خرسانة مسبقة الجهد / رابع انشائية

University Of Technology
Building and Construction Eng. Dept.
Final Exam-First Attempt 2015- 2016

Subject : Prestressed Concrete — Class: 4^{th} year
Branch :Structural Division — Time : 3 Hours
Examiner : Dr. Eyad & Dr. Nisreen — Date : 12 /6 /2016

Answer Only Four Questions

Q1: For the simply supported prestressed pretensioned concrete beam shown in Fig.(1), determine and plote the stress distribution at midspan for initial and service stages.

Q2: For the pretensioned concrete beam shown in Fig..(1), determine the elastic shortening (ES), creep (CR), and shrinkage (SH) losses.

Q3: For the prestressed concrete beam shown in Fig.(1), determine the uniformly distributed load (w, kN/m) that can be carried by the beam in addition to the given loads if,

a) Tensile stress in the bottom fiber is zero.
b) Cracking in the bottom fiber at modulus of rupture (f_r= 5 N/mm^2)
c) Ultimate moment capacity of the beam section.

Q4: If the pretensioned concrete beam section shown in Fig.(1) is a preliminary design section, check the design for the following additional data.

1- Allowable compressive strength of initial stage ($f_{ci} = 15\ N/mm^2$)
2- Allowable tensile strength at initial stage ($f_{ti} = 2.5\ N/mm^2$)
3- Allowable compressive strength at service stage ($f_{cs} = 15\ N/mm^2$)
4- Allowable tensile strength at service stage ($f_{ts} = 3\ N/mm^2$)

Area of strand = 92.9 mm^2 / strand.

Q5: For the pretensioned concrete beam shown in Fig.(1), design for shear at distance (d) from the face of support, if the following additional data is given.

Yield strength of shear reinforcement $f_y = 400\ N/mm^2$, use $\emptyset\ 8\ mm$ stirrups.

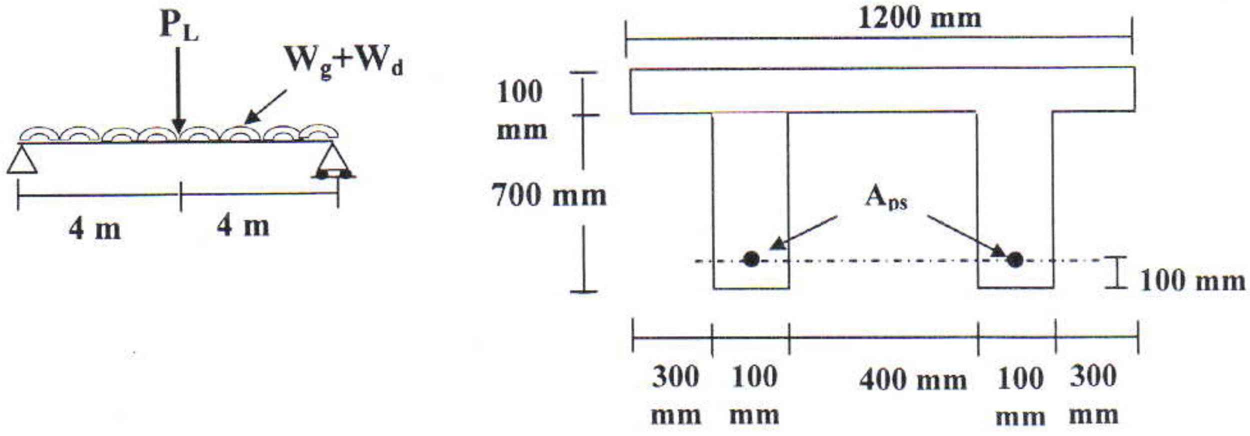

Fig.(1)

Area of prestressed steel ($A_{ps} = 800\ mm^2$)
Initial prestress stress ($f_{si} = 1200\ \ N/mm^2$)
Ultimate tensile stress ($f'_s = 1860\ \ N/mm^2$)
Initial compressive strength of concrete ($f'_{ci} = 36\ \ N/mm^2$)
Compressive strength of concrete at 28 days ($f'_c = 49\ \ N/mm^2$)
$\gamma_c = 25\ kN/m^3$

