الجامعة التكنولوجية

قسم هندسة البناء والإنشاءات

المرحلة الأولى

العـدد : ٥١م

التاريخ : ١ /٦/ 2016

الى / السيد معاون رئيس القسم

م/ الاجابة النموذجية لمادة (الرياضيات 2)

تحية طيبة ....

نرفق لكم طياً نسخة من الأسئلة الخاصة بمادة الرياضيات 2 و للإمتحان النهائي للفصل الدراسي الثاني – الدور الأول و للعام الدراسي 2015 – 2016 و الذي تم اجراءه بتاريخ 2016/6/01 مع الاجابة النموذجية الخاصة بها.

مع التقدير

أ.م.د. قيس جواد فريح

مسؤول المرحلة الأولى

2016 /6/ ٠١

نسخة منه الى/

- ملف اللجنة الامتحانية

وحدة الانترنيت / يرجى نشرها على الموقع

٢/٦

University of Technology

Building and Construction Eng. Dept.

Final Exam 2^{nd} course / 2015-2016

Subject : Mathematics

Class: 1^{st} Year

Time: 3 hrs.

Note: Answer Five Questions only

Q. No. 1: (20 marks)

A) Evaluate the following integral $\int_0^{1/\sqrt{2}} 2\,x\ \sin^{-1}(x^2)\,dx$

B) Find the area bounded by $y = x^2 - 2x$ and the line y=x .

Q. No. 2: (20 marks)

A) Evaluate the following integral $\int \frac{dx}{\sqrt{x^2-16}}$

B) using vectors show that the triangle ABC is right at A, if A (3,-2), B (1,-1) and C (8,8).

Q. No. 3: (20 marks)

A) Evaluate the following integral $\int \frac{2x-3}{\sqrt{3+2x-x^2}}\,dx$

B) Find dy/dx if : $y = \pi^{tanhx} + 2^{logx} + x^3 tan^{-1}(3)^x$.

Q. No. 4: (20 marks)

A) Evaluate the following integral $\int sin^3 x\, cos^3 x\, dx$

B) If v_1= 3i+2j-2k & v_2= 2i+2j+k, find the scalar (dot) and vector (Cross) product.

Q. No. 5: (20 marks)

A) Evaluate the following integral $\int \frac{dx}{x^2-2x-3}$

B) Find the volume of the solid generated by rotating the region bounded by $y=x^2$ and $y=x$ about $y=-2$.

Q. No. 6: (20 marks)

A) Evaluate the following integral $\int \frac{dx}{\sqrt{x}\ (1+\sqrt{x})}$.

B) Solve the equation : $\sinh(x^2) - 3\cosh(x^2) = -3$.

رياضيات كورس ثاني / 2016

Q.1 A) $\int 2x \sin^{-1} x^2 \, dx = \sin^{-1} x^2 * x^2 - \int x^2 * \frac{2x}{\sqrt{1-x^4}} \, dx$

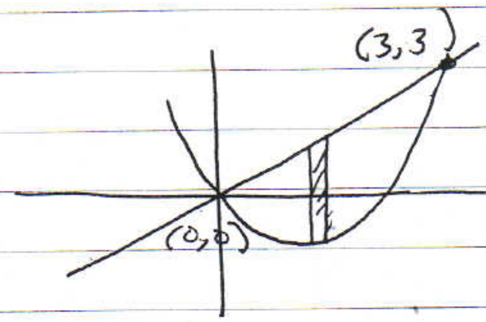
$\underbrace{2x \, dx}_{dv}$, $\sin^{-1} x^2 = u$

$I = x^2 \sin^{-1} x^2 - 2 \int \frac{x^3}{\sqrt{1-x^4}} \, dx$

$I = x^2 \sin^{-1} x^2 + \frac{1}{2} \int \frac{-4x^3}{\sqrt{1-x^4}} \, dx = x^2 \sin^{-1} x^2 + \sqrt{1-x^4} \Big|_0^{1/\sqrt{2}}$

$= \left(\frac{1}{2} \sin^{-1} \frac{1}{2} + \sqrt{1 - \frac{1}{4}}\right) - (0 + 1) = 1.1278 - 1 = 0.1278$

