Soil Mechanics Lectures

2016-2017

Third Year Students

Includes: Soil Formation, Basic Phase Relationship, soil classification, compaction, Consistency of soil, one dimensional fluid flow, two dimensional fluid flows.

B&CED الجامعة التكنولوجية/ قسم هندسة البناء والانشاءات
University of Technology

Chapter One

Soil Formation & Basic RelationShips

Chapter One

Soil Formation and Basic-Relation ships

Soil

Is any uncemented or weakly cemented accumulation of mineral particles formed by weathering of rocks, the void between the particles containing water/ or air. Weak cementation can be due to carbonates or oxides precipitated between the particles or due to organic carbonates or oxides precipitated between the particles or due to organic matter.

Depending on the method of deposition, soils can be grouped into two

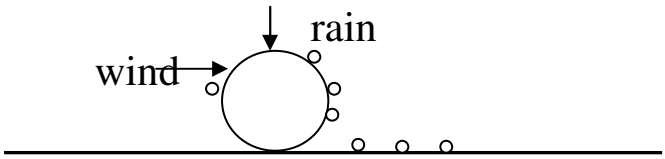
categories:

1- Residual soils:
The soils which remain at the place of disintegration of parent rock.
2- Transported soils :
The soils, which carried away from their place of disintegration to some other place by transporting agencies.

The transporting agencies may be classified as:

i) Water ii) wind iii)gravity iv) Ice

So in general soil is formed from disintegration of rocks over laying the earth crust.



Weathering

Which are usually results from atmospheric processes action on the rock at or near the earth surface.

1- Mechanical (Physical weathering):

All type of actions that cause a disintegration of the parent rocks by physical means such as, gravity, wind and water. The product of this type is rounded, sub rounded or granular, its products called coarse grained soil e.g. (gravel and sand) they present in nature in a single grain structure .

Coarse grained soil

- Sand & Gravel
- Cohesion less soil
- It properties are the same as parent rock.

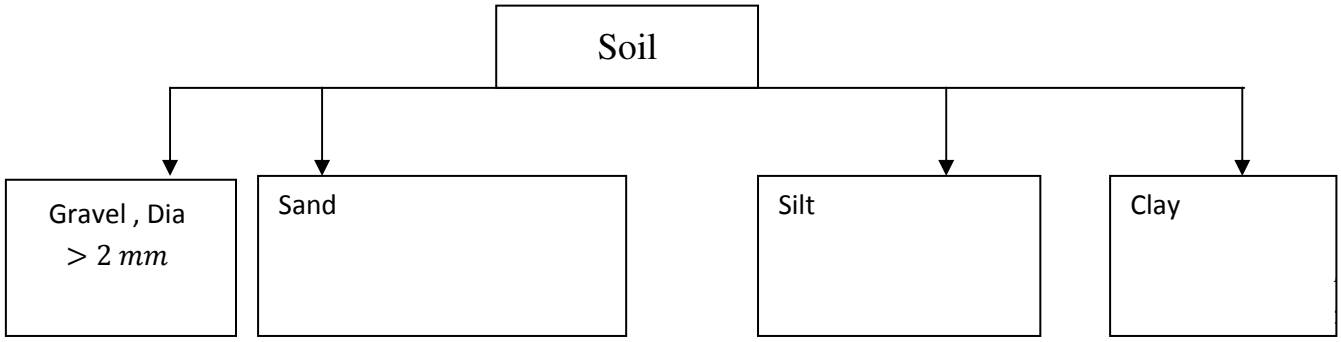
2- Chemical weathering

All types of chemical reactions that occur between the minerals of the rock and the environment (air, water ---et.) and will end up by disintegration of parent rock into fine grain particles; these products have different properties from the parent rock. They present in nature as a lumps of number of plate like particles.

The physical property of this product does not reflect the same properties of the parent rocks.

Fine grained soil

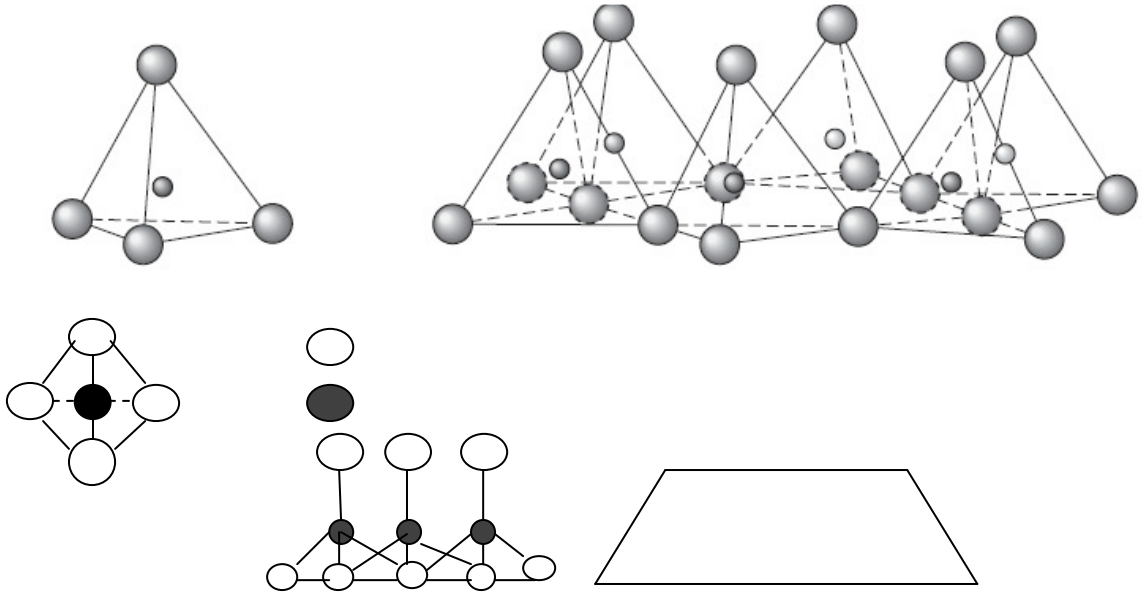
- Silt and clay
- Cohesive material
- Its properties do not reflect the same properties of the parent rocks.



Dia: equivalents diameter (mm)

Clay minerals: There are two basic structure units that form types of the minerals in the clay:

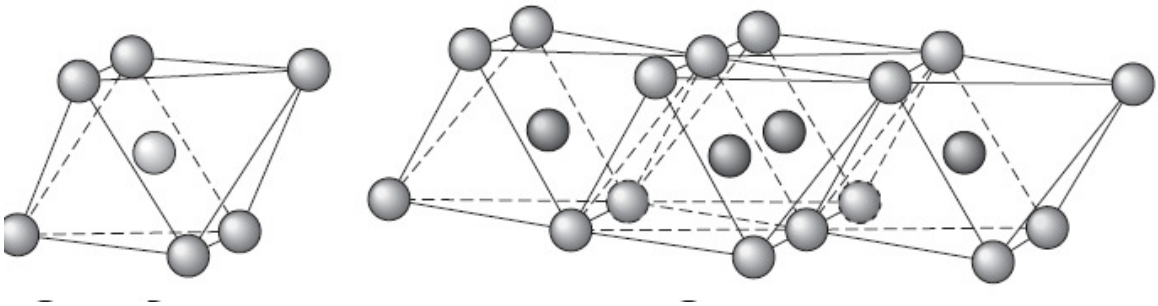
a) Tetrahedral Unit : Consists of four oxygen atoms (or hydroxyls, if needed to balance the structure) and one silicon a tom.

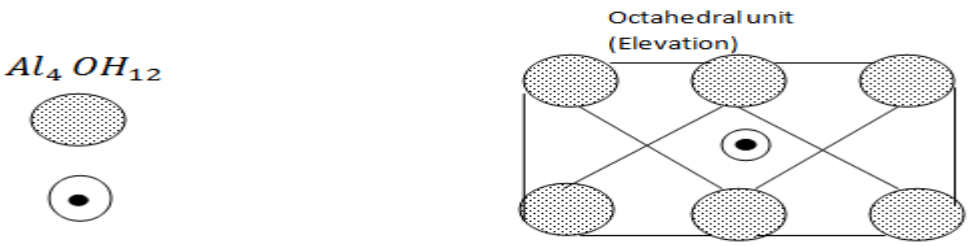


Elevation Tetrahedral sheet

هيدرات الرباعية تحتوي على جزيئة واحدة من السليكون + 4 جزيئات من ايون الاوكسجين.

b) Octahedral Unit (consist of six hydroxyl ion at apices of an octahedral enclosing an aluminum ion at the center).



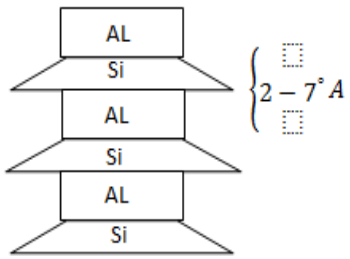


Formation of Minerals

The combination of two sheets of silica and gibbsite in different arrangements and condition lead to the formation of different clay minerals such as :

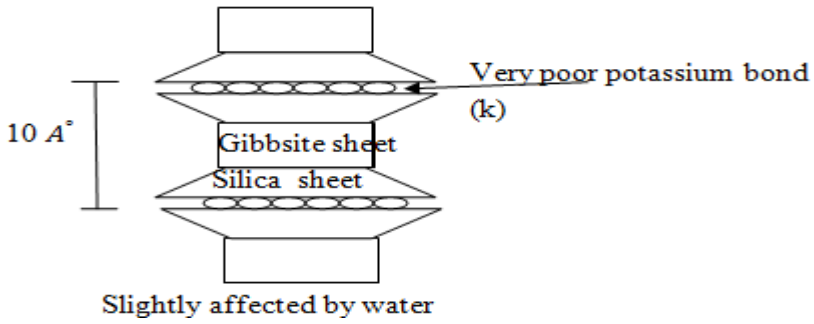
1- Kaolinite Mineral :

This is the most common mineral is the kaolin. The structure is composed of a single tetrahedral sheet and a single alumina octahedral sheet as shown in figure below:

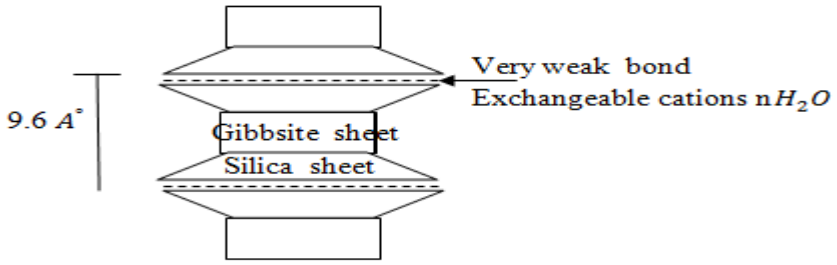


1- Strong Hydrogen Bond So not affected by water
2- And its also called China clay
3-

2- Illite has a basic structure consisting of two silica sheets with a central alumina sheet. There is a potassium bond between the layers.



3- Montmorillonite unit: The basic structural unit is similar to that of Illite.



Highly affected by water

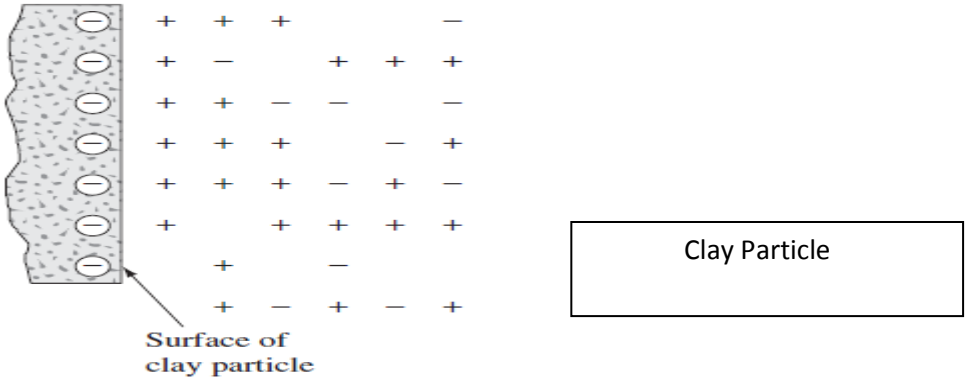
ويتكون عادة من سحق ال الايت

Highly affected by water with high shrinkage and Swell and it is called expansive soil.

وعادة تظهر هذه الحالة في المناطق الصحراوية وشبه الصحراوية

Clay Particle –water relations:

In nature every soil particle is surrounded by water. Since the centers of positive and negative charges of water molecules do not coincide, the molecules behave like dipoles. The negative charge on the surface of the soil particle therefore attracts the positive (hydrogen) end of the water molecules. More than one layer of water molecules sticks on surface with considerable force decrease with increase in the distance of the water molecule from the surface. The electrically attracted water surrounds the clay particle is known as the diffused double-layer of water. The water located within the zone of influence is known as the adsorbed layer as shown in figure:

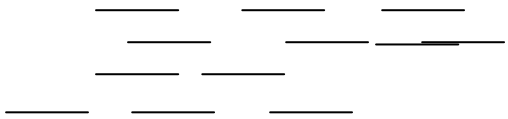


Diffuse double layer

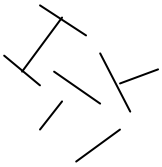
Adsorbed water layer surrounding a soil particle

Clay structures:

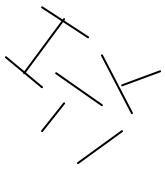
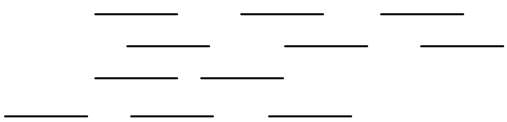
1) - Dispersed structure



2) - flocculated structure



Distinguish between flocculated and dispersed structures

Flocculated	**Dispersed**
	
More strength	Lower strength
Permeability is higher	permeability is less
Low compressibility	higher compressibility

Basic Relationships:

Wt = Ww + Ws

Where w_t: total weight of soil

w_w : Weight of water

w_s : Weight of solid

w_a : Weight of air ≈0

Volume Vt = Vv + Vs = Va + Vw + Vs

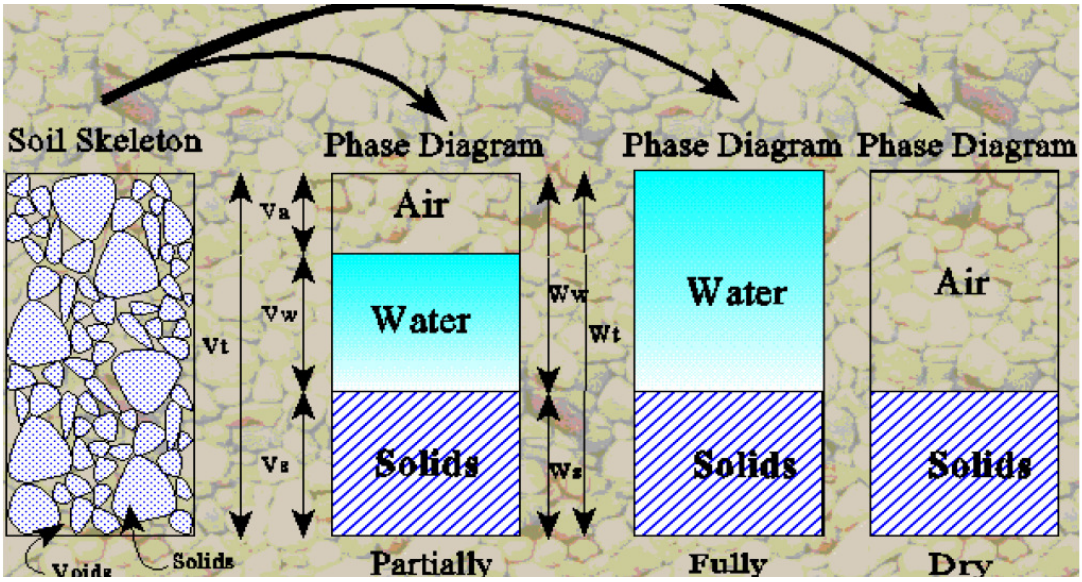
V_t : Total Volume

V_v: Volume of Void

V_a : Volume of air

V_w : Volume of water

V_s : Volume of Soild



1- **Unit Weight – Density**

$$\gamma_{soil} = \frac{\text{Total weight}}{\text{Total volume}} = \frac{w_t}{V_t}$$

2- **Water content %**

$$\omega_c\ \% = \frac{w_w}{w_s} * 100 \quad or \quad \omega_c = \frac{m_w}{m_s} * 100$$

3- **Void ratio , e**

$$e = \frac{V_v}{V_s}$$

4- **Porosity (n%)**

$$n\% = \frac{v_v}{v_t} * 100$$

5- **Air content A%**

$$A\% = \frac{v_a}{v_t} * 100$$

6- **Bulk Density (total density), ρ_t**

$$\rho_t = \frac{m_t}{v_t}$$

7- Dry density ,

$$\rho_{dry} = \frac{m_s}{v_t} \qquad (gm/cm^3) \quad or \ (\frac{kg}{m^3})$$

8- **Dry unit weight (γ_{dry})**

$$\gamma_{dry} = \frac{w_s}{v_t} \qquad (kN/m^3)$$

9- **Specific gravity , G_s**

$$G_s = \frac{\rho_s}{\rho_w} = \frac{m_s/v_s}{\rho_w} = \frac{m_s}{v_s * \rho_w}$$

$$G_s = \frac{\gamma_s}{\gamma_w} = \frac{w_s/v_s}{\gamma_w} = \frac{w_s}{v_s * \gamma_w}$$ **(its value range between 2.6- 2.85)**

10- Solid Density, ρ_s

$$\rho_s = \frac{m_s}{v_s} \quad , \quad \gamma_s = \frac{w_s}{v_s}$$

Some Useful Correlation:

1- $S.e = G_s . \omega_c$

2- $n = \frac{e}{1+e}$

3- $e = \frac{n}{1-n}$

4- $A = n(1-s)$

5- $A = \frac{e-\omega * G_s}{1+e}$

6- $\rho_t = \frac{G_s(1+\omega)}{1+e}\rho_w \quad or \quad \gamma_t = \frac{G_s(1+\omega)}{1+e}\gamma_w$

7- $\rho_t = \frac{G_s+s*e}{1+e}\rho_w \quad or \quad \gamma_t = \frac{G_s+s*e}{1+e}\gamma_w$

8- $\rho_s = \frac{G_s+e}{1+e}\rho_w \quad or \quad \gamma_s = \frac{G_s+e}{1+e}\gamma_w$

9- $\rho_{dry} = \frac{G_s}{1+e}\rho_w \quad or \quad \gamma_d = \frac{G_s}{1+e}\gamma_w$

10- $\rho_{eff.} = \acute{\rho} = \rho_{sat} - \rho_w$

11- $\gamma_{eff} = \acute{\gamma} = \frac{G_s-1}{1+e}\gamma_w$

Some typical values of void ratio, moisture content in a saturated condition, and dry unit weight for soils in a natural state are given in the following table:

Table 1- Void ratio, Moisture Content, and Dry Unit Weight for some Typical Soils in a Natural State.

Type of Soil	Void ratio	Natural moisture content in a saturated state (%)	Dry unit weight ,γ_d (kN/m^3)
Loose uniform sand	0.8	30	14.5
Dense uniform sand	0.45	16	18

Loose angular-grained silty sand	**0.65**	**25**	**16**
Dense angular-grained silty sand	**0.4**	**15**	**19**
Stiff clay	**0.6**	**21**	**17**
Soft clay	**0.9-1.4**	**30-50**	**11.5-14.5**

Note: the weight of one kilogram mass is 9.806 Newton

1 kg = 9.806 N

Example- 1: In its condition a soil sample has a mass of 2290 g and a volume of $1.15*10^{-3}$ m^3. After being completely dried in an oven the mass of the sample is 2035g. The value of G_s for the soil is 2.68. Determine the bulk density, unit weight, water content, void ratio, porosity, degree of saturation and air content.

Solution:

$$\rho_t = \frac{M}{V} = \frac{2.290}{1.15*10^{-3}} = 1990\ kg/m^3 = 1.99\frac{Mg}{m^3}$$

Unit weight ,$\gamma = \frac{Mg}{V} = 1990 * 9.8 = 19500\ N/m^3 = 19.5\ kN/m^3$

Water content , $\omega = \frac{Mw}{Ms} = \frac{2290-2035}{2035} = 0.125\ or\ 12.5\%$

$$\gamma_t = \frac{G_s(1+\omega_c)}{1+e}\gamma_w$$

$$19.5 = \frac{2.68(1+.125)}{1+e} * 10$$

e = 0.538

Porosity, n= $\frac{e}{1+e} = \frac{0.538}{1.538} = 0.3490\ \sim 0.35$

$$S.e = G_s.\omega_c$$

Degree of saturation , S= $\frac{0.125*2.68}{0.538} = 62.267\%$

Air content, A = n (1- S) = 0.35(1-.62)= 0.132

Example 2: a moist soil has these values : $V = 7.08 * 10^{-3}\ m^3$, m = 13.95 kg , $\omega =$ 9.8 %, $G_s = 2.66$. Determine:
ρ , ρ_d , e , n , $S(\%)$, *volume occupied by water and volume occupiesd* by soild?

Solution:

$$\rho = \frac{m}{V} = \frac{13.95}{7.08 * 10^{-3}} = 1970.3\ kg/m^3$$

$$\rho_d = \frac{\rho_{wet}}{1+\omega} = \frac{1970.3}{1+0.098} = 1794.4 \frac{kg}{m^3}$$

$$\rho_d = \frac{G_s}{1+e}\rho_w$$

$$1794.4 = \frac{2.66}{1+e} * 1000 \longrightarrow e = 0.48$$

$$n = \frac{e}{1+e} = \frac{0.48}{1.48} = 0.324$$

$$S.e = G_s . \omega \longrightarrow S.\ 0.48 = 2.66 * 0.098 \longrightarrow S = 54.3\%$$

$$\rho_d = \frac{m_s}{V_t} \longrightarrow 1794.4 = \frac{m_s}{7.08 * 10^{-3}} \longrightarrow m_s = 12.7\ kg$$

$$m_w = m - m_s \longrightarrow m_w = 13.95 - 12.7 = 1.25\ kg$$

$$\therefore\ v_w = \frac{m_w}{\rho_w} = \frac{1.25}{1000} = 0.00125\ m^3$$

$$v_s = \frac{m_s}{G_s \rho_{ww}} \longrightarrow v_s = \frac{m_s}{G_s \rho_w} = \frac{12.7}{2.66 * 1000} = 0.00478\ m^3$$

Example 3:In the natural state, a moist soil has a volume of 0.0093 m^3 and weighs 177.6 N. The oven dry weight of the soil is 153.6 N. If $G_s = 2.71$. Calculate the moisture content, moist unit weight, dry unit weight, void ratio, porosity and degree of saturation.

Solution: $\omega_c = \frac{w_w}{w_s} = \frac{177.6-153.6}{153.6} = 15.6\ \%$

$$\gamma_t = \frac{W}{V} = \frac{177.6}{0.0093} = 19096 \frac{N}{m^3} = 19.1\ kN/m^3$$

$$\gamma_d = \frac{W_s}{V} = \frac{153.6}{0.0093} = 16516\frac{N}{m^3} \sim 16.52\ kN/m^3$$

$$e = \frac{V_v}{V_s}, \quad V_s = \frac{W_s}{G_s\gamma_w} = \frac{0.1536}{2.71*10} = 0.0058\ m^3$$

$$\therefore V_v = 0.0093 - 0.0058 = 0.0035\ m^3$$

$$e = \frac{0.0035}{0.0058} = 0.6 \longrightarrow n = \frac{e}{1+e} = \frac{0.6}{1+0.6} = 0.375$$

$$S.e = G_s.\omega \longrightarrow S.\ 0.6 = 2.71 * 0.156 \longrightarrow S = 70.46\%$$

Example 4: A soil specimen has a volume of 0.05 m^3 and a mass of 87.5 kg. If the water content is 15% and specific gravity is 2.68. Determine 1) void ratio 2) porosity 3) dry unit weight 4) saturated unit weight 5) degree of saturation.

Solution:

$$\rho_t = \frac{m_t}{v_t} = \frac{87.5}{0.05} = 1750\ kg/m^3$$

$$w_c = \frac{m_w}{m_s} = 0.15 = \frac{87.5 - m_s}{ms} \longrightarrow \quad m_s = 76\ kg$$

$$v_s = \frac{m_s}{G_s\rho_w} = \frac{76}{2.68 * 1000} = 0.028\ m^3$$

$$e = \frac{v_v}{v_s} = \frac{0.0216}{0.028} = 0.77, \quad n = \frac{e}{1+e} = \frac{0.77}{1+0.77} = 0.43$$

$$\gamma_{dry} = \frac{G_s}{1+e}\gamma_w = \frac{2.68}{1+0.77}10 = 15.14\ kN/m^3$$

$$\gamma_{sat} = \frac{G_s + e}{1+e}\gamma_w = \frac{2.68 + 0.77}{1+0.77}10 = 19.49\ kN/m^3$$

$$S.e = G_s.\ \omega_c \longrightarrow S*0.77 = 2.68*0.15 \longrightarrow S = 52.2\%$$

Example 5: Show that $\gamma_{sat} = \gamma_{d+(\frac{e}{1+e}*\gamma_w)}$

Solution: take the right hand side :

$$\gamma_{d+\left(\frac{e}{1+e}*\gamma_w\right)} = \frac{G_s}{1+e}*\gamma_w + \frac{e}{1+e}*\gamma_w = \frac{G_s+e}{1+e}*\gamma_w = \gamma_{sat}$$

Example 6: Given mass of wet sample = 254 gm, void ratio = 0.6133, volume of air = 1.9 cm^3, mass of solid =210 gm. Determine degree of saturation, air content and dry unit weight.

Solution: mt = 254 gm, ms = 210 g $\longrightarrow m_w = 254 - 210 = 44\ gm$

$$v_w = \frac{m_w}{\rho_w} = \frac{44}{1} = 44\ cm^3$$

$$v_v = v_w + v_a = 44 + 1.9 = 45.9$$

$$0.6133 = \frac{45.9}{v_s} \rightarrow \therefore v_s = 74\ cm^3$$

$$S = \frac{v_w}{v_v} = \frac{44}{45.9} = 95.8\% \rightarrow A = n(1-s) = \frac{0.6133}{1+0.6133}(1-0.95) = 0.019$$

$$\rho_{dry} = \frac{m_s}{v_t}$$

$$v_t = v_w + v_{air} + v_s = 44 + 1.9 + 74.84 = 120\ cm^3$$

$$\therefore \rho_{dry} = \frac{210}{120} = 1.75\frac{gm}{cm^3} \rightarrow \gamma_{dry} = 17.5\ kN/m^3$$

Example 7: A soil specimen is 38 mm in diameter and 76 mm long and its natural condition weighs 168 gm when dried completely in an oven the specimen weighs 130.5 gm. The value of $G_s = 2.73$. what is the degree of saturation of the specimen?

Solution: Dia = 38 mm = 3.8 cm

L= 76 mm = 7.6 cm

$$v_t = (\frac{3.8}{2})^2\pi * 7.6 = 86.192\ cm^3$$

$$m_w = 168 - 130.5 = 37.5\ gm$$

$$v_w = \frac{37.5}{1} = 37.5\ cm^3$$

$$v_s = \frac{w_s}{G_s * \gamma_w} = \frac{130.5}{2.73 * 1} = 47.8\ cm^3$$

$$v_a = 86.192 - (37.5 + 47.80) = 0.889 cm^3$$

$$v_v = v_w + v_a = 37.5 + 0.889 = 38.389 cm^3$$

$$\therefore S = \frac{v_w}{v_v} = \frac{37.5}{38.389} = 97.6\%$$

Example 8: Given: mass of wet sample =254.1gm, void ratio = 0.6133, volume of air = 1.9 cm^3, mass of solids = 210 gm. Determine: Degree of saturation, Air content, dry unit weight.

