1}{x}}$

$\ln y = \frac{1}{x}\ln\tan x$

$$\frac{1}{y} * y' = \frac{1}{x} * \frac{1}{\tan x} * \sec^2 x + \ln\tan x * -x^{-2}$$