



**University of Technology**  
**Building and Construction Eng. Dept.**  
**FINAL Exam –2015/2016**

**Subject :Theory of Structures**  
**Branch : Highway & Bridges Eng.**  
**Examiner : Dr. Qays Abdul-Majeed**

**Class: 3<sup>rd</sup> year**  
**Time : 3.0 Hours**  
**Date : 29 /05/2016**

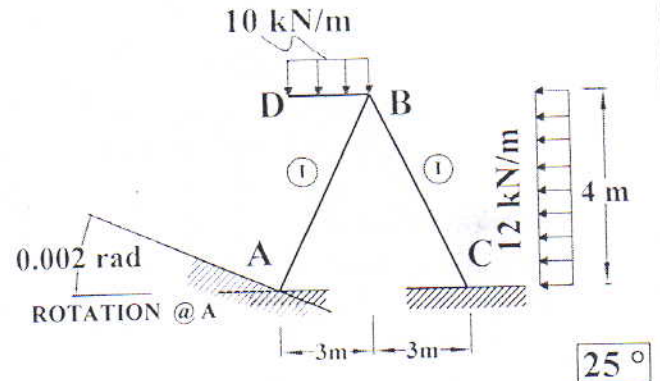


**ANSWER 4 QUESTIONS ONLY**

**Q 1 :-For the Frame shown :**

- Discuss the stability and determinacy.
- Use Slope-Deflection Method to find end moments.
- Draw the bending moment diagram for member AB.

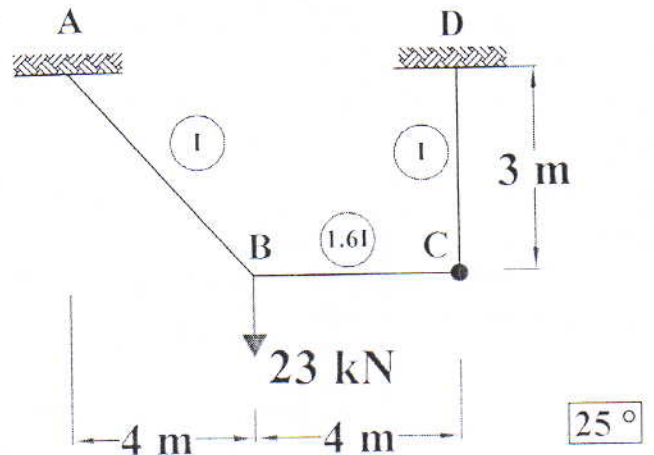
known that  $EI = 10^4 \text{ kN.m}^2$



**Q 2 :-For the structure shown :**

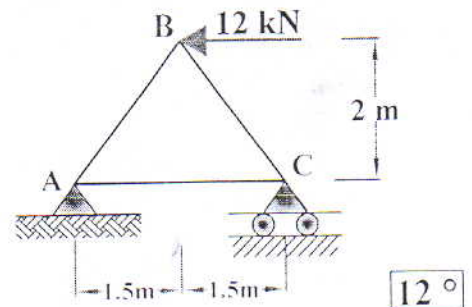
- Check the stability and determinacy.
- By using Moment Distribution Method find end moments.
- find Axial force in member CD.

known that  $EI = 10^4 \text{ kN.m}^2$

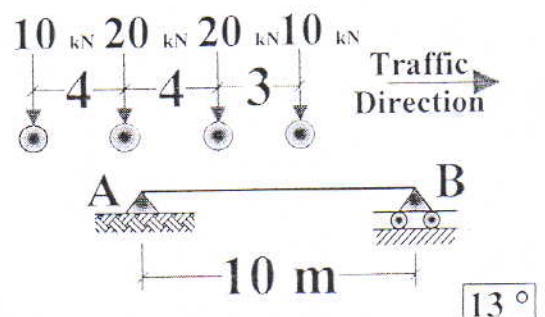


**Q3 :**

- Find the horizontal deflection at B " $\Delta B_h$ " due to a horizontal force "12 kN" shown in the figure.  
 known that  $AE = 10^4 \text{ kN}$ .