Solution:

Mass of water = 254.1 -210= 44.1 gm

Volume of water = $\frac{w_w}{\gamma_w} = 44.1\ cm^3$

$$e = \frac{v_v}{v_s} \rightarrow 0.6133 = \frac{v_v}{v_s} = \frac{v_w + v_a}{v_s} = \frac{44.1+1.9}{v_s}$$

$$0.6133 = \frac{46}{v_s} \rightarrow v_s = 75.\ cm^3$$

$$S = \frac{v_w}{v_v} = \frac{44.1}{46} = 95.8\%$$

$$A = \frac{e}{1+e}(1 - s) = \frac{0.6133}{1+0.6133}(1 - 0.958) = 0.015\ 7$$

$$v_t = v_v + v_s = 46 + 75 = 121\ cm^3$$

$$G_s = \frac{w_s}{v_s \gamma_w} = \frac{210}{75 * 1} = 2.8$$

$$\gamma_{dry} = \frac{G_s}{1+e}\ \gamma_w \rightarrow \gamma_{dry} = \frac{2.8}{1+0.6133} * 10 = 17.355\ kN/m^3$$

$$\text{Or } \rho_{dry} = \frac{m_s}{v_t} = \frac{210}{121} = 1.7355\ gm/cm^3$$

$$\therefore \gamma_{dry} = \rho_{dry} * g = 1.7355 * 10 = 17.355\ kN/m^3$$

Example 9: A soil specimen have void ratio of 0.7 , $G_s = 2.72$. Calculate the dry unit weight, unit weight and water content at degree of saturation of 75%.

Solution :

$$\gamma_{dry} = \frac{G_s}{1+e}\gamma_w = \frac{2.72}{1+0.7} * 10 = 16\frac{kN}{m^3},$$

$$\gamma_{sat} = \frac{2.72 + 0.7}{1 + 0.7} * 10 = 20.11\ kN/m^3$$

$$\gamma_b = \acute{\gamma} = \gamma_{eff} = \gamma_{sat} - \gamma_w = 20.11 - 10 = 10.11\ kN/m^3$$

$$\gamma_{at\ s=75\%} = \frac{2.72 + 0.75 * 0.7}{1 + 0.7} * 10 = 19.1\ kN/m^3$$

$$0.75 * 0.70 = 2.72 * \omega \qquad \rightarrow \qquad \omega = 19.3\%$$

Example 10 : Prove that S. e = $G_s.\omega_c$

Take the right hand side;

$$G_s.\omega_c = \frac{w_w}{w_s} * \frac{w_s}{v_s\gamma_w}$$

$$= \frac{v_w\gamma_w}{v_s\gamma_w} * \frac{v_v}{v_v} = \frac{v_w}{v_v} * \frac{v_v}{v_s} = S * e$$

Example 11: Show that $\gamma_{dry} = \frac{\gamma_t}{1+\omega}$

$$\frac{\gamma_t}{1+\omega} = \frac{w_t/v_t}{1+w_w/w_s} = \frac{w_t/v_t}{\frac{w_s+w_w}{w_s}} = \frac{\frac{w_t}{v_t}}{\frac{w_t}{w_s}} = \frac{w_s}{w_t}$$

Example 12 : Prove that $n = \frac{e}{1+e}$

$$\frac{e}{1+e} = \frac{v_v/v_s}{1+v_v/v_s} = \frac{v_v/v_s}{\frac{v_s+v_v}{v_s}} = \frac{v_v}{v_t} = e$$

References :

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2- Soil mechanics

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3- Soil Mechanics

Basic Concepts and Engineering Applications

A.Aysen

Chapter Two
Soil Consistancy

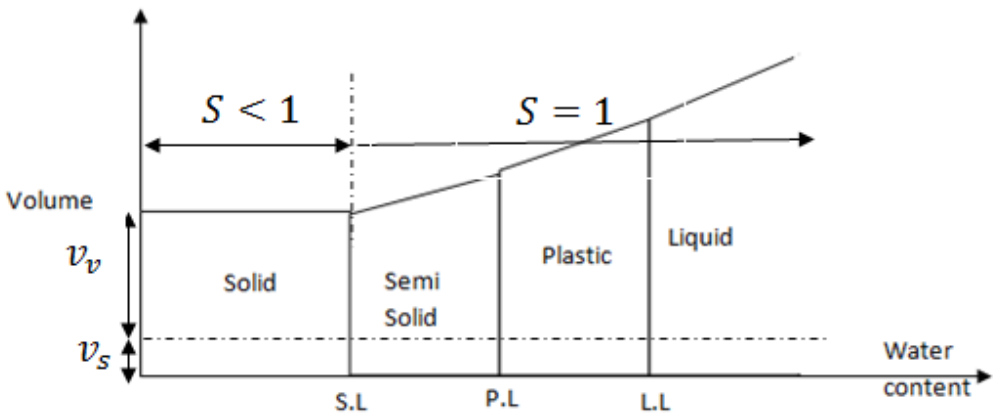
Chapter Two

Plasticity of Fine Grained Soils

Plasticity is the ability of a soil to undergo unrecoverable deformation at constant volume without cracking or crumbling. It is due to the presence of clay minerals or organic material.

Consistency limits (Atterberg limits):

Atterberg, a Swedish scientist developed a method for describing the limit consistency of fine grained soils on the basis of moisture content. These limits are liquid limit, plastic limit and shrinkage limit.



Liquid limit (L.L): is defined as the moisture content in percent at which the soil changes from liquid to plastic state.

Plastic Limit (P.L.): The moisture contents in % at which the soil changes from plastic to semi solid state.

Shrinkage Limit (S.L.): The moisture contents in % at which the soil changes from semi solid to solid state.

Plasticity Index (P.I.): it is the range in moisture content when the soil exhibited its plastic behavior:

$$P.I. = L.L - P.L.$$

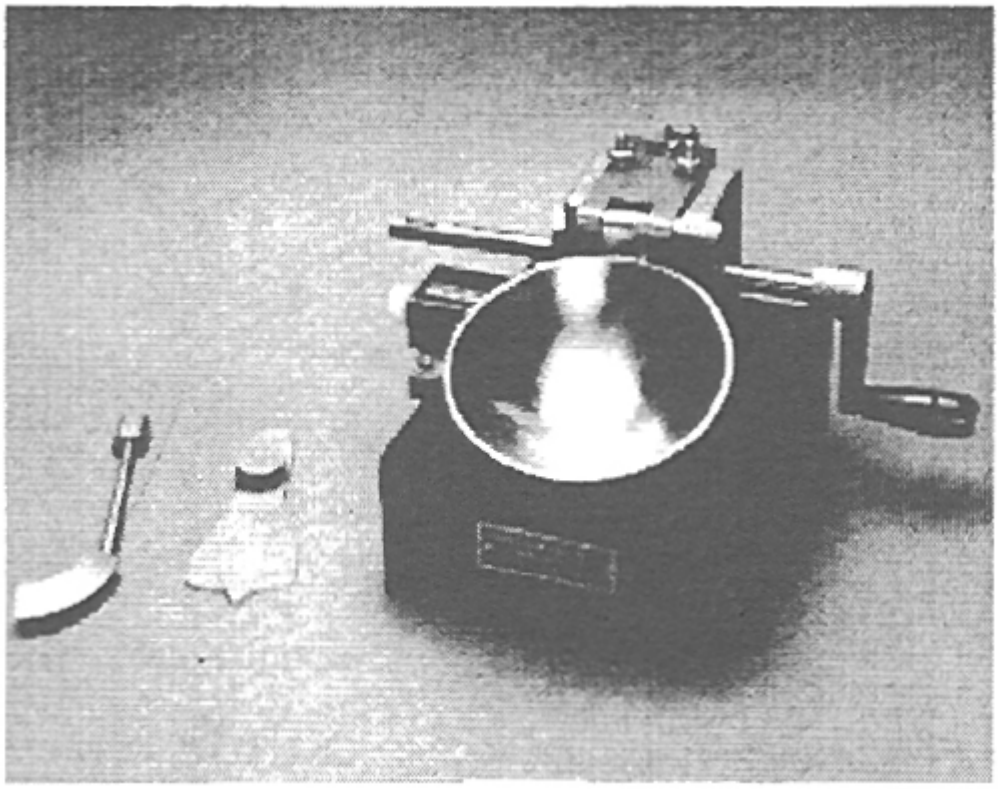
Liquidity Index (L.I. or IL) : a relation between the natural moisture contents (ω_n) and (L.L.) and (P.L.) in form:

$$L.I. = \frac{\omega_n - P.L}{L.L. - P.L.}$$

If LI > 1 Then the soil at Liquid state

If LI = 1 then the soil at L.L.

If $LI < 1$ then the soil below L.L.



Liquid limit device with grooving tools (Wykeham Farrance).

Activity: is the degree of plasticity of the clay size fraction of the soil and is expressed as:

$$Activity = \frac{P.I}{\%\ of\ clay\ size\ particles}$$

كلما زادت الفعالية كلما دلت على لدونة التربة عالية

Plasticity Chart: based on Atterberg limits, the plasticity chart was developed by Casagrande to classify the fine grained soil.

Some useful notes:

v_s : Constant at all stages

Degree of saturation (S %) at S.L. and up to =100%

Degree of Saturation in the region from S.L. and below $< 100\%$

$v_{tdry} = v_{t\ at\ S.L.}$--------

$v_{vdry} = v_{v\ at\ S.L.}$

$e_{dry} = e_{S.L.}$

Relative Density: is the ration of the actual density to the maximum possible density of the soil it is expressed in terms of void ratio.

$$RD(\%) = \frac{e_{max} - e_n}{e_{max} - e_{min}} * 100$$

Or $$RD(\%) = \frac{\gamma_{dmax}}{\gamma_{dn}} * \frac{\gamma_{dn} - \gamma_{dmin}}{\gamma_{dmax} - \gamma_{dmin}} * 100$$

e_{max}: The void ratio of the soil in its loosest condition

e_{min}: The void ratio of the soil in its densest condition

e_n: The void ratio of the soil in its natural condition

γ_{dmax}: Maximum dry unit weight (ate_{min})

γ_{dmin}: Minimum dry unit weight (ate_{max})

γ_{dn}: Natural dry unit weight (ate_n)

RD	Description
0 - $\frac{1}{3}$	loose
$\frac{1}{3} - \frac{2}{3}$	medium
$\frac{2}{3} - 1$	Dense

Example 1: for a granular soil, given,$\gamma_{dry} = 17.3\frac{kN}{m^3}$, relative density = 82%, $\omega = 8\%$ and $G_s = 2.65$. If $e_{min} = 0.44$. what would be e_{max} ? what would be the dry unit weight in the loosest state?

Solution:

$$\gamma_{dry} = \frac{G_s}{1+e_n} * 10 \Longrightarrow 17.3 = \frac{2.65}{1+e_n} * 10$$

$$\therefore \quad e_n = 0.53 \Longrightarrow RD = \frac{e_{max}-e_n}{e_{max}-e_{min}} * 100$$

$$0.82 = \frac{e_{max}-0.53}{e_{max}-0.44} \Longrightarrow \therefore \quad e_{max} = 0.94$$

$$\therefore \gamma_{dry}\,(at\ loosest) = \frac{G_s}{1+e_{max}}\gamma_w = \frac{2.65}{1+0.94} * 10$$

$$= 13.65\ kN/m^3$$

Example 2: a granular soil is compacted to moist unit weight of 20.45 kN/m^3 at moisture content of 18% . What is relative density of the compacted soil? Given, $e_{max} = 0.85$, $e_{min} = 0.42$ and $G_s = 2.65$?

Solution:

$$\gamma = \frac{G_s(1+\omega_c)}{1+e_n}\gamma_w \Longrightarrow 20.45 = \frac{2.65(1+0.18)}{1+e} * 10$$

$$\therefore e_n = 0.52 \Longrightarrow RD = \frac{e_{max}-e_n}{e_{max}-e_{min}} =$$

$$RD = \frac{0.85-0.52}{0.85-0.42} * 100 = 76.74\%$$

Example 3: A dry sample of soil having the following properties, L.L. = 52%, P.L. = 30%, $G_s = 2.7$, e= 0.53. Find: Shrinkage limit, d_{ry} density, dry unit weight, and air content at dry state.

Solution

Dry sample $\Longrightarrow e_{dry} = e_{shrinkage} = 0.53$

$\therefore S.e_{s.l} = G_s . \omega_{c_{s.l}} \Longrightarrow$ 1*0.53 =2.7 *S.L

S.L.= 19.6%

$$\rho_{dry} = \frac{G_s}{1+e}\rho_w \Longrightarrow \rho_{dry} = \frac{2.7}{1+0.53} 1 = 1.764 \frac{gm}{cm^3}$$

$$\therefore \gamma_{dry} = \rho_{dry} * g = 1.764 * 10 = 17.64 \ kN/m^3$$

Case is dry $\Longrightarrow$ s=0

$$\therefore A = n = \frac{e}{1+e} = \frac{0.53}{1+0.53} = 0.346$$

$$\therefore A = 34.6\%$$

Example 4: A saturated soil sample has a volume of 20 cm^3 at its L.L Given L.L= 42% , P.L.= 30% , S.L.= 17% , $G_s = 2.74$. Find the min. volume the soil can attain.

The minimum volume occurs at S.L. or at dry state.

$$v_t = v_v + v_s$$

v_s: is constant along all state.

At L.L.

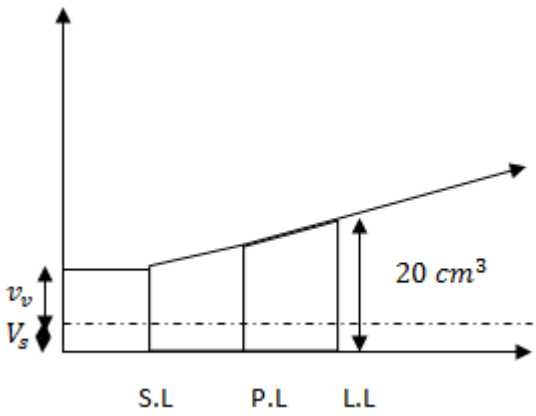
$$S.e = G_s.\omega_c$$

1*e= 2.74 *0.42

$e_{L.L.}$= 1.1508

$$e = \frac{v_v}{v_s} = \frac{20 - v_s}{v_s}$$

$$= 1.1508$$



$\therefore\ v_s = 9.3\ cm^3$

$$\therefore S.e = G_s\ .\omega_{S.L.}$$

$e_{S.L.} = 0.4658$

$e = \frac{v_v}{v_s} \Longrightarrow$

$v_{tS.L.} = v_{vS.L.}$

$= 4.33 + 9.3$

$\therefore v_{v\ L.L.}$ = 20- 9.3 =10.7cm^3

1* $e_{S.L.} = 2.74 * 0.17$

$v_{v\ S.L.} = 0.465 * 9.3 = 4.33\ cm^3$

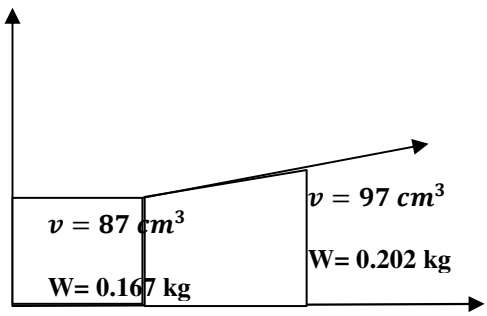
$= 13.63\ cm^3$

Example 5: A sample of saturated clay had a volume of 97 cm^3 and a mass of (0.202 kg). When completely dried at the volume of the sample was (87 cm^3) and it's mass (0.167 kg). Find

a) - initial water content. b)- shrinkage limit c)- specific gravity

Solution:

$$\rho_t = \frac{m_t}{v_t} = \frac{0.202}{97} = 2.08\ gm/cm^3$$



$$2.08 = \frac{G_s + e}{1 + e} * 1 ----(1)$$

$$\omega_c = \frac{m_w}{m_s} = \frac{202.167}{167} = 21\%$$

$$S.e = G_s.\ \omega_c$$

$$1*e = G_s * 0.21 ---(2)$$

Solving (1) and (2)

$$e = 0.565\ \ and\ G_s = 2.69$$

At dry state : $\rho_{dry} = \frac{ms}{vt} = 1.96\ gm/cm^3$

But $\rho_{dry} = \frac{G_s}{1+e}\ \rho_w \Longrightarrow 1.92 = \frac{2.69}{1+e_{S.L}} * 1$

$\therefore e_{S.L.} = 0.4$

$S.e_{S.L} = 2.69 *\ \omega_{S.L.} \Longrightarrow 1 * 0.4 = 2.69 * \omega_{S.L.}$

$\omega_{S.L.} = 15\%$

Chapter Three
Soil Compaction

Chapter three

Soil Compaction

Soil compaction is one of the most critical components in the construction of roads, airfield, embankments and foundations. The durability and stability of a structure are related to the achievement of proper soil compaction. Structural failure of roads, airfield and the damage caused by foundation settlement can often be traced back to the failure to achieve proper soil compaction.

Compaction of soil:

Compaction is the process of increasing the density of a soil by packing the particles closer together with a reduction in the volume of air only. Compaction increases the dry density and decreases the void ratio.

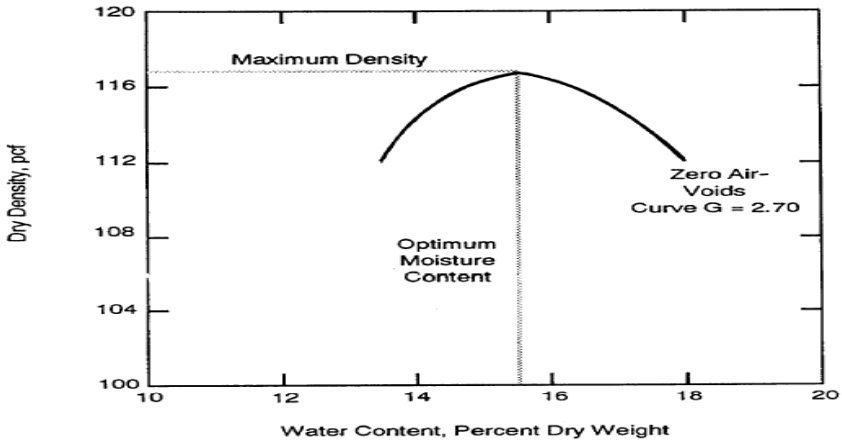
Purpose of compaction:

1- Increase shear strength of soil
2- Reduce void ratio thus reduce permeability
3- Controlling the swell-shrinkage movement
4- Reduce settlement under working load
5- Prevent the buildup of large water pressure

Factors affecting compaction:

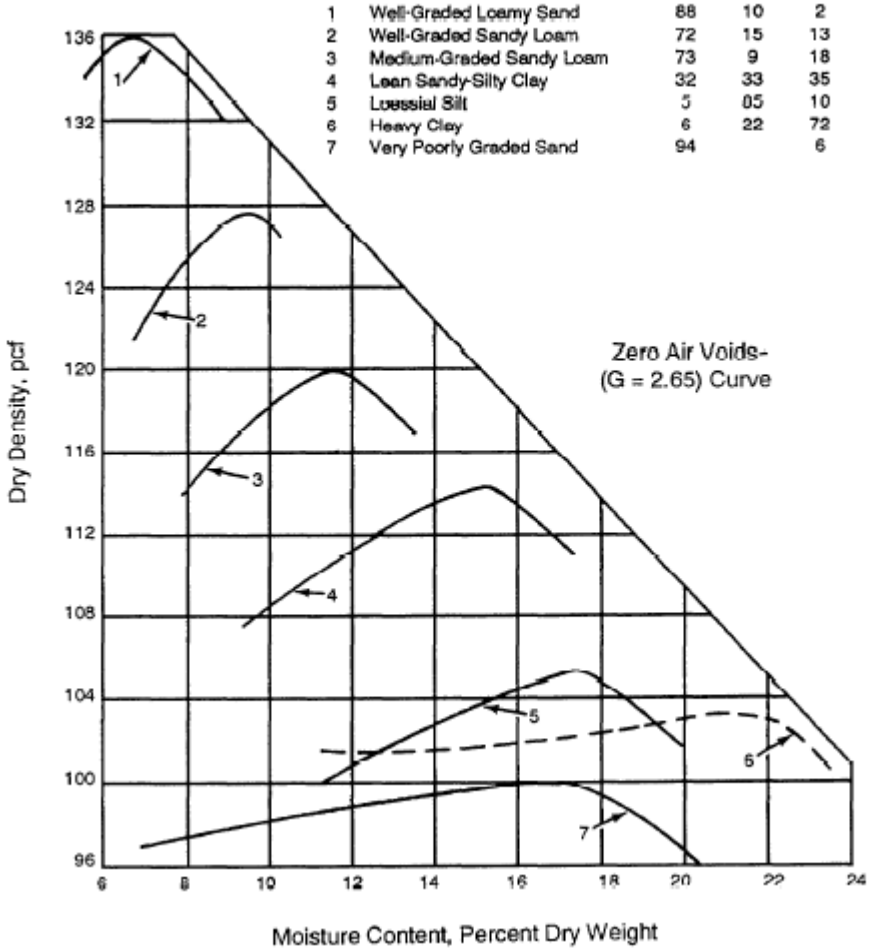
- Water content
- Type of soil
- Compaction energy or effort

All these factors are shown in the following figures:

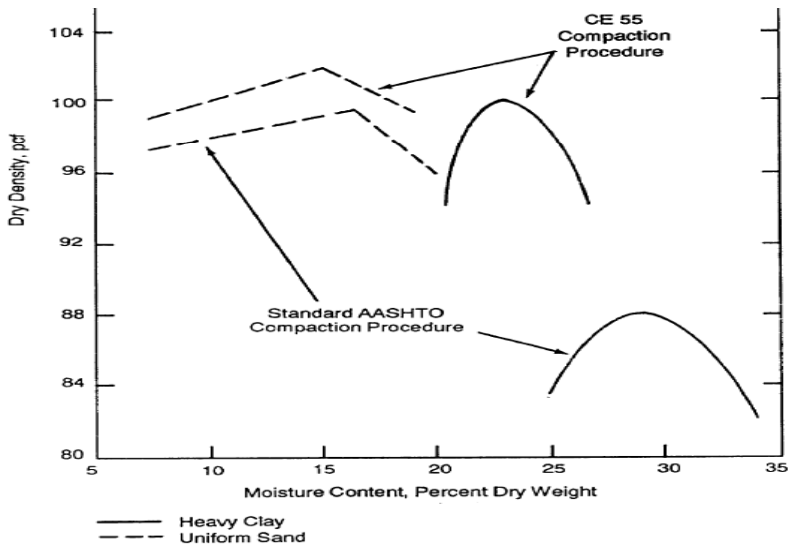


Soil Texture and Plasticity Data

No	Description	Sand	Silt	Clay	LL	PI
1	Well-Graded Loamy Sand	88	10	2	16	NP
2	Well-Graded Sandy Loam	72	15	13	16	0
3	Medium-Graded Sandy Loam	73	9	18	22	4
4	Lean Sandy-Silty Clay	32	33	35	28	9
5	Loessial Silt	5	85	10	26	2
6	Heavy Clay	6	22	72	67	40
7	Very Poorly Graded Sand	94		6	NP	



The effect of types of soil on the dry density using the same compaction Energy.

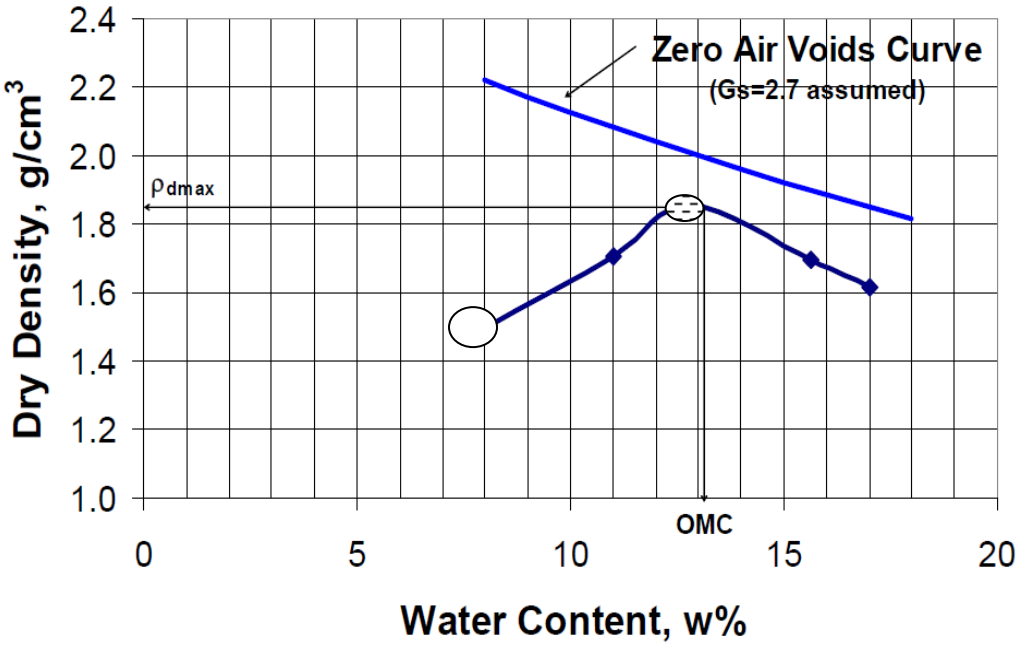


Different in compaction energy and types of soil

Theory of compaction:

Compaction is the process of reducing the air content by the application of energy to the moist soil. From compaction test we can find:

1- There is a unique relationship between the water content and the dry density for specific compaction energy.
2- There is one water content (O.M.C.) (Optimum moisture content) at which the max dry density is achieved
The two above points can be clearly shown through the following Figure:



Compaction curve

Compaction Test

The compaction test is performed to determine the relationship between the moisture content and the dry density of a soil for specific compactive effort. The compactive effort is the amount of mechanical energy that is applied to the soil mass. Several different methods are used to compact soil in field, and some examples include tamping, kneading, vibration, and static load compaction. This test will be carried out by using impact compaction method using the type of equipment and methodology developed by R.R.Proctor in 1933, therefore , the test is also known as the proctor test.

Two types of compaction tests are routinely performed: (1) The standard Proctor and (2) The modified Proctor test.

Type of test	No. of layer	No. of blows per layer	Volume of mold (cm^3)	Weight of hammer (kg)	Height of drops cm
Standard Proctor	3	25	1000	2.5	30
Modified Proctor	5	25	1000	4.5	45

Compaction Effort = $\frac{wt\ of\ hammer * drops\ height * No.of\ blows * No.of\ layer}{Volume\ of\ mold}$

Test Procedure :

1- a sufficient quantity of air-dried soil in large mixing pan (say 3 kg)
2- Determine the weight of the compaction mold with its base (without the collar).
3- Start with initial water such (3% of Soil weight)
4- Add the water to the soil and mix it thoroughly into the soil until the soil gets uniform color (see figure B and C).
5- Assemble the compaction mold to the base, place soil in the mold and compact the soil in the number of equal layers specified by the type of compaction method (see photo D and E).
   The number of drops per layer is dependent upon the type of compaction. The drops should be applied at a uniform rate not exceeding around 1.5

seconds per drops, and the rammer should provide uniform coverage of the specimen surface.

6- The soil should completely fill the cylinder and the last compacted layer must extend slightly above the collar joint. If the soil below the collar joint at the completion of the drops, the test point must be repeated.
7- Carefully remove the collar and trim off the compacted soil so that it is completely even with the top of the mold.(see photo F).
8- Weigh the compacted soil while it's in the mold and to the base, and record the mass (see Photo G). Determine the wet mass of the soil by subtracting the weight of the mold and base.
9- Remove the soil from the mold using a mechanical extruder (see Photo H) and take the soil moisture content samples from the top and bottom of the specimen (see Photo i). Determine the water content.
10- Place the soil specimen in the large tray and break up the soil until it appears visually as if it will pass through the #4 sieve, add 3% more water on the soil and remix as in step 4. Repeat step 5 through 9 until a peak value is reached followed by two slightly lesser compacted soil masses.

