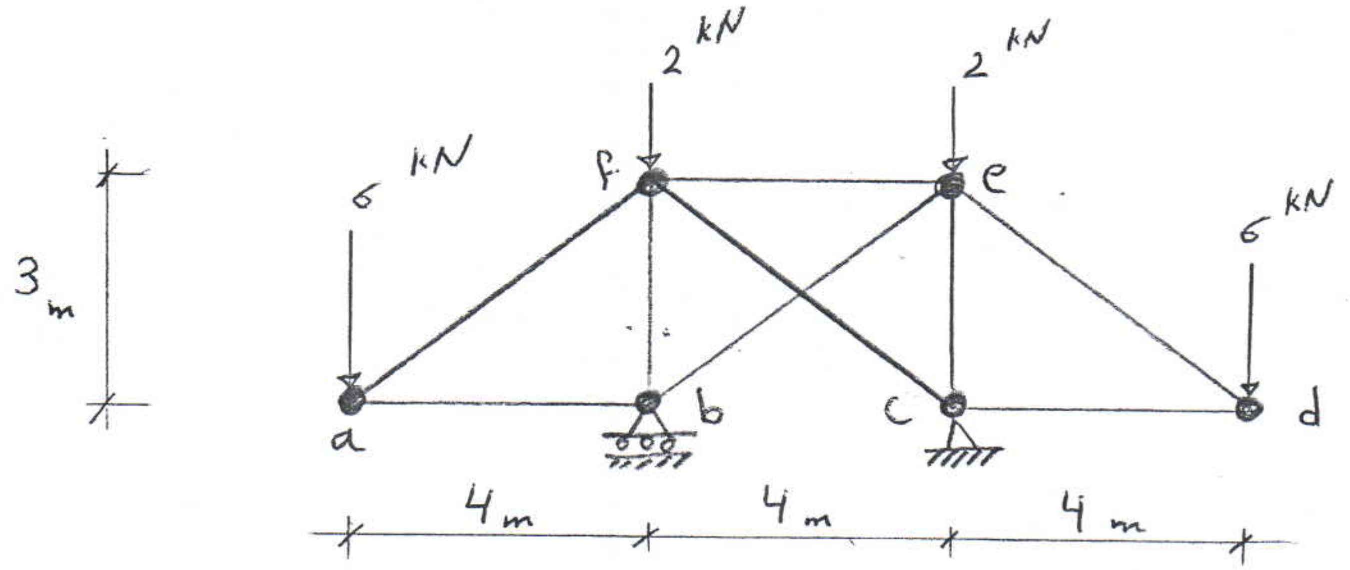
University Of Technology
Building and Construction Eng. Dept.
Final Exam 2014-2015

Subject : Structural Analysis **Class:** Third Class
Branch : Building& construction management **Time :** 3 Hours
Examiner : Dr. Zeyad M. Ali **Date :** 13 / 6 / 2015

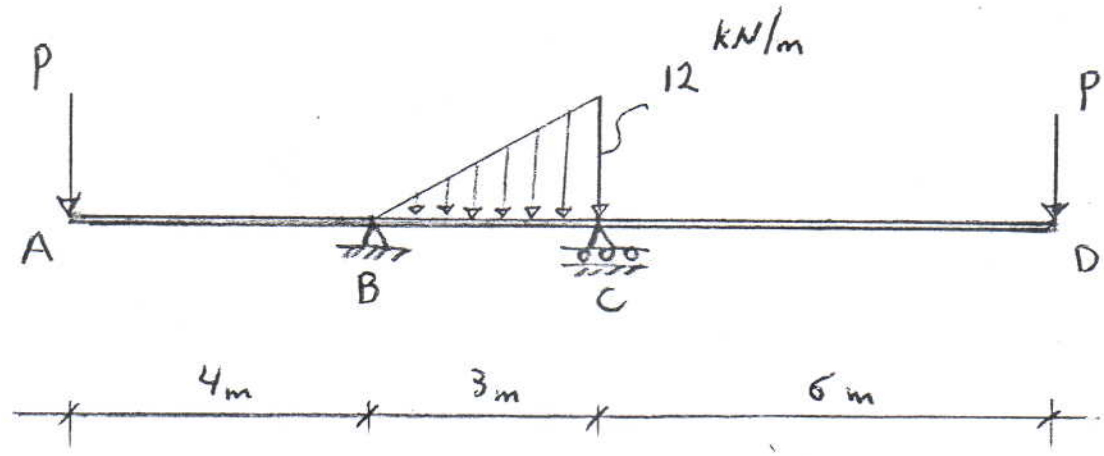
Note : **Answer Four questions only.**

Q1/For the plan loaded truss shown in the fig. , Determine the rotation of member (af) by the virtual work method ? Assume EA = 1000 kN where :-