Supper imposed uniformly distributed dead load (W_d)= 20 kN/m
Supper imposed concentrated live load (P_L) = 100 kN
Assumed total losses= 20%
Pretensioned normal weight concrete
K_{cr} = 2.0
Straight tendon
Relative humidity (RH)= 60%

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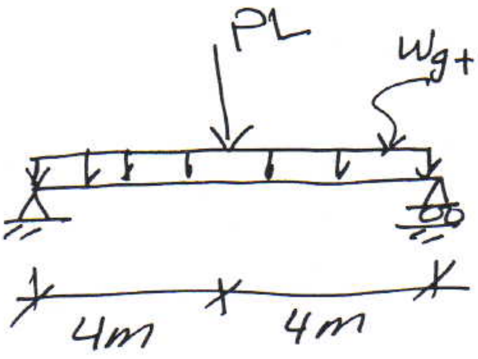
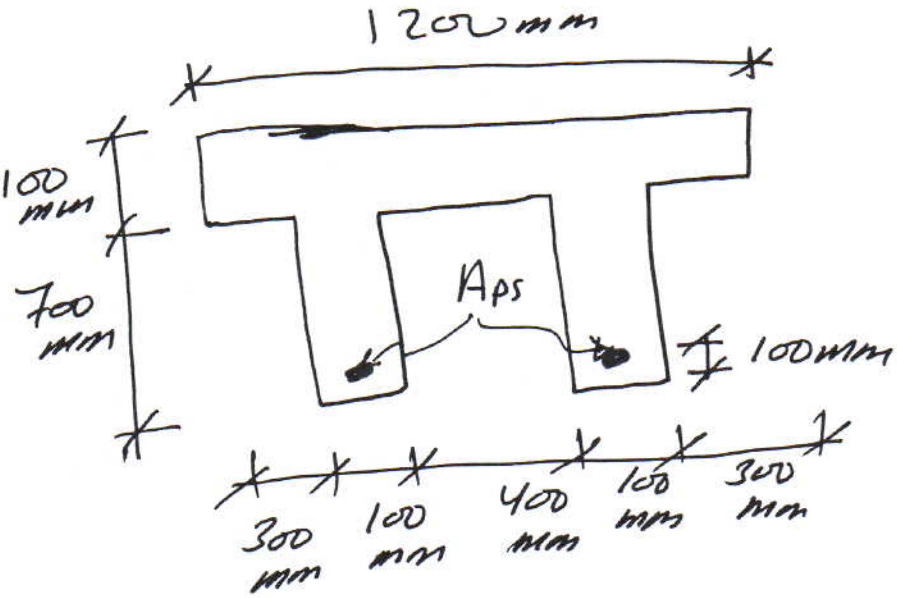
Q1





$$F_i = A_{ps} \times f_{si} = 800 \times 1200 \times 10^{-3} = 960\ kN$$

$$R = 1 - L = 1 - 0.2 = 0.80$$

$$F_{se} = R \times F_i = 0.8 \times 960 = 768\ kN.$$

$$A_g = 1200 \times 100 + 2 \times 100 \times 700 = 2.6 \times 10^5\ mm^2$$

$$y_t = \frac{\Sigma A \cdot y}{\Sigma A} = \frac{1200 \times 100 \times 50 + 2 \times 100 \times 700 \times 450}{2.6 \times 10^5} = 265.38\ mm$$

$$y_b = 800 - 265.38 = 534.62\ mm.$$

$$e = y_b - \text{cover} = 534.62 - 100 = 434.62\ mm.$$

$$I = \Sigma (I_c + A \cdot d^2).$$

$$= \frac{1200(100)^3}{12} + (1200 \times 100)(50 - 265.38)^2$$

$$+ 2\left[\frac{100(700)^3}{12} + (100 \times 700)(450 - 265.38)^2\right]$$

$$I = 1 \times 10^8 + 55.67 \times 10^8 + 2(28.58 \times 10^8 + 23.86 \times 10^8)$$

