الميكانيك الهندسي

المرحلة الاولى

جميع الفروع

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– Chapter one –

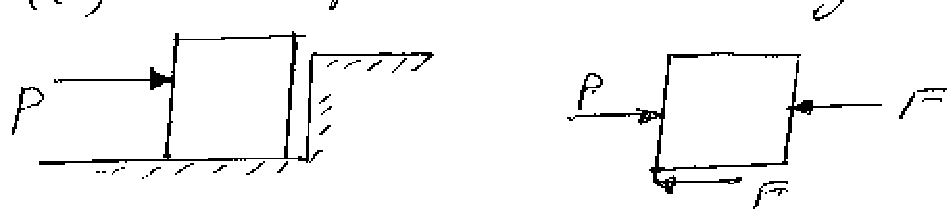
Introduction

Mechanics is that branch of physical science which considers the motion of bodies, with rest being considered a special case of motion. In engineering mechanics attention is directed primarily to the external effects of a system of forces acting on a rigid body.

A Force may be defined as the action of one body on another body which changes or tends to change the motion of the body acted on.

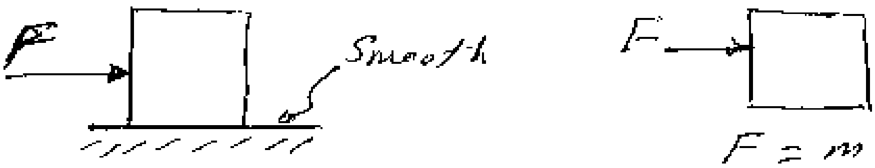
– The external effect of a force on a body is either:

(i) development of resisting forces (reactions)



(Equilibrium)

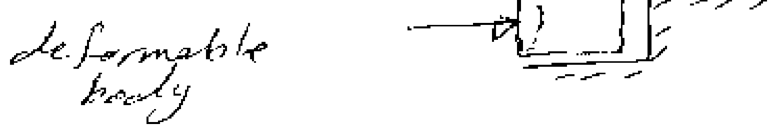
(ii) Acceleration of the body.



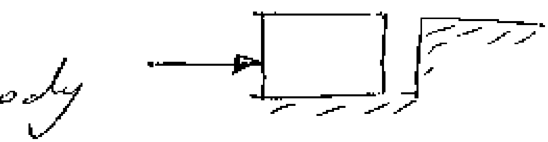
$F = m.a.$

(not Equilibrium)

- IF the internal effects of a force system are to be considered or when changes in the shape of the body are important, the problem becomes one of strength of materials.



- Mechanics deals only with rigid bodies, which are bodies in which all particles remain at fixed distance from each other.

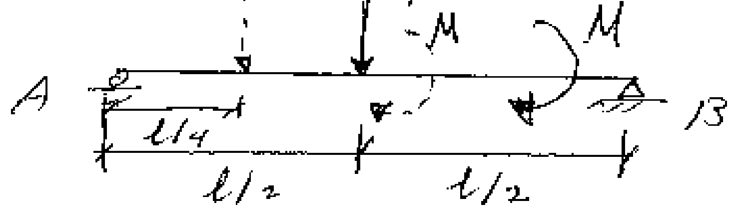


Can be represented by a directed line segment and that conforms to the parallelogram law of addition. Force, velocity, acceleration are examples of vector quantities.

Vector quantities can be divided into:-
(i) Free vector
(ii) Localized vector.

(i) Free vector - has specified slope and sense but does not act through a particular point.
For example, the velocity of wind & the moment of a couple.

(ii) Localized vector - has a definite line of action (acts through a particular point). ex. the force (P) on the beam shown in the figure below is a localized vector, while the moment (M) is a free vector.



Force system:- is a number of forces acting in a given situation and can be classified according to the arrangement of the lines of action of the forces of the system.
The forces may be:-

- Coplanar or non-coplanar.
- Concurrent or non-concurrent.
- parallel or non-parallel.
- Collinear or non-collinear.

The resultant of a force system: is the simplest force system which can replace the original system without changing its external effect on a rigid body.

When the force system acting on a body has a resultant equal to zero, the body is in equilibrium and the problem is one of statics

$(F = ma \,,\; F = 0 \,,\; m \neq 0 \;\; \therefore a = 0)$

When the resultant is different from zero, the problem is one of dynamics

$(F = ma,\; F \neq 0 \;\; \therefore a \neq 0)$.

1:1 SCALAR and VECTOR QUANTITES

A scalar quantity is one which has only magnitude, such as mass, volume, time, etc...

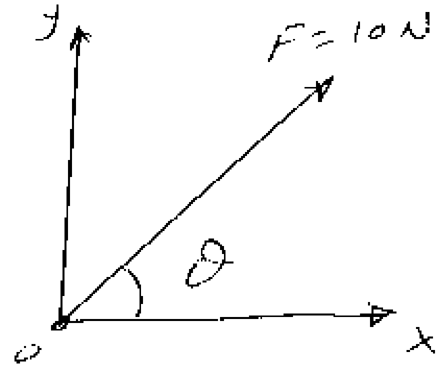
A vector quantity is one which involves both magnitude and direction so that it

1-2 Forces

The characteristics of a force (which describe its external effect on a rigid body) are:-

(i) its magnitude
(ii) its direction (Sense and slope)
(iii) The location of any point on its line of action.

As an example, to define force (F) shown in figure below



$|F|$ = magnitude (10 N)
θ = slope
o = point of application

Principle of transmissibility

The external effect of a force on a rigid body is independent on the point of application of the force along its line of action

- Force can be resolved into its Components at any point on its line of action.

