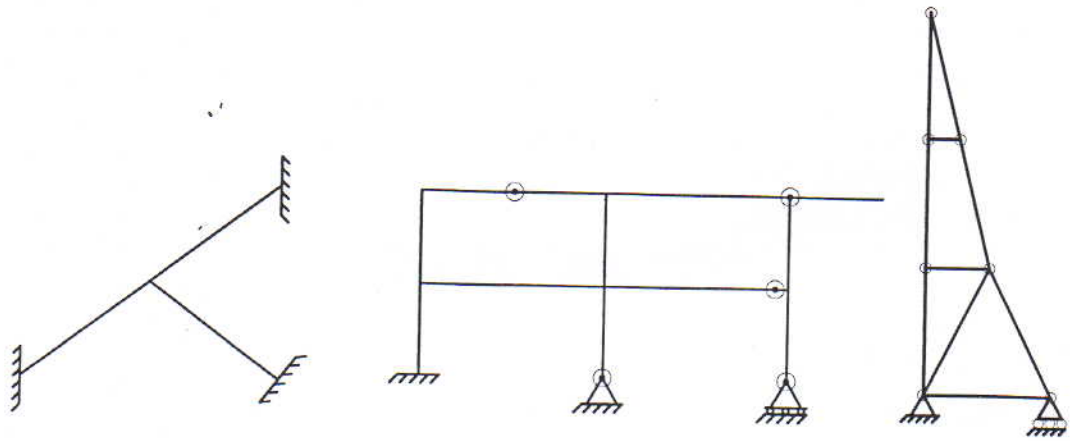
Theory of Structures (I) - BCE | May 29, 2016 | Time: 3 Hours
Third Year | Final Exam – First Attempt | **Closed Book & Notes**

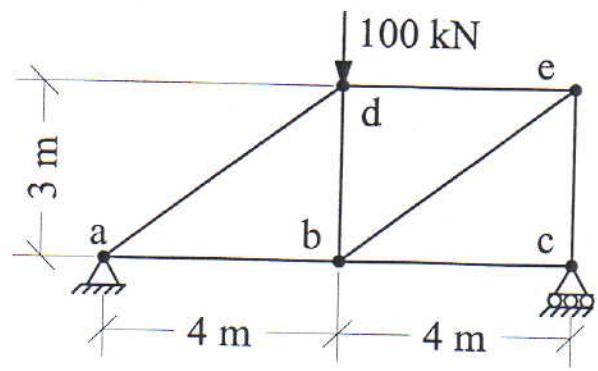
Answer 4 questions only. ثالث انشائية

Q1:

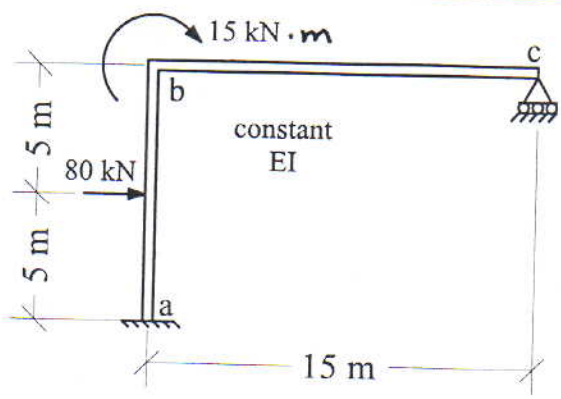
a. Write briefly about the structural analysis and design; discuss safety, economy and serviceability for the structures and the main role of engineer to provide these requirements.
b. Sometimes it is important to use an inactive members in a truss, define inactive these members and why it should be used for?
c. What is the stiffness and flexibility mean for structural engineer?
d. Discuss the stability and determinacy of the 3 structures shown:



Q2: For the structural truss shown, using the Unit Load Method, find the vertical deflection of joint B in terms of EA. EA = constant for all members.



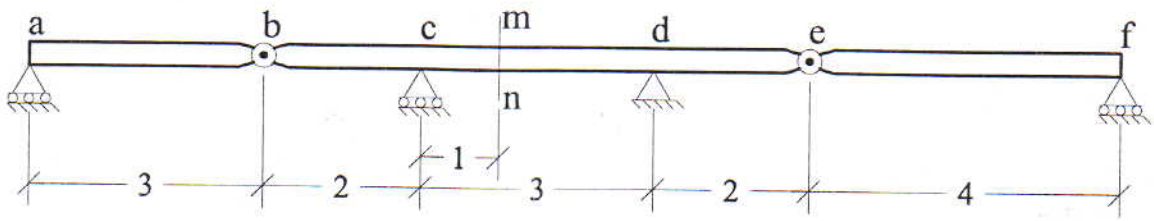
Q3: For the structural frame shown, using the Consistent Deformation Method, draw the bending moment diagram for member ~~cd~~ bc.