$$I = 161.55 \times 10^8 = 1.62 \times 10^{10}\ mm^4.$$

$$w_g = A_g \times \gamma_c = 2.6\times10^5 \times 25\times10^{-6} = 6.5\ kN/m$$

$$M_G = \frac{w_g . L^2}{8} = \frac{6.5\times(8)^2}{8} = 52\ kN.m$$

$$M_D = \frac{w_d \times L^2}{8} = \frac{20\ (8)^2}{8} = 160\ kN.m$$

$$M_L = \frac{P_L \times L}{4} = \frac{100\times 8}{4} = 200\ kN.m$$

$$M_s = M_D + M_L = 160 + 200 = 360\ kN.m$$

$$M_t = M_G + M_L = 52 + 360 = 412\ kN.m.$$

Initial stage

$$f_{top} = -\frac{F_i}{A} + \frac{F_i . e . y_t}{I} - \frac{M_G . y_t}{I}$$

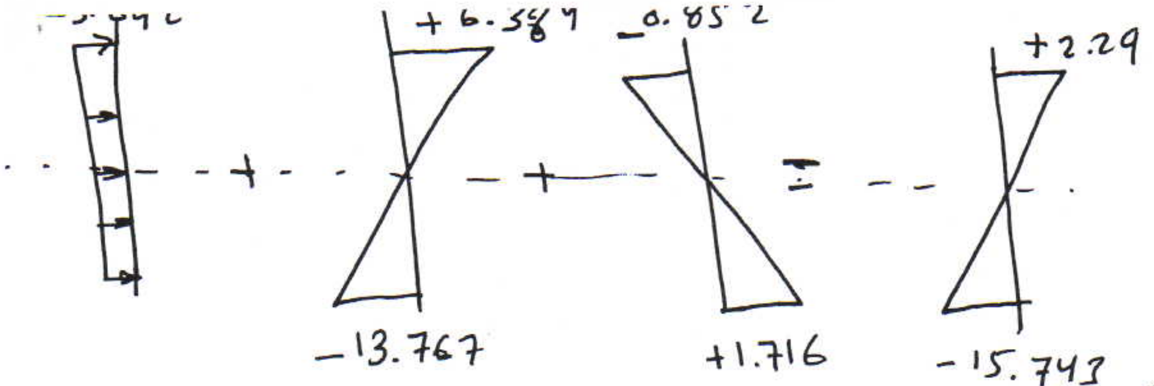
$$= -\frac{960\times10^3}{2.6\times10^5} + \frac{960\times10^3\times434.62\times265.38}{1.62\times10^{10}} - \frac{52\times10^6\times265.38}{1.62\times10^{10}}$$

$$= -3.692 + 6.834 - 0.852 = +2.29\ N/mm^2\ (\text{ten.})$$

$$f_{bot} = -\frac{F_i}{A} - \frac{F_i . e . y_b}{I} + \frac{M_G . y_b}{I}$$

$$= \frac{-960\times10^3}{2.6\times10^5} - \frac{960\times10^3\times434.62\times534.62}{1.62\times10^{10}} + \frac{52\times10^6\times534.6}{1.62\times10^{10}}$$

$$= -3.692 - 13.767 + 1.716 = -15.743\ N/mm^2\ (\text{Com}$$



stress distribution at initial stage

<u>service stage</u>

$$f_{top} = -\frac{F_{se}}{A} + \frac{F_{se}.e.y_t}{I} - \frac{M_t.y_t}{I}$$

$$= -\frac{768\times10^3}{2.6\times10^5} + \frac{768\times10^3\times434.62\times265.38}{1.62\times10^{10}} - \frac{412\times10^6\times265.38}{1.62\times10^{10}}$$