Analysis:

1- Calculate the moisture content of each compacted soil specimen.
2- Compute the wet density in grams per cm^3 of the compacted soil by dividing the wet mass by the volume of the mold used.
3- Compute the dry density using the weight density and the water content determined in step 1. Use the following formula:

$$\rho_{dry} = \frac{\rho_{wet}}{1 + \omega_c}$$

4- Plot the dry density values on the y-axis and the moisture contents on the x-axis. Draw a smooth curve connecting the plotted points.
5- On the same graph draw a curve of Saturation line (Zero air void line) using the following Equation :

$$\rho_{dry} = \frac{G_s}{1+e}\rho_w$$

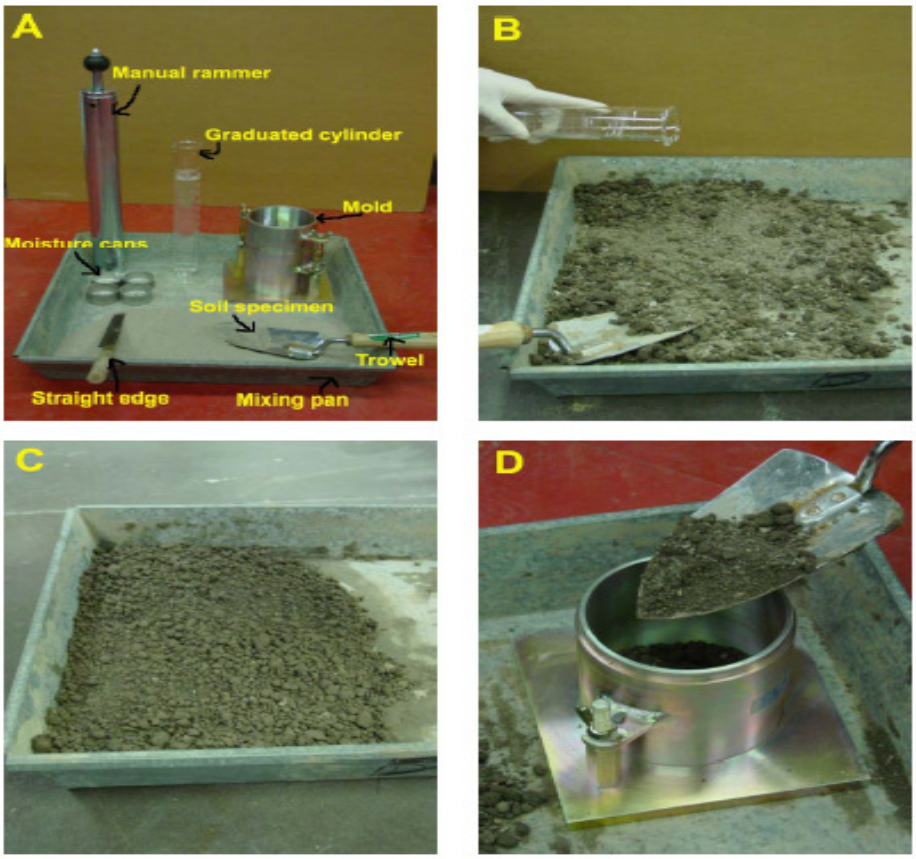
$S.e = G_s.\omega_c \Longrightarrow$ For S= 1 $\therefore$ $e = G_s\,\omega_c$

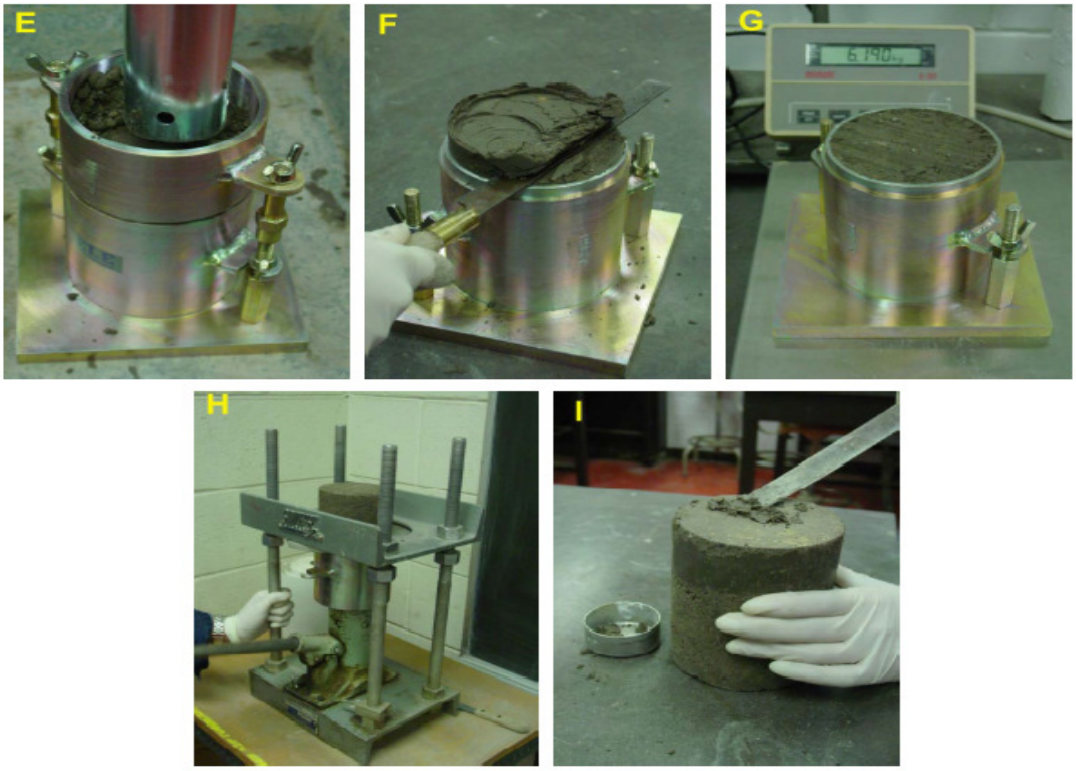
Assume values of water content and find dry density, then plot the zero air void line which must be parallel to the moist side of compaction curve and never intersect it , If so that mean there is some error.

To plot Air content line (A%) use the following equation:

$$\rho_{dry} = \frac{G_s(1-A)}{1+\omega_C G_s}\rho_w$$

The following Figures give the steps used in the test:



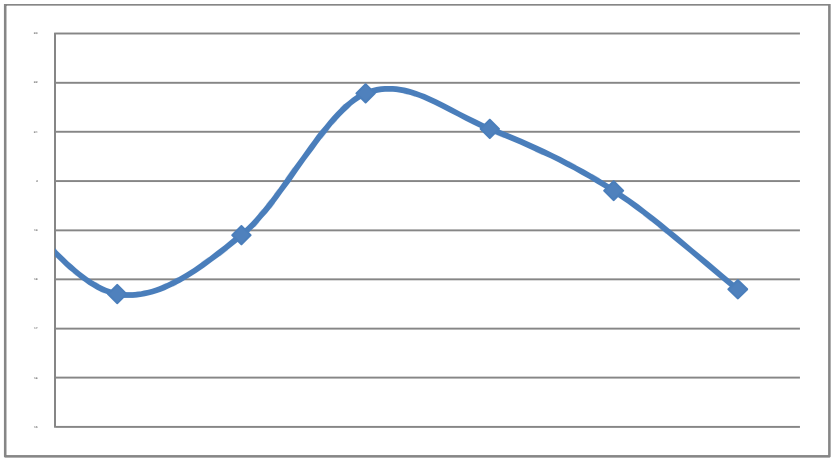


Compaction Equipments:

1- Sheep's foot roller (for cohesive soil)
2- Pneumatic roller (many different soils)
3- Vibratory rollers (mainly for granular material)
4- Grid rollers
5- Power Rammer
6- Vibratory plates.

Compaction Of cohesion less soil:

Moisture content has little or no influence on the granular soils (except when the soil is fully saturated) .



Their state of compaction can be obtained by relating dry density to the minimum and maximum dry densities and as in the following equation:

$$RD = \frac{e_{max} - e_{natural}}{e_{max} - e_{min}} = \left(\frac{\gamma_{dmax}}{\gamma_d}\right)\frac{\gamma_d - \gamma_{min}}{\gamma_{max} - \gamma_{min}}$$

Where RD : Relative density

Compaction in the field:

The results of laboratory tests are not directly applicable to the field compaction, because

1- The Laboratory tests are carried out on material smaller than 20 mm size.
2- Compactive efforts are different and apply in different method.

Relative compaction:

Or Degree of compaction is a means of comparing the field density with Laboratory results and is defined as the ratio of the dry density in the field to the maximum dry density in the Laboratory and in most construction works, the degree of compaction is specified as 95 % or more.

$$\boldsymbol{Relative\ \ Compaction\ R.C = \frac{\gamma_{dry}\, field}{\gamma_{max}\, at\ lab.} \geq 95\%\ or\ as\ specify\ in\ the\ works}$$

So by using sand replacement method, find dry density at field then check the R.C

The Optimum moisture content can be useful in field as follows :

If $\omega_{c\ field} < \omega_{opt}$ then add water and compact the soil

If $\omega_{c\,field} = \omega_{opt}$ then compact the soil directly

If $\omega_{c\,field} > \omega_{opt}$ then either postponed the compaction to other time or add some additive (such as cement or lime) to accelerate evaporation of extra water.

Measurement of field Density

1- **Core cutter**
2- **Sand Replacement method**
3- **Air-Ball on method**
4- **Penetrating Needle**
5- **Radiation Technique.**

Example 1 :

The following results were obtained from a standard compaction test. Determine the Optimum moisture content and maximum dry density. Plot the curves of 0%, 5% and 10% air content and gives the value of air content at the maximum dry density. Given the volume of standard mold is 1000 cm^3 and $G_s = 2.7$.

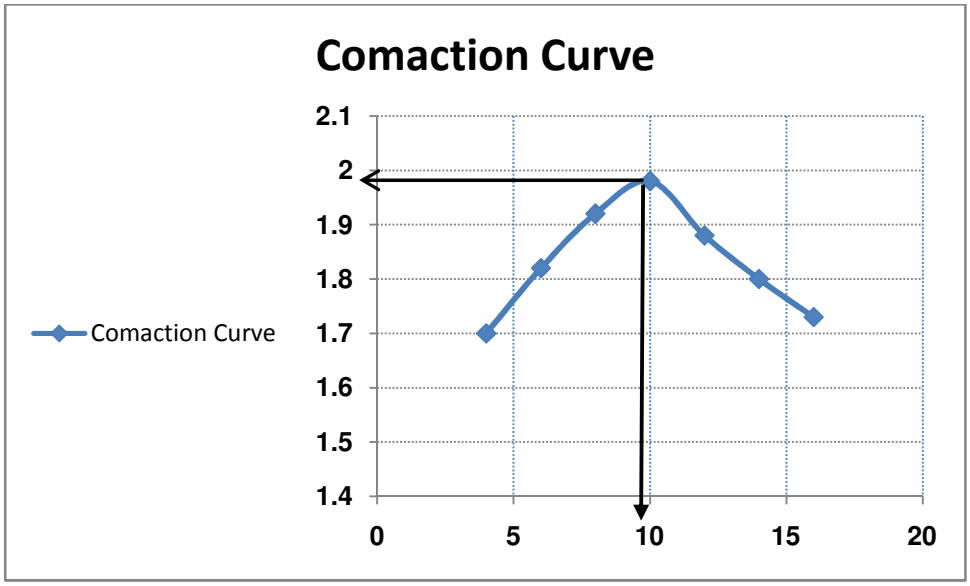
Mass (gm)	1768	1929	2074	2178	2106	2052	2007
Water content (%)	4	6	8	10	12	14	16

Solution :

Calculate dry density for each test and tabulate the results.

$\omega(\%)$	4	6	8	10	12	14	16
ρ_{wet}	1.768	1.929	2.074	2.178	2.106	2.052	2.007
$\rho_{dry}\ gm/cm^3$	1.7	1.82	1.92	1.98	1.88	1.8	1.73

$\omega(\%)$	A %	4	6	8	10	12	14	16
$\rho_{dry}\ gm/cm^3$	0	2.44	2.32	2.22	2.13	2.04	1.96	1.88
$\rho_{dry}\ gm/cm^3$	5	2.32	2.2	2.11	2.02	1.94	1.86	1.79
$\rho_{dry}\ gm/cm^3$	10	2.20	2.09	2.00	1.92	1.84	1.76	1.69



From Figure: The $\gamma_{dry\,max} = 1.98\ kN/m^3$,

and the Optimum Moisture content = 10%.

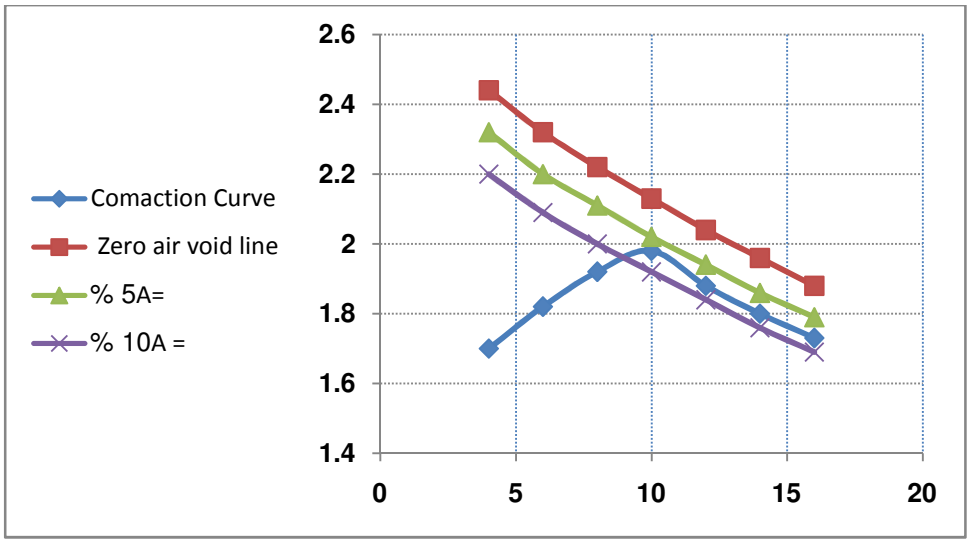


Figure show the Zero air void line and a line of 5 and 10% air content

Chapter Four
Soil Classification

Chapter Four

Soil Classification

Classification of soil is the separation of soil into classes or groups each having similar characteristics and potentially similar behaviour. A classification for engineering purposes should be based mainly on mechanical properties: permeability, stiffness, strength. The class to which a soil belongs can be used in its description.

The aim of a classification system is to establish a set of conditions which will allow useful comparisons to be made between different soils. The system must be simple. The relevant criteria for classifying soils are the ***size distribution*** of particles and the ***plasticity*** of the soil. Particle Size Distribution for measuring the distribution of particle sizes in a soil sample, it is necessary to conduct different **particle-size tests**. **Wet sieving** is carried out for separating fine grains from coarse grains by washing the soil specimen on a 75 micron sieve mesh.

1- **Dry sieve analysis** is carried out on particles coarser than 75 micron. Samples (with fines removed) are dried and shaken through a set of sieves of descending size. The weight retained in each sieve is measured. The cumulative percentage quantities finer than the sieve sizes (passing each given sieve size) are then determined.
The resulting data is presented as a distribution curve with **grain size** along x-axis (log scale) and **percentage passing** along y-axis (arithmetic scale).

U.S. Standard sieve sizes

Sieve no.	Opening (mm)
4	4.750
6	3.350
8	2.360
10	2.000
16	1.180
20	0.850
30	0.600
40	0.425
50	0.300
60	0.250
80	0.180
100	0.150
140	0.106
170	0.088
200	0.075
270	0.053

A set of sieves for a test in the laboratory

Hydrometer (Sedimentation) analysis is based on the principle of sedimentation of soil grains in water. When a soil specimen is dispersed in water, the particles settle at different velocities, depending on their shape, size, and weight. For simplicity, it is assumed that all the soil particles are spheres, and the velocity of soil particles can be expressed by *Stokes' law*, according to which:

$$v = \frac{\rho_s - \rho_w}{18\eta} D^2$$

where
v = velocity
ρ_s = density of soil particles
ρ_w = density of water

η = viscosity of water
D = diameter of soil particles

$$D = \sqrt{\frac{18\eta v}{\rho_s - \rho_w}} = \sqrt{\frac{18\eta}{\rho_s - \rho_w}}\sqrt{\frac{L}{t}}$$

where $v = \dfrac{\text{distance}}{\text{time}} = \dfrac{L}{t}$

Note that

$$\rho_s = G_s \rho_w$$

If the units of η are (g · sec)/cm^2, ρ_w is in g/cm^3, L is in cm, t is in min, and D is in mm, then

$$\frac{D\,(\text{mm})}{10} = \sqrt{\frac{18\eta\,[(\text{g}\cdot\text{sec})/\text{cm}^2]}{(G_s - 1)\rho_w\,(\text{g/cm}^3)}}\sqrt{\frac{L\,(\text{cm})}{t\,(\text{min}) \times 60}}$$

or

$$D = \sqrt{\frac{30\eta}{(G_s - 1)\rho_w}}\sqrt{\frac{L}{t}}$$

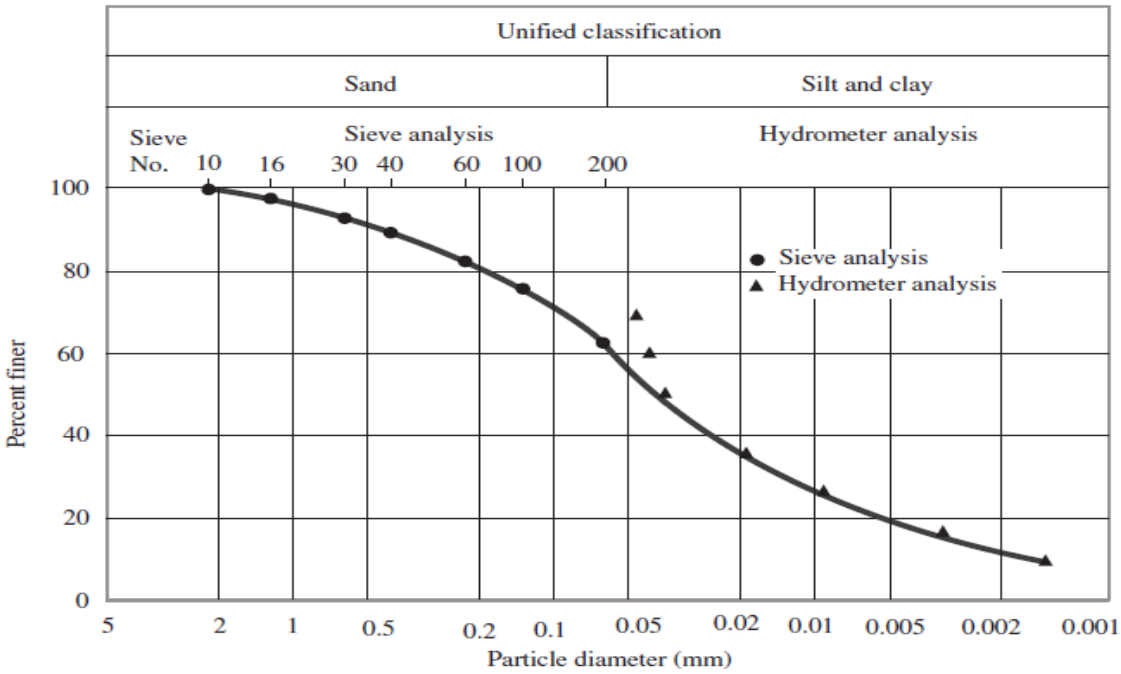
Assuming ρ_w to be approximately equal to 1 g/cm^3, we have

$$D\,(\text{mm}) = K\sqrt{\frac{L\,(\text{cm})}{t\,(\text{min})}}$$

where $K = \sqrt{\dfrac{30\eta}{(G_s - 1)}}$

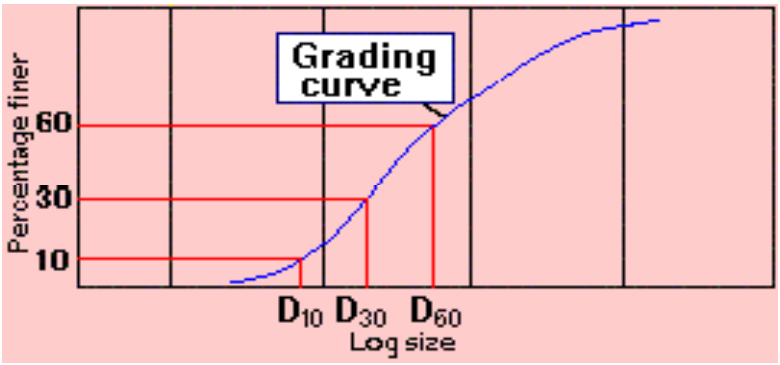
In this method, the soil is placed as a suspension in a jar filled with distilled water to which a deflocculating agent is added. The soil particles are then allowed to settle down. The concentration of particles remaining in the suspension at a particular level can be determined by using a hydrometer. Specific gravity readings of the solution at that same level at different time intervals provide information about the size of particles that have settled down and the mass of soil remaining in solution. The results are then plotted between **% finer (passing)** and **log size**. Grain-Size Distribution Curve The size

The results are then plotted between **% finer (passing)** and **log size**. Grain-Size Distribution Curve the size distribution curves, as obtained from coarse and fine grained portions, can be combined to form one complete **grain-size distribution curve** (also known as **grading curve**). A typical grading curve is shown.



Grain-size distribution curve

From the complete grain-size distribution curve, useful information can be obtained such as: **1. Grading characteristics**, which indicate the uniformity and range in grain-size distribution. **2. Percentages** (or **fractions)** of gravel, sand, silt and clay-size. Grading Characteristics A grading curve is a useful aid to soil description. The geometric properties of a grading curve are called **grading characteristics**.



To obtain the grading characteristics, three points are located first on the grading curve. D60 = size at 60% finer by weight
D30 = size at 30% finer by weight
D10 = size at 10% finer by weight
The grading characteristics are then determined as follows:

1. Effective size = D10

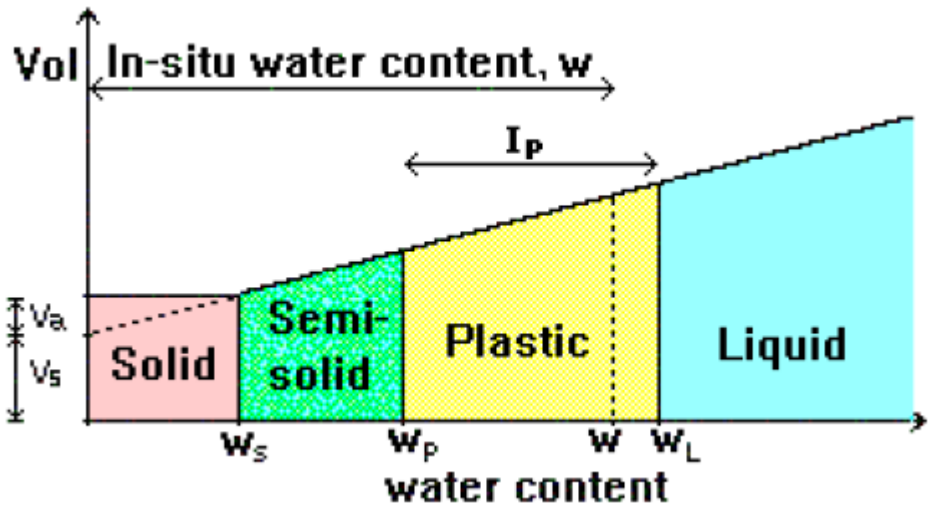
2. Uniformity coefficient, Cu, $C_u = \frac{D_{60}}{D_{10}}$

3. Curvature coefficient, Cc , $C_c = \frac{(D_{30})^2}{D_{60}D_{10}}$

If Cu **> 4 for gravel and** Cu **> 6 for sand and** Cc **between 1 and 3** indicates a well-graded soil (GW for gravel and SW for sand). i.e. a soil which has a distribution of particles over a wide size range

The **consistency** of a fine-grained soil refers to its firmness, and it varies with the water content of the soil.

A gradual increase in water content causes the soil to change from ***solid*** to ***semi-solid*** to ***plastic*** to ***liquid*** states. The water contents at which the consistency changes from one state to the other are called **consistency limits** (or **Atterberg limits**). The three limits are known as the shrinkage limit (**WS**), plastic limit (**WP**), and liquid limit (**WL**) as shown. The values of these limits can be obtained from laboratory tests. (as explained in chapter 3)



Classification Based on Grain Size The range of particle sizes encountered in soils is very large: from boulders with dimension of over 300 mm down to clay particles that are less than 0.002 mm. Some clay contains particles less than 0.001 mm in size which behave as colloids, i.e. do not settle in water.

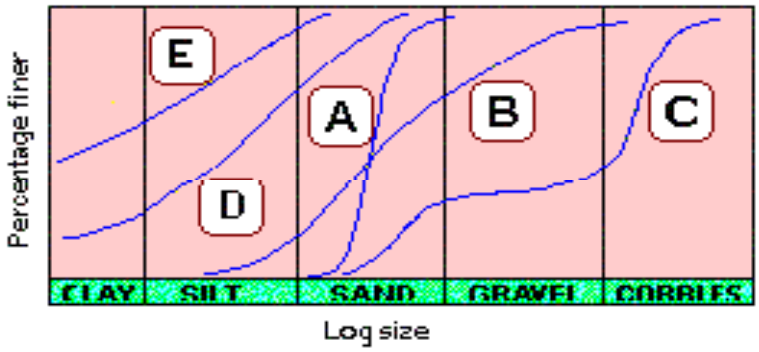
According to grain Size analysis:

Coarse soils	**Gravel size (G)**	***Coarse***	**20 - 80 mm**
		Fine	**4.75 - 20 mm**
	Sand size (S)	***Coarse***	**2 - 4.75 mm**
		Medium	**0.425 - 2 mm**
		Fine	**0.075 - 0.425 mm**
Fine soils	**Silt size (M)**		**0.002 - 0.075 mm**
	Clay size (C)		**< 0.002 mm**

Gravel, sand, silt, and clay are represented by **group symbols G, S, M, and C** respectively. Physical weathering produces very coarse and coarse soils. Chemical weathering produces generally fine soils.

Coarse-grained soils are those for which more than 50% of the soil material by weight has particle sizes greater than 0.075 mm. They are basically divided into either gravels (G) or sands (S). According to **gradation**, they are further grouped as well-graded (**W**) or poorly graded (**P**). If **fine soils** are present, they are grouped as containing silt fines (**M**) or as containing clay fines (**C**). For example, the combined symbol **SW** refers to well-graded sand with no fines. Both the position

and the shape of the grading curve for a soil can aid in establishing its identity and description. Some typical grading curves are shown.

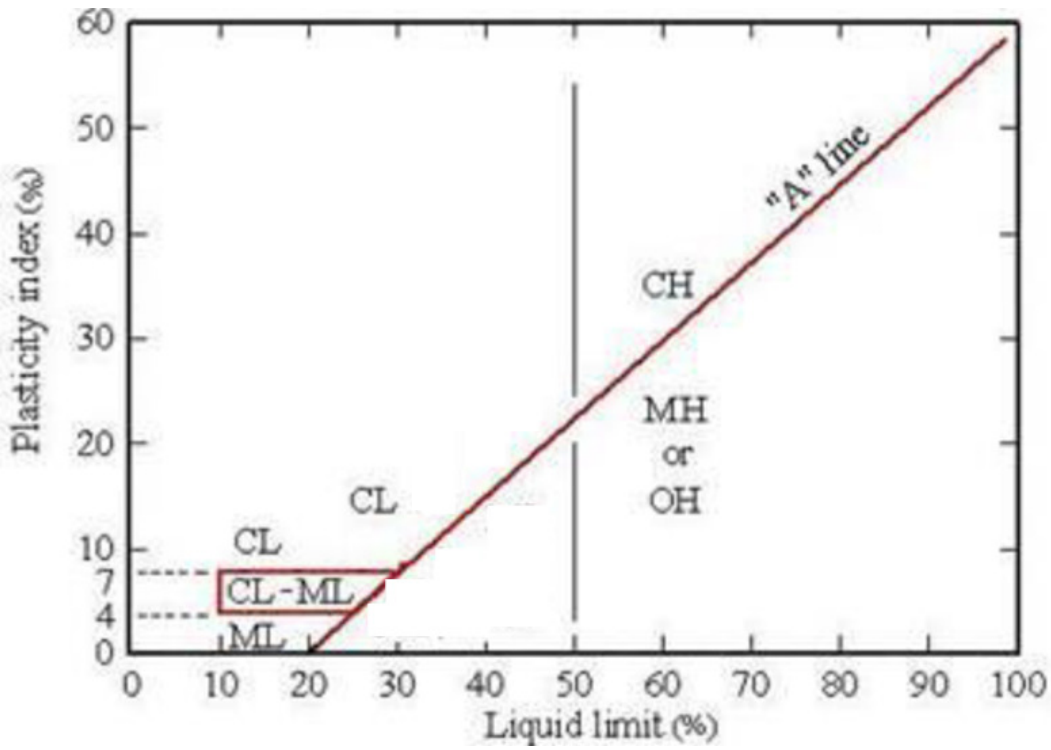


Curve A- poorly-graded SAND
Curve B - a well-graded GRAVEL-SAND (i.e. having equal amounts of gravel and sand)

Fine-grained soils are those for which more than 50% of the material has particle sizes less than 0.075 mm. Clay particles have a **flaky** shape to which water adheres, thus imparting the property of **plasticity**.