Q4:

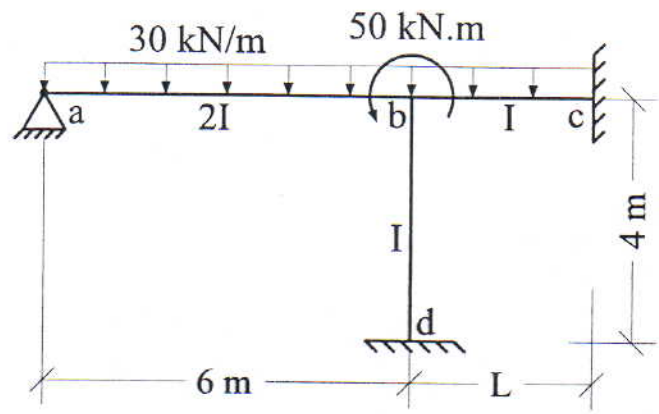
a. For the structural beam shown below, draw the influence line for: Ra; Rc; Mc; Md, Vc_{right}, Mmn; Vmn, Vd_{left} and Ve.

b. Find the value of Mmn if the structural beam is subjected to infinite uniformly distributed load (w = 30 kN/m).



Q5:

For the structural frame shown, use the Slope Deflection Method; find the span length (L) such that the rotation at joint (b) will not exceed 4.22° (in counterclockwise). The frame members have a modulus of elasticity (E) = 2×10^5 N/mm^2 and the moment of inertia (I) = 30×10^5 mm^4.



Notes:

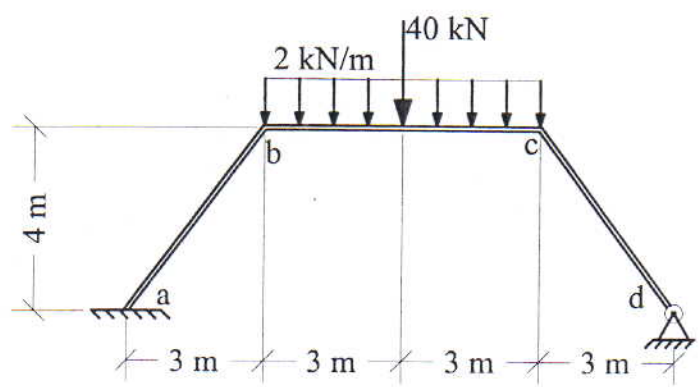
- π rad. / 180° = 1.
- The fixed end moments (FEM) for the uniformly distributed load at the member ends equal $\pm w\ell^2/12$.
- Use iteration method to solve the equation, start with L = 1 m, L ≤ 3 m.

Q6:

Using the Moment Distribution Method, find the reactions at support a (a_x & a_y) due to the applied loading shown. EI = constant. If it needed, try up to three cycles only.

Notes:

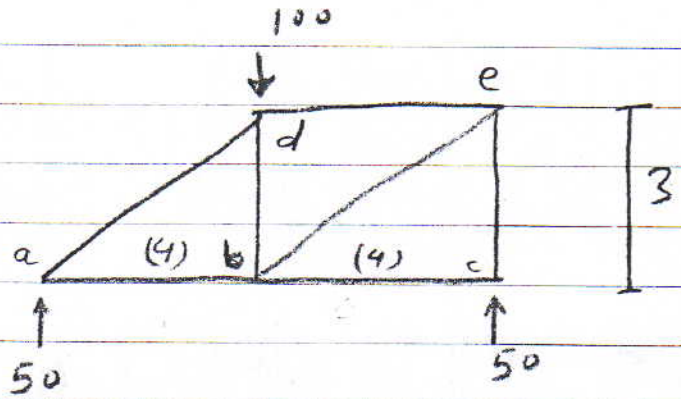
- The fixed end moments (FEM) for the uniformly distributed load at the member ends equal $\pm w\ell^2/12$.
- The fixed end moments (FEM) for the midspan concentrated load at the member ends equal $\pm p\ \ell/8$.