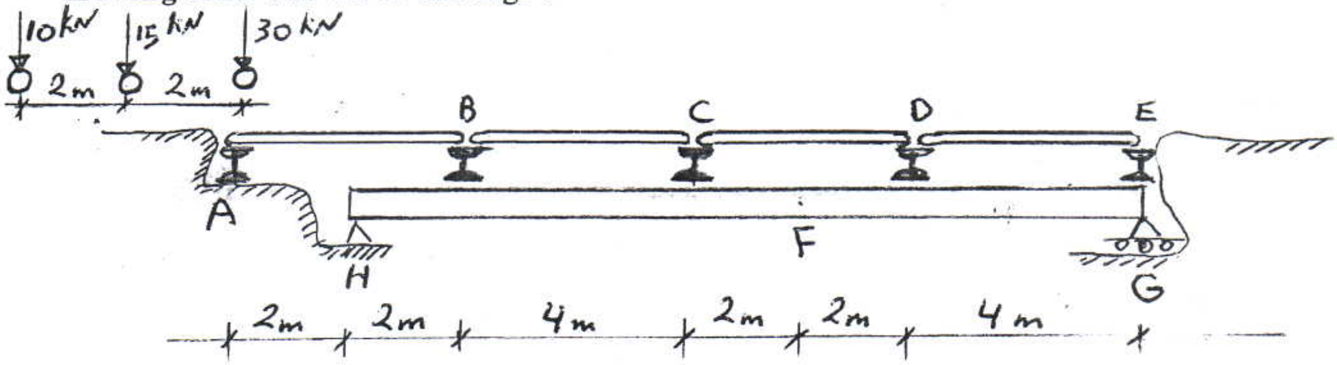
A = the gross section area of member (m^2) (constant)
E = modulus of elasticity of member (kN/m^2) (constant)



Q2/ Determine the value of load (P) which introduce a vertical deflection at (A & D) equal to (2.25 mm)? Given EI =1000 kN.m^2 (constant) .

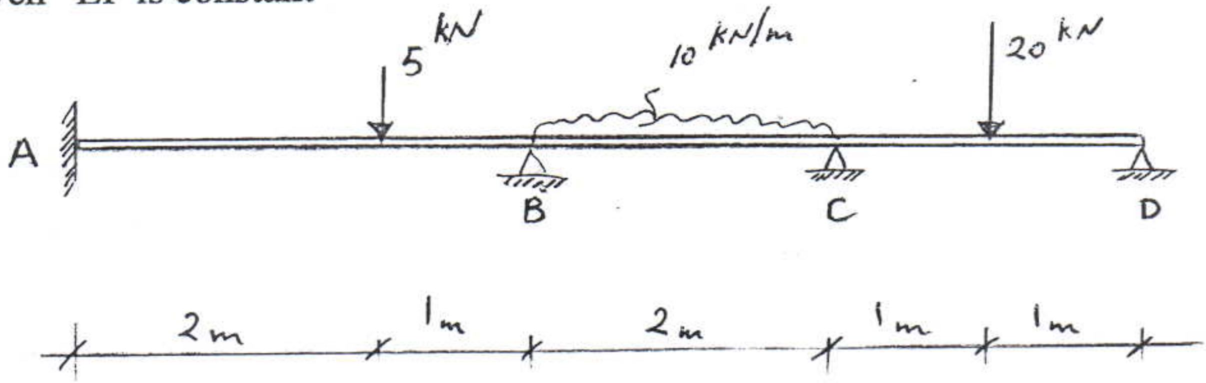


Q3/Determine the max.-moment at section (F) for the plan beam girder due the **moving load** shown in the fig. ?

