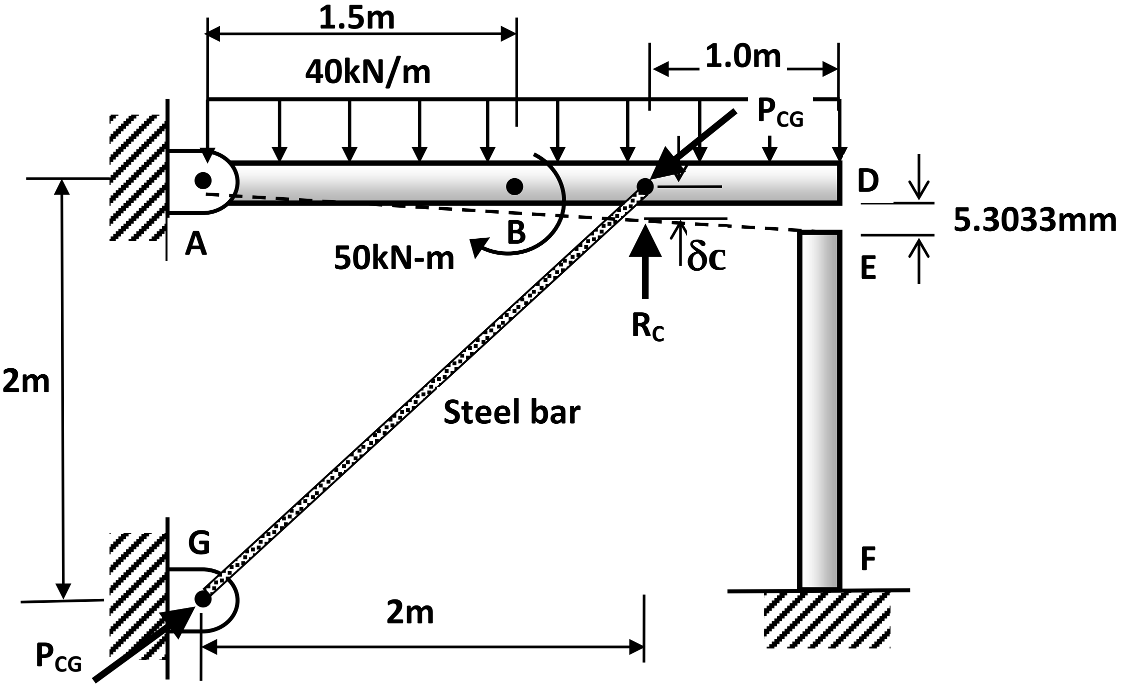
UNIVERSITY OF TECHNOLOGY
BUIILDING AND CONSTRUCTION DEPARTMENT
Final Examination – First Attempt – 2015

Class : Second Year — Time : 3 hours
Subject : **Strength of Materials** — Date : 7- 6 - 2015

Answer (4) Questions Only, All Questions have Equal marks

Q1 (25%): For the structure shown below, members AD, EF are rigid beams, Find the diameter of the steel bar CG (Est=200GPa) in order to cause contact between D and E (or to closed the gap 5.3mm) due to applied loads on member AD.



Q1) Member ABCD:

$M_A = 0$ + ↻ $-40*3*1.5 - 50 + 2*Rc = 0 \rightarrow Rc = 115$ kN

$P_{CG} = 115 * \sqrt{2} = 162.63$ kN

$\delta c = 5.3033 * 2/3 = 3.5355$ mm

$\delta_{CG} = 3.5355 * \sqrt{2} = 5$ mm

$\delta = PL / A E$ for steel bar CG ($L = (2^2 + 2^2)^{1/2} = 2.828$m)