Composition and Resolution of forces

Composition:- is the process of replacing a force system by its resultant

Resolution:- is the process of replacing a force by its Components.

1- Resolving a force along rectangular Components

a. In plane force (two dimensional)

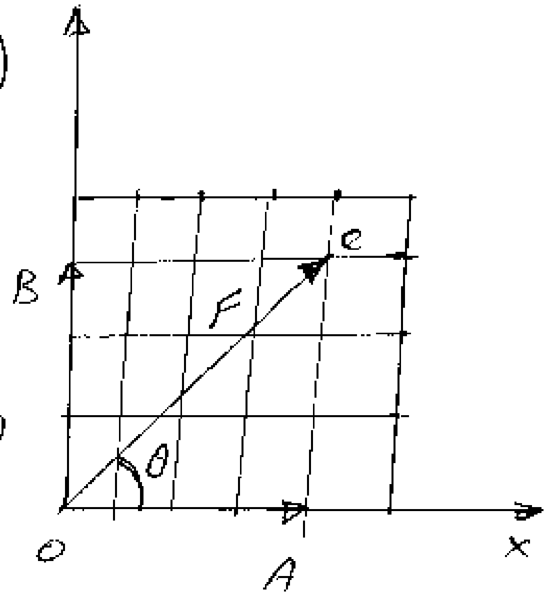
For most purposes, rectangular Components of a force are more useful.

$F_x = OA = F\cos\theta = \left(\frac{4}{5}\right)F$

$= 0.8F$ to the right through (o)

$F_y = OB = F\sin\theta = \frac{3}{5}F$

$F_y = 0.6F$ upward through (o)



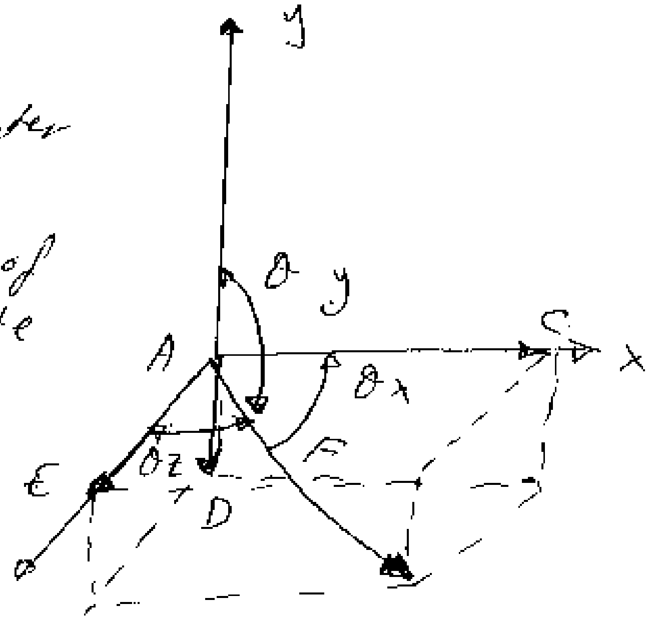
Note: The two Components of a force must intersect on the action line of the force. In other words oA and Ac are note the Components of (F).

b For a force in space (3 dimensional), the resolution is as following

$F_x = AC = F \cos \theta_x$
$F_y = AD = F \cos \theta_y$
$F_z = AE = F \cos \theta_z$

- The angles $\theta_x, \theta_y, \theta_z$ are the angles between the resultant force and the positive coordinate axes.

- The cosines of the angles θ_x, θ_y & θ_z are called <u>direction cosines</u>.



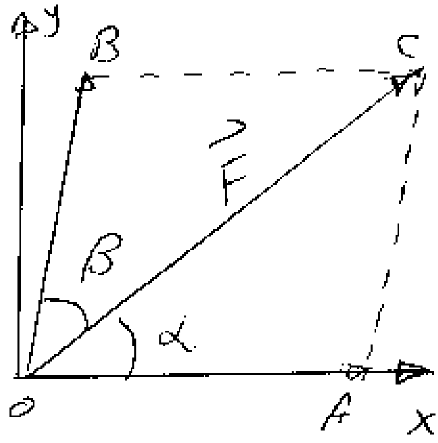
- If the angle is greater than 90° the cosine is negative & the sense of the component is opposite the +ve direction of the axis.

- Note that the three components always intersect at a point on the line of action of the resultant force.

2 - Resolving a force along non-rectangular Components:

a) graphically by drawing the parallelogram to any convenient scale

b) Using Law of Sines



$$\frac{OA}{\sin\beta} = \frac{\vec{F}}{\sin(180-\alpha-\beta)}$$

$F_x = (F_{OB})_x + (F_{OA})_x$ -- (1)

c

$F_y = (F_{OB})_y$

(c) Resolve the force $\vec{F}$ into rectangular Components and equate each of the rectangular Components of the force to the Sum of the Components of the rectangular Components of $\vec{OA}$ and $\vec{OB}$.

- The following example illustrates this third method of procedure.

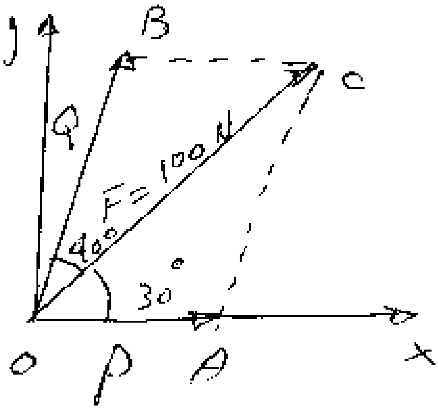
Ex: Resolve the 100 N force along OA & OB.