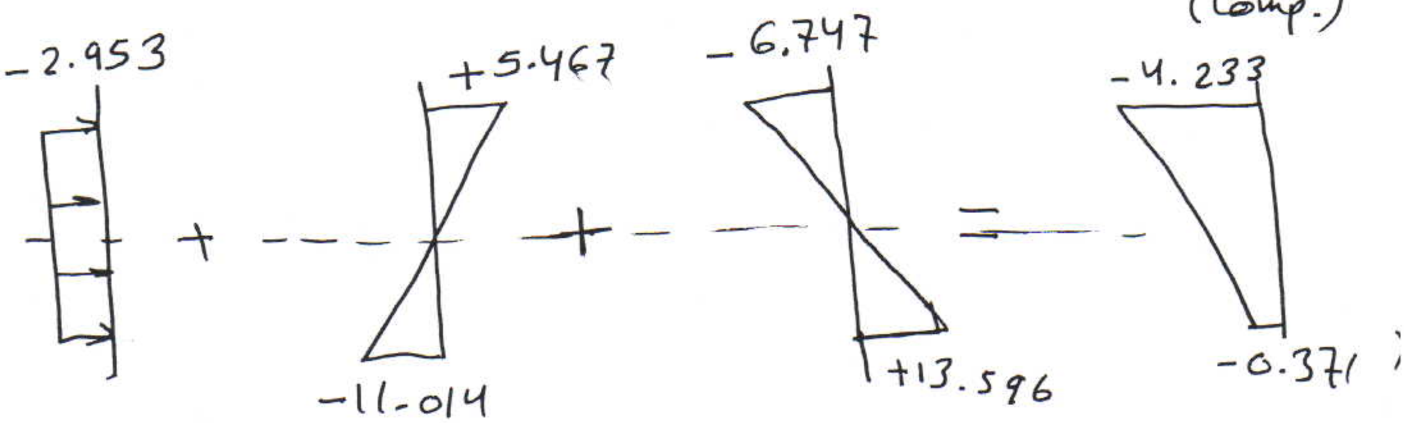
$$= -2.953 + 5.467 - 6.747 = -4.233\ N/mm^2_{(Comp)}$$

$$f_{bot} = -\frac{F_{se}}{A} - \frac{F_{se}.e.y_b}{I} + \frac{M_t.y_b}{I}$$

$$= -\frac{768\times10^3}{2.6\times10^5} - \frac{768\times10^3\times434.62\times534.62}{1.62\times10^{10}} + \frac{412\times10^6\times534.6}{1.62\times10^{10}}$$

$$= -2.953 - 11.014 + 13.596 = -0.371\ N/mm^2$$

(Comp.)



stress distribution at service stage

Q2:

__Elastic shortening ES__

$$ES = Kes \cdot ni \left(\alpha \frac{Fi}{A} + \alpha \frac{Fi \cdot e^2}{I} - \frac{M_G \cdot e}{I} \right)$$

$Kes = 1.0$, $\alpha = 0.9$ (pretensioned).

$$E_{ci} = 4730 \sqrt{f'_{ci}} = 4730 \sqrt{36} = 28380 \ N/mm^2$$

$$ni = \frac{E_s}{E_{ci}} = \frac{200\,000}{28380} = 7.04$$

$$\therefore ES = 1 \times 7.04 \left(0.9 \frac{960 \times 10^3}{2.6 \times 10^5} + 0.9 \frac{960 \times 10^3 (434.62)^2}{1.62 \times 10^{10}} - \frac{52 \times 10^6 \times 434.62}{1.62 \times 10^{10}} \right)$$

$$= 7.04 \left(3.323 + 11.194 - 1.395 \right)$$

$$ES = 7.04 \times 13.122 = 92.379 \ N/mm^2$$

$$ES\% = \frac{ES}{f_{si}} \times 100 = \frac{92.379}{1200} \times 100 = 7.7\%$$

__Creep (CR)__

$$CR = K_{cr}\, n \left(f_{cir} - f_{cds} \right)$$

$K_{cr} = 2.0$

$$E_c = 4730 \sqrt{f'_c} = 4730 \times \sqrt{49} = 6.04$$

$$f_{cir} = \alpha \frac{Fi}{A} + \alpha \frac{Fi \cdot e^2}{I} - \frac{M_G \cdot e}{I}$$

$$f_{cir} = 13.122 \ N/mm^2$$

$$f_{cds} = \frac{M_D . e}{I} = \frac{160 \times 10^6 \times 434.62}{1.62 \times 10^{10}} = 4.293 \ N/mm^2$$

$$\therefore CR = 2 \times 6.04 (13.122 - 4.293) = 106.65 \ N/mm^2$$

$$CR\% = \frac{CR}{f_{si}} \times 100 = \frac{106.65}{1200} \times 100 = 8.89\%$$