$5 = 162.63 *1000 * 2.828 *1000/(A* 200000)$

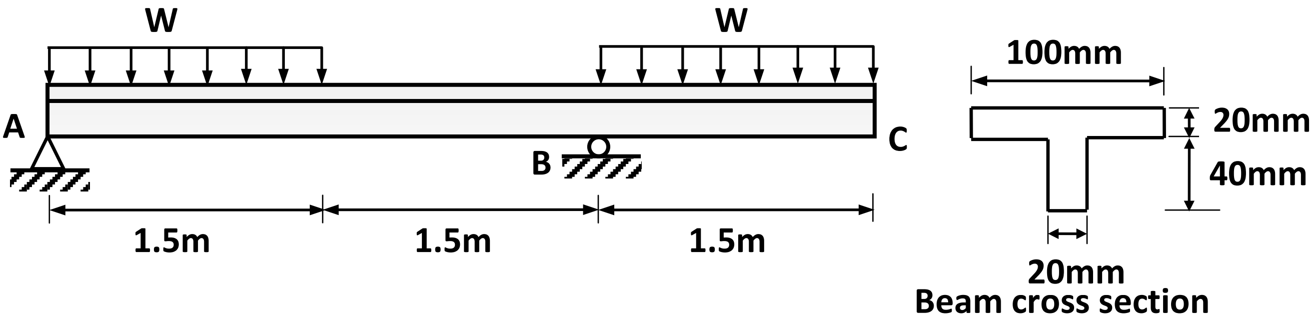
A = 460mm2

$A = \pi * d^2 /4$

$460 = \pi * d^2 / 4 \rightarrow d = 24.2$ mm

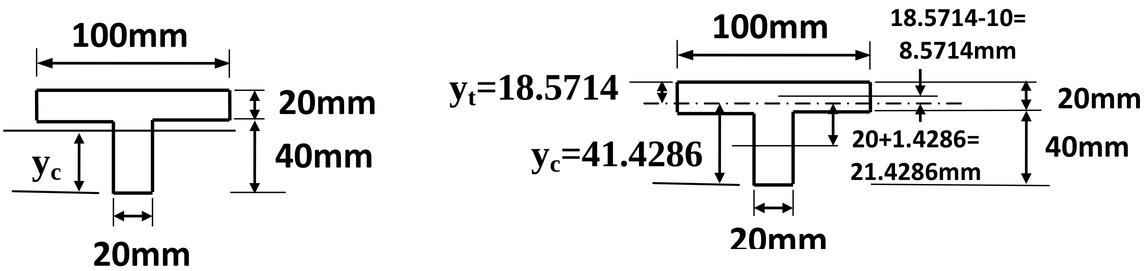
Q2 (25%) :

For the beam ABC loaded shown below having the (T) cross-section, if the allowable stress in tension is 150N/mm^2 and the allowable stress in compression is 100N/mm^2. Estimate the value of the allowable uniform distributed load (W).



$$y_c = \frac{20*(20*40)+(10+40)(100*20)}{40*20+100*20} = \frac{16000+100000}{2800} = 41.4286mm$$

y_t = 60 – 41.4286 = 18.5714 mm



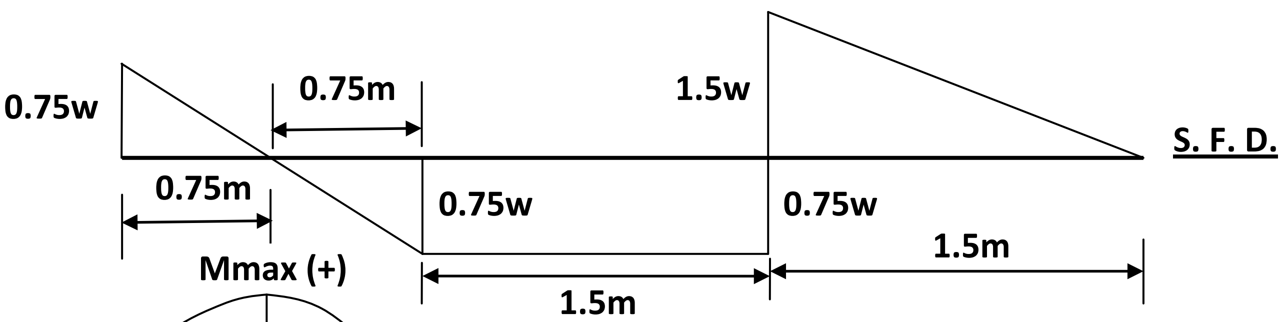
$$Ix = 100*20^3/12 + (100*20)*8.5714^2+20*40^3/12+(20*40)*21.4286^2$$

