University Of Technology
Building and Construction Eng. Dept.
Final Exam – First Attempt 2014- 2015

Subject : Prestressed Concrete | Class: 4^{th} year
Branch :Structural Division | Time 3 Hours
Examiner : Dr. Eyad K. & Dr. Nisreen S. | Date : 11/6 /2015

Answer Only Four Questions

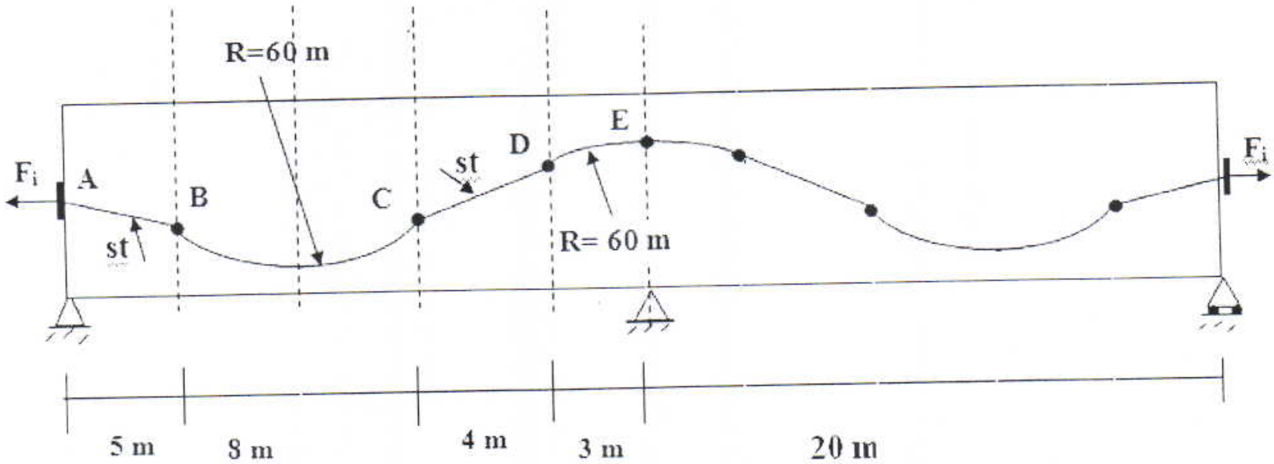
Q1: Using the following data, determine the safe uniformly distributed service load (w_s) that can be carried by a simply supported prestressed concrete beam;
Span (L)=15 m, A_g=194*10^3 mm^2 , I_g= 19.7*$10^9$$mm^4$, y_t=380 mm, y_b=520 mm, allowable tensile stress at initial and service stages (f_{ti} = f_{ts} =0) and allowable compressive stress at initial and service stages (f_{ci}=f_{cs} =14 N/mm^2), R=0.8 and γ_c =24 kN/m^3 .

Q2: For the symmetric post-tensioned beam shown in fig.(1) , determine the maximum friction losses using both exact and approximate approaches and assume μ = 0.3 and k= 0.0028.

Q3: For the simply supported beam shown in fig.(2), determine the ratio of ultimate to cracking additional uniformly distributed service load (w_s) that the beam can carry in addition to its selfweight.
Use the following data.
Under reinforced section; area of prestressed steel (A_{ps} =1000 mm^2), prestress stress after losses (f_{se} =900 N/mm^2), ultimate strength of prestressed steel (f'_s =1860 N/mm^2), f'_c = 40 N/mm^2 , γ_c =25 kN/m^3 , and modulus of rupture (f_r =4 N/mm^2).

Q4: Make preliminary design for a prestressed simply supported beam using the following data . overall depth (h=1000mm), effective prestress stress (f_{se}= 1000 N/mm^2). Span of beam (L=8 m), selfweight of beam (w_G= 15 kN/m), uniformly distributed service load (w_s= 60 kN/m), and allowable compressive stress of concrete (f_c= 12 N/mm^2).