Solution by using Law of sines (method (b))

$$\frac{P}{\sin 40} = \frac{100}{\sin(180-30-40)}$$

$\therefore P = 100\,\frac{\sin 40}{\sin 110} = 100 \times \frac{0.6427}{0.9396}$

$= 68.4N \longrightarrow$ through O



$$\frac{Q}{\sin 30} = \frac{100}{\sin 110}$$

$$\therefore Q = \frac{0.5 \times 100}{0.9396} = 53.2\ N \quad \angle 70^\circ \text{ through } O$$

by using method (3)

$F_x = 100 \cos 30 = 86.6\,N \rightarrow$ through o

$F_y = 100 \sin 30 = 50\ N \uparrow$ through o

$F_y = Q \sin(40+30)$ [P has no y component]

$50 = Q \sin 70$ --- (1)

$\therefore Q = 53.2\ N \quad \angle 70^\circ$ through o

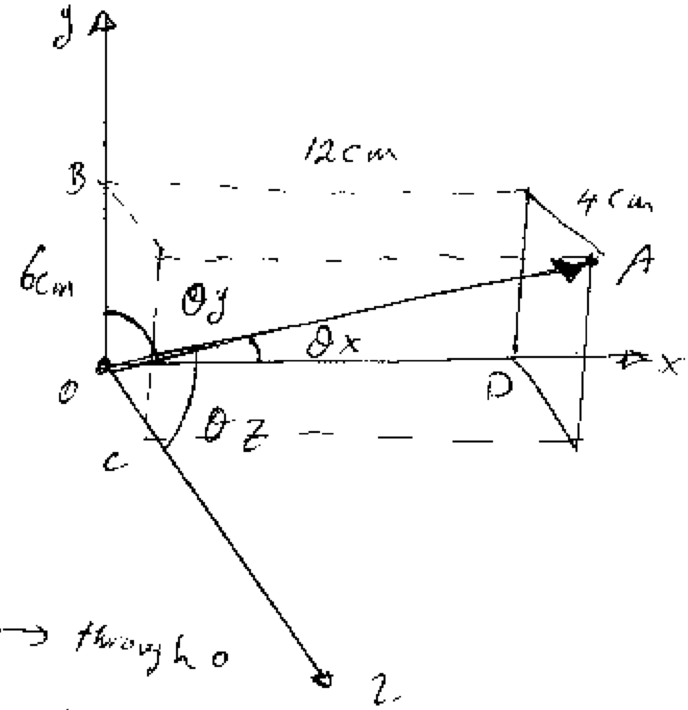
$F_y = Q \cos 70 + P$ --- (2)

$86.6 = 53.2 \cos 70 + P$

$\therefore P = 68.4\ N \longrightarrow$ through o.

Ex: Resolve the 500N force into its rectangular components.

1- Use direction Cosines



length of $OA = \sqrt{4^2 + 6^2 + 12^2} = 14$

$\cos \theta_x = \frac{12}{14}$, $\cos \theta_y = \frac{6}{14}$

$\cos \theta_z = \frac{4}{14}$

$F_x = R \cos \theta_x$

$\therefore F_x = 500 \left(\frac{12}{14}\right) = 428.5\ N \rightarrow$ through o

$F_y = 500 \left(\frac{6}{14}\right) = 214.3N \uparrow$ through o

$F_z = 500 \left(\frac{4}{14}\right) = 142.9N \searrow$ through o

2- Use Scale factor

length of $oA = 14$

the Scale of F is

$\frac{500}{14} = 35.7$ N/cm

$F_x = \frac{500}{14} * 12 = 428.5$ N $\longrightarrow$ Through o

$F_y = \frac{500}{14} * 6 = 214.3$ N $\uparrow$ Through o

$F_z = \frac{500}{14} * 4 = 142.9$N $\searrow$ through o

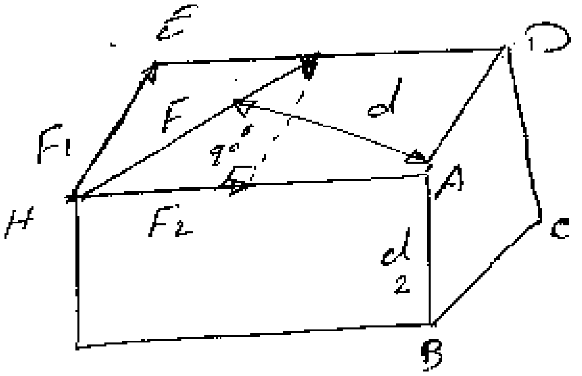
4- Moment of a force

The moment of a force is a measure of its tendency to turn or rotate a body about the moment axis.

- The moment of a force with respect to a line $\perp$ to a plane Containing the force is <u>the product of the force and the perpendicular distance from the force to the line or moment axis.</u>

- Line(AB) is $\perp$ to the plane ADEH.

- The magnitude of the moment of the force (F) is $(F \cdot d)$



<u>Note</u> $(F.d)$ is also the moment of the force F about point (A) in the plane (ADEH)

- If the force does not lie in a plane perpendicular to the moment axis, then the force may be resolved into two components one parallel to the axis and the other is in a plane perpendicular to the axis.

- The moment of (F) about the axis (Bc) may be found by resolving it into two components.