A **plasticity chart** , based on the values of liquid limit (**WL**) and plasticity index (**IP**),

The **'A' line** in this chart is expressed as **IP = 0.73 (WL - 20)**.



Depending on the point in the chart, fine soils are divided into **clays (C)**, **silts (M)**, or **organic soils (O)**. The organic content is expressed as a percentage of the mass of organic matter in a given mass of soil to the mass of the dry soil solids.

Soil classification using group symbols is as follows:

Group Symbol	Classification
Coarse soils	
GW	Well-graded GRAVEL
GP	Poorly-graded GRAVEL
GM	Silty GRAVEL
GC	Clayey GRAVEL
SW	Well-graded SAND
SP	Poorly-graded SAND
SM	Silty SAND
SC	Clayey SAND

Fine soils	
ML	SILT of low plasticity
MH	SILT of high plasticity
CL	CLAY of low plasticity
OH	Organic soil of high plasticity
CH	CLAY of high plasticity
OL	Organic soil of low plasticity

Activity "Clayey soils" necessarily do not consist of 100% clay size particles. The proportion of clay mineral flakes (< 0.002 mm size) in a fine soil increases its tendency to swell and shrink with changes in water content. This is called the **activity** of the clayey soil, and it represents the degree of plasticity related to the clay content.

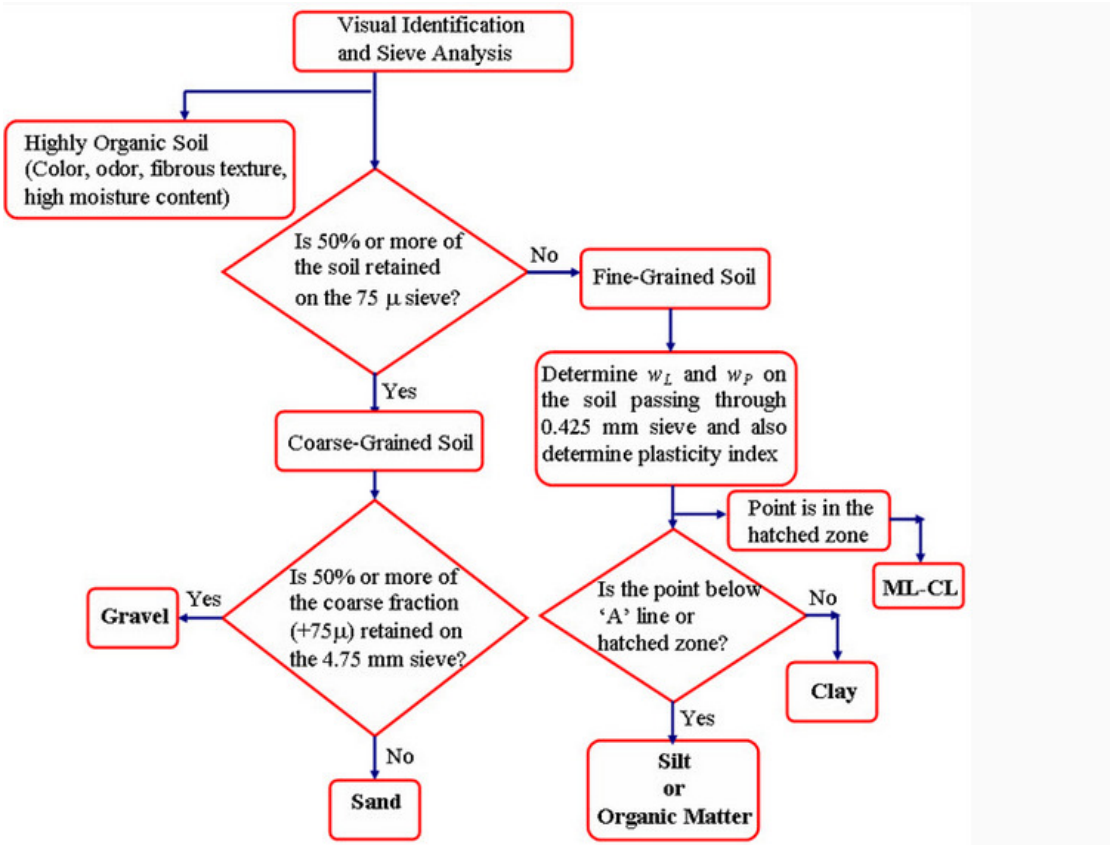
$$A = \frac{PI}{\%\ clay\ fraction\ (weight)}$$

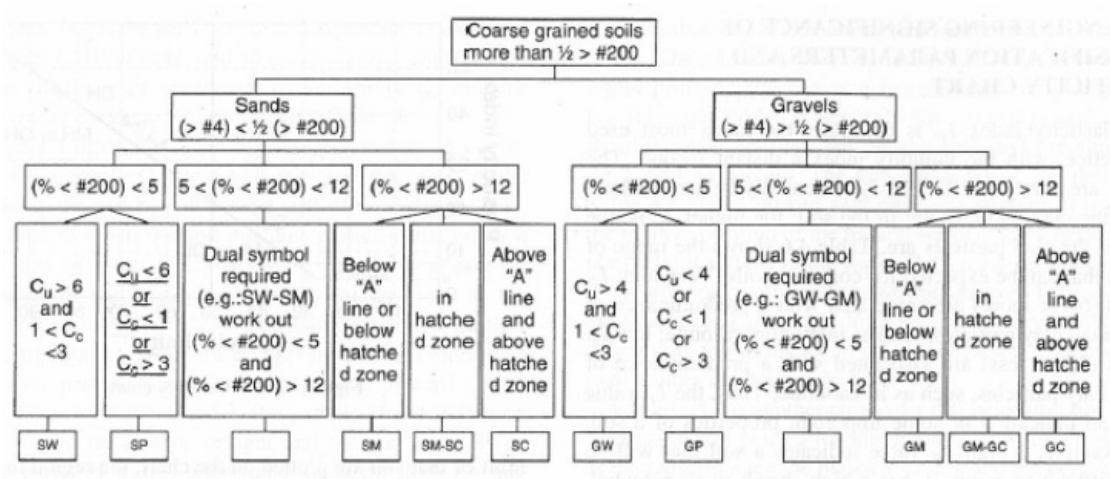
Where PI is plasticity index = $L_L - P_L$

Liquidity Index In fine soils, especially with clay size content, the existing state is dependent on the current water content (**w**) with respect to the consistency limits (or Atterberg limits). The **liquidity index (LI)** provides a quantitative measure of the present

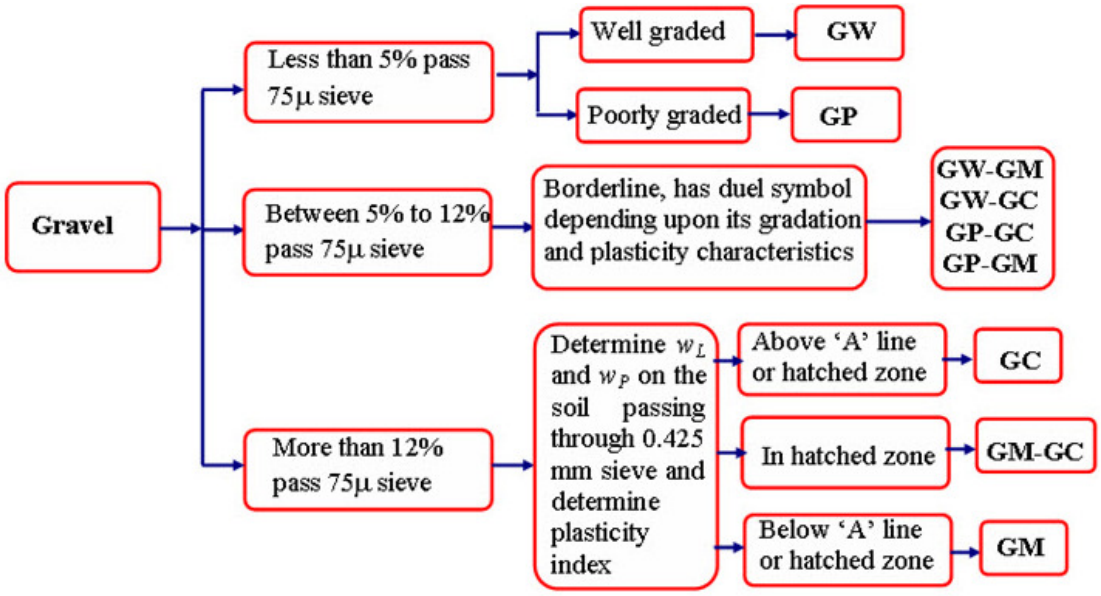
state.

$$\boldsymbol{L_I = \frac{\omega - P_L}{L_L - P_L}}$$

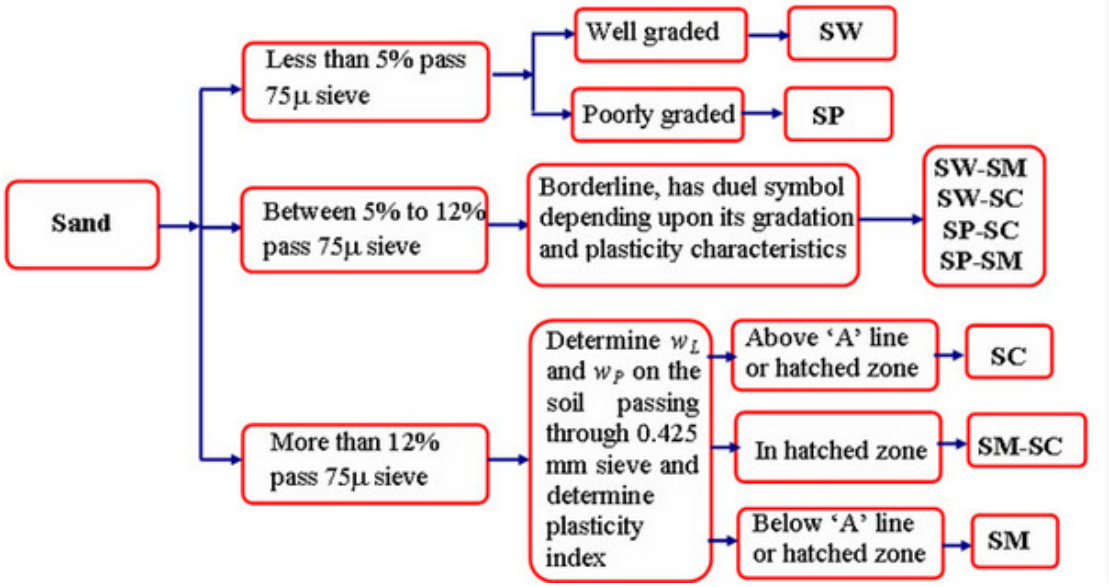

Visual Identification and Sieve Analysis
Highly Organic Soil (Color, odor, fibrous texture, high moisture content)
Is 50% or more of the soil retained on the 75 μ sieve?
No
Fine-Grained Soil
Yes
Coarse-Grained Soil
Determine w_L and w_P on the soil passing through 0.425 mm sieve and also determine plasticity index
Point is in the hatched zone
ML-CL
Is 50% or more of the coarse fraction (+75μ) retained on the 4.75 mm sieve?
Yes
Gravel
No
Sand
Is the point below 'A' line or hatched zone?
No
Clay
Yes
Silt or Organic Matter


Coarse grained soils more than ½ > #200
Sands (> #4) < ½ (> #200)
Gravels (> #4) > ½ (> #200)
(% < #200) < 5
5 < (% < #200) < 12
(% < #200) > 12
(% < #200) < 5
5 < (% < #200) < 12
(% < #200) > 12
Cu > 6 and 1 < Cc <3
Cu < 6 or Cc < 1 or Cc > 3
Dual symbol required (e.g.:SW-SM) work out (% < #200) < 5 and (% < #200) > 12
Below "A" line or below hatched zone
in hatched zone
Above "A" line and above hatched zone
Cu > 4 and 1 < Cc <3
Cu < 4 or Cc < 1 or Cc > 3
Dual symbol required (e.g.: GW-GM) work out (% < #200) < 5 and (% < #200) > 12
Below "A" line or below hatched zone
in hatched zone
Above "A" line and above hatched zone
SW
SP
SM
SM-SC
SC
GW
GP
GM
GM-GC
GC

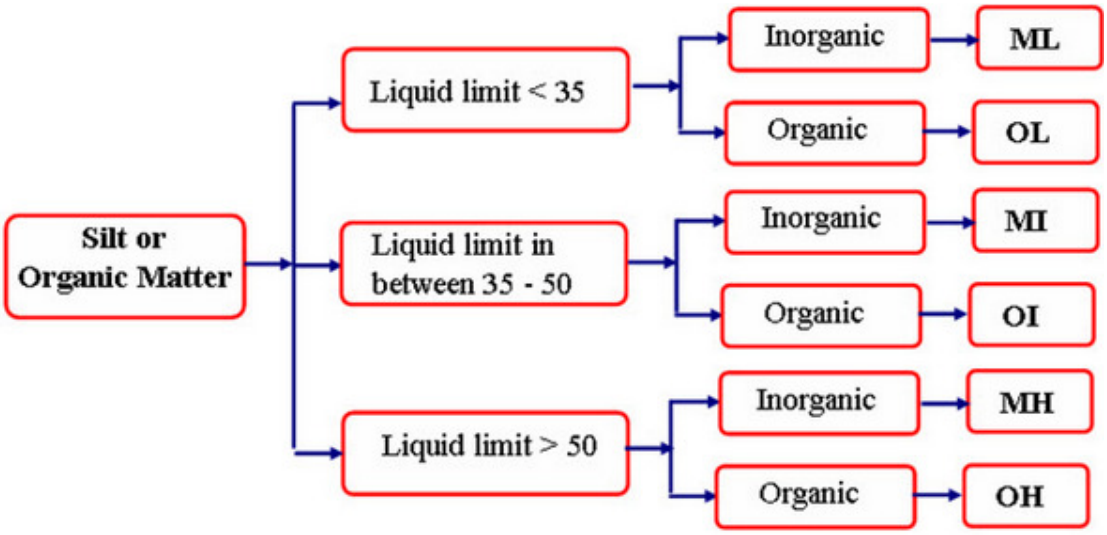
In Details Classification of Gravel:



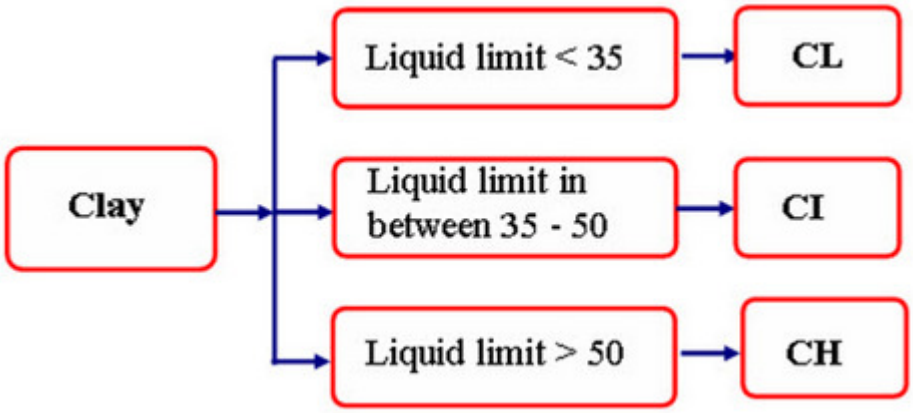
In Details Classification of Sand :



In Details Classification of Silt:



In Details Classification of Clayey Soil:



Unified Soil Classification System (USCS)

Fine-Grained Soil Classification, Plasticity Chart

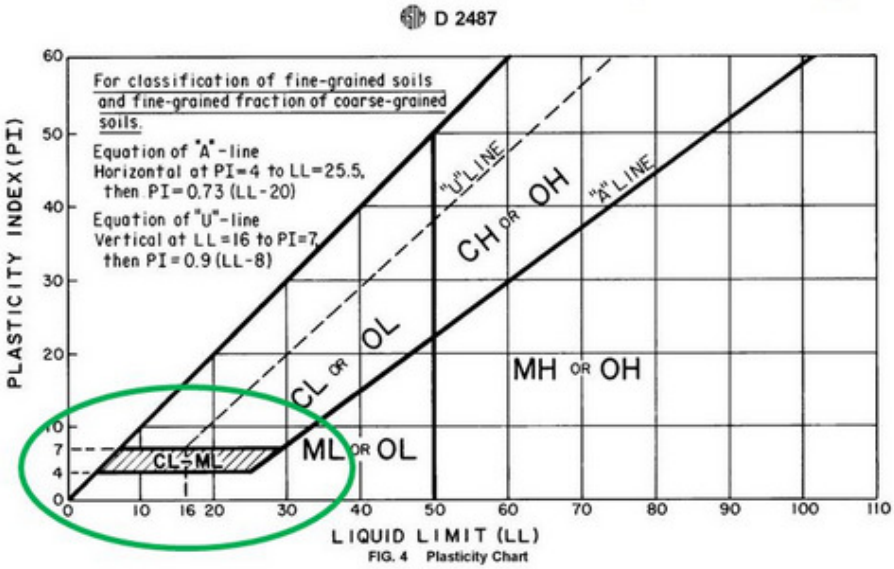


FIG. 4 Plasticity Chart

Example 1:

Following are the results of a sieve analysis. Make the necessary calculations and draw a particle –size distribution curve.

U.S.sieve size	Mass of soil retained on each sieve (g)
4	0
10	40
20	60
40	89
60	140
80	122
100	210
200	56
Pan	12

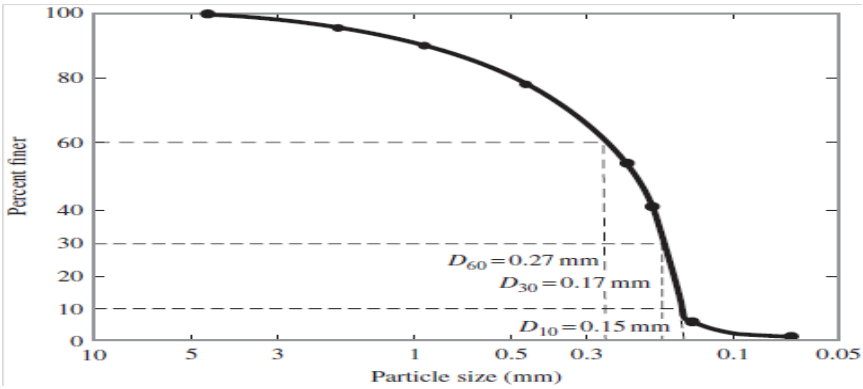
Solution:

Solution

The following table can now be prepared.

U.S. sieve (1)	Opening (mm) (2)	Mass retained on each sieve (g) (3)	Cumulative mass retained above each sieve (g) (4)	Percent finer[a] (5)
4	4.75	0	0	**100**
10	2.00	40	0 + 40 = 40	**94.5**
20	0.850	60	40 + 60 = 100	**86.3**
40	0.425	89	100 + 89 = 189	**74.1**
60	0.250	140	189 + 140 = 329	**54.9**
80	0.180	122	329 + 122 = 451	**38.1**
100	0.150	210	451 + 210 = 661	**9.3**
200	0.075	56	661 + 56 = 717	**1.7**
Pan	—	12	717 + 12 = 729 = ΣM	**0**

Solution:



Find C_U, C_C

$$C_u = \frac{D_{60}}{D_{10}} = \frac{0.27}{0.15} = 1.8$$

$$C_c = \frac{(D_{30})^2}{D_{60}D_{10}} = \frac{(0.17)^2}{0.27*0.15} = 0.71$$

% passing # 200 less than 50% so the soil is coarse , and since % passing # 4= 100 so the soil is sand and since Cu less than 6 , so the soil is SP.

Home work :1- Following are the results of a sieve analysis:

U.S. Sieve No.	Mass of soil retained on each sieve (g)
4	0
10	21.6
20	49.5
40	102.6

60	89.1
100	95.6
200	60.4
pan	31.2

I- Plote the grain –size distribution curve.

II- Calculate the uniformity coefficient ,C_U, and cofficient of gradation , C_C

A)-For a soil given:

$D10 = 0.1$ mm

$D30 = 0.41$ mm

$D60 = 0.62$ mm

B)-for a soil given:

$D10 = 0.082$ mm

$D30 = 0.29$ mm

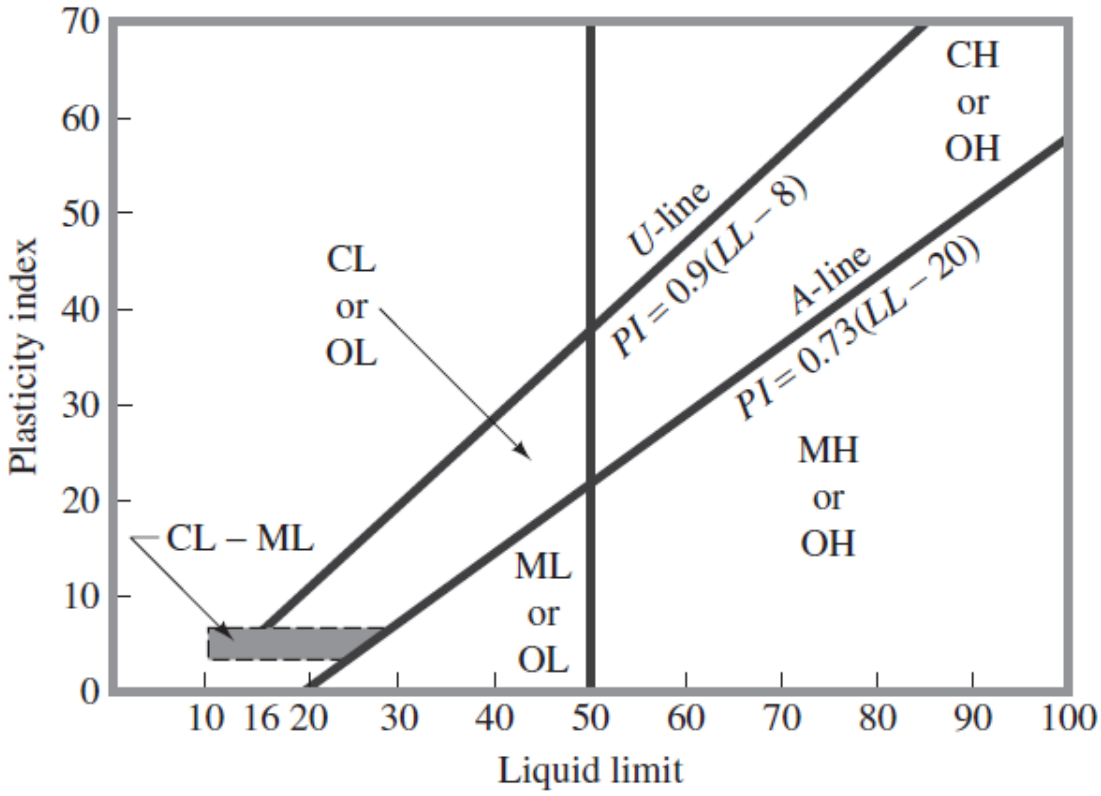
$D60$ 0.51 mm

Home work 1:

2- *Classify the following soil according to USCS*

U.S. Sieve No.	Mass of soil retained on the sieve in g
4	0
6	0
10	0
20	9.1
40	249.4
60	179.8
100	22.7
200	15.5
Pan	23.5

PI: 25, L.L. = 40


70
60
50
40
30
20
10
0
Plasticity index
10 16 20 30 40 50 60 70 80 90 100
Liquid limit
CH
or
OH
U-line
PI = 0.9(LL − 8)
A-line
PI = 0.73(LL − 20)
CL
or
OL
MH
or
OH
CL – ML
ML
or
OL

Chapter 5
Soil Permeability & Flow

Chapter Five

Soil Permeability and Flow

SOIL PERMEABILITY

A material is permeable if it contains continuous voids. All materials such as rocks, concrete, soils etc. are permeable. The flow of water through all of them obeys approximately the same laws. Hence, the difference between the flow of water through rock or concrete is one of degree. The permeability of soils has a decisive effect on the stability of foundations, seepage loss through embankments of reservoirs, drainage of sub grades, excavation of open cuts in water bearing sand, rate of flow of water into wells and many others.

Hydraulic Gradient

When water flows through a saturated soil mass there is certain resistance for the flow because of the presence of solid matter. However, the laws *of fluid mechanics* which are applicable for the flow of fluids through pipes are also applicable to flow of water through soils. As per *Bernoulli's*
equation, the total head at any point in water under steady flow condition may be expressed as

Total head = pressure head + velocity head + elevation head

Hydraulic Gradient

When water flows through a saturated soil mass there is certain resistance for the flow because of the presence of solid matter. The laws of fluid mechanics which are applicable for the flow of fluid through pipes are also applicable to flow of water through soils. The total head at any point in water under steady flow condition may be expressed as:

Total head = pressure head + velocity head + elevation head

The flow of water through a sample of soil of length L and cross-sectional area A as shown in figure1:

$$H_B = Z_B + \frac{P_B}{\gamma_w} + \frac{v_B^2}{2g}$$

$$H_C = Z_C + \frac{P_c}{\gamma_w} + \frac{v_c^2}{2g}$$

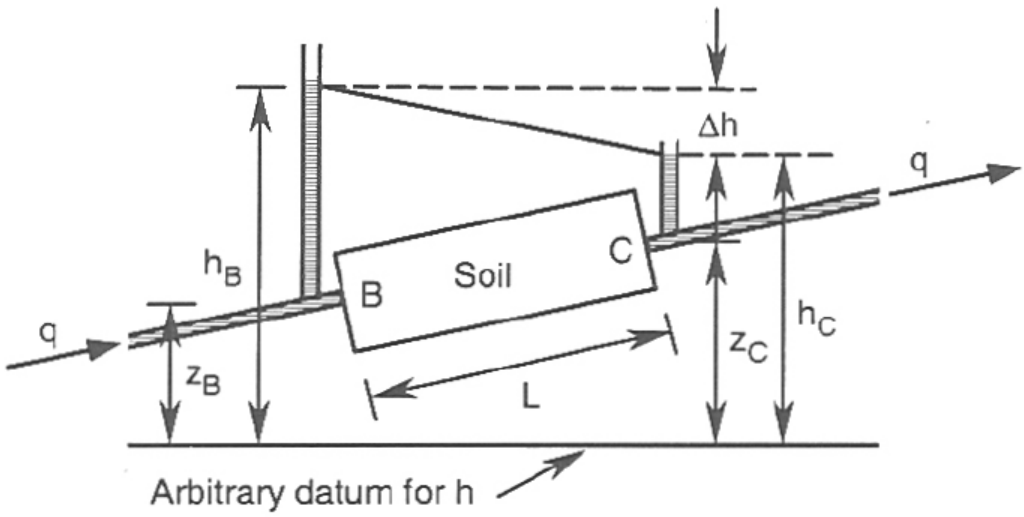


Figure (1) flow of water through a soil sample

For all practical purposes the velocity head is a small quantity and may be neglected.