Q5: For the beam shown in fig.(3), design for shear if the following data is given
Non-composite simply supported uniformly loaded beam. Straight tendon. Initial prestresss force (F_i =1200 kN), (M_u =700 kN.m), (V_u =300 kN), (V_i =140 kN) , (M_{max} =600 kN.m) , (V_d =100 kN) , (f_d = 4 N/mm^2)
Compressive strength of concrete (f'_c = 36 N/mm^2). Concrete cover = 100 mm, Use 8 mm stirrups. Yield strength of shear reinforcement (f_y = 400 N/mm^2).



St=straight

Fig. (1)

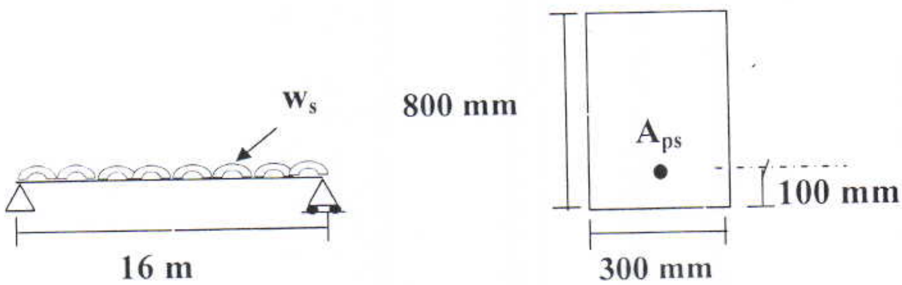


Fig. (2)

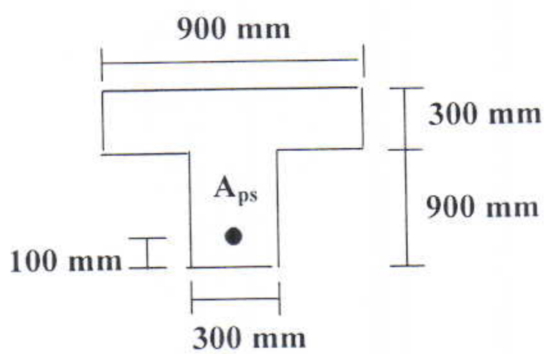


Fig.(3)

Notes

1-General

$$\overline{Y} = \frac{\sum A * Y}{\sum A}, \quad I = \sum(Ic + A * d^2)$$

2-Stresses

Initial stage

$Fi = A_{ps} * f_{si}$

$$f_{top} = -\frac{Fi}{A} + \frac{Fi * e * Yt}{I} - \frac{Mg * Yt}{I}, \qquad f_{bot} = -\frac{Fi}{A} - \frac{Fi * e * Yb}{I} + \frac{Mg * Yb}{I}$$

Service stage

$Fse = R * Fi$

$$f_{top} = -\frac{Fse}{A} + \frac{Fse * e * Yt}{I} - \frac{Mt * Yt}{I}, \qquad f_{bot} = -\frac{Fse}{A} - \frac{Fse * e * Yb}{I} + \frac{Mt * Yb}{I}$$

3-Losses

$$ES = K_{es} * n_i * \left(\alpha\frac{Fi}{A} + \alpha\frac{Fi * e^2}{I} - \frac{Mg * e}{I}\right), \quad n_i = \frac{Es}{Eci}, \quad Es = 200000\ N/mm^2$$

$Eci = 4730 * \sqrt{f'_{ci}}$

$$CR = K_{cr} * n * [f_{cir} - f_{cds}], \quad n = \frac{Es}{Ec}, \quad Ec = 4730 * \sqrt{f'c}$$

$$f_{cir} = \alpha\frac{Fi}{A} + \alpha\frac{Fi * e^2}{I} - \frac{Mg * e}{I}, \qquad f_{cds} = \frac{Md * e}{I}$$

$SH = 8.2 * 10^{-6} * K_{sh} * Es * (1 - 0.0024 * V/S) * (100 - RH)$

$$ANC = \frac{\Delta * Es}{L}$$

$Fx = Fs * e^{-(k * lx + \mu * \alpha)}$ ________________ exact

$Fx = Fs/(1 + k * lx + \mu * \alpha)$ __________ approximate

$$\alpha = 2 * \sin^{-1}\left(\frac{lx/2}{R}\right)$$

4-Cracking and Ultimate moment.

$$r = \sqrt{\frac{I}{A}}, \quad k_t = \frac{r^2}{Yb}, \quad k_b = \frac{r^2}{Yt}$$

$$M_1 = Fse * (e + k_t), \quad M_2 = \frac{fr * I}{Yb}, \quad M_{cr} = Fse * \left(e + \frac{r^2}{Yb}\right) + \frac{fr * I}{Yb}$$

$$kd = \frac{Aps * f's}{0.85 * f'c * b}, \quad A_{comp} = \frac{T}{0.85 * f'c}, \quad T = Aps * f's, \quad Mu = T * a$$

$l- \frac{L}{2}$

5- Preliminary and final elastic flexural design.