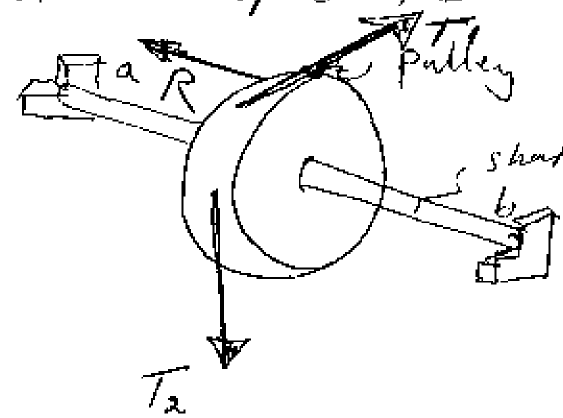
The moment of the component (F_1) about $Bc = 0$ $[F_1 // Bc]$

The moment of the component (F_2) about $Bc = F_2 d_2$

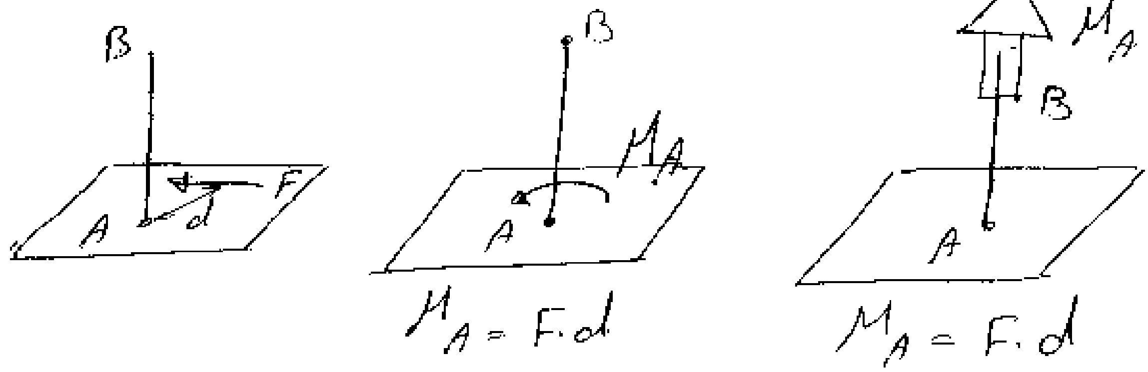
$\therefore$ The magnitude of the moment of (F) about (Bc) is $F_2 d_2$.

- A force // to an axis has no moment with respect to the axis,

Consider the shaft and pulley shown in figure below. The forces T_1, T_2 tend to rotate the pulley on the shaft ab, whereas the force R which is // to the shaft, tends to translate the pulley along the shaft (ab). So the moment of (R) with respect to (ab) is zero.



- The moment of a force acts either clockwise or counter clockwise about a particular moment axis, thus it is a vector quantity with a definite sense of rotation about a definite axis.
- The moment of a force is frequently represented as a vector along the moment axis by using a right-hand rule.



1-3 Principle of moments of forces

The principle of moments states that the moment of the resultant of the force system with respect to any axis is equal to the algebraic sum of the moments of the forces of the system with respect to the same axis.

Couples

A couple Consists of two forces which have equal magnitudes and parallel non collinear lines of action but which are opposite in sense.

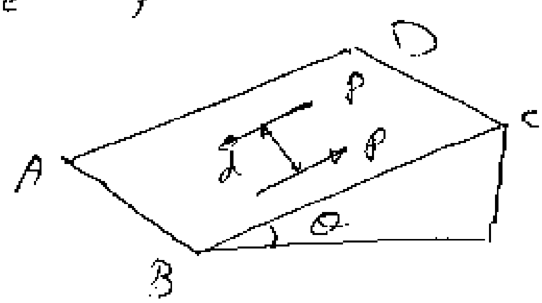
The Couple tends <u>only</u> to rotate the body on which it acts.

The properties of a couple are :-

a) The magnitude of the moment of the Couple.

b) The aspect or slope of the plane of the Couple

c) The Sense of rotation of the Couple.



- The moment of the Couple is the algebraic sum of the moments of its forces about any axis perpendicular to the plane of the Couple.

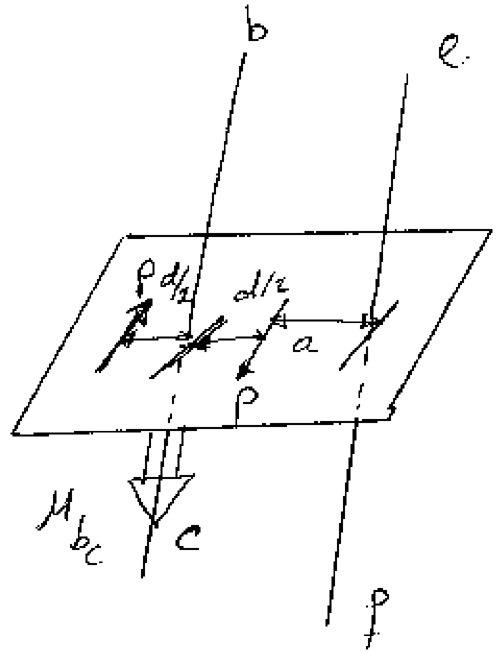
Note: The moment of the Couple is the same for all axes perpendicular to the plane.

$$+ \quad M_{bc} = P(d/2) + P(d/2)$$

$$M_{bc} = Pd$$

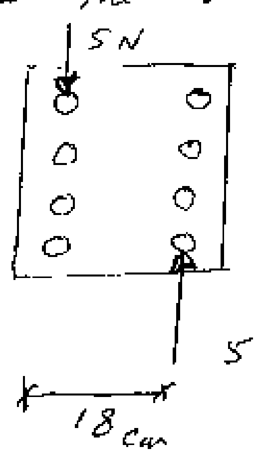
$$+ \quad M_{cf} = P(d+a) - Pa$$

$$M_{cf} = Pd$$

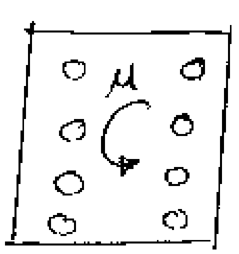


- A couple is a vector quantity having both magnitude (of moment) and direction (aspect of plane and sense of rotation).