Good Luck

Final Exam – First Attempt, 2016 – Solution:

Q2:

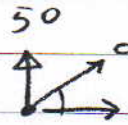


mem	L/EA	S (kN)	u	SuL/EA
ab	4/EA	+66.67	0.667	1600/9 EA
bc	4/EA	0	0	0
ad	5/EA	-83.33	-0.833	3125/9 EA
de	4/EA	-66.67	-0.667	1600/9 EA
be	5/EA	83.33	0.833	3125/9 EA
ce	3/EA	-50	-0.5	75/EA
bd	3/EA	-50	0.5	-75/EA

$\therefore \sum \frac{SuL}{EA}$

$= \boxed{\frac{1050}{EA} = \Delta_b}$

S: ⊛ joint a:

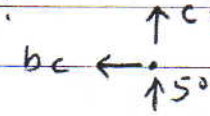


$\theta = \arctan(3/4) = 36.87°$ $\quad \sum F_y = 0 \Rightarrow 50 + ad \sin\theta = 0$

$ad = -83.33$ kN

$\sum F_x = 0 \Rightarrow ad\cos\theta + ab = 0 \Rightarrow ab = +66.67$ kN

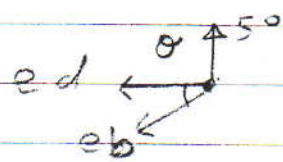
⊛ joint c:



$\Rightarrow \sum F_x = 0 \Rightarrow bc = 0$ and $\sum F_y = 0$

$ce = -50$ kN

⊛ joint e:

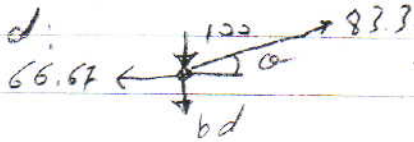


$\sum F_y = 0 \Rightarrow 50 = eb \sin\theta$

$eb = 83.33$ kN

$\sum F_x = 0 \Rightarrow ed + eb\cos\theta = 0 \Rightarrow ed = -66.67$

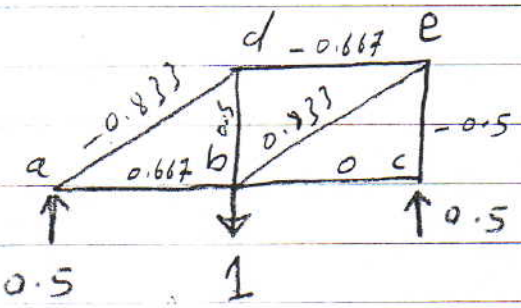
⊛ joint d:

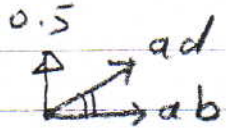


$\rightarrow \sum F_y = 0 \Rightarrow 100 + bd = 83.33 \sin\theta$

$bd = -50$ kN

4.





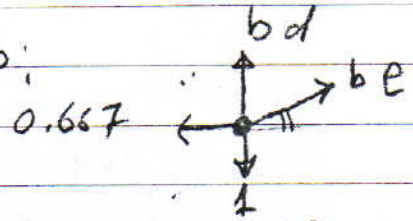
⊗Joint a;

$\Sigma F_y = 0$

$0.5 + ad \sin\theta = 0 \Rightarrow ad = -0.833$

$\Sigma F_x = 0 \Rightarrow +0.833 \cos\theta = ab \Rightarrow ab = 0.6667$

⊗joint b;

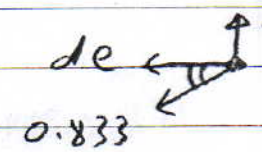


$\Sigma F_x = 0 \Rightarrow be \cos\theta = 0.667$

$be = 0.833$

$\Sigma F_y = 0 \Rightarrow 1 = 0.833 \sin\theta + bd \Rightarrow bd = +0.5$

*joint e;