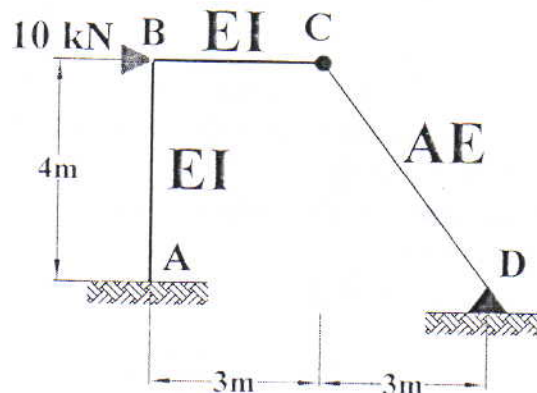
- Find absolute maximum moment for a simply supported beam of length of "10 m" due to moving load shown .



Q 4 :- For the structure shown :

- Check the stability and determinacy.
- By using Consistent Deformation Method find the force in link  $\overline{CD}$ .
- Draw the Axial & Shear force Diagram for member  $\overline{BC}$ .

known that  $EI = 10^4 \text{ kN.m}^2$  for AB & BC  
 $AE = 10^4 \text{ kN}$  for CD.

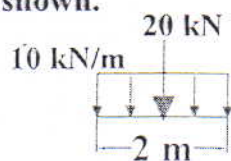


25°

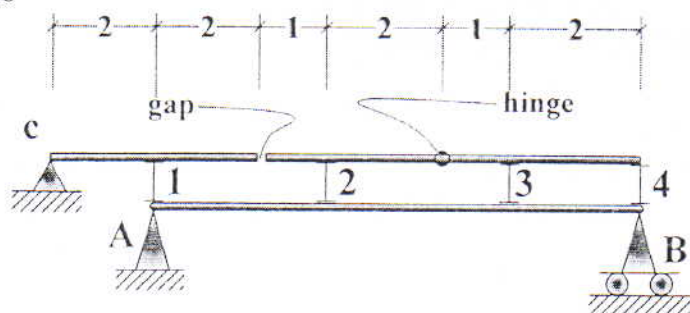
Q 5 :-

a) For the structure shown :

- Find maximum Reaction @ 3 due to moving load shown.

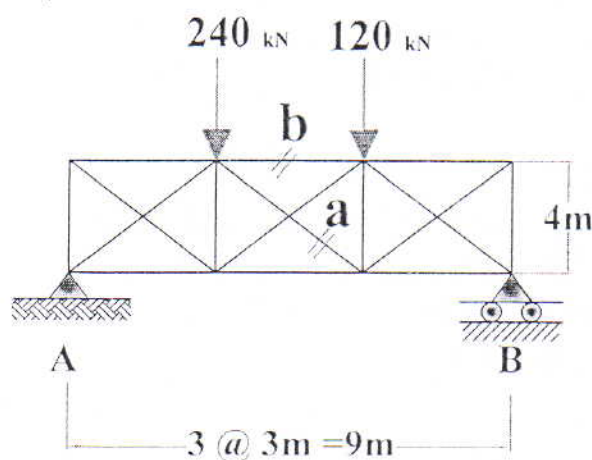


- Draw influence line for Shear between 2 & 3.



15°

- By using the Approximate Analysis find the forces in members (a & b).



10°



(1-8) حلول مسألة فرع الطرق وحسب طريقة المثلثات

عناصر الانشادات Q1

a) stable & Indet. to 3rd deg.

b)  $\sum M_B = 0$

$M_{BD} = 10(3)(1.5) = 45 \text{ kN.m}$

$\sum F_y = 0$

$V_B = 10(3) = 30 \text{ kN}$

$M_{AB}_F = \frac{4(10^4)(0.002)}{5} = 16 \text{ kN.m}$

$M_{BA}_F = \frac{2(10^4)(0.002)}{5} = 8 \text{ kN.m}$

$M_{BC}_F = -\frac{12(4)^2}{12} = -16 \text{ kN.m}$

$M_{CB}_F = \frac{12(4)^2}{12} = 16 \text{ kN.m}$

$k_{rel}$  AB BC

$\frac{I}{L}$   $\frac{1}{5}$   $\frac{1}{5}$  0

$K_{rel}$  1 1

معدلة التدرجات  
حسب المعنى B

$M_{AB} = 16 + 1(0 + \theta_B) = 16 + \theta_B$

$M_{BA} = 8 + 1(2\theta_B + 0) = 8 + 2\theta_B$

$M_{BC} = -16 + (2\theta_B + 0) = -16 + 2\theta_B$

$M_{CB} = 16 + (0 + \theta_B) = 16 + \theta_B$

$\sum M_{joint B} = M_{ext} = 0$

$M_{BA} + M_{BC} + M_{BD} = 0 \Rightarrow 8 + 2\theta_B - 16 + 2\theta_B + 45 = 0$