The water flows from the higher total head to lower total head. So the water will flow from point B to C.

$$H_B - H_C = (Z_B + \frac{P_B}{\gamma_w}) - (Z_C + \frac{P_c}{\gamma_w})$$

Where, Z_B and $Z_C = Elevation\ head,$ P_B and P_C = Pressure Head.

The loss of head per unit length of flow may be expresses as :

$$i = \frac{h}{L}$$

Where i is the hydraulic gradient.

Hydraulic gradient:

The potential drop between two adjacent equipotentials divided by the distance between them is known as the hydraulic gradient.

DARCY'S LAW

Darcy in 1856 derived an empirical formula for the behavior of flow through saturated soils. He found that the quantity of water q per sec flowing through a cross-sectional area of soil under hydraulic gradient / can be expressed by the formula

$q = kiA$

or the velocity of flow can be written as

$$V = \frac{q}{A}$$

Where k is termed the hydraulic conductivity (or coefficient of permeability) with units of velocity. The coefficient of permeability is inversely proportional to the viscosity of water which decreases with increasing temperature; therefore, permeability measurement at laboratory temperatures should be corrected to the values at standard temperature of 20^0C using the following equation.

$$k_{20} = \frac{\mu_T}{\mu_{20}} k_T$$

Where k_{20} : Coefficient of permeability at 20^0 C

k_T : Cofficient of permeability at Lab. Temperture^0 C

μ_T Viscosity of water at lab. Temperature

μ_{20} Viscosity of water at 20^0C

Temperature T(°C)	μ_T/μ_{20}	Temperature T(°C)	μ_T/μ_{20}
10	1.298	21	0.975
11	1.263	22	0.952
12	1.228	23	0.930
13	1.195	24	0.908
14	1.165	25	0.887
15	1.135	26	0.867
16	1.106	27	0.847
17	1.078	28	0.829
18	1.051	29	0.811
19	1.025	30	0.793
20	1.000		

Table (1) :The of $\frac{\mu_T}{\mu_{20}}$ at different temperature.

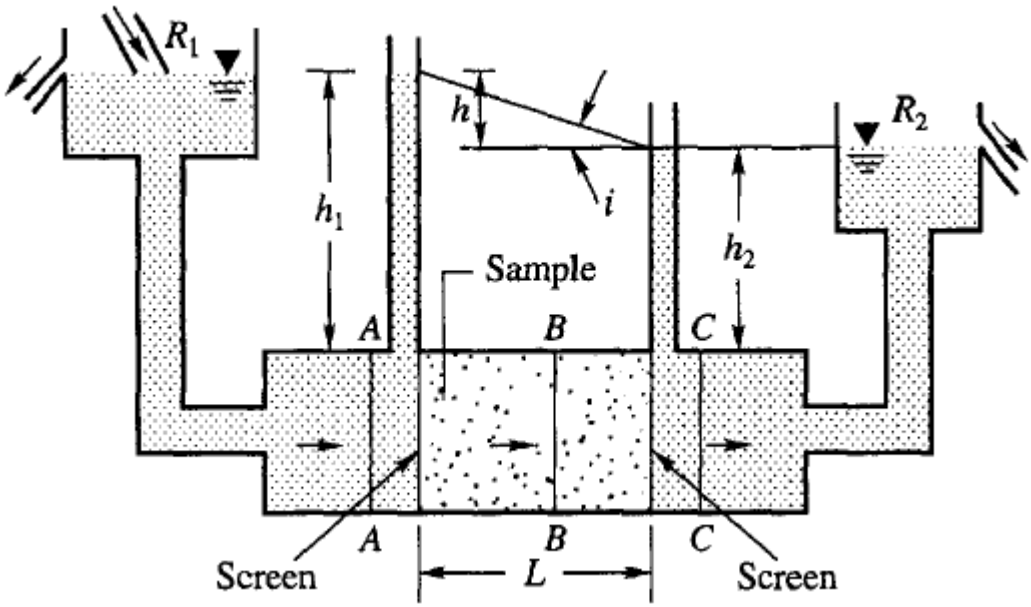
DISCHARGE AND SEEPAGE VELOCITIES:

Figure below shows a soil sample of length *L* and cross-sectional area *A*. The sample is placed in a cylindrical horizontal tube between screens. The tube is connected to two reservoirs *R1 and R2* in which the water levels are maintained constant. The difference in head between *R1* and *R2* is *h*. This difference in head is responsible for the flow of water. Since Darcy's law assumes no change in the volume of voids and the soil is saturated, the quantity of flow past sections *AA, BB* and *CC* should remain the same for steady flow conditions. We may express the equation of continuity as follows

qaa = qbb = qcc

If the soil be represented as divided into solid matter and void space, then the area available for the passage of water is only *Av*. If *vs.* is the velocity of flow in the voids, and v, the average velocity across the section then, we have

$$A_v v_s = Av \quad \text{or} \quad v_s = \frac{A}{A_v} v$$



Where A_v is the area of the void,

v_s is the seepage velocity,

v is the approach velocity

A: is the cross sectional area of the sample

$$v_s = \frac{A * L}{A_v * L} v = \frac{v_t}{v_v} v = \frac{v}{n}$$

Where n : is the porosity of the soil

METHODS OF DETERMINATION OF HYDRAULIC CONDUCTIVITY OF SOILS (Coefficient of permeability).

<u>Coefficient of permeability</u>

Laboratory methods:

1- Constant head permeability method
2- Falling head permeability method
3- Indirect determination from consolidation test

Field methods:

1- Pumping test
2- Bore hole tests

Laboratory methods:

1- Constant head permeability test:

The coefficient of permeability for coarse soils can be determined by means of the constant-head permeability test (Figure below): A steady vertical flow of water, under a constant total head, is maintained through the soil and the volume of water flowing per unit time (q):

A series of tests should be run, each at different rate of flow. Prior to running the test a vacuum is applied to the specimen to ensure that the degree of saturation under flow will be close to 100%.

2- Falling head permeability test:

For fine soils the falling-head test (Figure below) should be used. In the case of fine soils, undisturbed specimens are normally tested. The length of the specimen is l and the cross-sectional area A. the standpipe is filled with water and a measurement is made of the time (t_1) for water level (relative to the water level in the reservoir) to fall from h_0 to h_1. At any intermediate time t the water level in the standpipe is given by h and its rate of change by $-\frac{dh}{dt}$. At time t the difference in total head between the top and bottom of the specimen is h. then applying Darcy's law:

$$-a\frac{dh}{dt} = AK\frac{h}{l}$$

$$-a\int_{h_0}^{h_1}\frac{dh}{h} = \frac{AK}{l}\int_0^h dt$$

$$\therefore K = \frac{al}{At_1}ln\frac{h_0}{h_1} = 2.3\,\frac{al}{At_1}\log\frac{h_0}{h_1}$$

Ensure that the degree of saturation remains close to 100%. A series of tests should be run using different values of h_0 and h_1

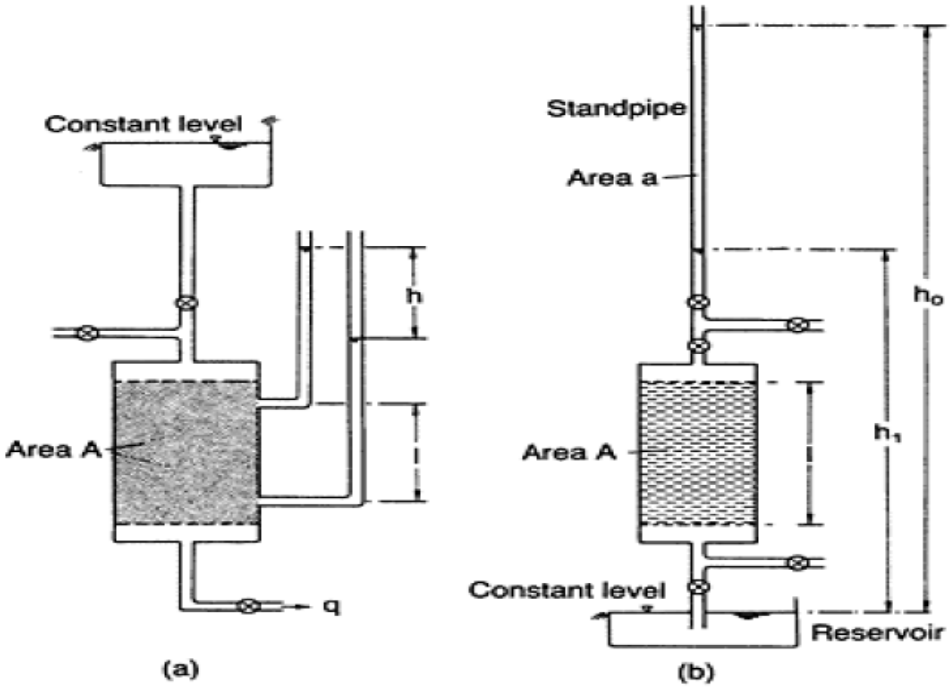


Figure: Laboratory Test (a) Constant Head (b) Falling Head

Example 1:

A constant head permeability test was carried out on a cylindrical of sand 4 in. in diameter and 6 in. in height . 10 in^3

Of water was collected in 1.75 min, under a head of 12 in. Compute the hydraulic conductivity in ft/year and the velocity of flow in ft/sec.

Solution:

$$k = \frac{Q}{Ai\ t}$$

$$Q = 10\ in^3\ , A = 3.14 * \frac{4^2}{4} = 12.56\ in^2$$

$$i = \frac{h}{L} = \frac{12}{6} = 2, t = 105\ sec$$

Therefore $k = \frac{10}{12.56*2*105} = 3.79 * \frac{10^{-3} in}{sec} = 31.58 * 10^{-5}\ ft/sec$

Velocity of flow $= ki = 31.58 * 10^{-5} * 2 = 6.316 * 10^{-4}\ ft/sec$

Example 2

A sand sample of 35 cm^2 cross sectional area and 20 cm long was tested in a constant head permeameter. Under a head of 60 cm, the discharge was 120 ml in 6 min. The dry weight of sand used for the test was 1120 g. and $G_s = 2.68$. Determine (a) the hydraulic conductivity in cm/sec. (b) the discharge velocity, and (c) the seepage velocity.

Solution:

$$k = \frac{QL}{\Delta hAt}$$

Q = 120 ml, t =6 min, $A = 35\ cm^2, L = 20\ cm, and\ h = 60\ cm$

$$k = \frac{120*20}{60*35*6*60} = 3.174 * 10^{-3} cm/sec$$

Discharge velocity, v= ki $= 31.74 * 10^{-3} * \frac{60}{20} = 9.52 * 10^{-3} cm/sec$

Seepage velocity v_s

$$\gamma_d = \frac{w_s}{v} = \frac{1120}{35 * 20} = 1.6\ gm/cm^3$$

$$\gamma_d = \frac{\gamma_w G_s}{1 + e}\ or \quad e = \frac{G_s}{\gamma_d} - 1$$

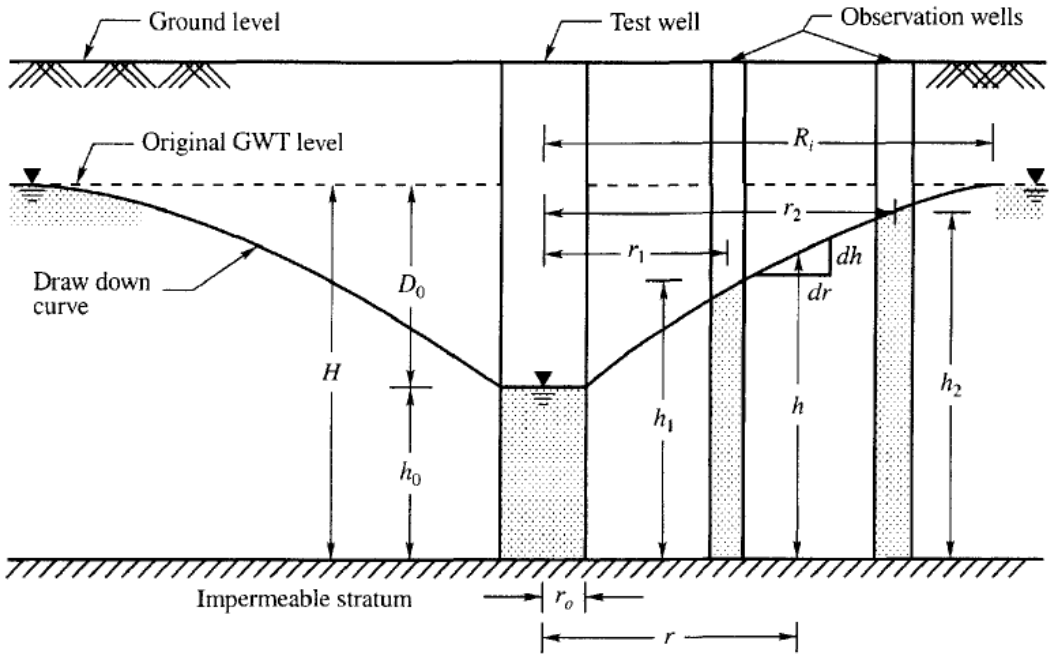
$$e = \frac{2.68}{1.6} - 1 = 0.675$$

$$n = \frac{e}{1+e} = 0.403$$

$$v_s = \frac{v}{n} = \frac{9.52 * 10^{-3}}{0.403} = 2.36 * 10^{-2} \quad cm/sec$$

DIRECT DETERMINATION OF K OF SOILS IN FIELD:

1- Field test in unconfined aquifer



Pumping Test in an unconfined aquifer

$$k = \frac{2.3q}{\pi(h_2^2 - h_1^2)} \log \frac{r_2}{r_1}$$

2- Field test in Confined aquifer

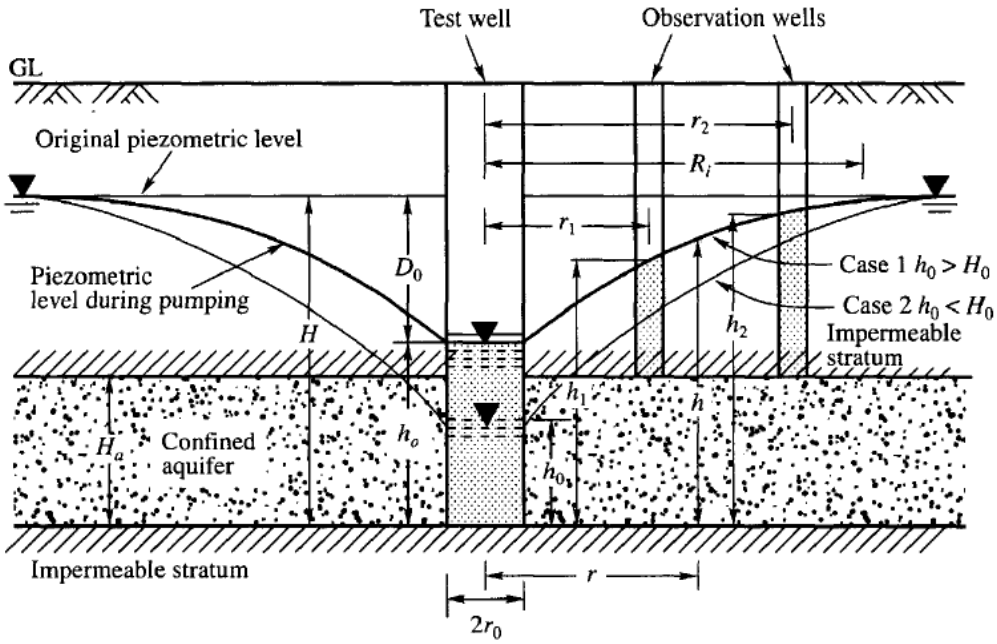
There are two cases :

Case 1- when $h_0 > H_0$

$$k = \frac{2.3q}{2\pi H_0 (h_2 - h_1)} \log \frac{r_2}{r_1}$$

Case 2-when $h_0 < H_0$

$$k = \frac{2.3q}{\pi(2HH_0 - H_0^2 - h_0^2)} \log\frac{R_i}{r_0}$$



Empirical Relations for Hydraulic Conductivity

Several empirical equations for estimating hydraulic conductivity have been proposed over the years.

Granular Soil

For fairly uniform sand (that is, a small uniformity coefficient), Hazen (1930) proposed an empirical relationship for hydraulic conductivity in the form:

$$k(cm/sec) = CD_{10}^2$$

C= a constant that varies from 1.0 to 1.5

$D10$ = the effective size (mm)

The above equation is based primarily on Hazen's observations of loose, clean, filter sands. A small quantity of silts and clays, when present in a sandy soil, may change the hydraulic conductivity substantially.

The accuracy of the values of k determined in the laboratory depends on the folloeing factors:

1- Temperture of the fluid
2- Viscosity of fluid

3- Trapped air bubbles present in the specimen
4- Degree of saturation
5- Migration of fines during testing
6- Duplication of field conditions in the laboratory.

The coefficient of consolidation of saturated cohesive soils can be determined by laboratory consolidation tests. This will be listed in details in "consolidation of soil".

Table -1 Typical Values of Hydraulic Conductivity of Saturated Soils

Typical values of permeability	
Gravel	$> 10^{-1}$ m/s
Sands	10^{-1} to 10^{-5} m/s
Fine sands, coarse silts	10^{-5} to 10^{-7} m/s
Silts	10^{-7} to 10^{-9} m/s
Clays	$< 10^{-9}$ m/s

Exercise:

1-

In a constant-head permeability test in the laboratory, the following are given: $L = 300$ mm and $A = 110$ cm^2. If the value of $k = 0.02$ cm/sec and a flow rate of 140 cm^3/min must be maintained through the soil, what is the head difference, h, across the specimen? Also, determine the discharge velocity under the test conditions.

2-

For a variable-head permeability test, the following are given:

- Length of the soil specimen = 20 in.
- Area of the soil specimen = 2.5 in.^{2}
- Area of the standpipe = 0.15 in.^{2}
- Head difference at time $t = 0$ is 30 in.
- Head difference at time $t = 8$ min is 16 in.

a. Determine the hydraulic conductivity of the soil (in./min)
b. What is the head difference at time $t = 6$ min?

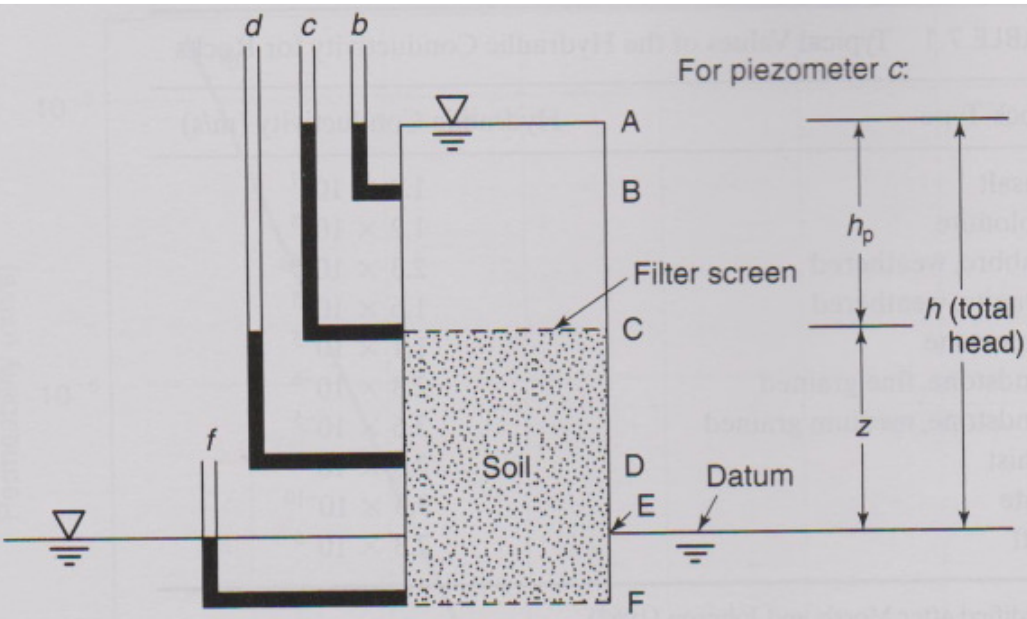
3-

The hydraulic conductivity k of a soil is 10^{-6} cm/sec at a temperature of 28° C. Determine its absolute permeability at 20° C given that, at 20° C, $\gamma_w = 9.789$ kN/m^3 and $\eta = 1.005 \times 10^{-3}$ N.s/m^2 (Newton second per meter squared).

HEADS AND ONE-DIMENSIONAL FLOW

There are three heads which must be considered in problem involving fluid flow in soil (Figure 4):

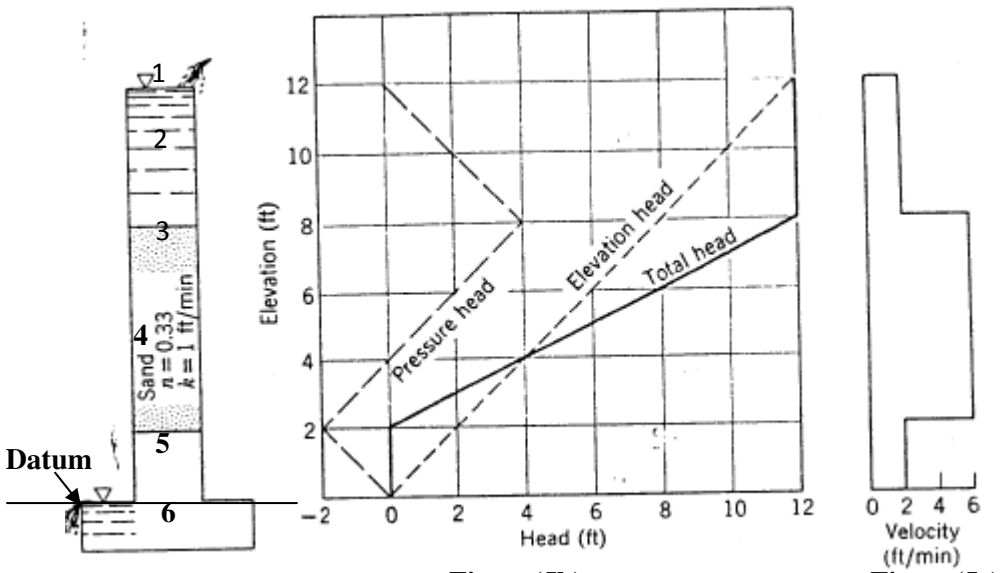
1- Pressure head (h_p) : is the pizometer reading = pore water pressure /unit weight of water
2- Elevation head at any point (h_e): is the vertical distance above or below some reference elevation or datum plane.
3- Total head, $h = h_p + h_e$



Point	Pressure Head	Elevation Head	Total Head	Head Loss through Soil
B	AB	BE	AE	0
C	AC	CE	AE	0
D	CD	DE	CE	1/2AE
F	EF	−EF	0	AE

Figure (4) illustration of types of head(after Taylor, 1948).

Example 1: For the Setup shown (Figure 5a), plot, ht, he, hp and the velocity of flow?



Figure(5a) **Figure(5b)** **Figure(5c)**

Points	ht(ft) (Figure 5b)	he(ft) (Figure 5b)	hp=ht-he(ft) (Figure 5b)	V(ft/min)* =Ki (Figure5c)
1	12	12	0	2
2	12	10	2	2
3	12	8	4	6
4	$=\frac{12+0}{2} = 6$	5	1	6
5	0	2	-2	6
6	0	0	0	2

$$^{*}i = \frac{h_{t\,at\,3} - h_{t\,at\,5}}{L_{3-5}} = \frac{12-0}{6} = 2$$

Approch velocity = ki= 1*2= 2 ft/min

Seepage velocity $=\frac{v}{n} = \frac{2}{0.333} = 6\ ft/min$

Example 2

For the setup shown(Figure 6a) , Draw, ht, he , hp and velocity of flow ?

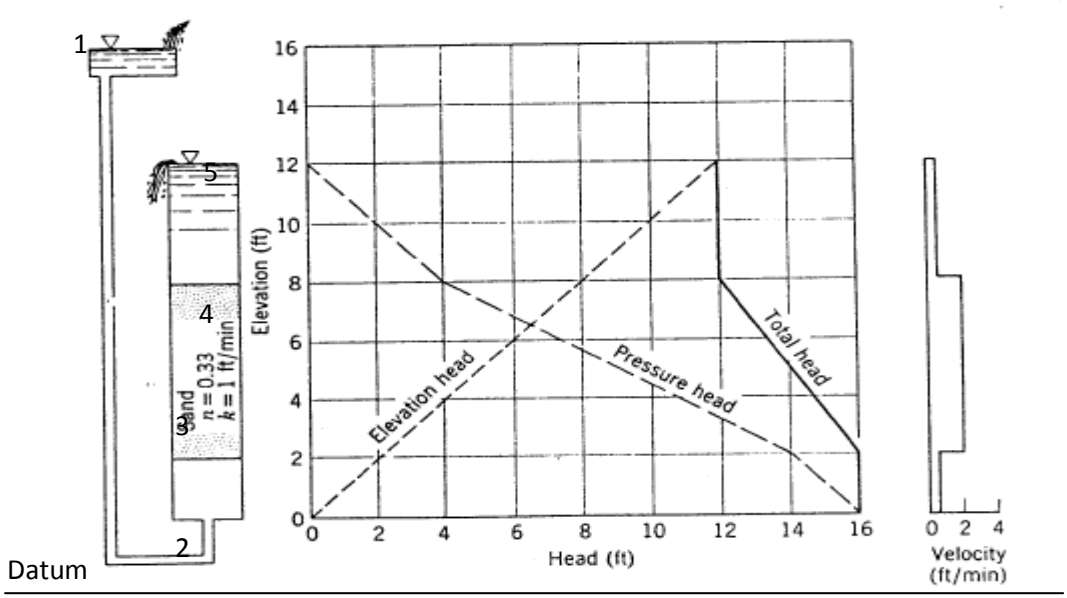


Figure (6)

1- Direction of flow is upward flow (look to the water'symbole usaually water flow from higher one to lower one)
2- List all point with direction of flow
3- Construct a table to solve the problem

Points	ht(ft) (Figure 6b)	he(ft) (Figure 6b)	hp=ht-he(ft) (Figure 6b)	V(ft/min)*=Ki (Figure6c)
1	16	16	0	
2	16	2	14	
3	$\frac{16+12}{2} = 14$	5	9	
4	12	8	4	
5	12	12	0	

$$^{*}i = \frac{h_{t\,at\,2} - h_{t\,at\,4}}{L_{2-4}} = \frac{16-12}{6} = 0.667$$

Approch velocity = ki= 1*0.667= 0.667 ft/min

Seepage velocity $= \frac{v}{n} = \frac{0.667}{0.333} = 2\ ft/min$

Example 3: For the setup shown(Figure below) , Draw, ht, he , hp and velocity of flow ?