Shrinkage (SH)

$$SH = 8.2 \times 10^{-6} \times K_{sh} \times E_s \left(1 - 0.0024 \frac{V}{S}\right)(100 - RH)$$

$K_{sh} = 1.0$

$E_s = 200\,000 \ N/mm^2$

$RH = 60\%$

$$\frac{V}{S} = \frac{A}{P} = \frac{2.6 \times 10^5}{(2 \times 1200) + (2 \times 800) + (2 \times 700)} = \frac{2.6 \times 10^5}{5400}$$

$$\frac{V}{S} = 48.15 \ mm.$$

$$SH = 8.2 \times 10^{-6} \times 1 \times 200\,000 \ (1 - 0.0024 \times 48.15)(100 - 60)$$

$$SH = 58.02 \ N/mm^2$$

$$SH\% = \frac{SH}{f_{si}} \times 100 = \frac{58.02}{1200} \times 100 = 4.83\%$$

Q3:

(a). $M_1 = F_{se}(e + k_t)$

$F_{se} = 768 \text{ kN}$; $e = 434.62 \text{ mm}$

$k_t = \dfrac{r^2}{y_b}$; $y_b = 534.62 \text{ mm}$.

$r^2 = \dfrac{I}{A} = \dfrac{1.62 \times 10^{10}}{2.6 \times 10^5} = 0.623 \times 10^5 \text{ mm}^2$

$\therefore k_t = \dfrac{0.623 \times 10^5}{534.62} = 116.53 \text{ mm}$

$\therefore M_1 = 768(434.62 + 116.53) \times 10^{-3} = 423.28 \text{ kN.m}$

$M_1 = M_t + \dfrac{w_1 (8)^2}{8}$

$423.28 = 412 + \dfrac{w_1 (8)^2}{8}$

$w_1 = 1.41 \text{ kN/m}$.

(b). $M_2 = \dfrac{f_r . I}{y_b} = \dfrac{5 \times 1.62 \times 10^{10}}{534.62} \times 10^{-6} = 151.509 \text{ kN.m}$

$M_{cr} = M_1 + M_2 = 423.28 + 151.509 = 574.789 \text{ kN.m}$

$M_{cr} = M_t + \dfrac{w_2 (8)}{8}$

$574.789 = 412 + \dfrac{w_2 (8)}{8}$

$w_2 = 20.34 \text{ kN/m}$

(c)

$T = A_{ps} \times f_s' = 800 \times 1860 \times 10^{-3} = 1488\ kN$

$A_{comp} = \dfrac{T}{0.85 f_c'} = \dfrac{1488 \times 10^3}{0.85 \times 49} = 35726.29\ mm^2$

$A_{flang} = 1200 \times 100 = 120\,000\ mm^2$

$\because A_{flang} > A_{comp}.$

$\therefore$ rectangular section.

$kd = \dfrac{A_{ps} \cdot f_s'}{0.85 f_c' b} = \dfrac{1488 \times 10^3}{0.85 \times 49 \times 1200} = 29.772\ mm$

$d = 800 - 100 = 700\ mm$

$a = d - \dfrac{kd}{2} = 700 - \dfrac{29.772}{2} = 685.114\ mm$

$M_u = T \cdot a = 1488 \times \dfrac{685.114}{1000} = 1019.45\ kN.m$

$M_u = M_t + \dfrac{w_3 (8)^2}{8}$

$1019.45 = 412 + \dfrac{w_3 (8)^2}{8}$

$w_3 = 75.93\ kN/m.$

Q4:

$$S_t = \frac{I}{y_t} = \frac{1.62\times10^{10}}{265.38} = 61.045\times10^{6}\ mm^3$$

$$S_b = \frac{I}{y_b} = \frac{1.62\times10^{10}}{534.62} = 30.302\times10^{6}\ mm^3$$

$$S_t = \frac{M_G + M_D + M_L}{R f_{ti} - f_{cs}} = \frac{[52+160+200]\times10^{6}}{0.8\times2.5+15}$$