$\therefore 4\theta_B = -37 \Rightarrow \theta_B = -9.25 \text{ (relative rotation)}$

$\therefore M_{AB} = 6.75 \text{ kN.m}, M_{BA} = -10.5 \text{ kN.m}$

$M_{BC} = -34.5, M_{CB} = 6.75 \text{ kN.m}$

سجل (1-8)

①  $V_A = V_B$  (due to pre)   
  $AB$  is D.C.

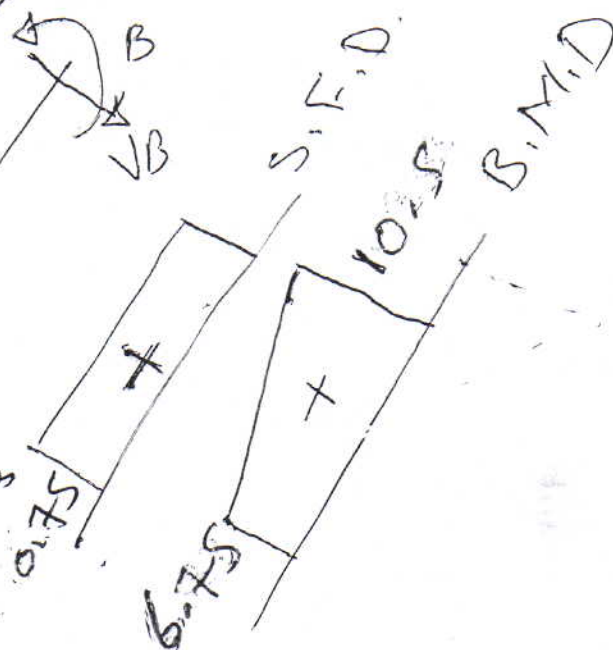
②  $\sum M_B = 0$

$$V_A = \frac{10.57 - 6.75}{5} = 0.75 \text{ kN}$$

$M_{AB} = 6.75 \text{ kN.m}$  (C.W.)

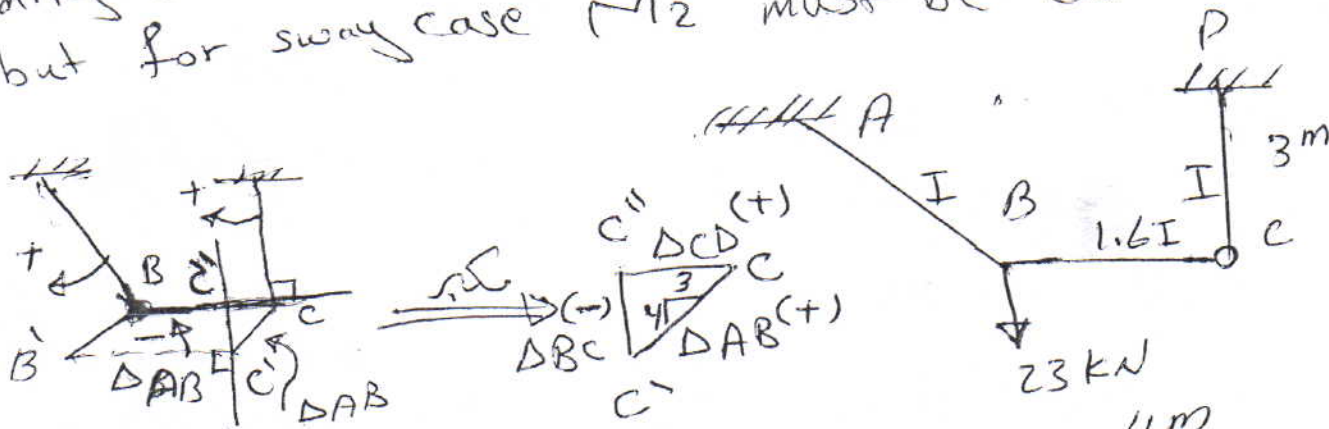
\* C.C.W. is positive   
 \* C.W. is negative

$M_{BA} = 10.5 \text{ kN.m}$  (C.C.W.)