$Fse = \frac{Mt}{0.65 * h}$, $Fse = \frac{Ms}{0.5 * h}$, $Aps = \frac{Fse}{fse}$, $Ac = \frac{Fse}{0.5 * fc}$

$S_t = \frac{I}{Yt}$, $S_b = \frac{I}{Yb}$, $S_t = \frac{(1-R) * Mg + Ms}{R * fti - fcs}$, $S_b = \frac{(1-R) * Mg + Ms}{fts - R * fci}$

$.f_{cent} = fti - \frac{Yt}{h} * (fti - fci)$, $Fi = A * |f_{cent}|$, $fsi = 0.7 * f_s'$

$.e = e_m = (fti - fcent) * \frac{St}{Fi} + \frac{Mg}{Fi}$; for rectangular section, $S_b = S_t = \frac{bh^2}{6}$

6-Shear

$V_{ci} = [\frac{\sqrt{fc'}}{20} * bw * d + V_d + \frac{Vi * Mcr}{M\max}] \geq \frac{\sqrt{fc'}}{7} * bw * d$

$V_{ci} = [\frac{\sqrt{fc'}}{20} * bw * d + \frac{Vu * Mct}{Mu}] \geq \frac{\sqrt{fc'}}{7} * bw * d$

$V_{cw} = 0.3 * (\sqrt{f'} + fpc) * bw * d + V_p$

$.d \geq 0.8 * h$

$Mcr = \frac{I}{Yten} * (0.5 * \sqrt{fc'} + fpe - fd)$

$Mcr = \frac{I}{Yten} (0.5 * \sqrt{f'} + fpe)$

$fpe = \frac{Fi}{A} + \frac{Fi * e * Yten}{I}$; $f_d = \frac{M_G . y_{ten}}{I}$

$fpc = \frac{Fi}{A}$

$\frac{Vu}{\phi} = Vc + Vs$

$S = \frac{Av * fy * d}{Vs}$, $S = \frac{3 * Av * fy}{bw}$

If $Vs \leq \frac{1}{3}\sqrt{fc'} * bw * d$ THEN S_{max} = min. { 0.75*h , 600 mm}

If $Vs > \frac{1}{3}\sqrt{fc'} * bw * d$ THEN S_{max}=min {0.375*h , 300 mm }

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التاريخ / / ٢٠١   الموضوع :

Q1: $W_g = A_g \cdot \gamma_c = 194\times10^{3}\times10^{-6}\times 24 = 4.66\ kN/m$

$M_G = \frac{4.66\times15^2}{8} = 130.95\ kN.m.$

initial stage:

$f_{ti} = f_{top} = -\frac{F_i}{A} + \frac{F_i.e.y_t}{I} - \frac{M_G.y_t}{I}$

$0 = -\frac{F_i}{194\times10^3} + \frac{F_i.e(380)}{19.7\times10^9} - \frac{130.95\times10^6\times380}{19.7\times10^9}$

$0 = -5.15\times10^{-6}F_i + 0.019\times10^{-6}F_i.e - 2.53$

$F_i = \frac{2.53\times10^6}{(0.019e - 5.15)}$ ----- (1)

$F_{ci} = f_{bot} = -\frac{F_i}{A} - \frac{F_i.e.y_b}{I} + \frac{M_G.y_b}{I}$

$-14 = -\frac{F_i}{194\times10^3} - \frac{F_i.e.(520)}{19.7\times10^9} + \frac{130.95\times10^6\times520}{19.7\times10^9}$

$-14 = -5.15\times10^{-6}F_i - 0.026\times10^{-6}F_i.e + 3.46$

$F_i = \frac{17.46\times10^6}{(0.026e + 5.15)}$ ---- (2)

eq(1) = eq (2)