$$S_t = 24.23\times10^{6}\ mm^3 < 61.045\times10^{6}\ mm^3 \quad \underline{o.k}$$

$$S_b = \frac{M_G + M_D + M_L}{f_{ts} - R f_{ci}} = \frac{412\times10^{6}}{3+0.8\times15} = 27.47\times10^{6}\ mm^3$$

$$< 30.302\times10^{6}\ mm^3 \quad \underline{o.k}$$

$$f_{cent} = f_{ti} - \frac{y_t}{h}(f_{ti} - f_{ci})$$

$$= 2.5 - \frac{265.38}{800}(2.5+15) = -3.305\ N/mm^2$$

$$F_i = A_g \cdot |f_{cent}| = 2.6\times10^{5}\times|-3.305|\times10^{-3} = 859.348\ kN.$$

$$f_{si} = 0.7\times f_s' = 0.7\times1860 = 1302\ N/mm^2$$

$$A_{ps} = \frac{F_i}{f_{si}} = \frac{859.349\times10^{3}}{1302} = 660.021\ mm^2$$

$$\text{No. of strands} = \frac{660.021}{92.9} = 7.104$$

Use 8 strands

$$e = (f_{ti} - f_{cent}) \frac{S_t}{F_i}$$

$$e = (2.5 + 3.305) \frac{61.045 \times 10^6}{859.348 \times 10^3}$$

$$e = 412.367 \text{ mm}$$

or

$$e = (f_{cent} - f_{ci}) \frac{S_b}{F_i}$$

$$e = (-3.305 + 15) \frac{30.302 \times 10^6}{859.348 \times 10^3}$$

$$e = 412.385 \text{ mm}$$

ok

Q_5:

Concrete shear resistance

a- flexural - shear

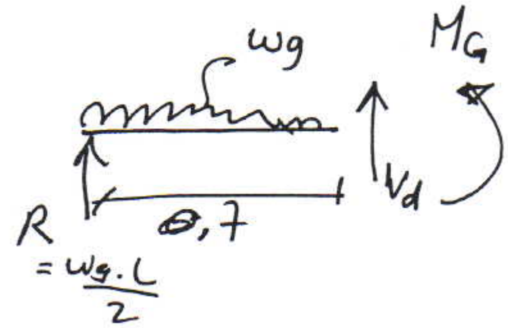
$$V_{ci} = \left[\frac{\sqrt{f_c'}}{20} b_w . d + V_d + \frac{V_i . M_{cr}}{M_{max}} \right] \geq \frac{\sqrt{f_c'}}{7} b_w . d$$

$d = 800 - 100 = 700 \text{ mm}$

$d = 0.8h = 0.8 \times 800 = 640 \text{ mm}$

use $d = 700$ mm.

$w_g = 6.5 \text{ kN/m}$



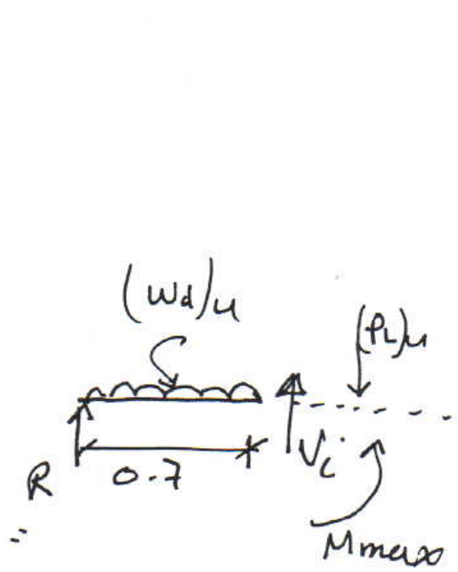
$$R = \frac{W_g . L}{2} = \frac{6.5 \times 8}{2} = 26 \text{ kN}$$