Q2 a) stable & Indet. to 2<sup>nd</sup> deg.

b) The structure has no external moment of loading cause moment. so  $M_1 = 0$  but for sway case  $M_2$  must be calculated



Let  $\Delta_{AB} = +\Delta$  so  $\Delta_{BC} = 3.8\Delta$

$\Delta_{CD} = +0.6\Delta$

Let  $EI\Delta = 25$

$M_{AB} = M_{BA} = -\frac{6EI\Delta}{5^2} = -\frac{6}{25}EI\Delta = -6$

$M_{BC} = M_{CB} = -\frac{6(1.6EI)(-3.8\Delta)}{4^2} = 0.48EI\Delta = 12$

$M_{CD} = M_{DC} = -\frac{6(EI)(0.6\Delta)}{4^2} = -0.45EI\Delta = -10$



حل المسألة الأولى من المرحلة الثالثة مركز العلاقة والحيوية  
(3-8)

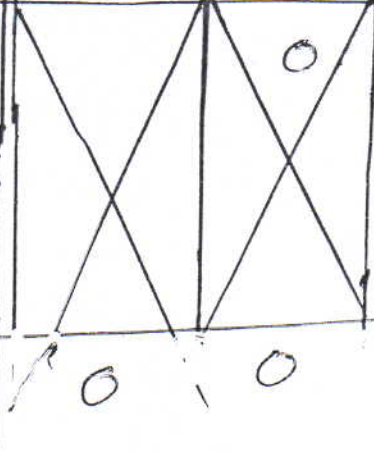
مسألة Q2

في الصيغة السابقة تم حساب العزوم حسب الزوايا  
الامتدادية ونجح في إيجاد  $EI\Delta = 25$  (أراي رقم 1) وكانت نتائج العزوم  
كما موضحة في الصفحة السابقة الآن نريد حساب التوزيع للمفصل B

$$\frac{k}{\Sigma k} = \frac{4(EI) = 0.8EI}{5} \quad \frac{3(1.6EI) = 1.2EI}{4} \quad \frac{2EI}{2EI}$$

$$D.F. = \frac{k}{\Sigma k} = \frac{0.8EI}{2EI} = 0.4 \quad \frac{1.2EI}{2EI} = 0.6$$

نحسب التوزيع

Joint	A	B		C <sub>L</sub>	C <sub>R</sub>	D
Member	AB	BA	BC	CB	CD	DC
D.F.	0	0.4	0.6	1	1	0
F.E.M	-6	-6	12	12	-10	-10
D.M	0	-2.4	-3.6	-12	10	0
C.O.M	-1.2	0	-6		0	5
D.M	0	+2.4	+3.6		0	0
C.O.M	1.2	0	0		0	-
D.M	0	0	0		0	-
$\Sigma M_2$	-6	-6	6	0	0	-5
Maximal	-6k	-6k	6k			-5k

(3-8)

(4-8)

مسألة Q2

ليجاد عتة ك نقطة في العتة

$$\sum M_O = 0$$

$$V_A(10) + V_D(6)$$

$$+ 23(4) - M_{AB} - M_{DC} = 0$$

أذن في  $V_A$  و  $AB$ 

$$\sum M_B = 0$$

$$V_A = \frac{M_{AB} + M_{BA}}{5}$$

$$\sum M_C = 0 \quad V_D = \frac{M_{DC}}{3}$$

الآن نفعل مع  $M_{AB} = -6k$   $M_{BA} = -6k$   $M_{DC} = -5k$   $M_{CB} = 24$

$$V_A = -\frac{12k}{5}, \quad V_D = -\frac{5k}{3}$$

نعمل بالعبارة السابقة

$$-\frac{12k}{5}(10) + (-\frac{5k}{3})(6) + 92 - (-6k) - (-5k) = 0$$

$$-24k - 10k + 92 + 6k + 5k = 0 \Rightarrow k = \frac{92}{23} = 4$$

$$\therefore M_{AB} = -24 \text{ kN.m}, \quad M_{BA} = -24 \text{ kN.m}, \quad M_{DC} = -20 \text{ kN.m}$$

$$M_{BC} = 24 \text{ kN.m}$$

ليجاد القوة المحورية في العتة  $CD$  نأخذ مقطع  $C$

$$\sum F_y = 0 \quad N_{CD} = V_{CB}$$

$$\sum M_B = 0 \Rightarrow V_{CB}(4) - 24 = 0 \Rightarrow V_{CB} = 6 \text{ kN} \quad \therefore N_{CD} = 6 \text{ kN}$$