$\frac{2.53\times10^6}{0.019e - 5.15} = \frac{17.46\times10^6}{0.026e + 5.15}$

$e = 381.16\ mm.$

$\therefore F_i = \dfrac{17.46 \times 10^6}{0.026 \times 381.16 + 5.15} = 1146500\, N$

$F_i = 1146.5\, kN$

$F_{se} = K\, F_i = 0.8 \times 1146.5 = 917.2\, kN.$

Service stage

$f_{ts} = f_{bot} = -\dfrac{F_{se}}{A} - \dfrac{F_{se}.\, e.\, y_b}{I} + \dfrac{M_t . y_b}{I}$

$0 = -\dfrac{917.2 \times 10^3}{194 \times 10^3} - \dfrac{917.2 \times 10^3 \times 381.16 \times 520}{19.7 \times 10^9} + \dfrac{M_t \times 520}{19.7 \times 10^9}$

$0 = -4.73 - 9.23 + 0.026 \times 10^{-6} M_t$

$M_t = 536.92 \times 10^6\ N.mm = 536.92\ kN.m.$

$f_{cs} = f_{top} = -\dfrac{F_{se}}{A} + \dfrac{F_{se}.\, e.\, y_t}{I} - \dfrac{M_t . y_t}{I}$

$-14 = -\dfrac{917.2 \times 10^3}{194 \times 10^3} + \dfrac{917.2 \times 10^3 \times 381.16 \times 380}{19.7 \times 10^9} - \dfrac{M_t . 380}{19.7 \times 10^9}$

$-14 = -4.73 + 6.74 - 0.019 \times 10^{-6} M_t$

$M_t = 829.99 \times 10^{-6}\ N.mm = 829.99\ kN.m$

$\therefore M_t = 536.92\ kN.m$ (the smaller).

$M_t = M_G + M_s$

$536.92 = 130.95 + M_s$

$M_s = 405.92\ kN.m.$

$M_s = \dfrac{w_s L^2}{8}$

$405.92 = \dfrac{w_s (15)^2}{8}$

$w_s = 14.43\ kN/m.$

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الموضوع : التاريخ / / ٢٠١

Q_2: (1) Exact approach.

$$\alpha = 2 \sin^{-1}\left(\frac{l_x/2}{R}\right) \qquad k = 0.0028$$

$$F_x = F_s . e^{-(kl_x + \mu\alpha)} \qquad \mu = 0.30$$

segment	L_x (cm)	kl_x	α (rad)	$\mu\alpha$	$e^{-(klx+\mu\alpha)}$	F_x
AB	5	0.014	ϕ	ϕ	0.9861	$0.9861F_i$
BC	8	0.0224	0.133	0.0399	0.9396	$0.9265F_i$
CD	4	0.0112	ϕ	ϕ	0.9889	$0.9162F_i$
DE	3	0.0084	0.05	0.015	0.9769	$0.895F_i$

$\therefore F_r$ Losses $= F_i - 0.895 F_i = 0.105 F_i$

$F_r\% = 10.5\%$

(2) Approximate losses.

$$F_x = F_s . \frac{1}{1 + kl_x + \mu\alpha}$$

Segment	$\frac{1}{1+kl_x+\mu\alpha}$	F_x
AB	0.9861	$0.9861 F_i$
BC	0.9414	$0.9284 F_i$
CD	0.9889	$0.9181 F_i$
DE	0.9771	$0.897 F_i$

F_r losses $= F_i - 0.897 F_i = 0.103 F_i$

$F_r\% = 10.3\%$

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Q3: $A = 300 \times 800 = 2.4 \times 10^5 \ mm^2$