- The methods used to indicate a couple are:

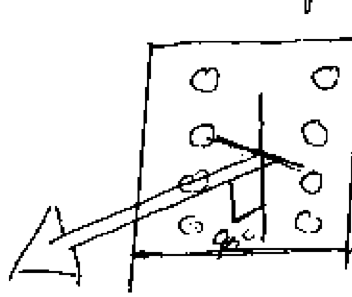


(a)



M = 90 N.cm

(b)



M = 90 N.cm

(c)

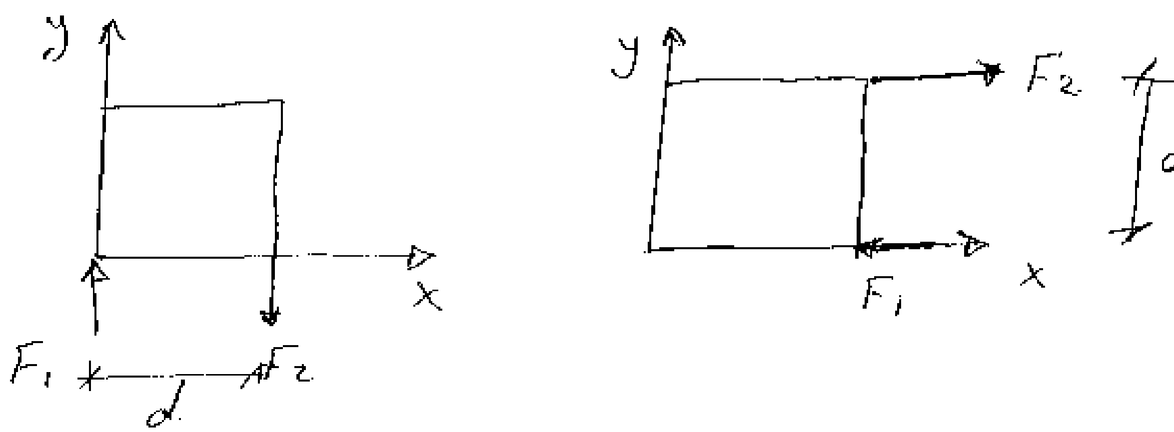
Vector representation of a couple in Fig. (c) is convenient for determining the resultant of a system of a couples in space.
A couple is frequently indicated by clockwise or counterclockwise arrow when coplanar force systems are involved (as in Fig. b).

Transformations of A Couple

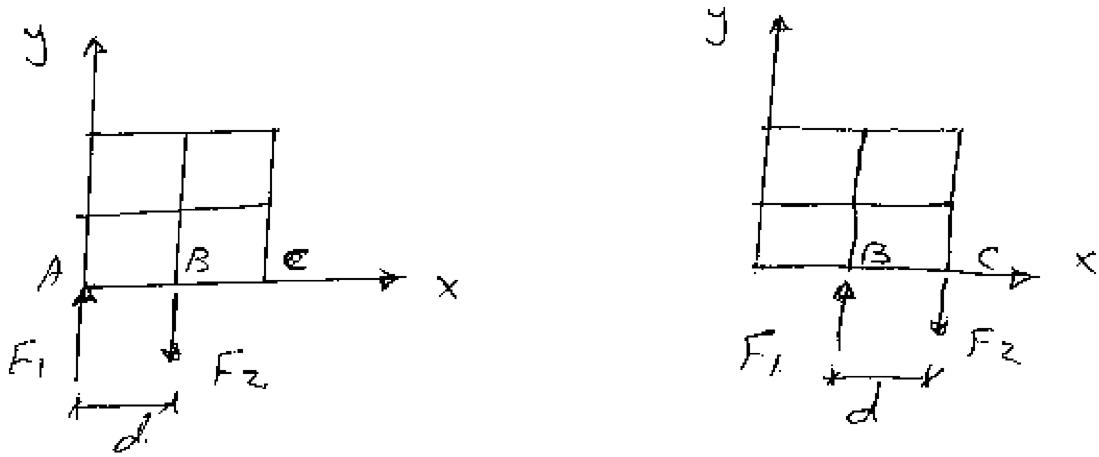
Transformations of a Couple are operations on the Couple that do not change any of its characteristics.

- The properties of a Couple are not changed if :-

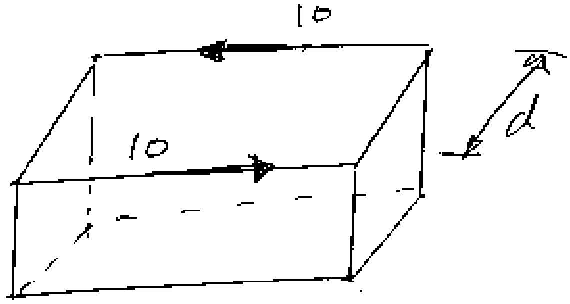
1- The Couple is rotated in its plane.