(4-8)

ans. ↑



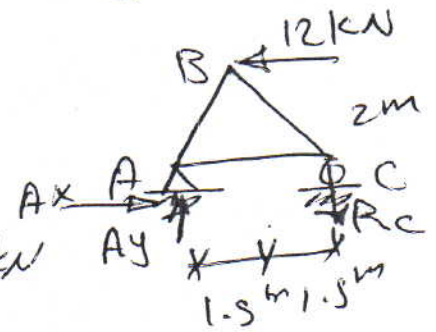
حل مسألة الدبر الدال انشاداً (5-8) الصب الثالث  
 فرع الطريقة والحبر

Q3 a) Req'd  $\Delta H)_B$

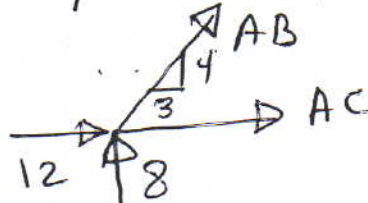
$$\Sigma M_A = 0$$

$$12(2) - R_C(3) = 0 \Rightarrow R_C = 8 \text{ kN} \downarrow$$

$$\Sigma F_y = 0 \quad A_y = 8 \text{ kN} \uparrow, \quad \Sigma F_x = 0 \quad A_x = 12 \text{ kN}$$



Joint A



$$\Sigma F_y = 0 \quad AB\left(\frac{4}{5}\right) + 8 = 0$$

$$\therefore AB = -10 \text{ kN (comp.)}$$

$$\Sigma F_x = 0 \quad 12 + AB\left(\frac{3}{5}\right) + AC = 0 \quad \therefore AC = -6 \text{ kN (comp.)}$$

Joint B

(لأن الصب AB قوي  
 ونسبة 10-4 أي صوب)



$$\Sigma F_y = 0 \quad 8 - BC\left(\frac{4}{5}\right) = 0$$

$$\therefore BC = 10 \text{ kN}$$

mem	S	L	S <sup>2</sup> L
AB	-10	2.5	250
BC	10	2.5	250
CD	-6	3	108
			608

$$\Delta H)_B = \frac{\Sigma uL}{AE}$$

$$u = S/12$$

$$\therefore \Delta H)_B = \frac{\Sigma S^2L}{12AE}$$

$$\Delta H)_B = \frac{608}{12 \times 10^4} = 5.0667 \times 10^{-3} \text{ m}$$

$$= 5.0667 \text{ mm}$$

b)

في النهاية تحذف الحمل البعيد 10 لأنه  
 لا ينفذ أي العارضة بطول 10 m

$$R = 20 + 20 + 10 = 50 \text{ kN}$$

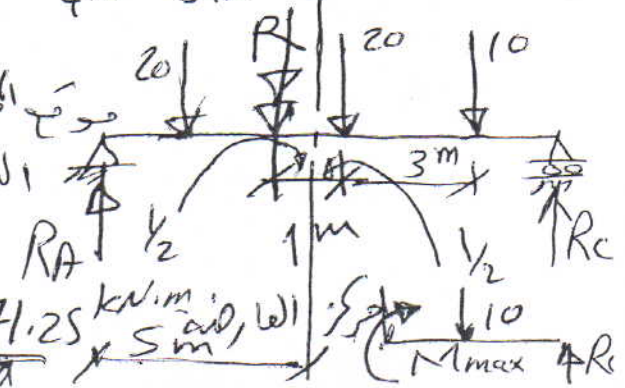
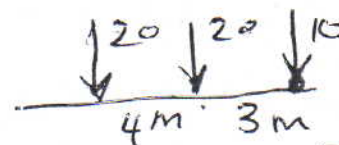
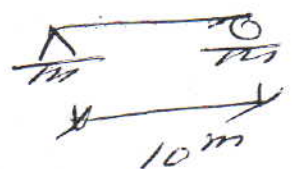
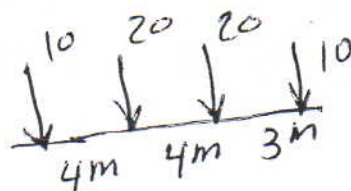
$$\Sigma M_C = 20(7) + 20(3) = 200 \text{ kN}\cdot\text{m}$$