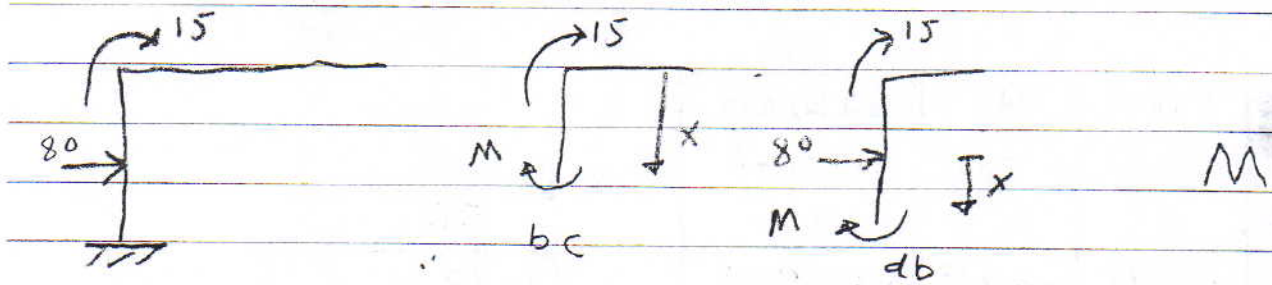


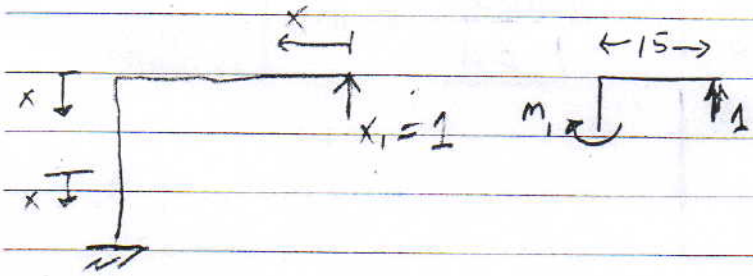
$\Rightarrow \Sigma F_x = 0 \Rightarrow 0.833 \cos\theta = -de$

$\therefore de = -0.667$

Q3:

Memb.	EI	orig.	limit	M	m_1	$M m_1$	m_1^2
ab	1	b	0-5	$-15-80x$	x	$-15x-80x^2$	x^2
bc	1	c	0-5	-15	15	-225	225
cd	1	d	0-15	0	15	0	225





$$\Delta_1 = \Delta_1' + \delta_{11} X_1$$

$$\Delta_1' = \int \frac{M m_1 dx}{EI} = \left[\int_0^5 (-15x - 80x^2)dx + \int_0^5 (-225)dx + 0\right]\frac{1}{EI}$$

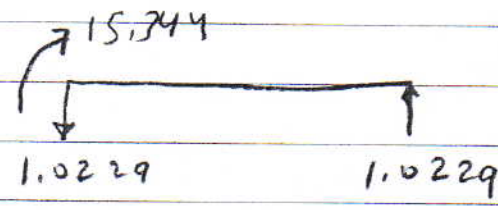
$$\Delta_1' = \frac{1}{EI}\left[-3520.833 - 1275\right] = \frac{-27875}{6EI} = -4645.833/EI$$

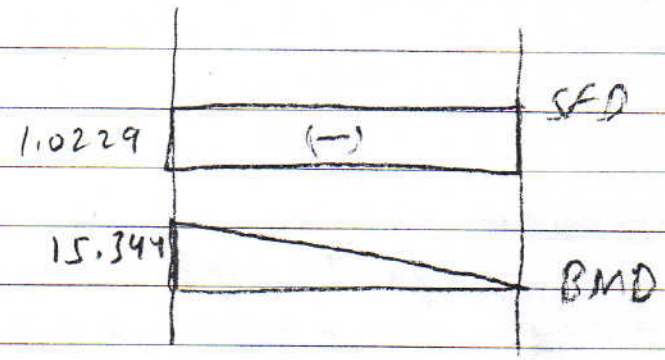
$$\delta_{11} = \int \frac{m_1 m_1 dx}{EI} = \left[\int_0^5 (x^2 + 225)dx + \int_0^{15} (225)dx\right]\frac{1}{EI}$$

$$= \frac{13625}{3EI} = 4541666/EI$$

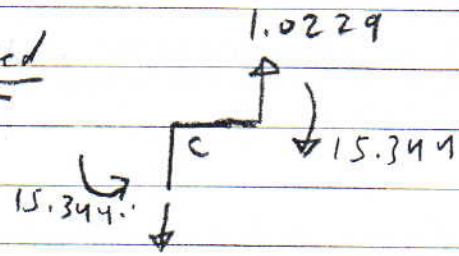
$$\text{at } \frac{13625}{3EI} X_1 = \frac{27875}{6EI} \Rightarrow X_1 = 1.0229 \text{ KN} \uparrow$$