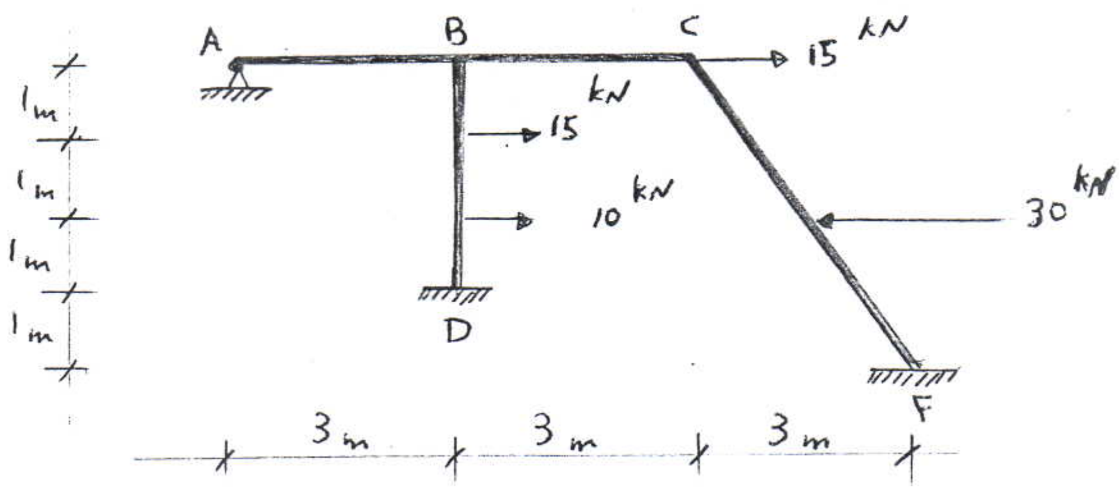


Q4/ Determine the ends moment for the plane continues beam shown in the fig. due to the yielding of support (A) = 0.02 rad. Ante clockwise & vertical settlement of (3 cm) at support (C) in addition to the load acting by the moment distribution method?

Given EI is constant

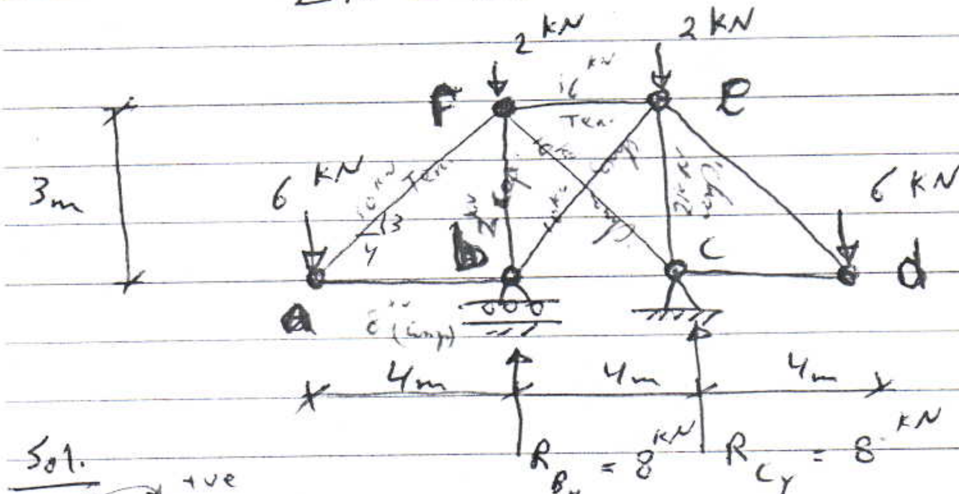


Q5/Determine the end moments by the slope-deflection method for the plane frame shown in the fig. ? Given EI = 300 $kN.m^2$ (constant) .



Q./ For the plane Loaded truss shown in the fig, find the rotation of member aF? Assume the gross section area of member (m^2) = A (constant) & Modulus of elasticity of member (kN/m^2) = E (Constant)

$$EA = 1000^{kN}$$



Sol.

$\circlearrowright^{+ve}\ \Sigma M_b = 0 \qquad 2\times4 + 6\times8 - R_c\times4 - 6\times4 = 0$

$\therefore R_{c_y} = \dfrac{8^{kN} + 24^{kN}}{4} = 8^{kN}$

$\uparrow^{+ve}\ \Sigma F_y = 0 \qquad R_{c_y} + R_{B_y} = 16^{kN}$

$\therefore R_{B_y} = 8^{kN}$

joint A

$\Sigma F_y = 0 \qquad 0.6\,F_{af} = 6 \qquad \therefore F_{af} = 10^{kN}$ (Ten.)

$\Sigma F_x = 0 \qquad 0.8\,F_{af} = 8 \qquad \therefore F_{ab} = 8^{kN}$ (Comp.)

joint b

$\Sigma F_x = 0 \qquad \therefore 0.8\,F_{be} = 8^{kN} \qquad F_{be} = 10^{kN}$ (Comp.)

$\Sigma F_y = 0 \qquad F_{bf} = 2^{kN}$ (Comp.)

joint f

$\Sigma F_y = 0 \qquad \therefore F_{fc} = 10^{kN}$ (comp.)