$$d = 200/50 = 4 \text{ m} \rightarrow \text{نقطة المحصلة}$$

البعد بين المحصلة والمركز =  $\frac{1}{2}$  متر

$$R_C = \frac{50(5 - \frac{1}{2})}{10} = 22.5 \text{ kN}$$

$$\therefore M_{abs)_{max}} = 22.5(5 - \frac{1}{2}) - 10(3) = 71.25 \text{ kN}\cdot\text{m}$$

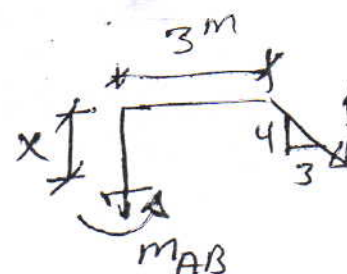
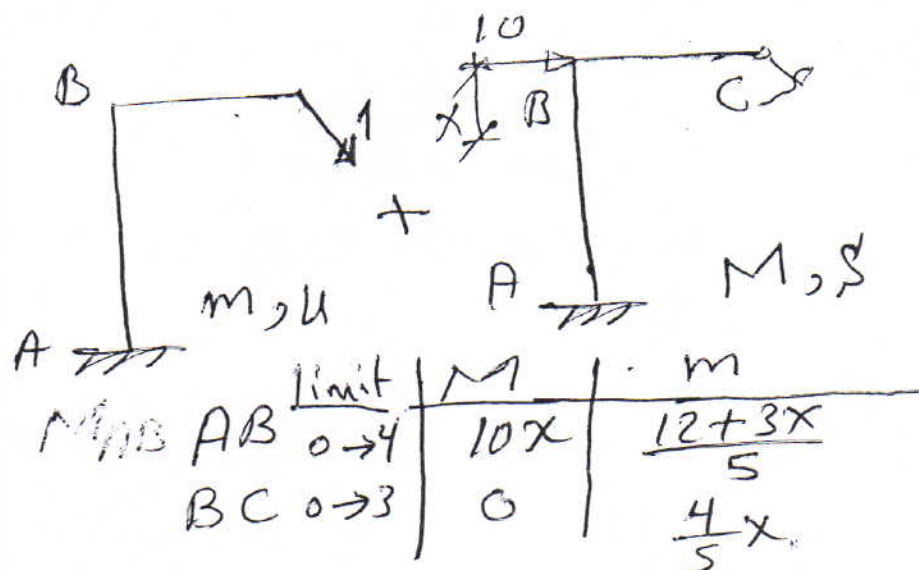
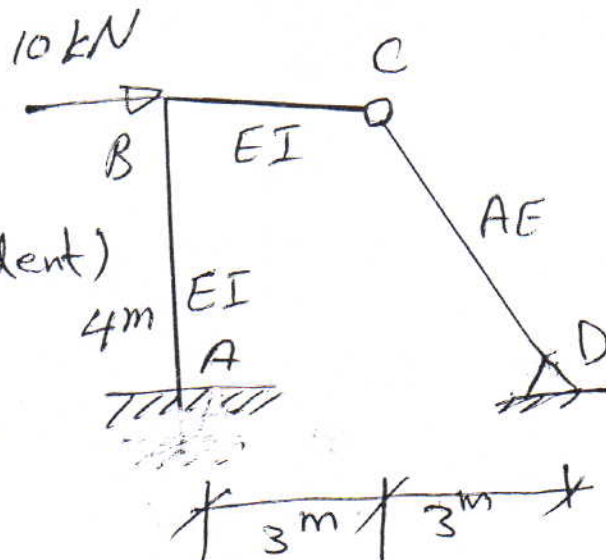


Q4

(6-8)

a) stable & Indet. to 1<sup>st</sup> deg.