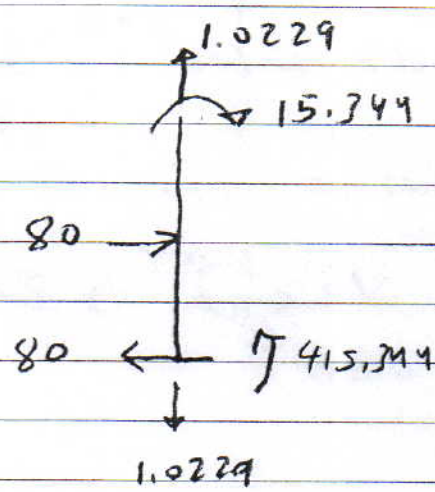
member cd



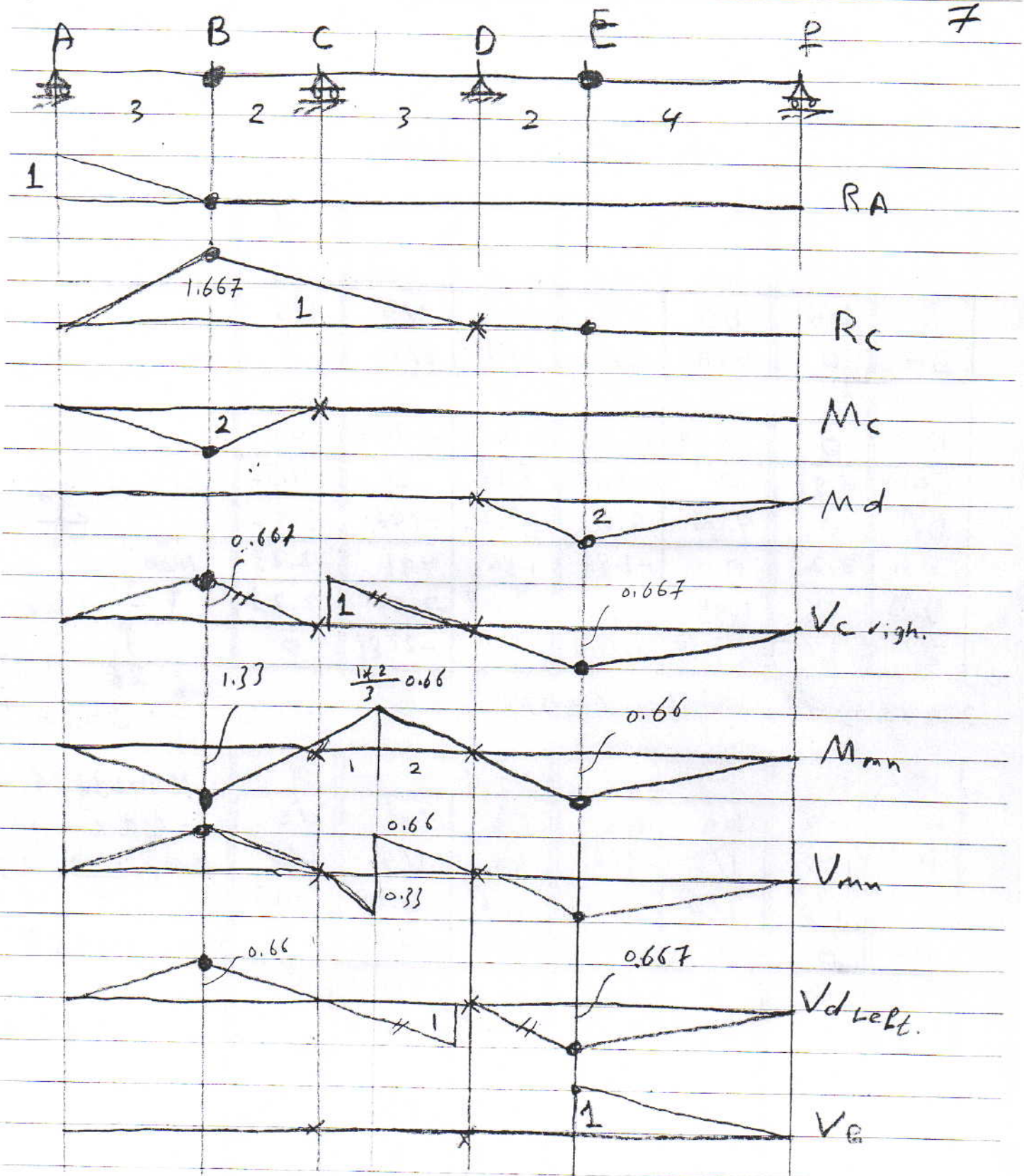


not required





Q4



$$M_{mn} = 30\ \frac{kN}{m}\left[\frac{-1.33\times5}{2} + \frac{0.667\times3}{2} - \frac{0.667\times6}{2}\right] = -130\ kN.m$$