$\therefore V_d = 26 - 6.5 \times 0.7 = 21.45 \text{ kN}$

$$M_G = 26 \times 0.7 - 6.5 \frac{(0.7)^2}{2} = 16.608 \text{ kN.m.}$$

$(P_L)_u = 1.6 \times 100 = 160 \text{ kN}$

$(W_d)_u = 1.2 \times 20 = 24 \text{ kN/m.}$



$$R = \frac{160}{2} + 24 \times \frac{8}{2} = 176 \text{ kN}$$

$V_i = 176 - 24 \times 0.7 = 159.20 \text{ kN}$

$$M_{max} = 176 \times 0.7 - 24 \frac{(0.7)^2}{2} = 117.32 \text{ kN.m}$$

$$M_{cr} = \left(\frac{I}{y_{ten}}\right)\left(0.5\sqrt{f_c'} + f_{pe} - f_d\right)$$

$$f_{pe} = \frac{F_i}{A} + \frac{F_i . e . y_{ten}}{I}$$

$$= \frac{960 \times 10^3}{2.6 \times 10^5} + \frac{960 \times 10^3 \times 434.62 \times 534.62}{1.62 \times 10^{10}}$$

$$f_{pe} = 3.697 + 13.767 = 17.464 \text{ N/mm}^2$$

$$f_d = \frac{M_G . y_{ten}}{I} = \frac{16.608 \times 10^6 \times 534.62}{1.62 \times 10^{10}} = 0.548 \text{ N/mm}^2$$

$$\therefore M_{cr} = \frac{1.62 \times 10^{10}}{534.62}\left(0.5\sqrt{49} + 17.464 - 0.548\right) \times 10^{-6}$$

$$M_{cr} = 618.644 \text{ kN.m.}$$

$$\therefore V_{ci} = \left[\frac{\sqrt{49}}{20} \times 2 \times 100 \times 700 \times 10^{-3} + 24.45 + \frac{159.2 \times 618.644}{117.32}\right]$$

$$V_{ci} = 909.932 \text{ kN} > \frac{\sqrt{49}}{7} \times 2 \times 100 \times 700 \times 10^{-3} = 140 \text{ kN}$$

o.k

b- web - shear

$$V_{cw} = 0.3\left[\sqrt{f_c'} + f_{pc}\right] b_w . d + V_p$$

$V_p = \phi$ (straight tendon)

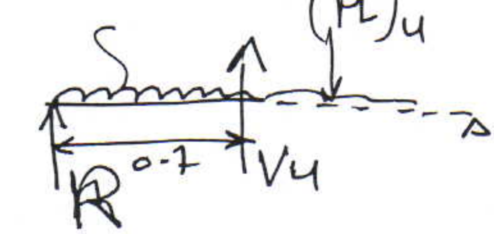
$f_{pc} = \frac{F_i}{A} = 3.697 \ N/mm^2$

$\therefore V_{cw} = 0.3\left[\sqrt{49} + 3.697\right] \times 2 \times 100 \times 700 \times 10^{-3} = 449.274$ kN

$\therefore V_c = 449.274$ kN , $\frac{V_c}{2} = 224.637$ kN $\quad$ control.

$R = \frac{160}{2} + 1.2(20+6.5) \times \frac{8}{2}$

$= 207.2$ kN



or; $R = 1.2 \times 26 + 176 = 207.2$ kN

$V_u = 207.2 - 1.2(20+6.5) \times 0.7$

$V_u = 184.94$ kN

$\frac{V_u}{\phi} = \frac{184.94}{0.75} = 246.587$ kN

$\frac{V_c}{2} < \frac{V_u}{\phi} < V_c$

use min reinf.

$A_v = 2 \times \frac{\pi}{4}(8)^2 = 100 \ mm^2$

$S = \frac{3A_v \cdot f_y}{b_w} = \frac{3 \times 100 \times 400}{2 \times 100} = 600$ mm

$F_{ult} = A_p \cdot f_s' = 800 \times 1860 \times 10^{-3} = 1488$ kN

$0.4 \times 1488 = 595.2$ kN $< F_i$ o.k

$S = \frac{80 \ A_v \cdot f_y \cdot d}{A_{ps} \ f_s'}\sqrt{\frac{b_w}{d}} = \frac{80 \times 100 \times 400 \times 700}{800 \times 1860}\sqrt{\frac{2 \times 100}{700}} = 804.65$ mm

use $\phi 8$ @ 600 mm.