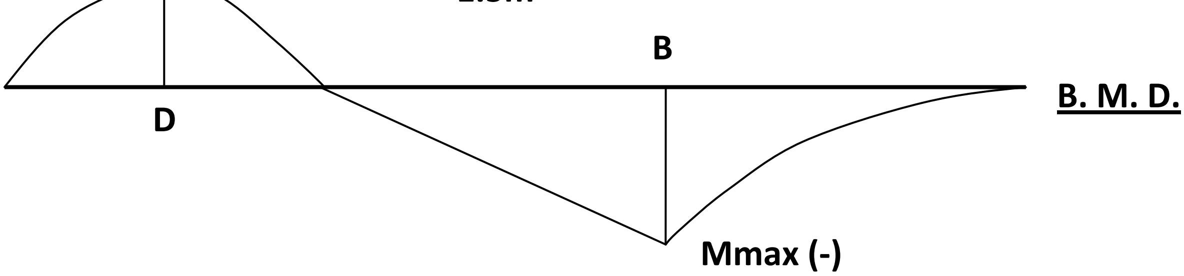
$$= 66666.67+146937.8+106666.67+367347.9 = 687619mm^4$$

$M_A = 0 \rightarrow - w * 1.5 *1.5/2 + 3R_B – w *1.5 * (3+1.5/2) = 0 \rightarrow R_B = 2.25w$

$\sum Fy = 0 \rightarrow R_A + R_B – 1.5w – 1.5w = 0$

$R_A + 2.25w – 3w = 0 \rightarrow R_A = 0.75w$





Mmax (+) = 0.75w* 0.75/2 = 0.28125w (kN.m)
Mmax (-) = 1.5w * 1.5/2 = 1.125w (kN.m)

$$\sigma = \frac{M * c}{I}$$

$at\ point\ D:$

$$150 = \frac{0.28125w * 10^6 * 41.4286}{687615} \Rightarrow w = 8.852 N/mm$$

$$100 = \frac{0.28125w * 10^6 * 18.5714}{687615} \Rightarrow w = 13.164 N/mm$$

$at\ point\ B:$

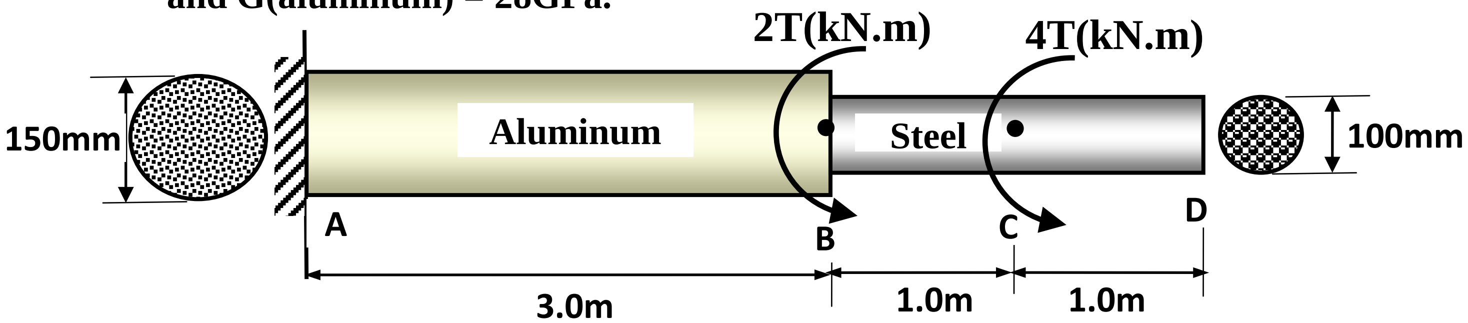
$$150 = \frac{1.125w * 10^6 * 18.5714}{687615} \Rightarrow w = 4.936 N/mm$$