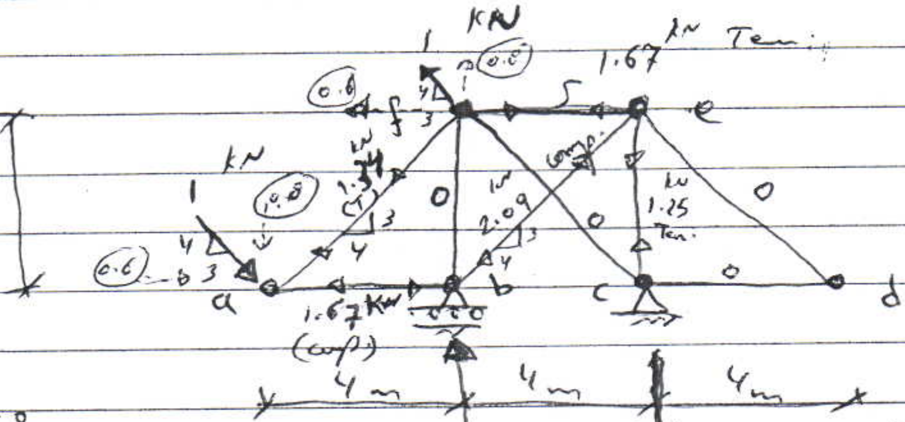
$\Sigma F_x = 0 \qquad F_{fe} = 16^{kN}$ (Ten.)

$\theta_{af} = ?$ $AE = 1000\ kN$

$\therefore\ 1 \cdot \theta_{af} = \sum \frac{S \cdot u \cdot L}{AE}$

joint e

$\sum F_y = 0 \quad \therefore F_{ce} = 2^{kN}$ Comp.



+ve

$\sum M_b = 0$

$0.6(3) + 0.8(4) = R_{cy}(4)$ $R_{by} = 1.25^{kN}$

$\therefore R_{cy} = 1.25^{kN} \downarrow$

+ve

$\uparrow \sum F_y = 0 \quad \therefore R_{cy} = -R_{by} = 1.25^{kN} \uparrow$

joint a

$\sum F_y = 0 \quad 0.6\, F_{af} = 0.8 \quad \therefore F_{af} = 1.34^{kN}$ Ten.

$\sum F_x = 0 \quad 0.6 + 0.8(1.34) = F_{ab} = 1.67^{kN}$ Comp.

joint b

$\sum F_x = 0 \quad 1.67^{kN} = 0.8\, F_{be} \quad \therefore F_{be} = 2.09^{kN}$ Comp.

$\uparrow \sum F_y = 0 \quad 1.25 - 0.6\, F_{be} = F_{bf} = 0$

joint c $\quad \sum F_y = 0 \quad \therefore F_{cb} = 1.25^{kN}$ Ten.

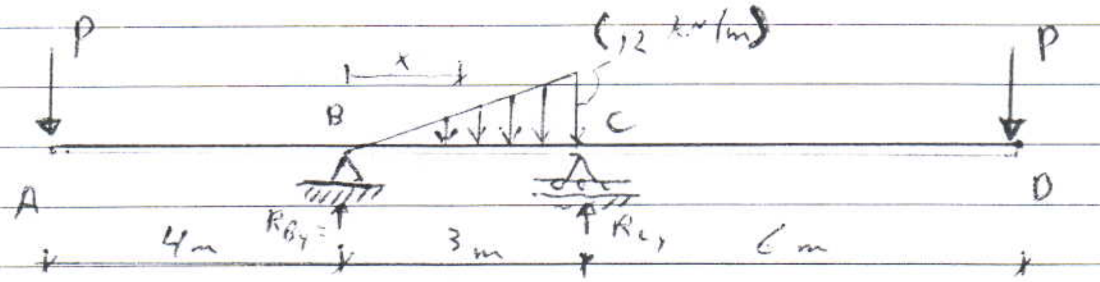
joint f $\quad \sum F_x = 0 \quad 0.6 + 0.8(1.34) = F_{fe} = 1.67^{kN}$

member	Length (m)	S (kN)	u (kN)	S.u.L
ab	4	-8	-1.67	53.44
af	5	10	1.34	67.0
fe	4	16	1.67	106.88
be	5	-10	-2.09	104.5
ce	4	-2	1.25	-10

$\sum S.u.L = 321.82 / EA = 0.3218$ rad.

Q_2 /

Find the value of Load (P) as shown in the fig which cause a vertical deflection at (A) & (D) equal to (2.25 mm)? Given $EI = 1 \times 10^3\ kN \cdot m^2$ (constant)