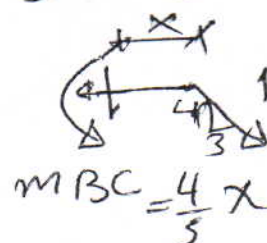
b) let  $X_1 = I_{CD}$  (redundant)



$$\Delta_{10} = \int \frac{M m dx}{EI} = \int_0^4 \frac{10x \left( \frac{12+3x}{5} \right) dx}{EI}$$

$$m_{AB} = 1 \left( \frac{4}{5} \right) (3) + 1 \left( \frac{3}{5} \right) x = \frac{12}{5} + \frac{3x}{5}$$

$$= \frac{2}{10^4} \left( \frac{12x^2}{2} + \frac{3x^3}{3} \right) \Big|_0^4 = 0.032 \text{ m}$$



$$S_{11} = \int \frac{m^2 dx}{EI} + \sum \frac{u^2 L}{AE} =$$

$$\begin{aligned} & \int_0^4 \frac{\left( \frac{12+3x}{5} \right)^2 dx}{EI} + \int_0^3 \frac{\left( \frac{4}{5}x \right)^2 dx}{EI} + \frac{1(5)}{AE} = \\ & = \frac{1}{10^4} \left[ \frac{1}{25} \int_0^4 (144 + 72x + 9x^2) dx + \frac{1}{25} \int_0^3 16x^2 dx + 5 \right] \\ & = \frac{1}{10^4} \left[ \frac{1}{25} \left[ 144x + 36x^2 + 3x^3 \right]_0^4 + \frac{16x^3}{75} \Big|_0^3 + 5 \right] \\ & = \frac{1}{10^4} \left[ \frac{1344}{25} + \frac{144}{25} + 5 \right] = 6.452 \times 10^{-3} \text{ m} \end{aligned}$$

$$\Delta_{10} + X_1 S_{11} = 0 \Rightarrow X_1 = \frac{-0.032}{6.452 \times 10^{-3}} = -4.96 \text{ kN (comp.)}$$

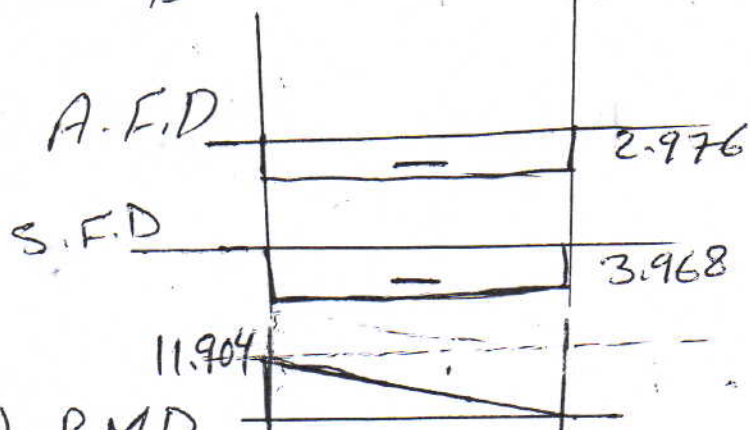


(7-8)

Q4 c)

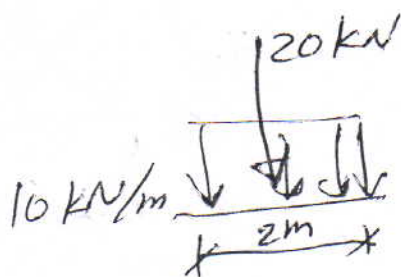
Q4 حل

$\sum M_B = 0$   
 $M_{BC} = 4.96 \left(\frac{4}{5}\right) (3) + V_B$   
 $= 11.904 \text{ kN.m}$   
 $\sum F_y = 0 \quad V_B = 3.968, \quad \sum F_x = 0 \quad N_B = 2.976 \text{ kN}$