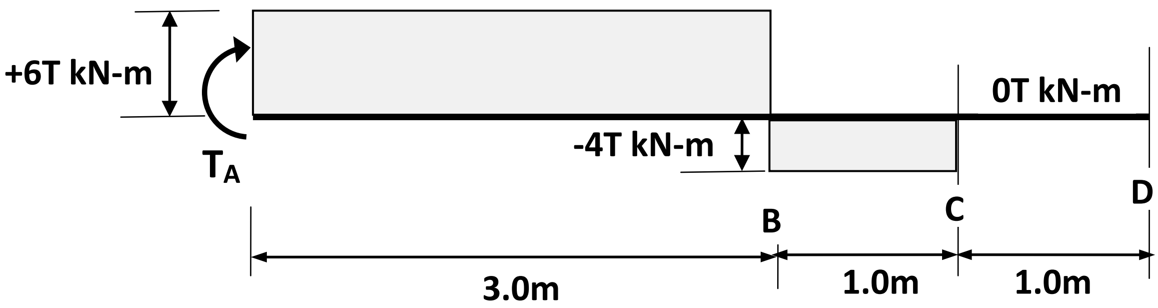
$$100 = \frac{1.125w * 10^6 * 41.4286}{687615} \Rightarrow w = 1.475 N/mm$$

$least\ value\ where: w = 1.475 N/mm$

<u>**Q3 (25%) :**</u>

For the compound shaft shown below, determine the maximum permissible value of (T) and the angle of twist at point (B) if shear stress in steel 100MPa and in aluminum 70MPa and the angle of rotation of free end (point D) is limited to (10°). Use G(steel) = 120GPa, and G(aluminum) = 28GPa.





<u>**Torque- Diagram**</u>

$\sum M_A = 0 \rightarrow T_A - 2T - 4T = 0 \rightarrow \therefore T_A = 6T$

$$\tau = \frac{T\ r}{J}$$

$$\tau_{Alum.} = 70 = \frac{6T * 10^6 * 75}{\frac{\pi(75)^4}{2}} = 9.054T \quad \Rightarrow T = 7.7314 kN.m$$

$$\tau_{Steel} = 100 = \frac{4T * 10^6 * 50}{\frac{\pi(50)^4}{2}} = 20.372T \quad \Rightarrow T = 4.9087 kN.m$$

$$\varphi = \frac{T\ L}{G\ J}$$

$$\varphi_D = \varphi_A + \varphi_{AB} + \varphi_{BC} + \varphi_{CD}$$

$$10^\circ(\frac{\pi}{180}) = 0 + \frac{6T * 10^6 * 3000}{28000(\frac{\pi(75)^4}{2})} + \frac{(-4T) * 10^6 * 1000}{120000(\frac{\pi(50)^4}{2})} + 0$$

$$\frac{\pi}{180} = 0.012934T - 0.003395T = 0.00954T$$

$$T = 18.295 kN.m$$

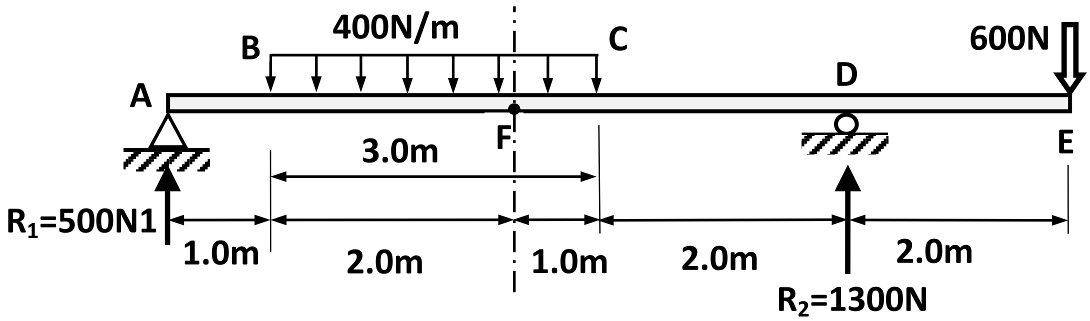
The least value of T = 4.9087kN.m

$$\varphi_B = 0 + \frac{6 * 4.9087 * 10^6 * 3000}{28000(\frac{\pi(75)^4}{2})} = 0.06337 radian$$

$$\varphi_B = 0.06337 * \frac{180}{\pi} = 3.63^\circ$$

Q4 (25%) :