$$\sum M_c = 0 \qquad P(6) + R_{By}(3) - P(7) - \frac{12 \times 3}{2}(1) = 0$$

$$\boxed{\therefore R_{By} = \frac{P+18}{3}}$$

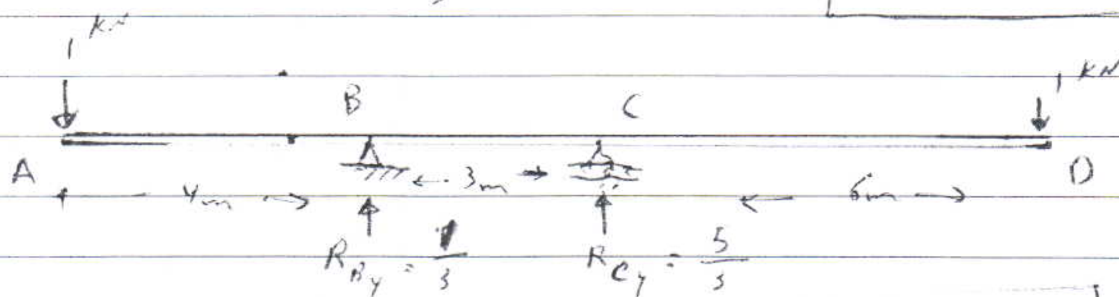
$$\frac{y}{x} = \frac{12}{3}$$

$$\boxed{y = 4x}$$

$$\overset{+ve}{\uparrow} \sum F_y = 0 \qquad R_{By} + R_{cy} = 2P + 18$$

$$\therefore R_{cy} = 2P + 18 - \frac{P+18}{3}$$

$$= \frac{6P + 54 - P - 18}{3} \quad = \boxed{\frac{5P+36}{3} = R_{cy}}$$



$$\sum M_c = 0 \qquad 1(6) + R_{By}(3) - 1(7) = 0 \qquad \therefore R_{By} = \frac{1}{3}\ kN$$

$$\sum F_y = 0 \qquad R_{By} + R_{cy} = 2 \qquad \therefore R_{cy} = 5/3\ kN$$

Sec.	origin	Limit	EI	M	m
AB	A	0 – 4	Const.	$-Px$	$-x$
BC	B	0 – 3	"	$-P(4+x) + \left(\frac{P+18}{3}\right)x - \boxed{\left(\frac{xy}{2}\right)\frac{x}{3}}$ $\left(\frac{2x^3}{3}\right)$	$-1(4+x) + \frac{x}{3}$
CD	D	0 – 6	"	$-Px$	$-x$

$\underline{M}$ $\qquad\qquad\qquad$ $\underline{m}$

for BC $\quad -4P - xP + \frac{xP}{3} + 6x - \frac{2}{3}x^3 \qquad -4 - x + \frac{x}{3}$

$\left(-4P - \frac{2}{3}xP + 6x - \frac{2}{3}x^3\right) \qquad \left(-4 - \frac{2}{3}x\right)$

$$Mm = +16P + \frac{8}{3}xP - 24x + \frac{8}{3}x^3 + \frac{8}{3}xP + \frac{4}{9}x^2P - 4x^2 + \frac{4}{9}x^4$$

$$\therefore 1.\Delta = \int_0^L \frac{M \cdot m}{EI}\,dx$$

$$(1.\Delta)EI = \int_0^L M.m\,dx$$

$$\left(\frac{2.25}{1000}\right)\times 10^3 = \int_0^4 Px^2\,dx + \int_0^3 16P\,dx + \int_0^3 \frac{8}{3}xP\,dx - \int_0^3 24x\,dx + \int_0^3 \frac{8}{3}x^3\,dx + \int_0^3 \frac{8}{3}xP\,dx + \int_0^3 \frac{4}{9}Px^2\,dx - \int_0^3 4x^2\,dx + \int_0^3 \frac{4}{9}x^4\,dx + \int_3^6 Px^2\,dx$$

$$22.5 = \frac{Px^3}{3}\Big|_0^4 + 16Px\Big|_0^3 + \frac{8}{6}Px^2\Big|_0^3 - 12x^2\Big|_0^3 + \frac{2}{3}x^4\Big|_0^3 + \frac{8P}{6}x^2\Big|_0^3 + \frac{4}{27}Px^3\Big|_0^3 - \frac{4}{3}x^3\Big|_0^3 + \frac{4x^5}{45}\Big|_0^3 + \frac{P}{3}x^3\Big|_0^6$$