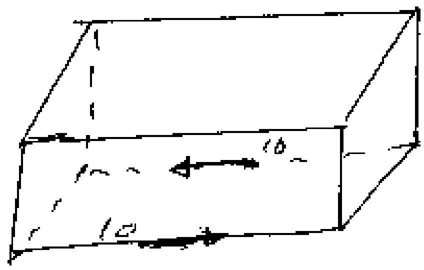


2- The Couple is moved to a parallel position in its plane.

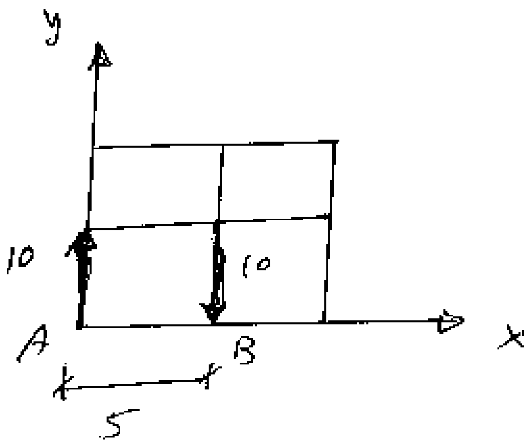


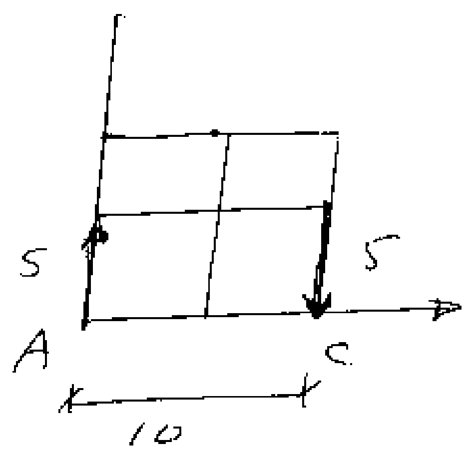
3 - The Couple is moved to a parallel plane.





4 - The distance between the forces of the Couple & the magnitude of the forces are changed, provided the moment remains the same.





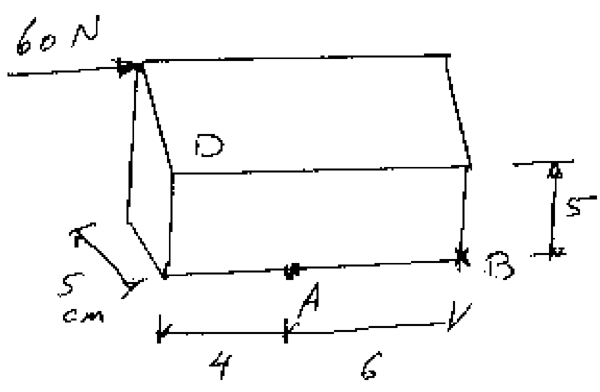
$$M = F \cdot d$$

$$F = \frac{M}{d} = \frac{50}{10} = 5$$

Resolution of A Force into A Force and A Couple

A force may be replaced by an equal parallel force through any other point of the body and a couple. This can be done by adding two equal collinear forces of opposite sense to a force system on a rigid body.

Ex: Replace the 60N force shown in the fig. by a force through (D) and a couple where forces act horizontally through A & B.



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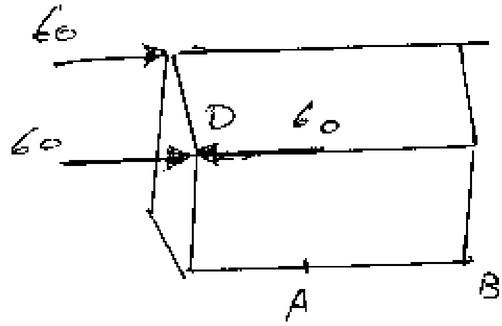

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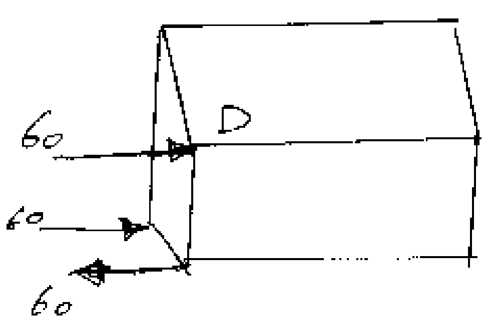
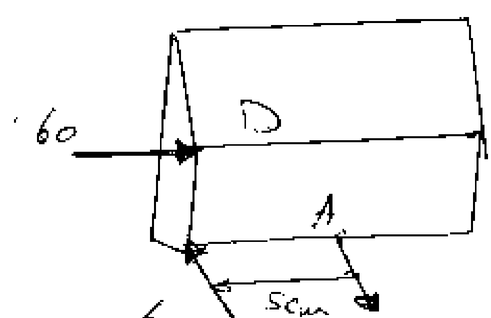
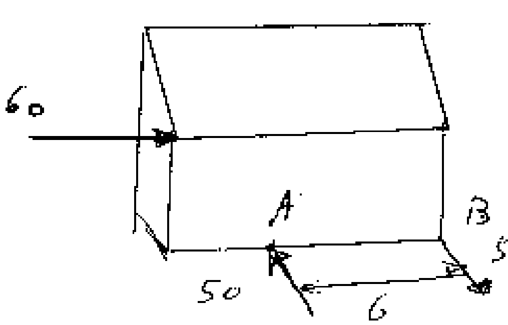




$M_C = 60 \times 5$

$M_C = 300$



$F \times 6 = 300$

$\therefore F = 50\ N$

Chapter two
Resultants of Force Systems

2. Introduction

A resultant of a force system is defined as the simplest force system which can replace the original system without changing its external effect on rigid body.