$y_t = y_b = \frac{800}{2} = 400 \ mm$

$e = 400 - 100 = 300 \ mm.$

$I = \frac{300\,(800)^3}{12} = 1.28 \times 10^{10} \ mm^4$

$k_t = \frac{h}{6} = \frac{800}{6} = 133.33 \ mm.$

$w_g = A \cdot \gamma_c = 2.4 \times 10^5 \times 25 \times 10^{-6} = 6 \ kN/m$

$M_g = \frac{6 \times (16)^2}{8} = 192 \ kN.m.$

$F_{se} = A_{ps} \cdot f_{se} = 1000 \times 900 \times 10^{-3} = 900 \ kN.$

$M_{cr} = F_{se}\,(e + k_t) + \frac{f_r \cdot I}{y_b}$

$= 900\,(300 + 133.33) \times 10^{-3} + \frac{4 \times 1.28 \times 10^{10}}{400} \times 10^{-6}$

$= 390 + 128 = 518 \ kN.m$

$\therefore M_s = 518 - 192 = 326 \ kN.m$

$M_s = \frac{w_s \cdot L^2}{8}$

$326 = \frac{w_s \cdot (16)^2}{8} \Rightarrow (w_s)_{cr} = 10.19 \ kN/m$

$T = 1000 \times 1860 \times 10^{-3} = 1860 \ kN$

$kd = \frac{1860 \times 10^3}{0.85 \times 40 \times 300} = 182.35 \ mm$

$d = 800 - 100 = 700 \ mm$

$a = d - \frac{kd}{2} = 700 - \frac{182.35}{2} = 608.82 \ mm$

$M_u = T \cdot a = 1860 \times 608.82 \times 10^{-3} = 1132.41 \ kN.m$

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Class

$M_s = 1132.41 - 192 = 940.41 \text{ kN.m}$

$$940.41 = \frac{w_s (16)^2}{8}$$

$$(w_s)_u = 29.39 \text{ kN/m}.$$

$$\frac{(w_s)_u}{(w_s)_{cr}} = \frac{29.39}{10.19} = 2.88.$$

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Q4: Make preliminary design for a prestress simply supported beam using the following dat.

Over all depth ($h = 1000$ mm), effective prestress stress ($f_{se} = 1000$ N/mm^2). span of beam ($L = 8$ m) self weight of beam ($W_G = 15$ kN/m), uniformly distributed service load ($W_s = 60$ kN/m) and allowable compressive stress of concrete ($f_c = \cancel{16}\ 12$ N/mm^2).

Sol.

$$M_G = \frac{15 \times 8^2}{8} = 120 \text{ kN.m.}$$

$$M_s = \frac{60 \times 8^2}{8} = 480 \text{ kN.m.}$$

$$M_t = 120 + 480 = 600 \text{ kN.m.}$$

$$\frac{M_G}{M_t} = \frac{120}{600} = 0.20$$

∴ Try both effect

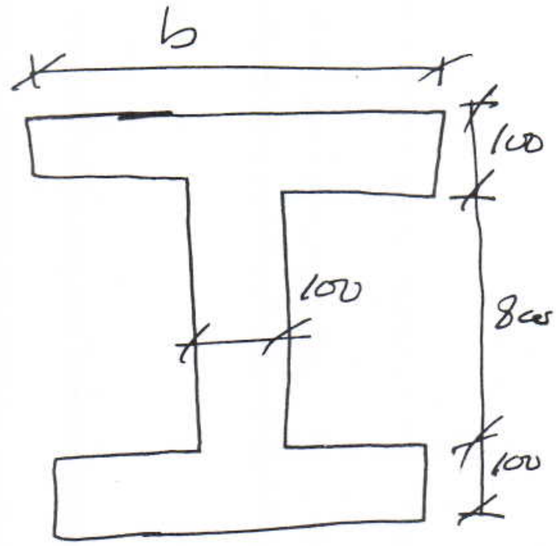
$$F_{se} = \frac{M_t}{0.65 h} = \frac{600 \times 10^3}{0.65 \times 1000} = 923.1 \text{ kN.}$$

$$F_{se} = \frac{M_s}{0.5 h} = \frac{480 \times 10^3}{0.5 \times 1000} = \underline{960 \text{ kN}}$$

control

$$A_c = \frac{F_{se}}{0.5 f_c} = \frac{960 \times 10^3}{0.5 \times 12} = 160000 \text{ mm}^2$$

$$A_{ps} = \frac{F_{se}}{f_{se}} = \frac{960 \times 10^3}{1000} = 960 \text{ mm}^2$$



$$160000 = 800 \times 100 + 2b \times 100$$

$$b = 400\ mm.$$

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التاريخ / / ٢٠١

الموضوع :

Q5.