$$= 21.333P + 48P + 12P - 108 + 54 + 12P + 4P - 36 + 21.6 + 72P$$

$$= 169.333P - 68.4$$

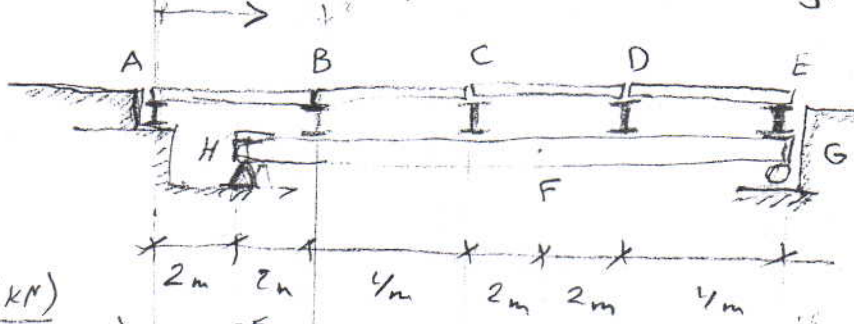
90.9

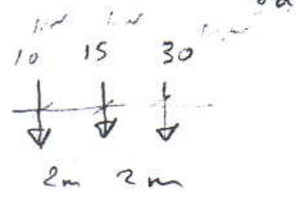
$$= 169.333P$$

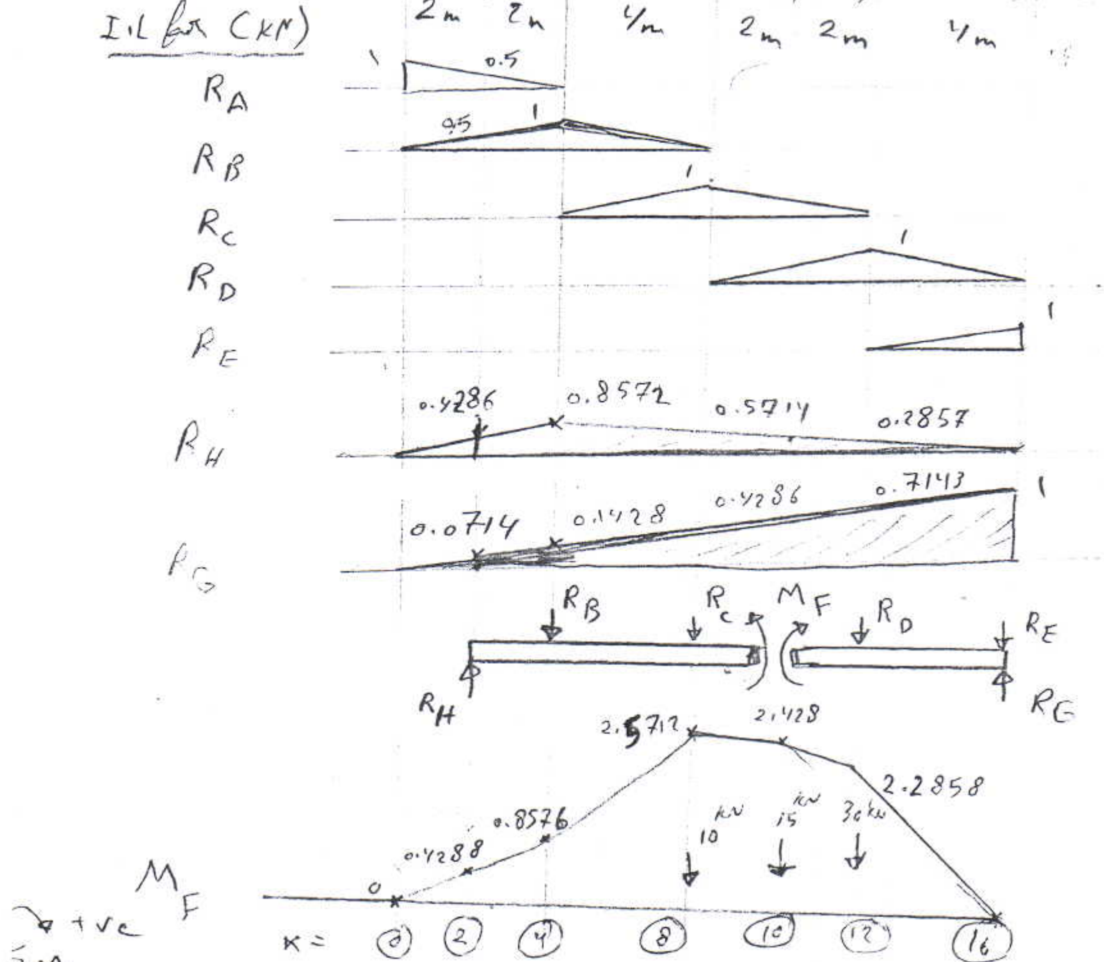
$$\therefore P = 0.5368 \text{ kN}$$

ind the max. moment ... of the Plane beam girder shown in the f... due to the moving loa...