Find the value EIy at the position midway between supports (point F), and at the end of overhanging portion (point A) for the beam shown



Solution:

$$EI\frac{d^2y}{dx^2} = M = 500.X - \frac{400}{2}(x-1)^2 + \frac{400}{2}(x-4)^2 + 1300(x-6)$$

$$EI\frac{dy}{dx} = 250.X^2 - \frac{200}{3}(x-1)^3 + \frac{200}{3}(x-4)^3 + 650(x-6)^2 + C_1$$

$$EIy = \frac{250}{3}.X^3 - \frac{50}{3}(x-1)^4 + \frac{50}{3}(x-4)^4 + \frac{650}{3}(x-6)^3 + C_1.X + C_2$$

B.C:

At: x=0 ⇒ $EIy = 0$ ⇒ $C_2 = 0$

x = 6 ⇒ $EIy = 0$

$$0 = \frac{250}{3}(6)^3 - \frac{50}{3}(5)^4 + \frac{50}{3}(2)^4 + 6C_1 \Rightarrow C_1 = -1308$$

At x = 3m:

Ignored (-ve) Ignored (-ve)

$$EIy = \frac{250}{3}.3^3 - \frac{50}{3}(2)^4 + \frac{50}{3}(x-4)^4 + \frac{650}{3}(x-6)^3 - 1308(3) = -1941$$

⇒ $\boxed{EIy = -1941}$

At x = 8m:

$$EIy = \frac{250}{3}.8^3 - \frac{50}{3}(7)^4 + \frac{50}{3}(8-4)^4 + \frac{650}{3}(8-6)^3 - 1308(8) = -1941$$

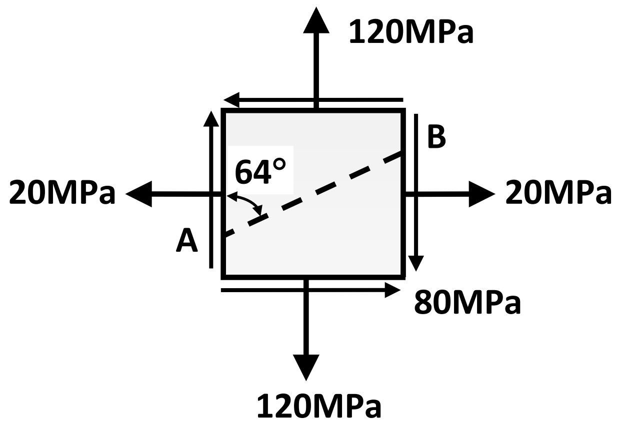
EIy=42666.67-40016.667+4266.667+1733.33-10464

EIy=-1814

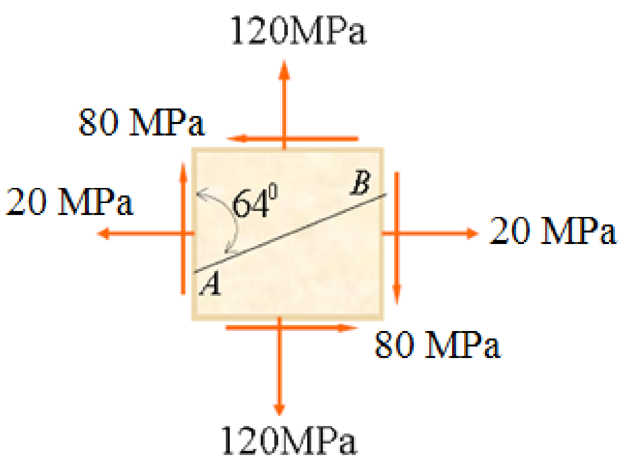
<u>Q5 (25%) :</u>

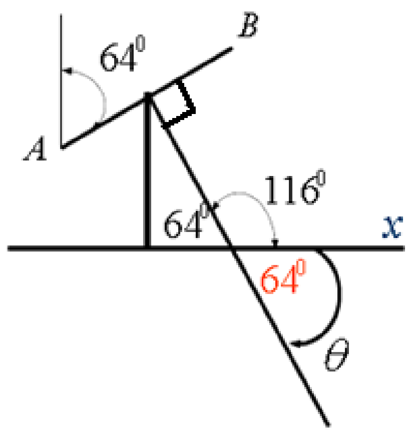
A structural element is shown below with normal x-stress = 20MPa, normal y-stress = 120MPa, shear stress = 80MPa, by Mohr's circle find:

a) **Principal stresses**
b) **Maximum shear stress.**
c) **Normal and shearing stresses on the inclined plane AB shown.**



<u>Solution:</u>





The given values for use in drawing Mohr's circle are:

$\sigma_x = 20\,\text{MPa}$

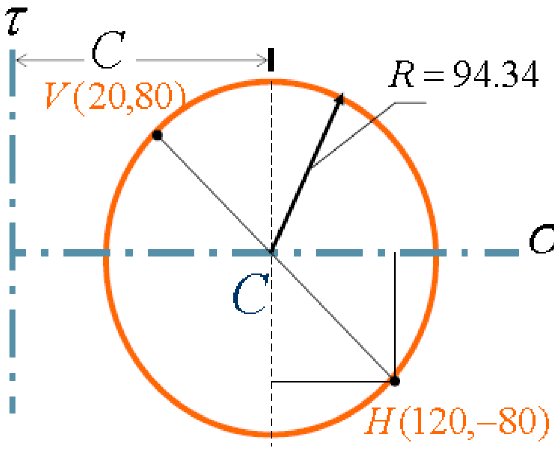
$\sigma_y = 120\,\text{MPa}$

$\tau_{xy} = \text{-}80\,\text{MPa}$

$\sigma_z = \sigma_{p3} = 0 \quad 2\theta = -2(64) = -128^0$

$$C = \frac{20+120}{2} = 70\,\text{MPa}$$

$$R = \text{radius} = \sqrt{(120-70)^2 + (-80)^2} = 94.34$$



The principal stresses and maximum shearing stress can be computed as

(a)

$\sigma_{p1} = C + R = 70 + 94.34 = 164.34\,\text{MPa}$

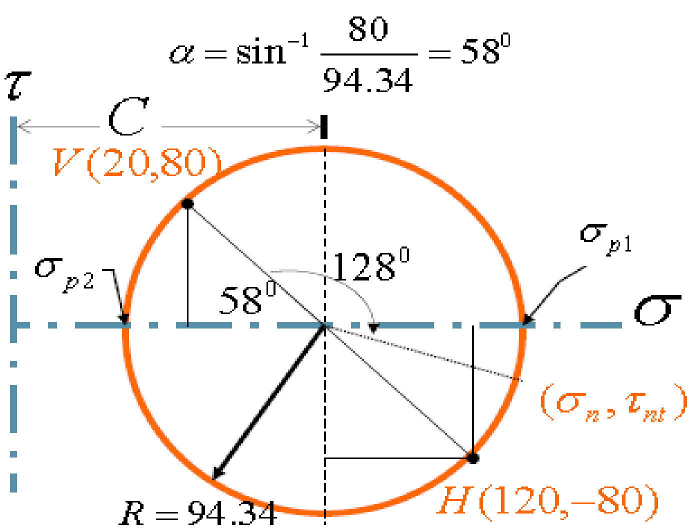
$\sigma_{p2} = C - R = 70 - 94.34 = -24.34\,\text{MPa}$

$\sigma_{p3} = 0$

$\tau_p = R = 94.34\,\text{MPa}$

Since σ_{p1} and σ_{p1} have opposite sign,

$\tau_{\max} = \tau_p = 94.34\,\text{MPa}$



The normal and shearing stress on inclined plane can be computed as

(b)

$\sigma_n = C + R\cos 6^0 = 70 + 94.34\cos 6^0$

$= \boxed{163.8\,\text{MPa}}$

$\tau_{nt} = R\sin 6^0 = 94.34\sin 6^0 = \boxed{9.86\,\text{MPa}}$