Figure (7a)

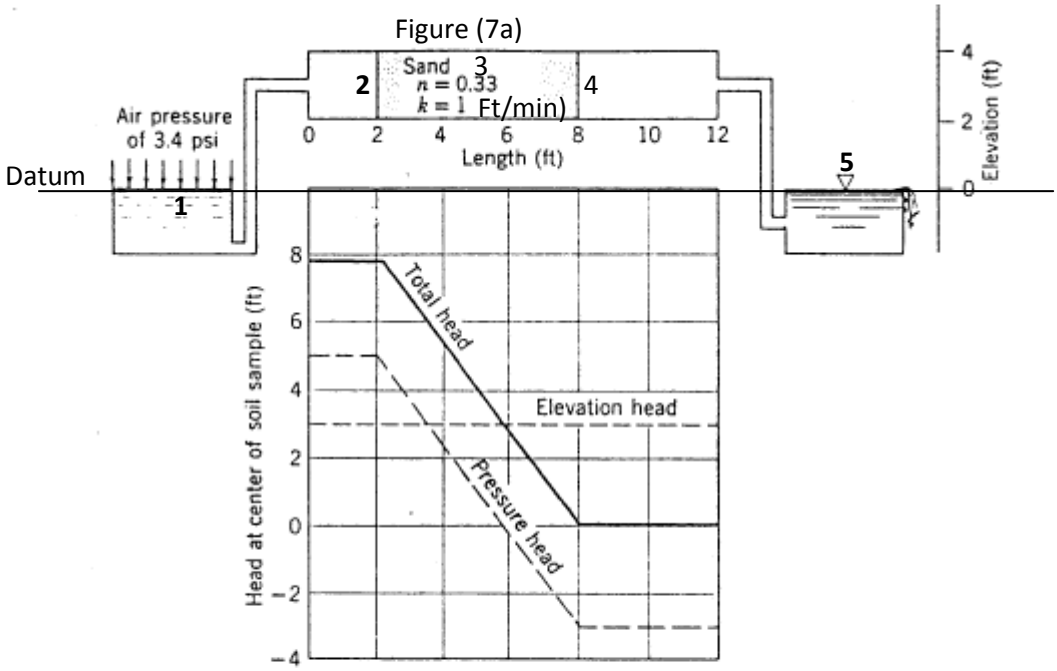


Figure heads vs horizontal distance

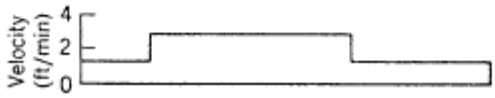


Figure (7c)

Solution:

1- Assume any arbitrary line representing the datum and let it at elevation =0(Figure 7a).
2- The flow will be in horizontal direction (elevation head is constant)
3- Construct the table

Since pressure =3.4 psi=3.4*144= 489.6 lb/ft^2

$$hp = \frac{pressure}{unit\ weight\ of\ water} = \frac{489.6}{62.4} = 7.84\ ft$$

Points	ht(ft)	he(ft)	hp(ft)
1	7.84	0	8
2	7.84	3	5.84
3	3.92	3	0.92
4	0	3	-3
5	0	0	0

$$i = \frac{h_{t\ at\ 2} - h_{t\ at\ 4}}{L_{2-4}} = \frac{7.84 - 0}{6} = 1.3$$

Approch velocity = ki= 1*1.3= 1.3 ft/min

Seepage velocity $= \frac{v}{n} = \frac{1.3}{0.333} = 3.9\ ft/sec$

Example 4

For the setup shown in figure 8:a) - Calculate the pressure head, elevation head, total head and head loss at points B, C,D and F in centimeter of water. b)-Plot the heads versus the elevation.

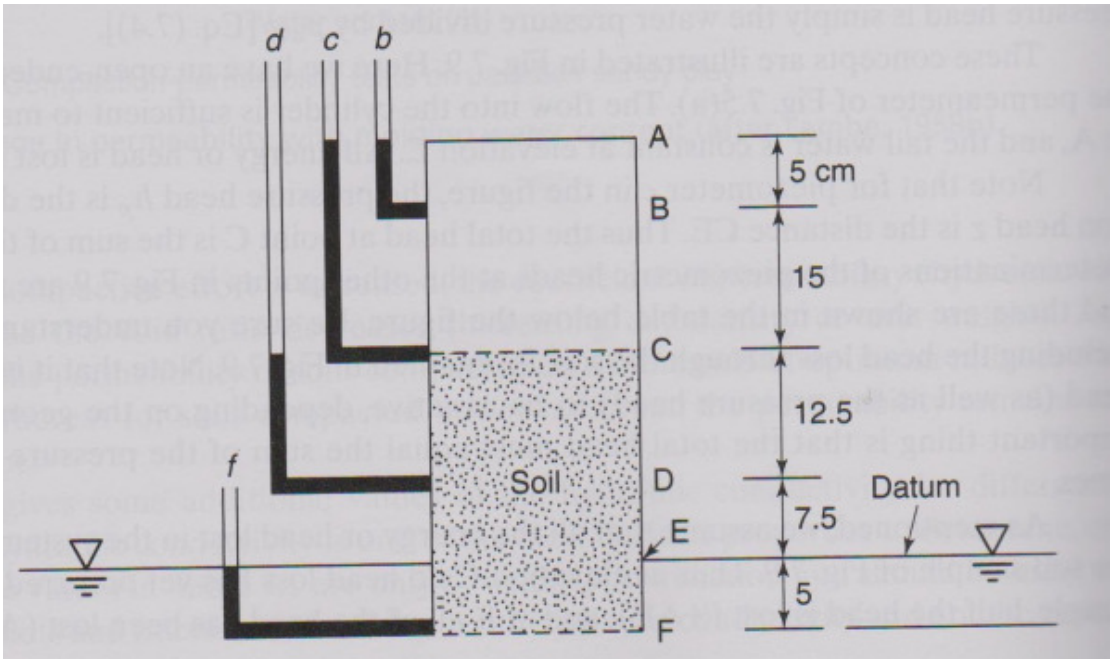


Figure shows the Set up of example 4

Solution (1):

points	ht(cm)	he(cm)	hp(cm)	Head loss
B	40	35	5	0
C	40	20	20	0
D	20	7.5	12.5	20
F	0	-5	5	40

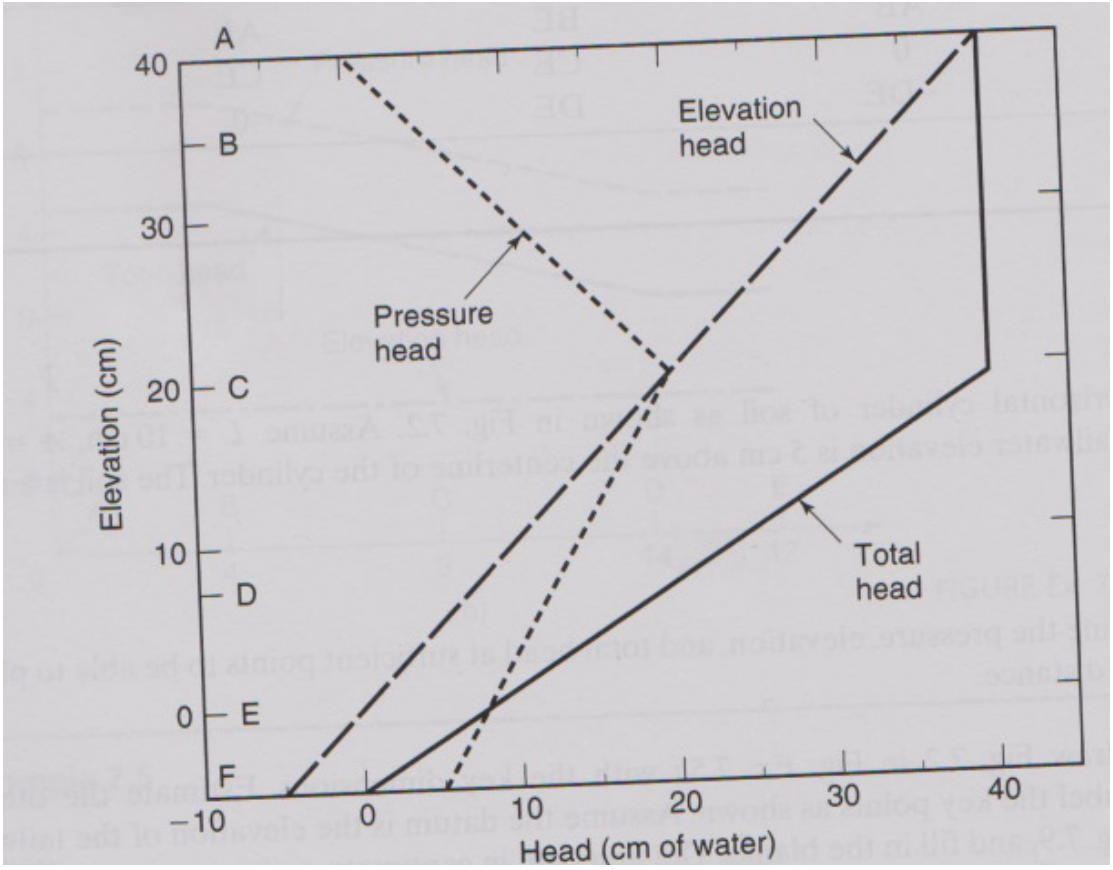
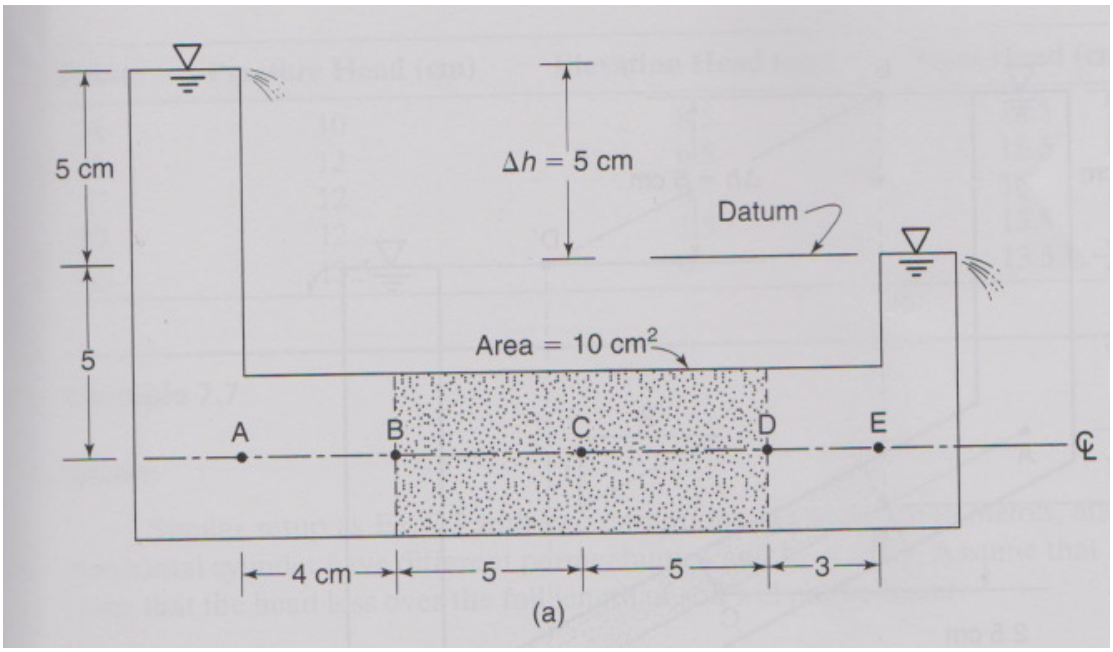


Figure (8)Example 4

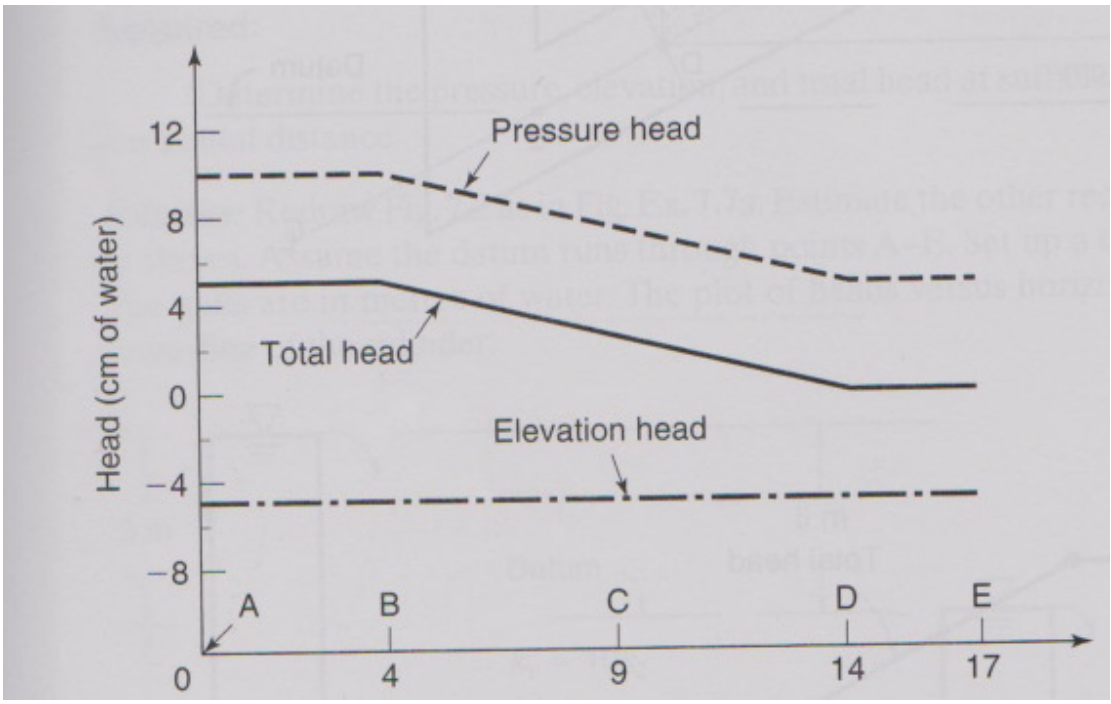
Example (5): for the setup shown Calculate and plot total head, elevation head and pressure head.



Example 5

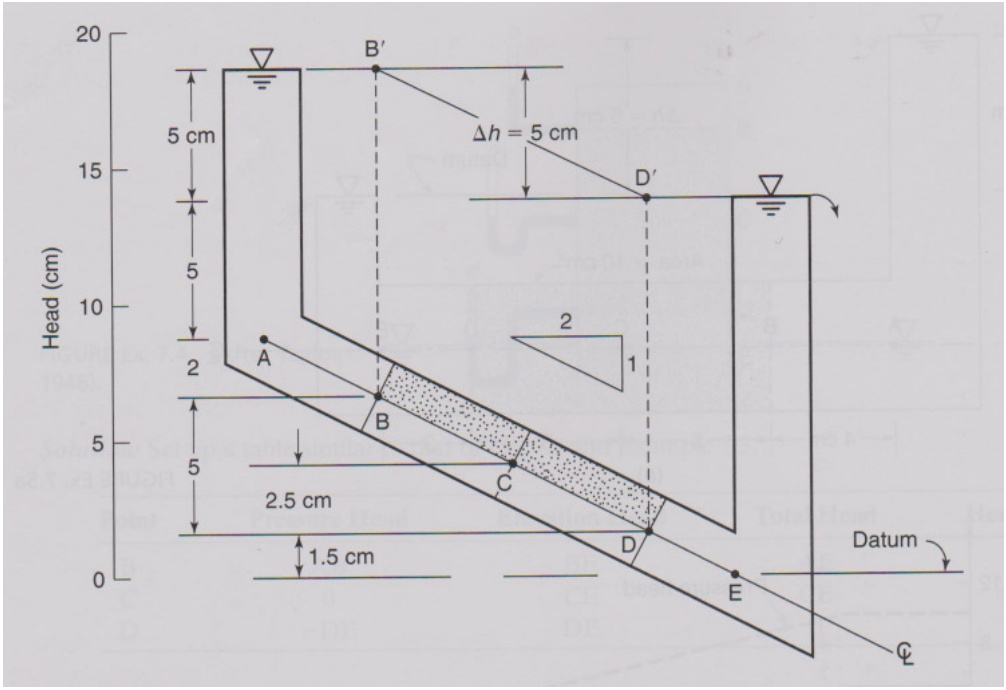
Solution example 5:

Points	Ht(cm)	He(cm)	Hp(cm)	Head loss
A	5	-5	10	0
B	5	-5	10	0
C	2.5	-5	7.5	2.5
D	0	-5	5	5

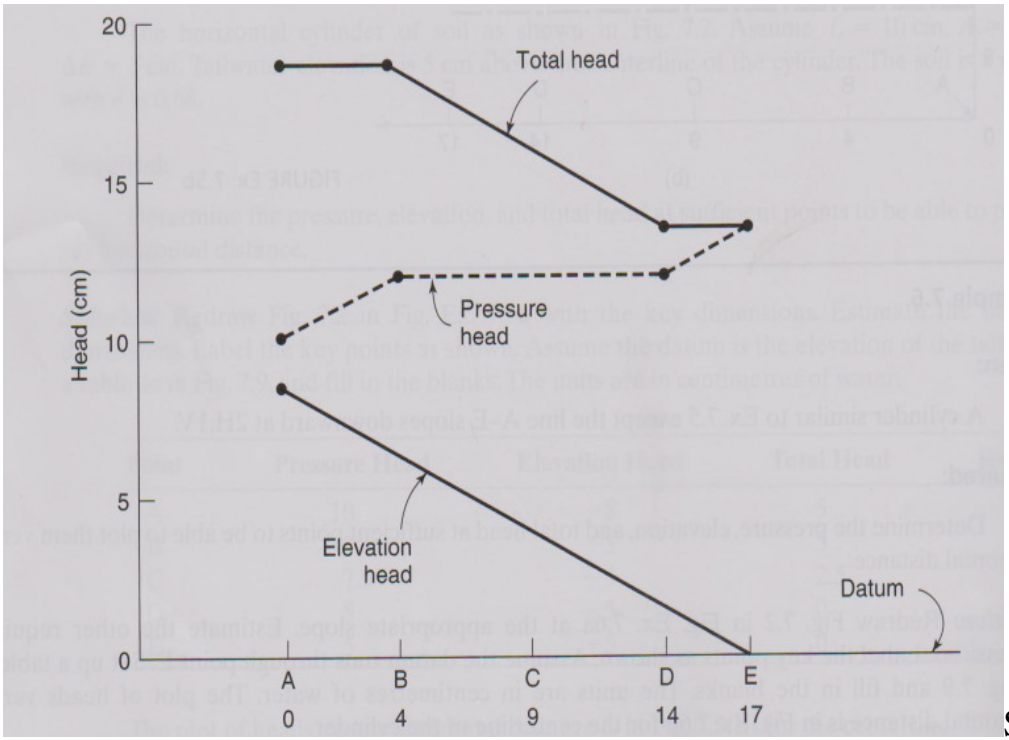


Solution Example 5

Example -6 : For the set up shown , draw the variation of total head, pressure head and elevation head along points A,B,C,D and E.



Example 6

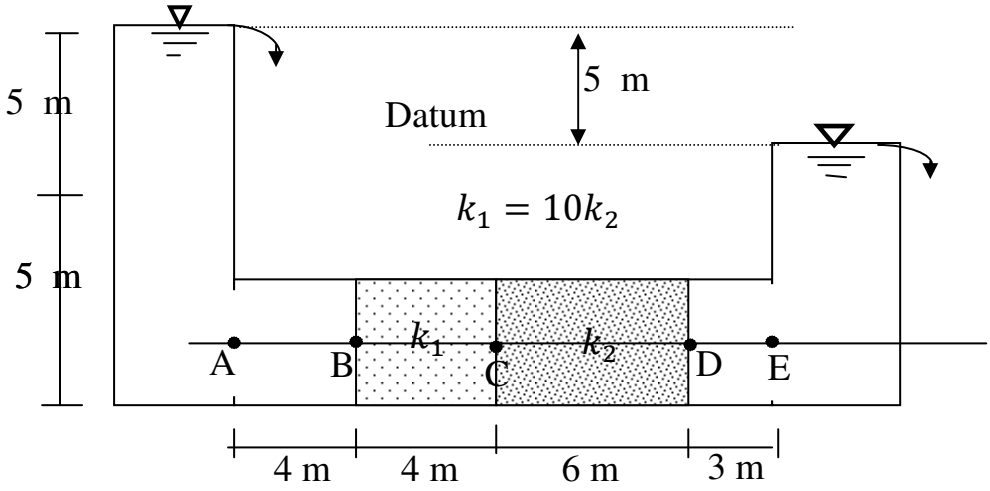


Solution of Example 6

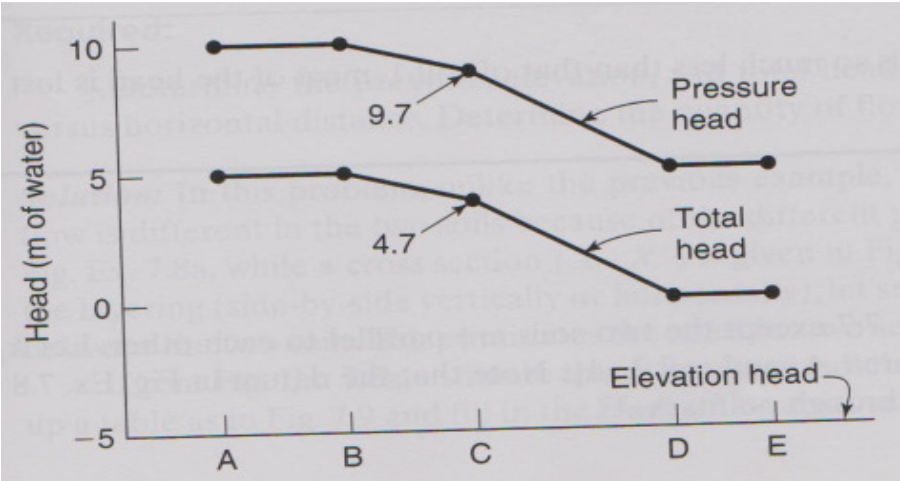
Solution of Example 6

point	Total Head(cm)	Elevation Head(cm)	Pressure Head(cm)	Head Loss (cm)
A	18.5	8.5	10	0
B	18.5	6.5	12	0
C	16	4	12	2.5
D	13.5	1.5	12	5
E	13.5	0	13.5	5

Example 7: For the setup shown, Find total head (ht) , Elevation head (he) and Pressure head(hp) for the soil the setup shown.



Example 7 setup



series. Thus the quantity of flow in one soil has to be the same as in the second soil. So,

$$q_1 = k_1 i_1 A_1 = q_2 = k_2 i_2 A_2$$

Since the areas are the same, $q_{1,2} = k_1 i_1 = k_2 i_2$ with $k_1 = 10k_2$ and $i = \Delta h/l$. Substituting,

$$q_{1,2} = 10k_2 \frac{\Delta h_1}{L_1} = k_2 \frac{\Delta h_2}{L_2}$$

Also, the total head loss, $\Delta h = \Delta h_1 + \Delta h_2$. So, $\Delta h_1 = \Delta h - \Delta h_2$, and we obtain

$$q_{1,2} = 10k_2 \frac{(\Delta h - \Delta h_2)}{L_1} = k_2 \frac{\Delta h_2}{L_2}$$

Rearranging and multiplying out,

$$L_2 10k_2 \Delta h - L_2 10k_2 \Delta h_2 = k_2 \Delta h_2 L_1$$

Rearranging and canceling out the k_2's,

$$10L_2 \Delta h = \Delta h_2 (L_1 + 10L_2)$$

Solving for Δh_2,

$$\Delta h_2 = \frac{10L_2 \Delta h}{L_1 + 10L_2}$$

$$= \frac{10 \times 6\text{ m} \times 5\text{ m}}{(4\text{ m} + 10 \times 6\text{ m})} = \frac{300\text{ m}^2}{64\text{ m}}$$

$$= 4.69\text{ m}$$

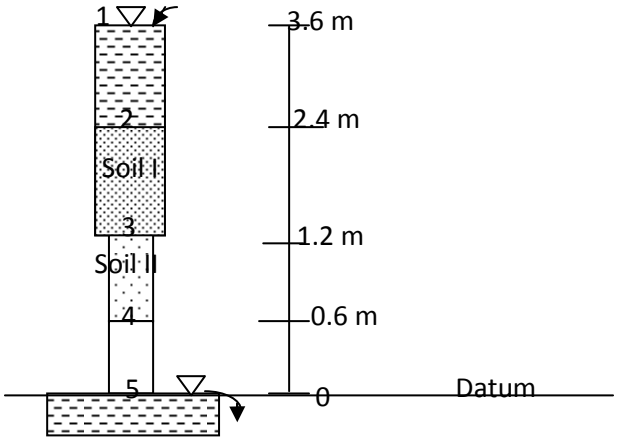
$$= 4.69\text{ m}$$

$$\therefore \Delta h_1 = \Delta h - \Delta h_2 = 5 - 4.69 = 0.31\text{ m}$$

Point	Pressure Head (m)	Elevation Head (m)	Total Head (m)	Head Loss (m)
A	10	−5	5	0
B	10	−5	5	0
C	9.7	−5	4.7	0.31
D	5	−5	0	5
E	5	−5	0	5

Because the permeability of soil 2 is so much less than that of soil 1, most of the head is lost in soil 2.

Example 8:For Setup shown: Soil I , A= 0.37 m^2 , n=0.5 and k= 1 cm/sec, Soil II, A= 0.186 m^2 , n=0.5 and k=0.5 cm/sec.



Points	ht(m)	he(m)	hp(m)
1	3.6	3.6	0
2	3.6	2.4	1.2
3	2.4	1.2	1.2
4	0	0.6	-0.6
5	0	0	0.0

Solution :

1- qI=qII

$$k_I i_I\, A_I = k_{II} i_{II}\, A_{II}$$

$$= \frac{1\ cm/sec}{100} * \frac{\Delta h_I}{(2.4-1.2)} * 0.37 = \frac{0.5\ \text{cm/sec}}{100} * \frac{\Delta h_{II}}{(1.2-0.6)} * 0.1 ----(1)$$

2- $\Delta h_I + \Delta h_{II} = 3.6 ---------- ---------(2)$

From Equation(1) ----------$\Delta h_I = 0.502\ \Delta h_{II}$

Substitute in Equation 2--------- $0.502\Delta h_I + \Delta h_{II} = 3.6$

$$\therefore\ \Delta h_{II} = 2.4\ m$$

$$\Delta h_I = 1.2\ m$$

Approach Velocity= ki

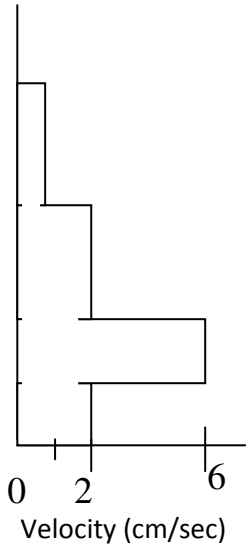
Approach velocity for soil I=$k_I i_I = 1 * \frac{1.2}{(2.4-1.2)} = 1\ cm/sec$

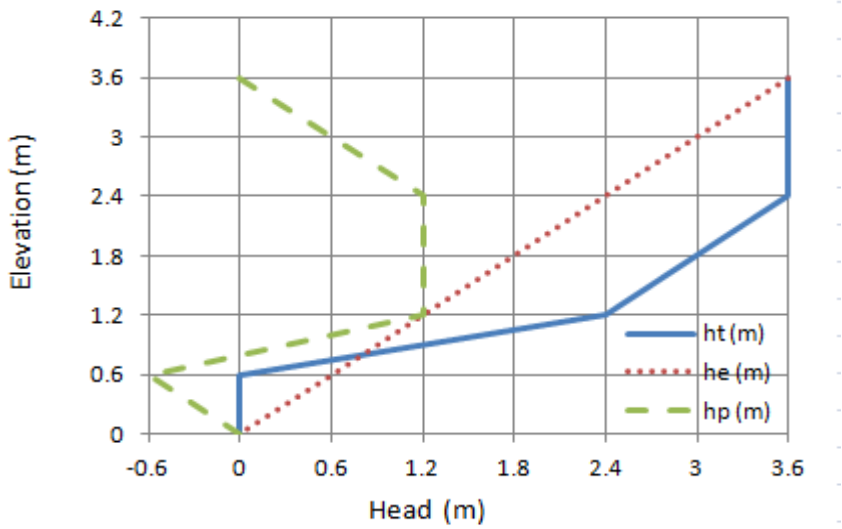
Seepage velocity $= \frac{v}{n} = \frac{1}{0.5} = 2\ cm/sec$

For soil II

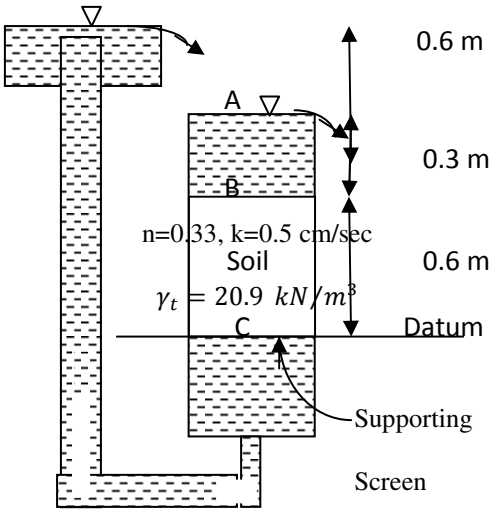
Approach velocity=$k_{II} i_{II} = 0.5 * \frac{2.4}{(1.2-0.6)} = 2\ cm/sec$

Seepage velocity (II) = $\frac{v}{n} = \frac{2}{0.333} = 6cm/sec$

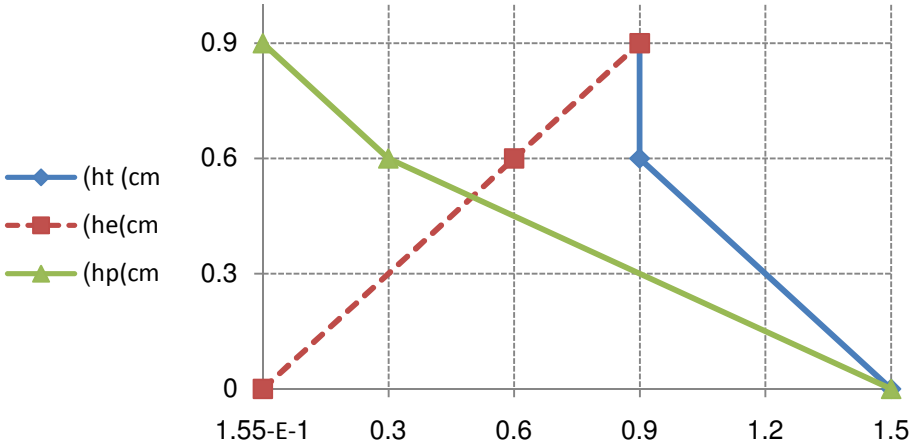




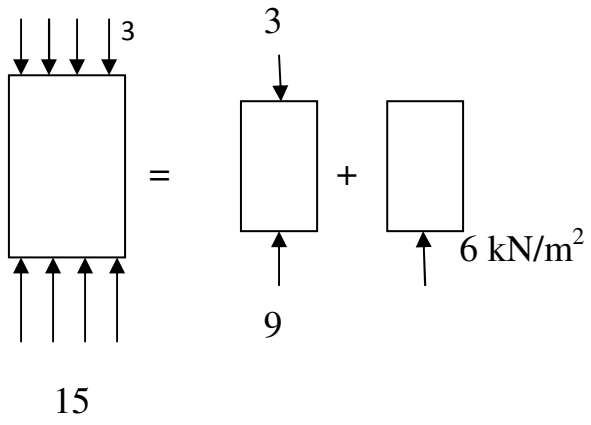
Example 9: For the setup shown draw ht, he, hp and find the seepage force .