Q.1 B) $A = \int_0^3 (y_2 - y_1) \, dx$



$A = \int_0^3 (x - x^2 + 2x)$

$= \int_0^3 (3x - x^2) \, dx$

$= \frac{3x^2}{2} - \frac{x^3}{3} \Big|_0^3$

$= \left(\frac{27}{2} - \frac{27}{3}\right) - 0 = \frac{27}{6}$ Square unit

$x^2 - 2x = x$

$x^2 - 3x = 0$

$x(x-3) = 0$

$x = 0 \Rightarrow y = 0$

$x = 3 \Rightarrow y = 3$

Q.2 A) $\int \frac{dx}{\sqrt{x^2 - 16}}$ let $x = 4 \sec\theta$, $dx = 4 \sec\theta \tan\theta \, d\theta$

$I = \int \frac{4 \sec\theta \tan\theta \, d\theta}{\sqrt{16 \sec^2\theta - 16}} = \int \sec\theta \, d\theta$

$= \ln|\sec\theta + \tan\theta|$

but $\sec\theta = \frac{x}{4}$

$I = \ln\left|\frac{x}{4} + \frac{\sqrt{x^2 - 16}}{4}\right| + C$

Q.2 B) $\overrightarrow{AB} = -2i + j$, $\overrightarrow{AC} = 5i + 10j$

$\overrightarrow{AB} \cdot \overrightarrow{AC} = -10 + 10 = 0$

$\therefore \overrightarrow{AB} \perp \overrightarrow{AC}$

Q.3 A) $\displaystyle\int \frac{2x-3}{\sqrt{-(x^2-2x-3}}\,dx = \int \frac{2x-3}{\sqrt{-(x^2-2x+1-4)}}\,dx$

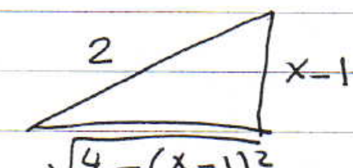
$\displaystyle = \int \frac{2x-3}{\sqrt{-\left[(x-1)^2-4\right]}} = \int \frac{2x-3}{\sqrt{4-(x-1)^2}}\,dx$

$\displaystyle = \int \frac{(4\sin\theta + 2 - 3) * 2\cancel{\cos}\theta\, d\theta}{\cancel{\sqrt{4-4\sin^2\theta}}}$ let $x-1 = 2\sin\theta$

$x = 2\sin\theta + 1$

$dx = 2\cos\theta\, d\theta$

$\displaystyle = -4\cos\theta - \theta = -4 * \frac{\sqrt{4-(x-1)^2}}{4} - \sin^{-1}\frac{x-1}{2}$



$\sin\theta = \dfrac{x-1}{2}$

Q 3 B) $\displaystyle y' = \pi^{\tanh x} * \ln\pi \cdot \operatorname{sech}^2 x + 2^{\log x} * \ln 2 \cdot \frac{1}{\ln 10} * \frac{1}{x}$

$\displaystyle + x^3 * \frac{3^x \ln 3}{1+(3^x)^2} + \tan^{-1} 3^x * 3x^2.$

Q 4 A) $\displaystyle\int \sin x(1-\cos^2 x)\cos^3 x\, dx$

$\displaystyle = \int \sin x \cos^3 x\, dx - \int \sin x \cos^5 x\, dx$

$\displaystyle = -\frac{\cos^4 x}{4} + \frac{\cos^6 x}{6} + C$

Q4 B) $V_1 \cdot V_2 = 6 + 4 - 2 = 8$

$$V_1 \times V_2 = \begin{vmatrix} i & j & k \\ 3 & 2 & -2 \\ 2 & 2 & 1 \end{vmatrix} = 6i - 7j + 2K$$