Q.3/







↻ +ve

$\Sigma M_F = 0$

$R_H(8) - R_B(6) - R_C(2)$ $\boxed{0 \le x \le 10}$

x (m)

0 : $M_F = 0$ kN.m

2 : $M_F = 0.4286(8) - 0.5(6) - 0 = 0.4288$

4 : $M_F = 0.8572(8) - 1(6) - 0 = 0.8576$

8 : $M_F = 0.5714(8) - 0 - 1(2) = 2.5712$

10 : $M_F = 0.4285(8) - 0 - 0.5(2) = 2.428$

↺ +ve

$\Sigma M_F = 0$

$M_F + R_D(2) + R_E(6) - R_G(6) = 0$

$M_F = (R_G - R_E)6 - R_D(2)$ $\boxed{10 \le x \le 16}$

x (m)

$x = 10, \; M_F = (0.5714)6 - 0.5(2) = 2.4284$

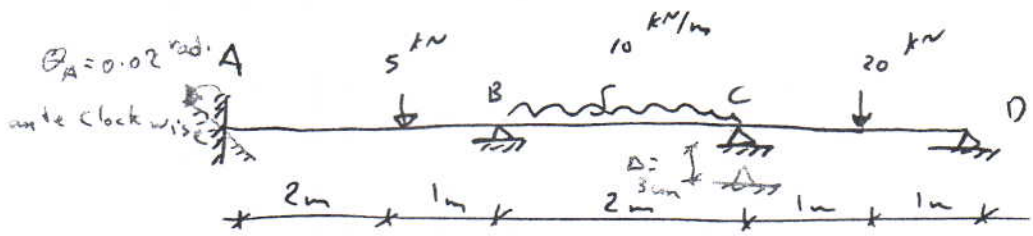
$x = 12, \; M_F = (0.7143)6 - 1(2) = 2.2858$

$x = 16, \; M_F = 0$

$(M_F)_{max.} = 10(2.5712) + 15(2.428) + 30(2.2858)$

$= 130.706$ kN.m

Q4/



k value

span AB $= \dfrac{4EI}{L_{AB}} = \dfrac{4}{3} \times 6 = \dfrac{8}{4} = 2$

span BC $= \dfrac{3EI}{L_{BC}} = \dfrac{3}{2} \times 6 = \dfrac{9}{4} = 2.25$

" CD $= \dfrac{3EI}{L_{CD}} = \dfrac{3}{2} \times 6 = \dfrac{9}{4} = 2.25$

$\dfrac{2EI}{L}$ value when EI Const.

span AB $= \dfrac{2}{3} = 0.67$

" BC $= \dfrac{2}{2} = 1$

" CD $= \dfrac{2}{2} = 1$

F.E.M (kN.m)

$M_{FAB} = -\dfrac{5 \times 2 \times (1)^2}{(3)^2} + 0.67(-0.04) = -1.138$

$M_{FBA} = \dfrac{5 \times 1 \times (2)^2}{(3)^2} + 0.67(-0.02) = 2.209$

$M_{FBC} = -\dfrac{10(2)^2}{12} + 1\left(-3\dfrac{0.03}{2}\right) = -3.378$

$M_{FCB} = \dfrac{10(2)^2}{12} + 1\left(-3\dfrac{0.03}{2}\right) = 3.289$

$M_{FCD} = -\dfrac{20(2)}{8} + 1\left(3\dfrac{0.03}{2}\right) = -4.955$

$M_{FDC} = \dfrac{20(2)}{8} + 1\left(3\dfrac{0.03}{2}\right) = 5.045$

…t	A	B		C		D
…ber	AB	BA	BC	CB	CD	DC
	2	2	2.25	2.25	2.25	2.25
$F = k/\Sigma k$	—	0.47	0.53	0.5	0.5	1
F.E.M	-1.138	2.209	-3.378	3.289	-4.955	5.045
Bal.	—	0.549	0.620	0.833	0.833	-5.045
C.O.M	0.274	—	0.4165	0.31	-2.522	0.4165
Bal.	—	-0.1957	-0.2207	1.106	1.106	-0.4165
C.O.M	0.0978	—	0.553	-0.1103	-0.208	0.553
Bal.	—	-0.26	-0.293	0.1591	0.1591	-0.553
ΣM	-0.9618	2.30	-2.30	5.5868	-5.5868	0