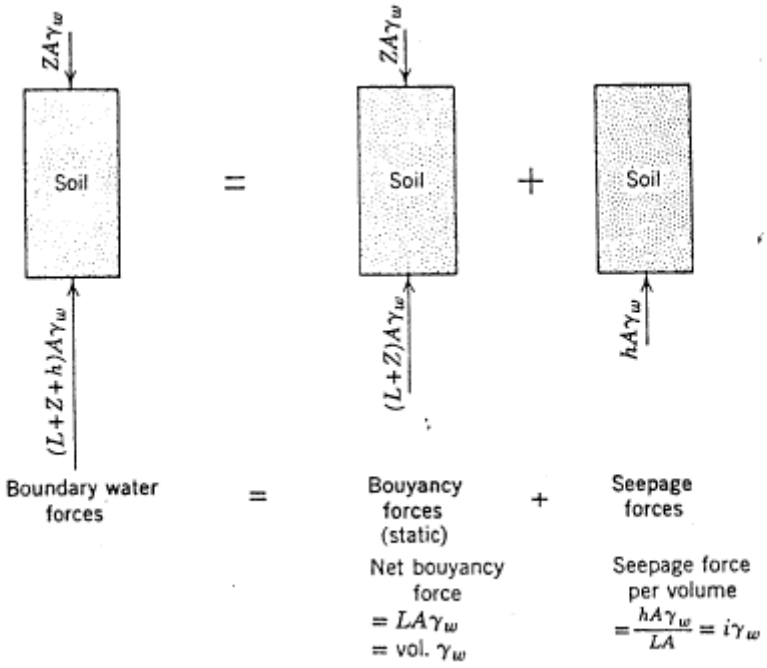
Points	ht(cm)	he(cm)	hp(cm)
A	0.9	0.9	0
B	0.9	0.6	0.3
C	1.5	0.0	1.5



Elevation (cm)	$\Delta\sigma_{v\ (kN/m^2)}$	$\sigma_v(\frac{kN}{m^2})$	$u(\frac{kN}{m^2})$	
0.9		0	0	0
	0.3 *10=3kN/m^2			
0.6		3	0.3*10=3	0
	0.6*20.9 = 12.54			
0		15.54	1.5*10=15	0.54



Water pressure on soil sample (a) Boundary water pressure (b) Buoyancy water pressure(static) (c) Pressure lost in seepage.



Water Force on Soil:

$$j = \frac{Seepage\ force}{Volume\ of\ soil} = \frac{hA\gamma_w}{LA} = i\,\gamma_w$$

Seepage forces usually act with direction of flow.

Quick Condition:

The shear strength of cohesionless soil is directly proportional to the effective stress. When a cohesionless soil is subjected to a water condition that results in zero effective stress, the strength of the soil becomes zero and quick condition exists.

Quick condition: occurs in upward flow(for cohesionless soil) and when the total stress equals to pore water pressure .

$$\acute{\sigma}_{effect} = 0 = LA\gamma_w - hA\gamma_w = 0$$

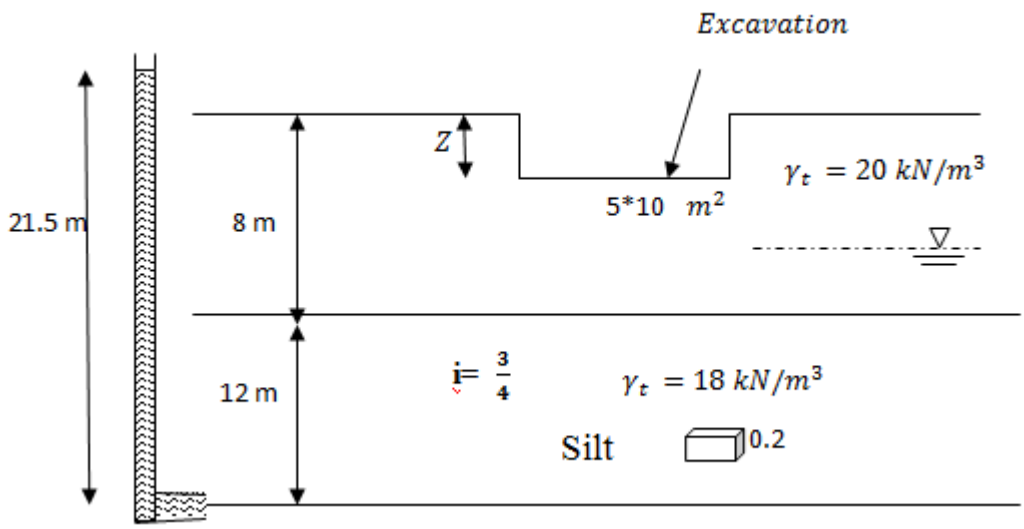
$$\frac{h}{L} = i = \frac{\gamma_b}{\gamma_w} = i_c$$

i_c: The gradient required to cause a quick condition, termed critical gradient.

Example 10: Excavation is been carried out as shown in the figure. Find: 1- the depth Z that could caused boiling at the bottom of clay layer.

2-The depth (Z) for the factor of safety against boiling equal to 2 at the bottom of the clay layer.

3- What is the thickness of the raft foundation that should be used before boiling occurs. If an uplift pressure of 60 kN/m^2 at the bottom of excavation exist($\gamma_{concrete} = 25\ kN/m^3$).
4- Find the seepage force at an element of 0.2 m cube located at the center of silt layer.



Solution:

$$ht_1 = 21.5$$

1- $i = \frac{3}{4} = \frac{\Delta h_{1--2}}{10\ m} = \frac{21.5 - ht_2}{10} - - - - - - \therefore\ ht_2 = 14\ m$

$$\therefore\ hp_2 = ht_2 - he_2 = 14 - 10 = 4\ m$$

To find Z

F. down = F. upward

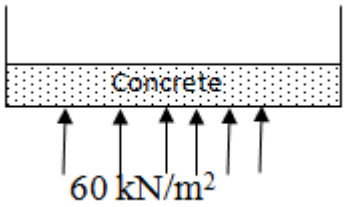
A(8-z) *20 = 4.0 *10 *A $\longrightarrow$ z= 6 m

2- $$F.S = \frac{down\ ward\ force}{up\ ward\ force}$$

$$2 = \frac{(8 - z) * 20 * A}{4 * 10 * A}$$

$$\therefore Z = 4\ m$$

3- F down ward = F upward



t*5*10*25 = 60*5*10

$\therefore t = \frac{60}{25}$= 2.4 m (thickness of concrete)

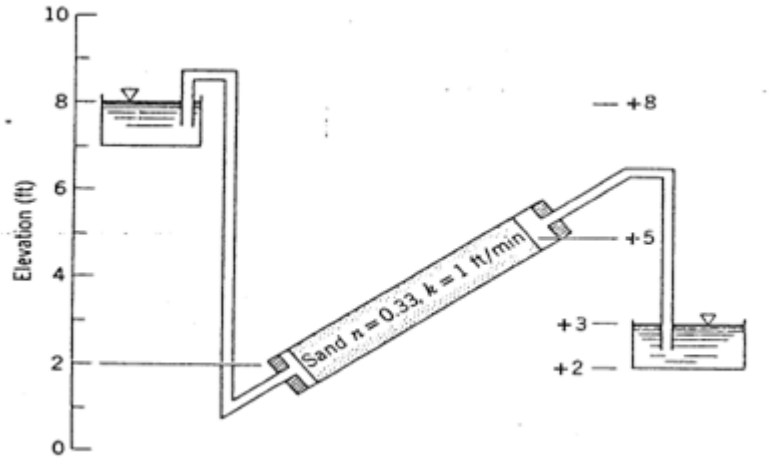
4- Seepage force = iγ_w $volume$

$= \frac{3}{4}$ *10* $(0.2)^3$ = 0.06 kN

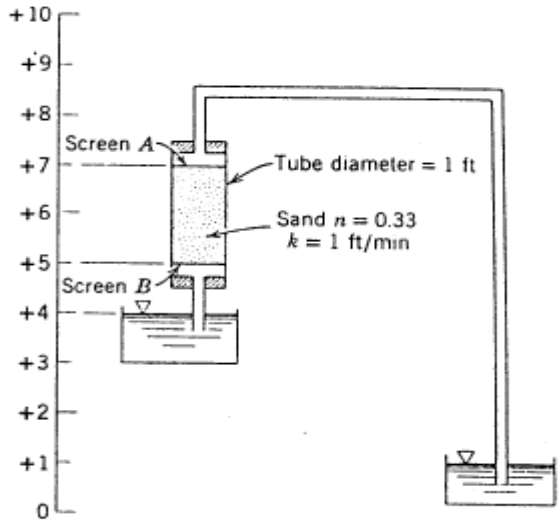
Summary of Main Points:

1- In soils $v = ki$
2- There are three heads of importance to flow through porous media: elevation head (h_e), pressure head (h_p)and total head (h_t).
3- Flow depends on difference in total head.
4- The seepage force per a volume of soil is i*$\gamma_w$and acts in the direction of flow.
5- "Quick", refers to a condition where in a cohesion less soil loses its strength because the upward flow of water makes the effective stress become zero.

Exercise 1: For the setup shown. Plot to scale elevation head, pressure head, total head and seepage velocity versus distance along the sample axis.



Exercise 2: For the setup shown, compute the vertical force exerted by the soil on screen A and that on screen B. Neglect friction between the soil and tube. G= 2.75.



Exercise 3: For the set up shown: Steady vertical seepage is occurring. Make scaled plot of elevation versus pressure head, pore pressure, seepage velocity, and vertical effective stress. Determine the seepage force on a 1 ft cube whose center is at elevation -15 ft. G for all soils = 2.75.

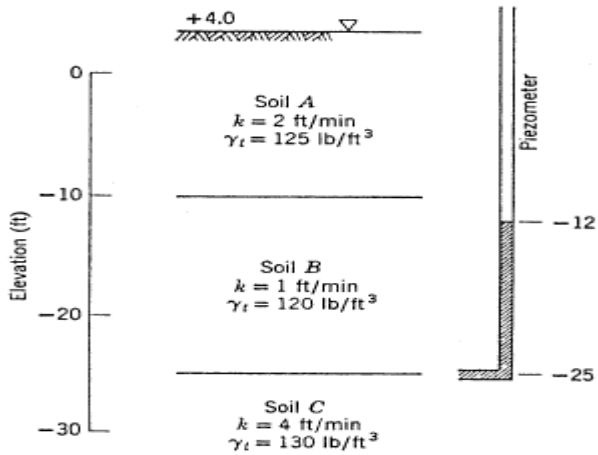
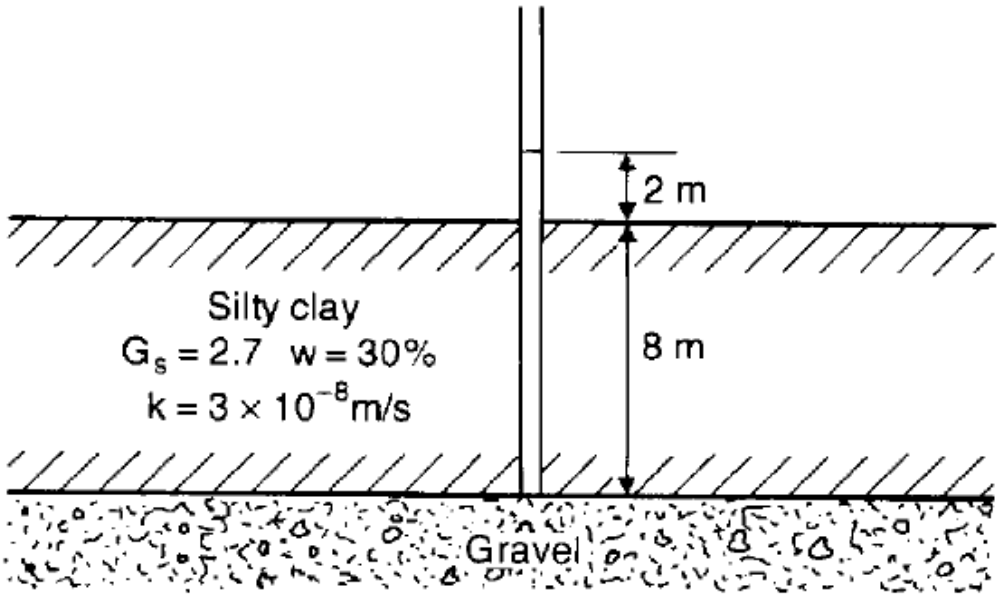


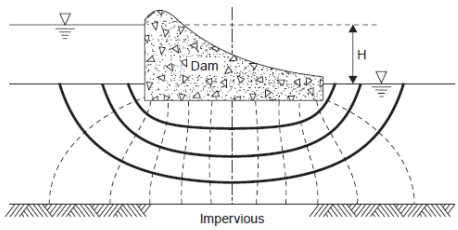
Fig. P17.6

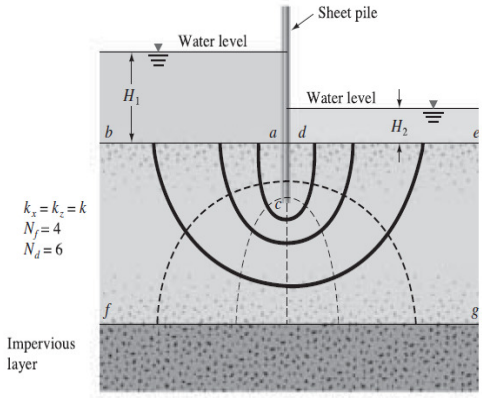
Exercise 4: An 8 m thick layer of silty clay is overlaying a gravel stratum containing water under artesian pressure. A stand –pipe was inserted into the gravel and water rose up the pipe to reach a level 2m above the top of the clay. The clay has a particle specific gravity of 2.7 and a natural moisture content of 30 percent. The permeability of the silty clay is $3*10^{-8}$ m/sec. It proposed to excavate 2 m into the soil in order to insert a wide foundation which, when constructed, will exert a uniform pressure of 100 kN/m^2 on to its supporting soil. Determine (a) The unit rate of flow of water through the silty clay in m^3 per year before the work commences. (b) the factor of safety against heaving: i) at the end of excavation ii) after construction of the foundation?



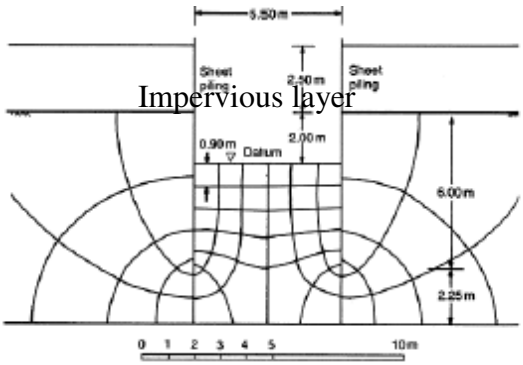
Two Dimensional fluids Flow

Most problems of flow are two dimensional flows, e.g. are shown in Fig. below:



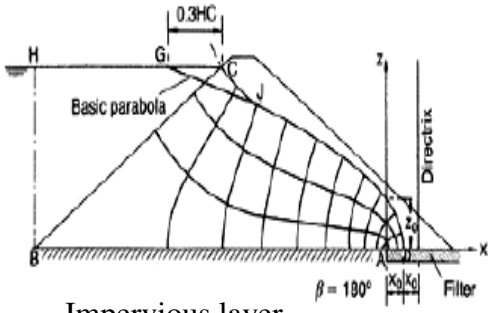


Impervious layer



Impervious layer

Impervious layer



Impervious layer

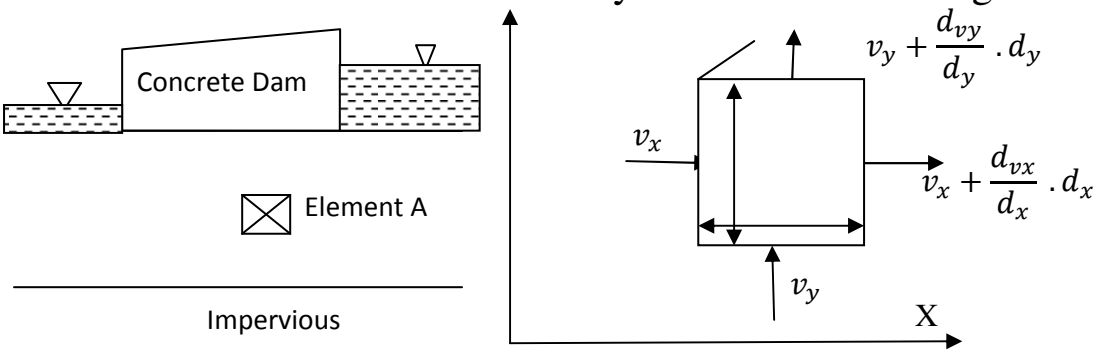
The purpose of studying the flow in two Dimension are :

1- To find the amount of seepage per meter length (i.e. rate of flow).
2- Pressure distribution (pore water pressure)
3- Stability against piping or boiling.
4- Pizometer levels of selected point required.

Seepage Theory:

The general case of seepage in two dimensions will now be considered. In same principle used in one dimensional problem applied (Darcy's law & Continuity flow state).

Consider the two dimensional steady state flows in the fig.



Take element A with dimension dx , dy and dz

Rate of flow (q_{in}) the flow entering the element

$$(v_x + \frac{d_{v_x}}{d_x} dx)dydz + (v_y + \frac{d_{v_y}}{d_y} dy)dxdz$$

Since the flow is steady so $q_{in} = q_{out}$

$$v_x \text{dy dz} + v_y \text{dx dz} = (v_x + \frac{d_{v_x}}{d_x} dx)dydz + (v_y + \frac{d_{v_y}}{d_y} dy)dxdz$$

By simplification

$$\frac{dv_x}{dx} + \frac{dv_y}{dy} = ---------- (1)$$

Darcy's law = $v_x = -k \frac{dh}{dx}$

$$\therefore \frac{dv_x}{dx} = -k \frac{d^2h}{dx^2}$$

$$v_y = -k\,\frac{dh}{dy}$$

$$\therefore \frac{dv_y}{dy} = -k\,\frac{d^2h}{dy^2}$$

Sub. In equation (1)

$$-k\,\frac{d^2h}{dx^2} + \left(-k\,\frac{d^2h}{dy^2}\right) = 0$$

$$\frac{d^2h}{dx^2} + \frac{d^2h}{dy^2} = 0 \qquad \textit{Laplace equation}$$

Consider a function $\emptyset(x, y)$ so that

$$v_x = \frac{d\emptyset}{dx}$$

$$v_y = \frac{d\emptyset}{dy}$$

$$v_x = \frac{d\emptyset}{dx} = -k\,\frac{dh}{dx}$$

$$v_y = \frac{d\emptyset}{dy} = -k\,\frac{dh}{dy}$$

$$\emptyset(x, y) = -kh(x, y) + c$$

Where c is a constant

Thus if the function $\emptyset(x, y)$ is given a constant value equal to $\emptyset_1$ & it will represent a curve a long which the value of total head (h_1) is constant. If the function $\emptyset(x, y)$ is given a series of constant value $\emptyset_1$, $\emptyset_2$, $\emptyset_3$ etc a family of curves, such curves are called equipotentials and this will corresponding to total head h_1 , h_2 , h_3 ---------h_n from the total differ nation.

$$d\emptyset = \frac{d\emptyset}{dx} . dx + \frac{d\emptyset}{dy} dy$$

$$0 = v_x \text{ dx} + v_y dy$$

$$-v_x \text{dx} = v_y dy$$

The second function ψ(x,y) called the flow line

$$v_x = -\frac{d\psi}{dy}$$

$$v_y = -\frac{d\psi}{dx}$$

$$\frac{dv_x}{dx} = -\frac{d^2\psi}{dxdy}$$

$$\frac{dv_y}{dy} = -\frac{d^2\psi}{dydx}$$

$$\frac{dv_x}{dx} + \frac{dv_y}{y} = 0$$

$$\frac{d^2\psi}{dxdy} - \frac{d^2\psi}{dydx} = 0$$

$\therefore$ $\psi(x, y)$ Satisfy the Laplace equation

A gain a series of ψ using $\psi_1 , \psi_2 , \psi_3 - - - - \psi_n$

Is selected and this function

$$\psi = \frac{d\psi}{dx} dx + \frac{d\psi}{dy} dy$$

$$0 = v_y dx + (-v_x) dy$$

$$v_y dx = v_x dy$$

$$\frac{dy}{dx} = \frac{v_y}{v_x}$$

Flow net:

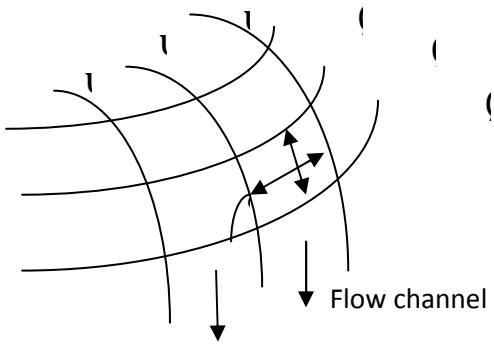
The graphical representation of the Laplace equation is represented by the two families of curve:

1- Equipotential lines: A series of lines of equal total head e.g. $h_1 , h_2 , h_3 - - - - - - - h_n$
2- Flow lines: A family of the rate of flow between any two adjacent flow lines is constant.

For isotropic soil:

The flow net is formed by a mesh of the intersection of two lines with the following limitation

1- Each element is a curvilinear square ψ_1



$$\frac{b}{l} \cong 1$$

Summary of the main points:

1- Laplace equation govern's the steady state flow in two dimensions
2- The solution is represented by two families of curve
   a- Equipotential lines

b- Flow lines

3- The intersection of the two lines is represented by a flow net of square elements

4- Each element is a curvilinear square with dimension $\frac{b}{l} \cong 1$

5- The rate of flow is expressed /m length by

$$q = kh\,\frac{Nf}{Nd}$$

Q: rate of flow/m

Where k: Coefficient of permeability

H: Total head difference between the first and last equipotential lines

$Nf: No.of\ flow\ channels$

$Nd: No.of\ drops\ (equipotential\ drops)$

Steps in drawing a flow net:-

The first step is to draw in one flow line, upon the accuracy of which the final correctness of the flow net depends. There are various boundary conditions that help to position the first flow line, including:

1- Buried surface (e.g. the base of the dam, sheet pile) which are flow lines as water cannot penetrate into such surface.

2- The junction between a permeable and impermeable material which is also a flow line : for flow net purpose a soil that has a permeability of one-tenth or less the permeability of the other may regard as impermeable.

3- The horizontal ground surface on each side of the dam which are equipotential lines.

The procedure is as follows:

a- Draw the first flow line hence establish the first flow channel

b- Divide the first flow line into squares ($b \cong L$)

c- Project the equipotentials beyond the first flow channel, which give an indication of the size of the square in the next flow channel.

d- With compasses determine the position of the next flow line; draw this line as a smooth curve and complete the squares in the flow channel formed.

e- Project the equipotentials and repeat the procedure until the flow net is completed.