When the resultant is zero, then the body is in equilibrium and the original force system in this case called a balanced.

Type of force system	Possible resultant
Concurrent, coplanar	Force
Non Concurrent Coplanar	Force or a Couple
Concurrent non Coplanar	Force
parallel non Coplanar	Force or a Couple
Couples	Couple.
Non Concurrent, non parallel non Coplanar	Force or a Couple or a force and a couple

1 Resultant of a Concurrent, Coplanar Force System

- The resultant of a Concurrent, Coplanar force system is a single force passing through the point of Concurrence.

Procedure for Solution:

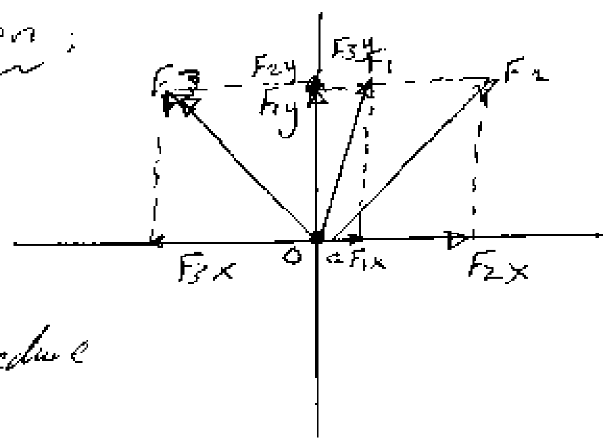


Fig. 1

To Find the resultant of a force system shown in Fig. (1, we must Follow the following procedure

1- Each force in the force System is resolved into a pair of rectangular Components.

2- Find the Components of the resultant in (x) and (y) direction (Rx and Ry) which are equal to the algebraic Sum of the x & y Components of the forces, respectively

$R_x = \Sigma F_x$ and $R_y = \Sigma F_y$

R_x and R_y: The Components of the resultant in x and y direction respectively.

ΣF_x and ΣF_y :- The algebraic Sum of the x & y Component of the force system.

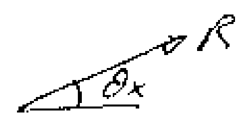
3- Find The magnitude of the resultant.

$$R = \sqrt{R_x^2 + R_y^2}$$

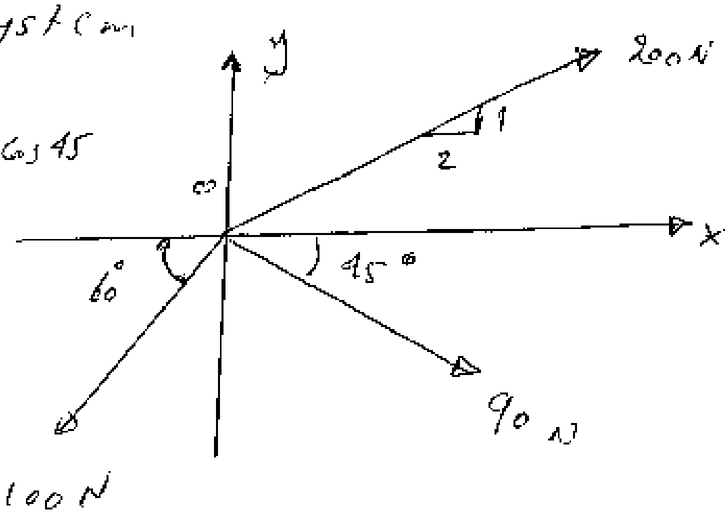
4- Find the slope of the resultant

$$\tan \theta_x = \frac{R_y}{R_x}$$

Where θ_x is the angle between the resultant and the x-axis, and the sense can be determined from the components R_x and R_y.



Ex: Determine the resultant of the concurrent coplanar force system



$(\xrightarrow{+})$ $R_x = \Sigma F_x = -100 \cos 60 + 90 \cos 45 + 200\left(\frac{2}{\sqrt{5}}\right)$

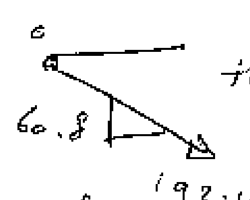
$R_x = 192.4\,N$ $\overset{o}{\longrightarrow}$

$(+\uparrow)$ $R_y = \Sigma F_y = -100 \sin 60 - 90 \sin 45 + 200\left(\frac{1}{\sqrt{5}}\right)$

$R_y = -60.8$

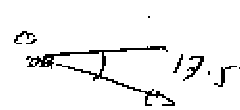
$R_y = 60.8\,N \downarrow^{o}$

$\therefore R = \sqrt{R_x^2 + R_y^2} = 201.77$ through o



or $\theta_x = \tan^{-1} \dfrac{60.8}{192.4} = 17.5°$

$\therefore R = 201.77$ through o.