Q5:

$$FEM_{ab} = \frac{-wl^2}{12} = \frac{-30*6^2}{12} = -90$$

$$\theta_b = -4.22^{\circ}_{eq} * \frac{\pi\ rad}{180^{\circ}_{eg}} = -0.07365\ rad.$$

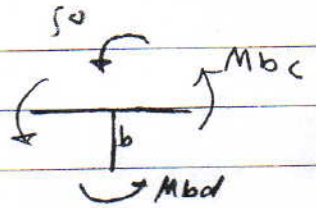
$$EI = 2*10^5 \frac{N}{mm^2} * 30*10^5 mm^4 = 6*10^{11} N.mm^2 * \frac{kN}{1000mm} * \frac{m^2}{(1000)^2 mm^2}$$

$$EI = 600\ kN.m^2$$

$$\sum M_{@a} = 0 \Rightarrow M_{ab} = 0 \Rightarrow \frac{2*600}{6}[4\theta_a - 2(0.07365)] - 90 = 0$$

$$\therefore \theta_a = 0.1493\ rad.$$

$$\sum M_{@b} = 0 \Rightarrow M_{ba} + M_{bd} + M_{bc} + 50 = 0$$



$$\frac{2*600}{6}[2\theta_a - 4\theta_b] + 90 + \frac{600}{4}[-4\theta_b] + \frac{600}{L}[-4\theta_b] - \frac{30L^2}{12} = -50$$

$$\left(\frac{176.767}{L} + 2.5L^2 = 96.6165\right) \quad ; \frac{176.767}{L} + 2.5L^2 = R$$

L	R / 96.6165
1	1.855
2	1.018288
2.1	0.9853
2.05	1.0012 ←

$\therefore L = 2.05\ m$

$(L_{exact} = 2.0537\ m)$

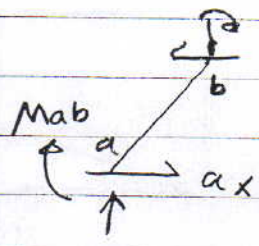
Q 6: $FEM = \mp\left(\frac{wl^2}{12} + \frac{Pl}{8}\right) = \mp\left(\frac{2*6^2}{12} + \frac{40*6}{8}\right)$

$= \mp 36$

either:

1- not using the modified MDM:

J	a	b		c		d
m	ab	ba	bc	cb	cd	dc
K	1/5	1/5	1/6	1/6	1/5	1/5
DF	0	6/11	5/11	5/11	6/11	1
FEM	0	0	-36	36	0	0
DM	0	19.64	16.36	-16.36	-19.64	0
CoM	9.82	0	-8.18	8.18	0	-9.82
DM	0	4.46	3.72	-3.72	-4.46	9.82
CoM	2.23	0	-1.86	1.86	4.91	-2.23
DM	0	1.01	0.85	-3.08	-3.69	2.23
Σ	12.05	25.11	-25.11	22.88	-22.88	0



$\Sigma M_{@b} = 0$

$Mab + 3*26 = 4ax$

$\therefore ax = \frac{Mab}{4} + 19.5$

$ax_1 = 22.5 \uparrow$

$ax_2 = 22.6 \uparrow$

or 2- using the modified MDM

J	a	b		c		d
m	ab	ba	bc	cb	cd	dc
K	1/5	1/5	1/6	1/6	1/5	1/5
K'	1/5	1/5	1/6	1/6	0.15	0.15
DF	0	6/11	5/11	0.526	0.474	1
FEM	0	0	-36	36	0	0
DM	0	19.64	16.36	-18.936	-17.064	0
CoM	9.82	0	-9.468	8.18	0	
DM	0	5.16	4.3	-4.3	-3.88	
CoM	2.58	0	-2.15	2.15	0	
DM	0	1.17	0.98	-1.13	-1.02	
Σ	12.4	25.97	-25.97	21.96	-21.96	0