kN.m

$\therefore M_{AB} = -0.9618$

$M_{BA} = 2.3$

$M_{BC} = -2.3$

$M_{CB} = 5.5868$

$M_{CD} = -5.5868$

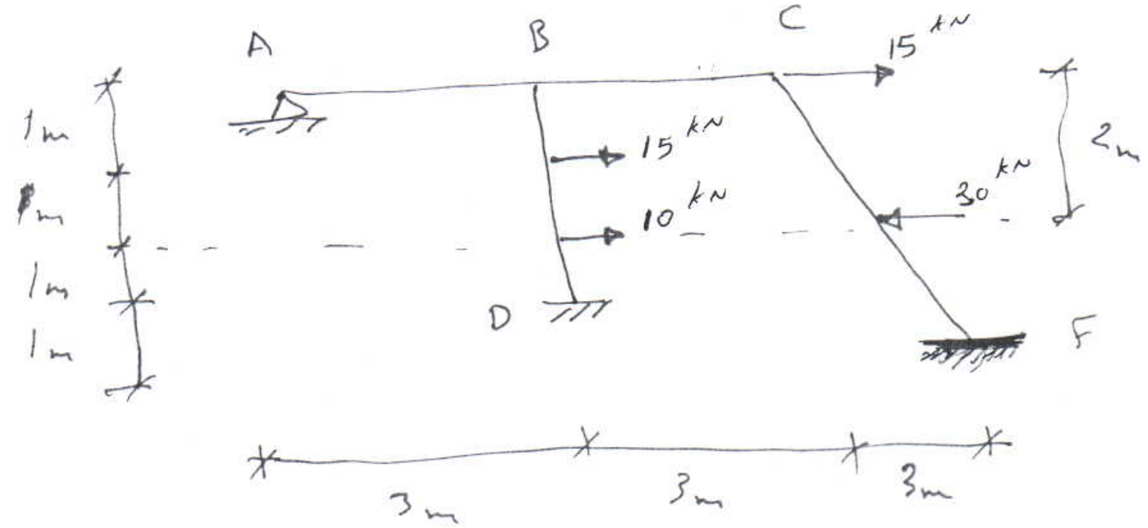
$M_{DC} = 0$

$EI = 300 \ kN.m^2$

Constant.

Q5/

Determine the end moments by the slope-deflection method for the plane frame shown in the fig? Given $EI = 300 \ kN.m^2$ Constant



Sol.

1) $\frac{2EI}{L}$ values

AB, BC & BD : 200 kN.m

CF = 120 kN.m

2) M_F value.

$M_{ABF} = M_{BAF} = M_{BCF} = M_{CBF} = 0$ kN.m

$$M_{DBF} = -\frac{10 \times 1 \times 2^2}{3^2} - \frac{15 \times 2 \times 1^2}{3^2} = -7.778$$

$$M_{BDF} = \frac{15 \times 1 \times 2^2}{3^2} + \frac{10 \times 2 \times 1^2}{3^2} = 8.889$$

$$M_{FCF} = \frac{30 \times 4}{8} = 15$$

$$M_{CFF} = -15$$

3) slope - def. eq's

kN.m

$M_{AB} = 200 (2\theta_A + \theta_B)$ — $M_{AB} = 0$

$M_{BA} = 200 (2\theta_B + \theta_A)$ — $M_{BA} = -3.923$

$M_{BC} = 200 (2\theta_B + \theta_C)$ — $M_{BC} = 0.268$

$M_{CB} = 200 (2\theta_C + \theta_B)$ — $M_{CB} = 8.384$

$M_{DB} = -7.778 + 200(\theta_B)$ $\qquad M_{DB} = -10.394$

$M_{BD} = 8.889 + 200(2\theta_B)$ $\qquad M_{BD} = 3.657$

$M_{FC} = 15 + 120(\theta_c)$ $\qquad M_{FC} = 18.30$

$M_{CF} = -15 + 120(2\theta_c)$ $\qquad M_{CF} = -8.40$

4) eq's of Condetion

$M_{AB} = 0$ — (a)

$M_{BA} + M_{BC} + M_{BD} = 0$ — (b)

$M_{CB} + M_{CF} = 0$ — (c)

$\therefore \boxed{400\,\theta_A + 200\,\theta_B = 0}$ — (a)

$400\,\theta_B + 200\,\theta_A + 400\,\theta_B + 200\,\theta_c + 8.889 + 400\,\theta_B = 0$ — (b)

$\boxed{200\theta_A + 1200\,\theta_B + 200\,\theta_c = -8.889}$ — (b)

$400\,\theta_c + 200\,\theta_B - 15 + 240\,\theta_c = 0$ — (c)

$\boxed{200\,\theta_B + 640\,\theta_c = 15}$ — (c)

$\therefore \underline{200\,\theta_B} = 15 - 640\,\theta_c$ from (c)

sub. into (a)

$400\,\theta_A + 15 - 640\,\theta_c = 0$

$\therefore \underline{400\,\theta_A} = 640\,\theta_c - 15$

sub. into (b)

$320\,\theta_c - 7.5 + 90 - 3840\,\theta_c + 200\,\theta_c = -8.889$

$-3320\,\theta_c = -91.389$

$\therefore \theta_c = 0.0275$ rad.

$\therefore \theta_B = -0.01308$ rad.

$\theta_A = 6.5429 \times 10^{-3}$ rad.