$A = 2 \times 900 \times 300 = 5.4 \times 10^5\ mm^2$

$\bar{y} = \frac{900 \times 300 (1050) + 900 \times 300 (450)}{5.4 \times 10^5} = 750\ mm$

$y_b = 750\ mm$, $y_{top} = 450\ mm$.

$e = 750 - 100 = 650\ mm$.

$I = \frac{900 (300)^3}{12} + (300 \times 900)(1050 - 750)^2 + \frac{300 (900)^3}{12} + (300 \times 900)(450 - 750)^2$

$I = 6.89 \times 10^{10}\ mm^4$

(a) flexural shear

$V_{ci} = \left[\frac{\sqrt{f_c'}}{20} b_w . d + V_d + \frac{V_i M_{cr}}{M_{max}} \right] \geq \frac{\sqrt{f_c'}}{7} b_w d$

$d = 1200 - 100 = 1100\ mm$

$d = 0.8 h = 0.8 \times 1200 = 960\ mm$

$\therefore d = 1100\ mm$.

$M_{cr} = \frac{I}{y_{ten}} (0.5 \sqrt{f_c'} + f_{pe} - f_d)$

$f_{pe} = \frac{F_i}{A} + \frac{F_i . e . y_{ten}}{I}$

$= \frac{1200 \times 10^3}{5.4 \times 10^5} + \frac{1200 \times 10^3 \times 650 \times 750}{6.89 \times 10^{10}}$

$= 2.22 + 8.49 = 10.71\ N/mm^2$

$\therefore M_{cr} = \frac{6.89 \times 10^{10}}{750} \left(0.5\sqrt{36} + 10.71 - 4 \right) \times 10^{-6}$

$M_{cr} = 892.03 \ kN.m.$

$\therefore V_{ci} = \left[\frac{\sqrt{36}}{20} \times 300 \times 1100 \times 10^{-3} + 100 + \frac{140 \times 892.03}{600} \right]$

$V_{ci} = 407.14 kN > \frac{\sqrt{36}}{7} \times 300 \times 1100 \times 10^{-3} = 282.86 \ kN$ o.k

for non. composite uniformly loaded beam

$V_{ci} = \left[\frac{\sqrt{f_c'}}{20} b_w.d + \frac{V_u M_{ct}}{M_u} \right] \geq \frac{\sqrt{f_c'}}{7} b_w.d$

$M_{ct} = \frac{I}{y_{ten}} \left(0.5\sqrt{f_c'} + f_{pe} \right)$

$= \frac{6.89 \times 10^{10}}{750} \left(0.5\sqrt{36} + 10.71 \right) \times 10^{-6}$

$= 1259.49 \ kN.m.$

$\therefore V_{ci} = \left[\frac{\sqrt{36}}{20} \times 300 \times 1100 \times 10^{-3} + \frac{300 \times 1259.49}{700} \right]$

$V_{ci} = 638.78 \ kN > 282.86 \ kN$ o.k

use $V_{ci} = 407.14 \ kN$.

(b) Web shear

$$V_{cw} = 0.3\left[\sqrt{f_c'} + f_{pc}\right] b_w \cdot d + V_p .$$

$V_p = \phi$ (straight tendon)

$$f_{pc} = \frac{F_i}{A} = 2.22 \ N/mm^2$$

$$\therefore V_{cw} = 0.3\left[\sqrt{36} + 2.22\right] \times 300 \times 1100 \times 10^{-3} + \phi$$

$$V_{cw} = 813.78 \ kN$$

$$\therefore V_c = 407.14 \ kN$$

$$\frac{V_u}{\phi} = \frac{300}{0.75} = 400 kN$$

$$\frac{V_c}{2} = 203.57 \ kN$$

$$\frac{V_c}{2} < \frac{V_u}{\varphi} < V_c$$

use min. reinf.

$$A_v = 2 \times \frac{\pi}{4}(8)^2 = 100.53 \ mm^2$$

$$s = \frac{3 A_v \cdot f_y}{b_w} = \frac{3 \times 100.53 \times 400}{300} = 402.12 \ mm$$

use $\phi 8$ @ 400.

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