Q.5 A) $\displaystyle\int \frac{dx}{(x+1)(x-3)} = \frac{A}{x+1} + \frac{C}{x-3}$

$$= \frac{A(x-3) + C(x+1)}{(x+1)(x-3)}$$

$1 = A(x-3) + C(x+1)$, when $x = 3 \Rightarrow C = \frac{1}{4}$

when $x = -1 \Rightarrow A = -\frac{1}{4}$

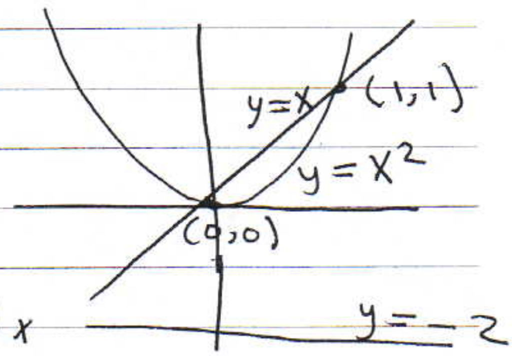
$$I = -\frac{1}{4}\ln|x+1| + \frac{1}{4}\ln|x-3| + c$$

let $x - 1 = 2\sec\theta$

$dx = 2\sec\theta\tan\theta\, d\theta$

or $\displaystyle\int \frac{dx}{x^2 - 2x + 1 - 1 - 3} = \int \frac{dx}{(x-1)^2 - 4}$

Q5. B) $\displaystyle V = \int_0^1 \pi r^2 dx - \int_0^1 \pi r^2 dx$



$$V = \int_0^1 \pi(y+2)^2 dx - \int_0^1 \pi(y+2)^2 dx$$

$$= \int_0^1 \pi(x+2)^2 dx - \int_0^1 \pi(x^2+2)^2 dx$$

$$= \int_0^1 \pi(x^2 + 4x + 4)\Big|_0^1 - \int_0^1 \pi(x^4 + 4x^2 + 4)\Big|_0^1$$

$$= \pi\left(\frac{x^3}{3} + 2x^2 + 4x\right)\Big|_0^1 - \pi\left(\frac{x^5}{5} + \frac{4x^3}{3} + 4x\right)\Big|_0^1$$

$$= \pi\left(\frac{1}{3} + 2 + 4\right) - \pi\left(\frac{1}{5} + \frac{4}{3} + 4\right)$$

$$\pi\,\frac{5 + 30 + 60}{15} - \pi\,\frac{3 + 20 + 60}{15}$$

$$= \pi\left(\frac{95}{15} - \frac{83}{15}\right) = \pi\left(\frac{12}{15}\right) = \frac{4}{5}\pi$$

Q.6 A) let $x = y^2$

$dx = 2y\,dy$

$$I = \int \frac{2y\,dy}{y(1+y)} = 2\int \frac{dy}{1+y}$$

$$= 2\ln|1+y|$$

$$= 2\ln|1+\sqrt{x}| + c$$

or

$$\int \frac{x^{-\frac{1}{2}}}{1+x^{1/2}}\,dx$$

$$\frac{1}{\left(\frac{1}{2}\right)} \int \frac{\left(\frac{1}{2}\right) x^{-\frac{1}{2}}}{1+x^{1/2}}\,dx$$

$$= 2\ln|1+\sqrt{x}| + c$$

Q.6 B)

$$\frac{1}{2}\left(e^{x^2} - e^{-x^2}\right) - \frac{3}{2}\left(e^{x^2} + e^{-x^2}\right) = -3$$

$$\frac{1}{2}e^{x^2} - \frac{1}{2}e^{-x^2} - \frac{3}{2}e^{x^2} - \frac{3}{2}e^{-x^2} = -3$$

$$-e^{x^2} - 2e^{-x^2} = -3$$

$$e^{x^2} + 2e^{-x^2} = 3$$

$$e^{x^2} + \frac{2}{e^{x^2}} = 3 \Longrightarrow \left(e^{x^2}\right)^2 + 2 = 3e^{x^2}$$

$$\left(e^{x^2}\right)^2 - 3e^{x^2} + 2 = 0$$

$$\left(e^{x^2} - 1\right)\left(e^{x^2} - 2\right) = 0$$

$$e^{x^2} = 1 \Longrightarrow x^2 = \ln 1 = 0 \Longrightarrow x = 0$$

$$e^{x^2} = 2 \Longrightarrow x^2 = \ln 2 = 0.693 \Longrightarrow x = 0.8325$$