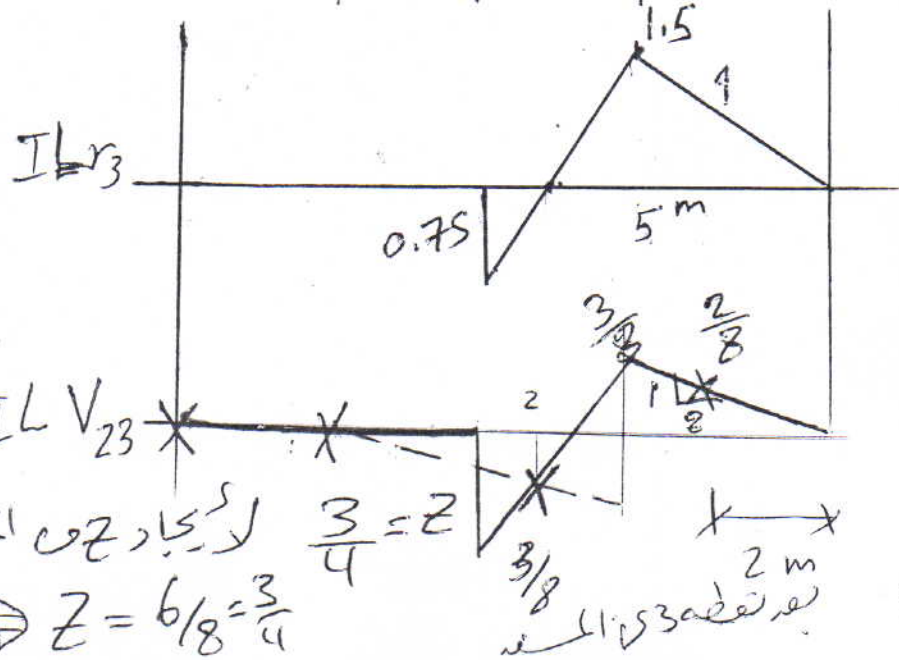
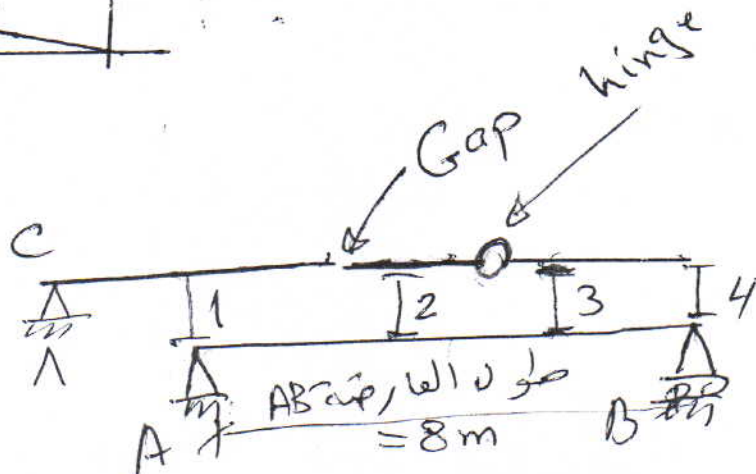
(غير مطلوب) B.M.D

Q5A



Max reaction at 3

$= 20(1.5) + 10(1.5)(2) \left(1 - \frac{2}{2(5)}\right)$   
 $= 54 \text{ kN}$



$\frac{1}{8} = \frac{1}{\text{طول العنبر}} = \text{ميل القف}$   
 $\therefore \text{نقطة 2 تنخفض}$   
 $\frac{3}{8} = 3 \times \frac{1}{8}$   
 $\frac{3}{8} + \frac{3}{8} = \frac{3}{4}$   
 $\frac{3}{4} = Z$   
 $\Rightarrow Z = \frac{6}{8} = \frac{3}{4}$

$$\sum \circledast 5 : b$$

(8-8)

$$\sum M_b = 0 \quad \textcircled{D}$$

$$A_y(9) - 240(6) - 120(3) = 0$$

$$\therefore A_y = 200 \text{ kN}$$

$$\sum F_x = 0 \quad A_x = \text{Zero}$$

$$\sum F_y = 0 \quad \uparrow \quad B_y = 240 + 120 - 200 = 160 \text{ kN}$$

$$(V+V) = 240 - 200 = 40$$

$$\therefore V = \frac{40}{2} = 20 \text{ kN} \quad \uparrow$$

$$a * \frac{4}{5} = V$$

$$\therefore a = \frac{20 * 5}{4} = 25 \text{ kN Comp.}$$

$$\sum M_o = 0 \quad \textcircled{+}$$

$$200(3) - a\left(\frac{3}{5}\right)(4) + 4b = 0$$

$$600 - 25 * \frac{12}{5} + 4b = 0$$

$$\therefore b = -135$$

$$= 135 \text{ kN Comp.}$$