Example:

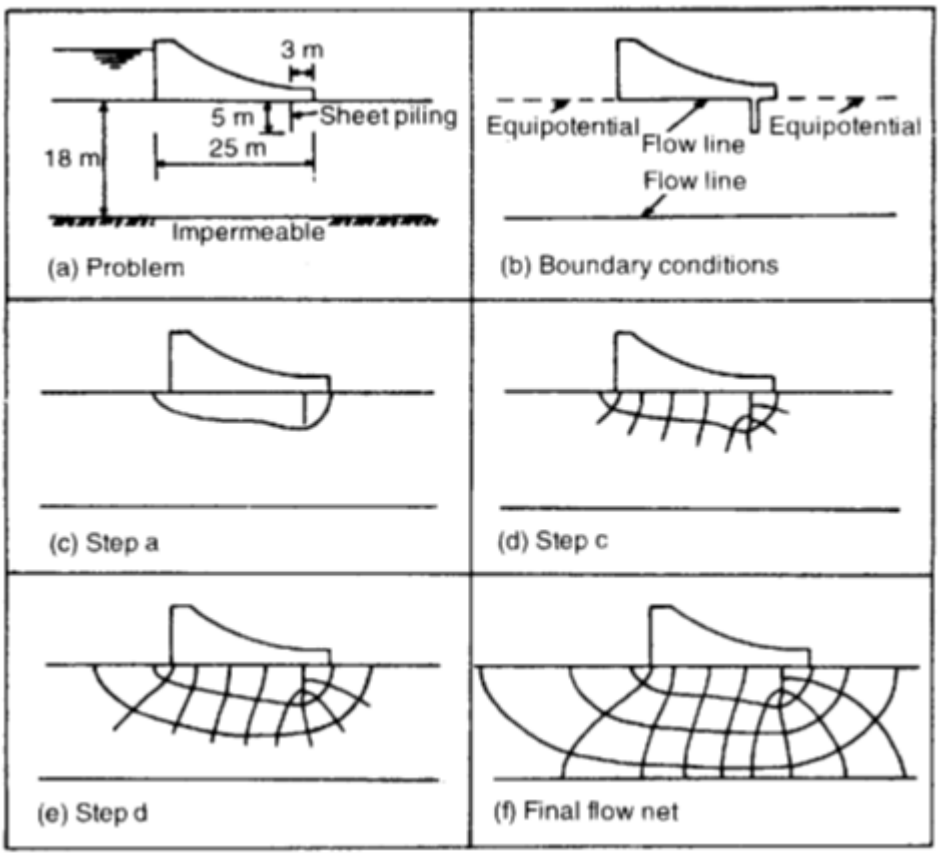


Figure example for flow net construction

Rate of flow

$$q = kiA$$

$$\Delta q = b * 1 * k * \frac{\Delta_{ht}}{L} \qquad this\ is\ for\ one\ flow\ channel$$

$$\Delta ht = \frac{H}{nd}$$

$$\Delta q = \frac{b}{L} * k * \frac{H}{Nd} \qquad (b \cong L)$$

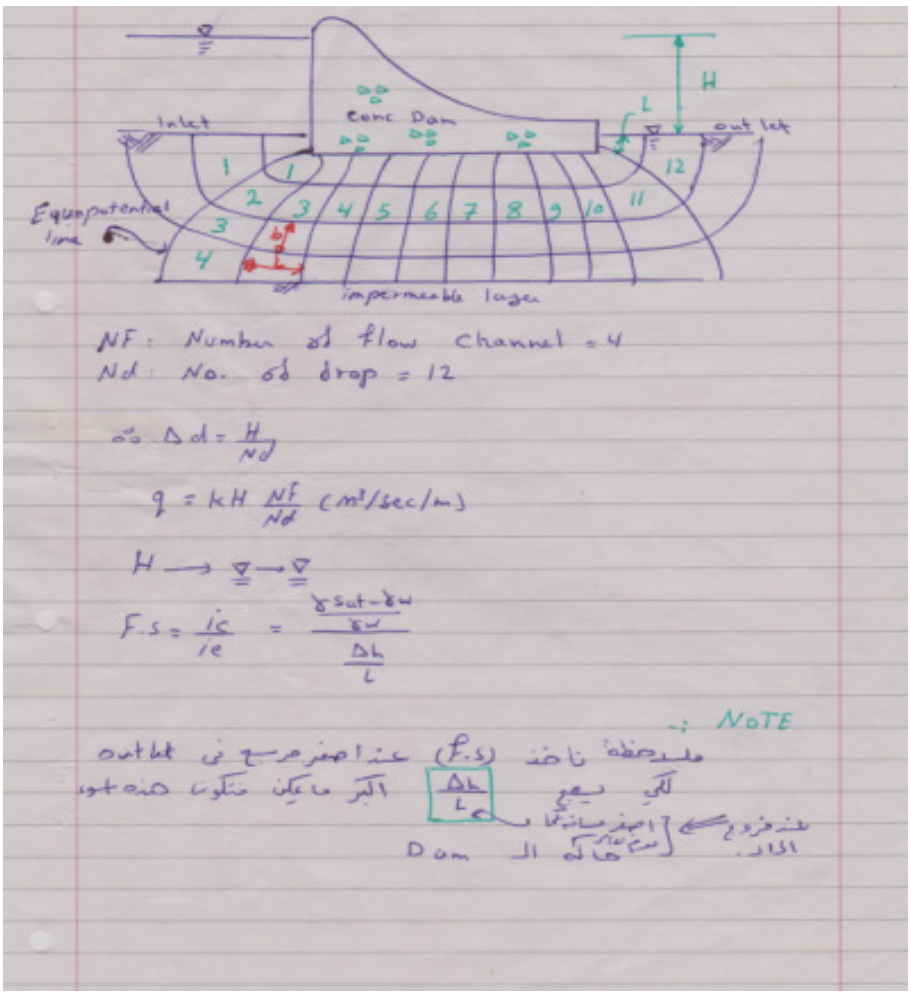
$$\Delta q = k\frac{H}{Nd}$$ this is for one channel

Assume No. of channel = Nf

$$\therefore q = \Delta q Nf = KH\frac{Nf}{Nd}$$

Where H= difference in water level (upstream and downstream.

Example 1:



Example 2

Example

Given the flow net in the fig.

The pore water pressure at point A is 200 kN/m².

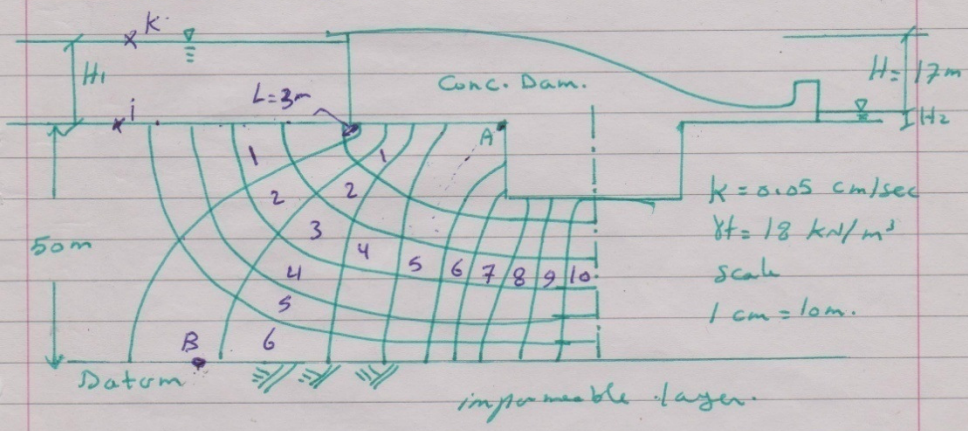
a. Find the quantity of seepage under the dam in m³/day/m.

b. Find H_1 & H_2

c. Find the total stress, pore water pressure and the effective stress at point B.

d. Factor of Safety against piping

Assume $\gamma_w = 10$ kN/m³.



Solution

$UA = h_p \gamma_w \Longrightarrow 200 = h_p \times 10 \Longrightarrow \therefore h_{p_A} = \frac{200}{10} = 20m$

$h_p = 20 \Longrightarrow h_{e_A} = 50m \Longrightarrow \therefore h_{t_A} = 50 + 20 = 70m.$

$h_{t_A} = H_1 + 50 - 4.4 * \frac{17}{20} = 70 \Longrightarrow \therefore H_1 = 23.74m.$

$\therefore H_2 = 23.74 - 17 = 6.74m$

$q = kH \frac{N_f}{N_d} = \frac{0.05}{100} * 17 * \frac{6}{20} * 24 * 3600 = 220.320$ m³/day/m

$\sigma_{Total_B} = 23.74 * 10 + 50 * 18 = 1137.4$ kN/m².

$U_{at\,B} = (73.74 - \frac{17}{20} * 1.66) * 10 = 723.3$ kN/m²

$\bar{\sigma}_{at\,B} = 1137.4 - 723.3 = 414.100$ kN/m².

$F.S = \frac{i_c}{i} = \frac{\frac{\gamma_{sat} - \gamma_w}{\gamma_w}}{\Delta H/L} = \frac{\frac{18-10}{10}}{0.85/3} = 2.823$ (10)

Example 3

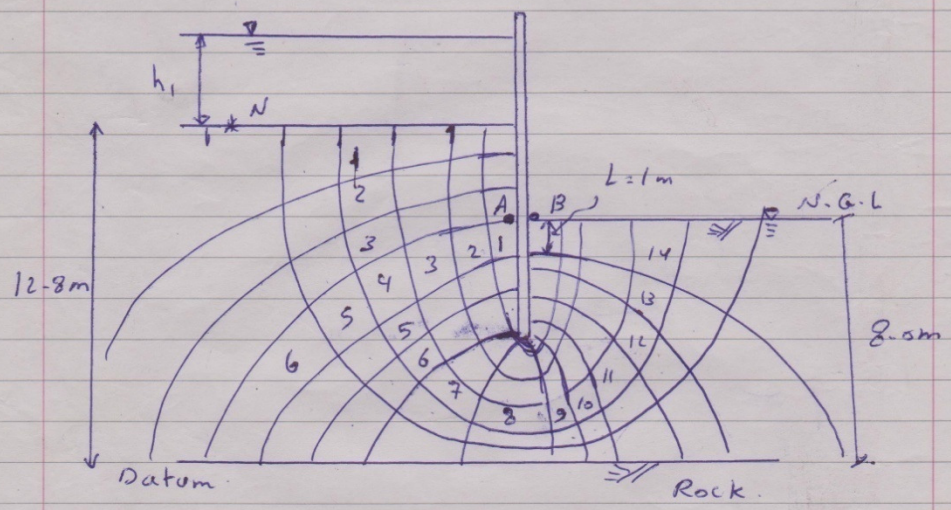
Example :

The fig shows a long a pile wall driven in sand with a saturated unit weight of 18 kN/m³ & k = 0.05 cm/sec . It is required to;

a- Estimate the maximum possible value of dimension h_1

b- Calculate the seepage loss (m³/day) per meter run of wall

c- Calculate h_t, h_e & h_p at points A & B.



Solution :

1- Assume $f.s = 1$

$$f.s = \frac{i_c}{i_e} = \frac{\frac{\gamma_{sat} - \gamma_w}{\gamma_w}}{\frac{\Delta h}{L}} \Rightarrow 1 = \frac{\frac{18-10}{10}}{\frac{\Delta h}{1}} \Rightarrow \Delta h = 0.8\ m.$$

∴ h_t at point N = 0.8 * 14 + 8.0 = 19.2 m.

∴ $h_1 = 19.2 - 12.8 = 6.4\ m$ maximum possible

2- $q = \frac{0.05}{100} * 24 * 3600 * \underbrace{11.2}_{\Sigma H} * \frac{6}{14} = 207.36\ m^3/day/m$

3- $h_{t_A} = 19.2 - 3 * 0.8 = 16.8\ m$

$h_{e_A} = 8\ m \Rightarrow 16.8 - 8 = 8.8\ m.$

$h_{t_B} = 8 \Rightarrow h_e = 8 \Rightarrow h_{p_B} = 0.0$

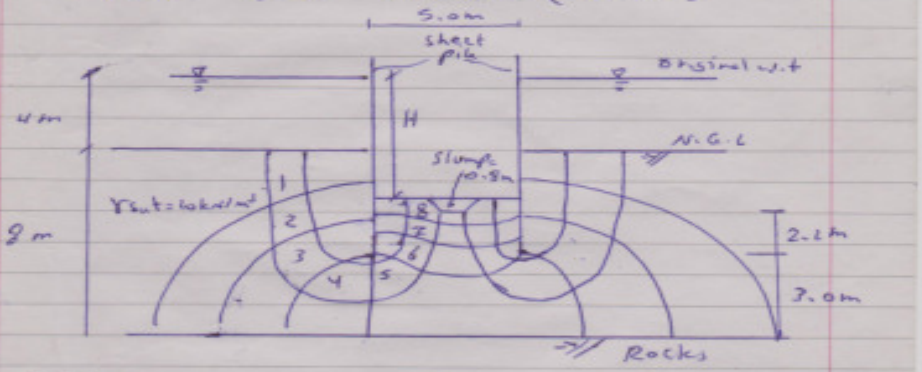
Example 4

Example: It is proposed to build a coffer dam as shown in figure below on soil with $k = 0.07$ m/sec. the coffer dam is 12 m long.

1- calculate the capacity of pump required to maintain the water table in the coffer dam constant (m^3/day).

2- Discuss the stability of the coffer dam against piping (check with F.S = 2.0)

3- What would the water table level be to have a safe structure (FS = 2.0)



Solution

1- $q = kH \frac{N_f}{N_d} = \frac{0.07}{100} * 60 * 60 * 2 * 6.8 * 8 * 12$

$= 3701.4$ m^3/day

2- $F.S = \frac{i_c}{i_e} = \frac{\frac{2.9-1.0}{1.0}}{\frac{6.8}{8}/0.8} = 0.94 < 2$

is not good (Not safe)

3- To find the safe of water level

$2 = \frac{\frac{2.9-1.0}{1.0}}{\frac{\Delta h}{0.8}} \Rightarrow \Delta h = 0.4$

$\therefore h_{tot\ at\ N.G.L} = h_{tot\ coffer\ dam} + 0.4 * 8$

$= (2.2 + 3.0) + 0.4 * 8 = 8.4$ m.

Water above N.G.L: $8.4 - 8 = 0.4$ m above N.G.L.

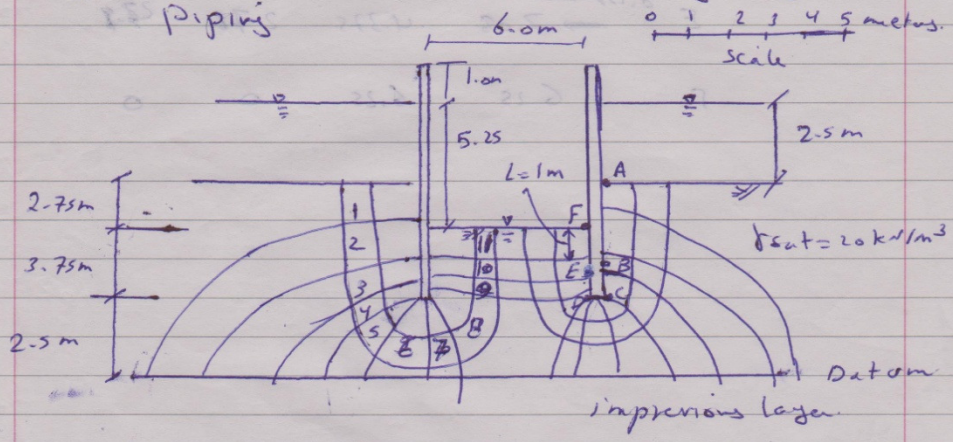
Example 6

Example

For the temporary coffendam shown below with $k = 4.3 * 10^{-5}$ m/sec & length of coffen dam = 10m

It is required to:

1. Calculate the capacity of the pump required in liter /min)
2. Calculate the water pressure kpa (kN/m²) at points A, B, C, D, E & F.
3. Calculate the factor of safety against piping



$$q = KH\frac{N_f}{N_d} * \text{length} = 4.3 * 10^{-5}\frac{m}{sec} * 60 * \frac{2.5}{11} \times 10 * 2 * 5.25$$

$$= 6156.818 * 10^{-5}\ m^3/min * 10^6 = 61568.18\ cm^3/min$$

$1 lit = 1000 cm^3$

$$= \frac{61568.18}{1000} = 61.56 \text{ littres/min}$$

$$F.S = \frac{i_c}{i_e} = \frac{\frac{20-10}{10}}{\frac{0.477}{1}} = 2.095 \simeq 2.1$$

$$\Delta h = \frac{5.25}{11} = 0.477$$

Points	h_t (m)	h_e (m)	h_p (m)	U (kN/m²)
A	11.5	9	2.5	25
B	10.3	4.375	5.92	59.2
C	11.5-4x0.477 = 9.59	2.5	7.09	70.9
D	9.59-3x0.477 = 8.159	2.5	5.65	56.5
E	8.159-1.25x0.477 = 7.08	4.375	2.71	27.1
F	6.25	6.25	0	0

Example 7

Example

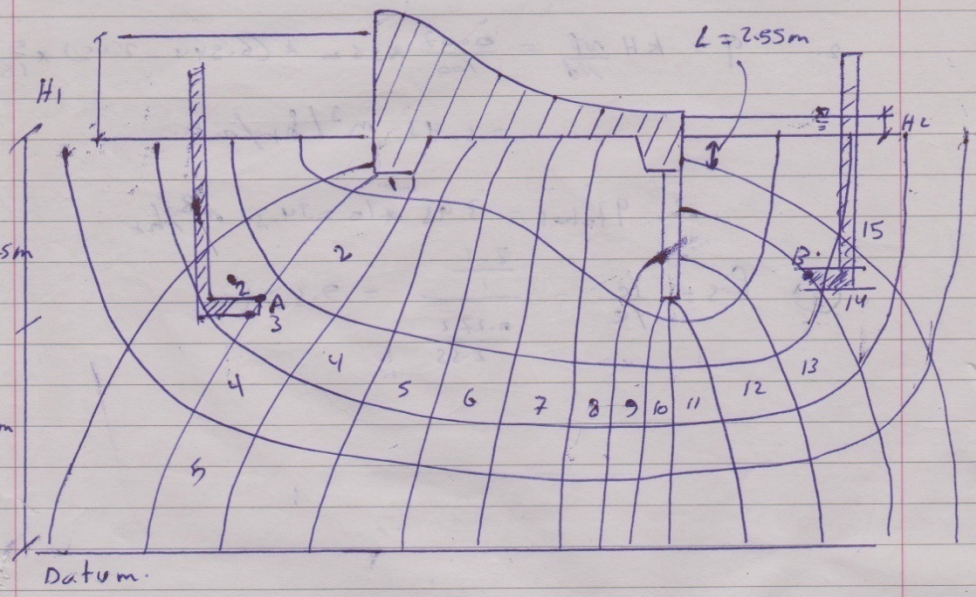
For the concrete dam shown in the figure below is 10 m long, and is constructed on a soil with a coefficient of permeability = 0.07 cm/sec & $\gamma_{sat} = 20\ kN/m^3$.

Required :-

a- If the piezometer in points A & B rise to elevation 13.5 m & 10.5 m respectively, what are the value of H_1 & H_2

b- What is the total quantity of seepage (m^3/hr) under neath the dam.

c- What is the factor of safety against piping (boiling)



Solution

$h_{pA} = 13.5\ m$ given $\longrightarrow h_{eA} = 9.6\ m \longrightarrow h_{tA} = 13.5 + 9.6$

$h_{tA} = 23.1\ m$

$h_{pB} = 10.5\ m$ given $\longrightarrow h_{eB} = 9.6\ m \longrightarrow h_{tB} = 10.5 + 9.6 = 20.1\ m$

$\therefore \Delta h_{f\ from\ A \to B} = 23.1 - 20.1 = 3\ m.$

3 m losses over (11) squares $\longrightarrow \Delta h = \frac{3}{11} = 0.272\ m$

$\therefore$ $h_{t\,at\ the\ surface} = h_{ta} + 2 * 0.272 = 23.1 + 2 * 0.272$

$= 23.644\ m.$

$\therefore$ $H_1 = 23.644 - (7.5 + 9.6) = 6.544\ m.$

$\rightarrow$ Find H_2

$h_{t\,B} - 2 * 0.272 = 20.1 - 2 * 0.272 = 19.556\ m.$

$\therefore$ $H_2 = 19.556 - (9.6 + 7.5) = 2.45\ m.$

2. $q = kH \frac{N_f}{N_d} = \frac{0.07}{100} * 360 * (6.544 - 2.45) * \frac{5}{15}$

$= 3.43\ m^3/hr/m$

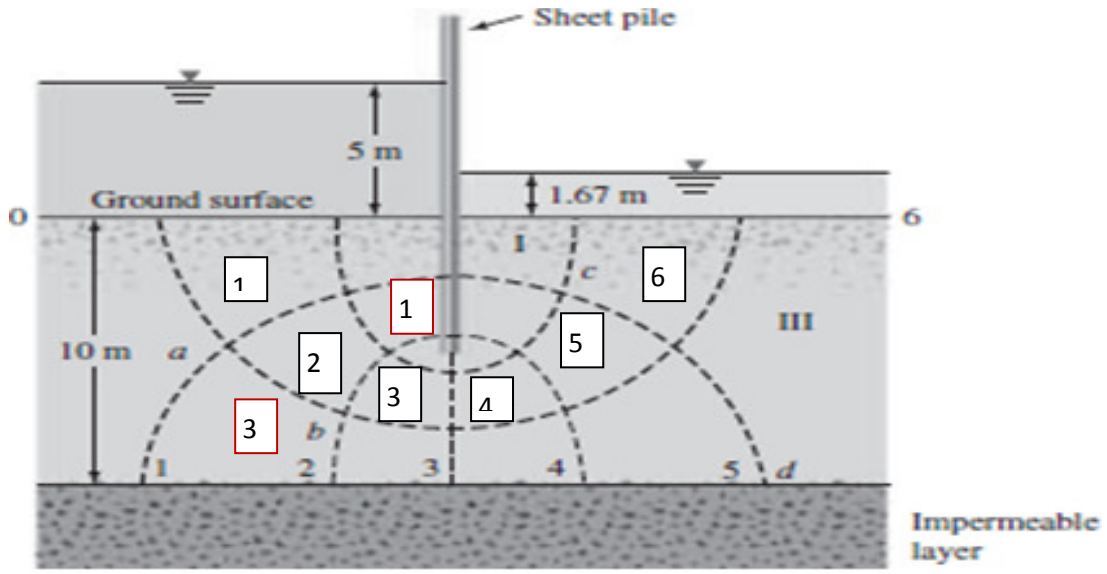
$\therefore$ $q_{total} = 3.43 * 10 = 34.3\ m^3/hr$

3. $F.S = \frac{i_c}{i_e} = \frac{\frac{2-1}{1}}{\frac{0.272}{2.55}} = 9.3 > 1$

Example 8:

A flow net for around a single row of sheet piles in a permeable soil layer is shown in figure below. Given that k= 5* 10 $^{-3}$ cm /sec.

a) **How high (above the ground surface)will the water rise if pizometers are placed at points a,b , c and d?**
b) **What is the rate of seepage under sheet pile?**



Solution:

From flow net $N_f = 3$, $N_d = 6$

$$\Delta h = \frac{5 - 1.67}{6} = 0.555$$

$$h_t \; at \; point \; a = 15 - 0.555 = 14.445 \; m$$

$$h_t \; at \; point \; b = 15 - 2 * 0.555 = 13.89 \; m$$

$$h_t \; at \; point \; c = h_t \; at \; point \; d = 15 - 5 * 0.555 = 12.225 \; m$$

So pizometer reading at point a = 14.445-10= 4.445 m above surface

So pizometer reading at point b = 13.89-10= 3.89 m above surface

So pizometer reading at point c = 12.225-10=2.25 m above surface

$$q = kH\frac{N_f}{N_d} = 5 * 10^{-5} m/sec * (5 - 1.67) * \frac{3}{6} = 8.325 * 10^{-5} \frac{m^3}{sec}/m$$

Example 9

A deposit of cohesion less soil with a permeability of $3*10^{-2}$ cm/sec has a depth of 10 m with an impervious ledge below. A sheet pile wall is driven into deposit to a depth of 7.5 m. The wall extends above the surface of the soil and 2.5 m depth of water acts on one side. Determine the seepage quantity per meter length of the wall.

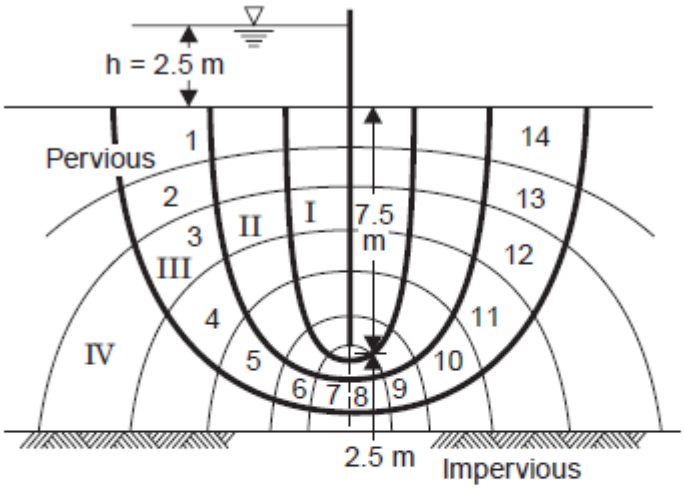


Fig. 6.28 Sheet pile wall (Example 6.6)

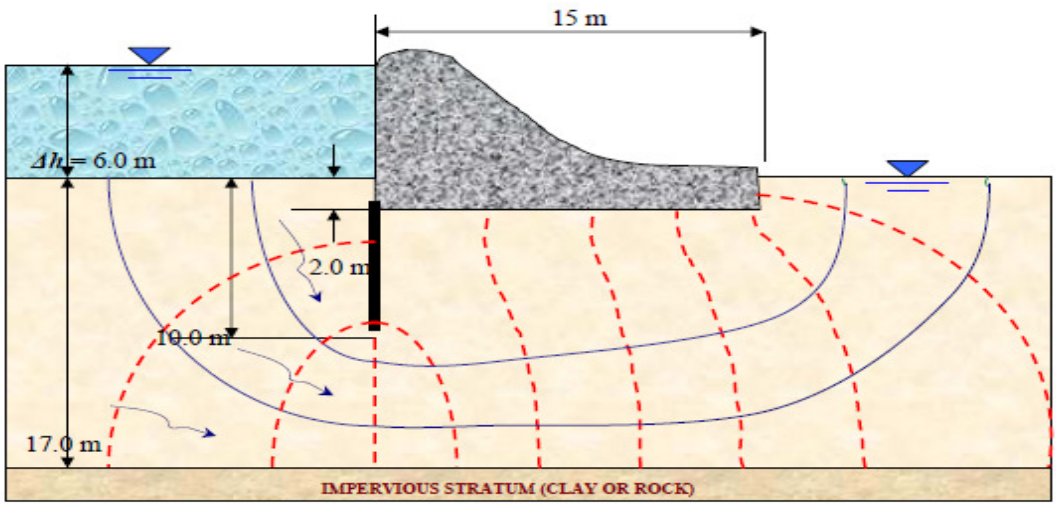
$$= 3 \times 10^{-4} \times 2.5 \times \frac{4}{14} \text{ m}^3/\text{sec/metre run}$$

$$= 2.143 \times 10^{-4} \text{ m}^3/\text{sec/ meter run}$$

$$= \mathbf{214.3 \text{ ml/sec/metre run.}}$$

Example 10-

For the flow net shown below includes sheet-pile cutoff wall located at the head water side of the dam in order to reduce the seepage loss. The dam is half kilometer in width and the permeability of the silty sand stratum is $3.5 *10^{-4}$ cm /sec. Find (a) the total seepage loss under the dam in liters per year , and (b) would the dam be more stable if the cutoff wall was placed under its tail-water side?



Solution:

a) Notice that $\Delta h = 6.0\ m$, the number of flow channels $N_f = 3\ and\ N_d = 10$ by using $q = k\Delta h \frac{N_f}{N_d}$

$q = (3.5*10^{-4}\frac{cm}{sec})\left(\frac{m}{100\ cm}\right)(6.0m)\left(\frac{3}{10}\right)$

$= 6.3 * 10^{-6} m^3/sec/m$

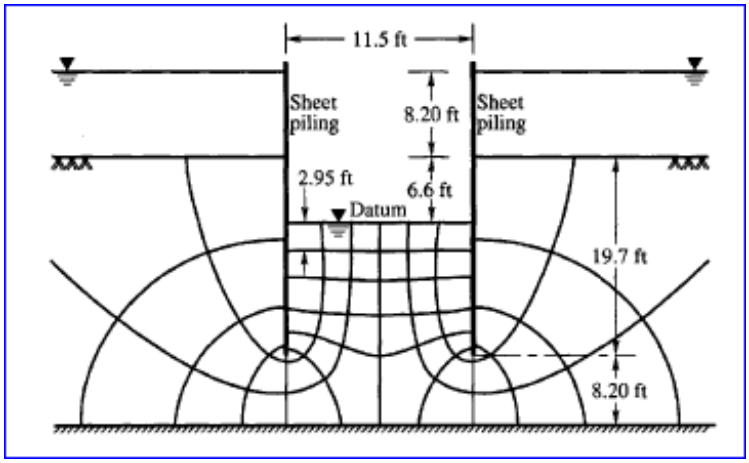
Since the dam is 500 meters wide, the total Q under the dam is

$Q = Lq = 500m\ (6.3*10^{-4} m^3/sec)(\frac{10^3 liters}{1m^3})\left(31.5 * 10^6\ \frac{sec}{year}\right) = 100\ \frac{million\ liters}{year}$

b) - No: Placing the cutoff wall at the toe would allow higher uplift hydrostatic pressure to develop beneath the dam.

Home work:

1. Two lines of sheet piles were driven in a river bed as shown in figure. The depth of water over the river bed is 8.20 ft. The trench level within the sheet piles is 6.6 ft. below the river bed. The water level within the sheet piles is kept at trench level by resorting to pumping. If a quantity of water flowing into the trench from outside is 3.23 ft^3/hour per foot length of sheet pile, what is the hydraulic conductivity of the sand? What is the hydraulic gradient immediately below the trench bed? *(Ans – 1 x 10^{-4} ft/sec, 0.50).*


11.5 ft
Sheet piling
8.20 ft
Sheet piling
2.95 ft
6.6 ft
Datum
19.7 ft
8.20 ft