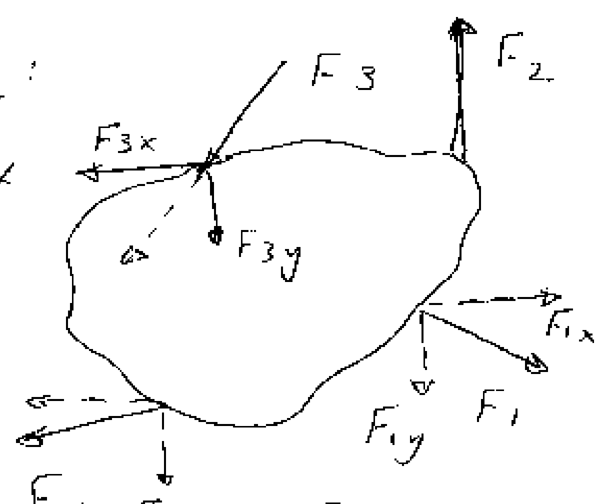


-2 Resultant of a Non Concurrent Coplanar force system

The resultant of this force system is either a single force or a couple.

Procedure for solution:

To determine the resultant analytically:-



1- Each force is resolved into rectangular components then:-

a- If the algebraic sum of the components in either x or y direction, or both is different from zero, the resultant is a force.

- The magnitude of the resultant force is obtained as following:-

$R_x = \Sigma F_x$, $R_y = \Sigma F_y$

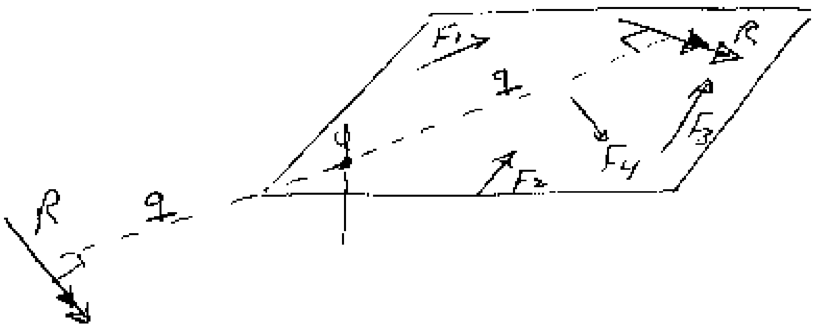
$R = \sqrt{R_x^2 + R_y^2}$

- The angle between the x-axis and the resultant is obtained from the relation

$\tan \theta_x = \frac{R_y}{R_x}$

- The distance between the resultant and a specified point is determined by the principle of moment

$R \cdot q = \Sigma M_o$

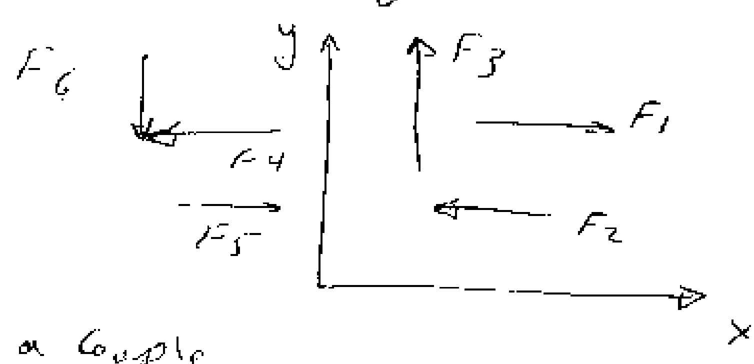


where q is $\perp$ distance from the moment axis through o to the resultant (R).

Note:. The direction of distance q is determined from the sense of (R) & of (ΣM_o).

b- If the algebraic sum of the components of the forces is zero in two different directions, the resultant is a couple in the plane of the forces

- The magnitude and the sense of rotation of the couple may be obtained as the algebraic sum of the moments of the forces with respect to any point in the plane.



Note :-

If $R_x = 0$
$R_y = 0$
$\therefore$ the resultant is a couple

If $R_x \neq 0$
$R_y = 0$
$\therefore$ the resultant is a force and it is // to x-axis

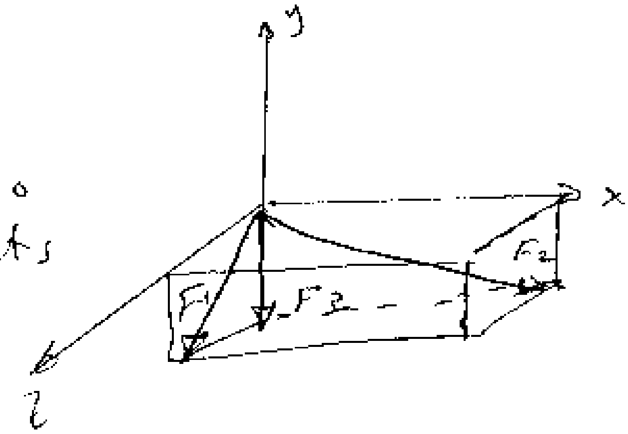
If $R_x = 0$
$R_y \neq 0$
$\therefore$ the resultant is a force and it is // to y-axis

2-3 Resultant of a Concurrent, Non Coplanar Force System

The resultant of any set of Concurrent forces must be a force passing through the point of Concurrence of the forces of the system.

Procedure of Solution

The Components of the resultant in three perpendicular directions are obtained as following. -



1- Resolve each force into a rectangular Components

$R_x = \Sigma F_x$

$R_y = \Sigma F_y$

$R_z = \Sigma F_z$

∴ the magnitude of the resultant is

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$

2- To indicate the resultant (R) in space a rectangular parallelepiped is drawn with one Corner at the point of Concurrence, and with edges parallel to the Components of the resultant R_x, R_y and R_z

The resultant (R) is indicated in space as following :-

for example, if $R_x = 175\ N \rightarrow$

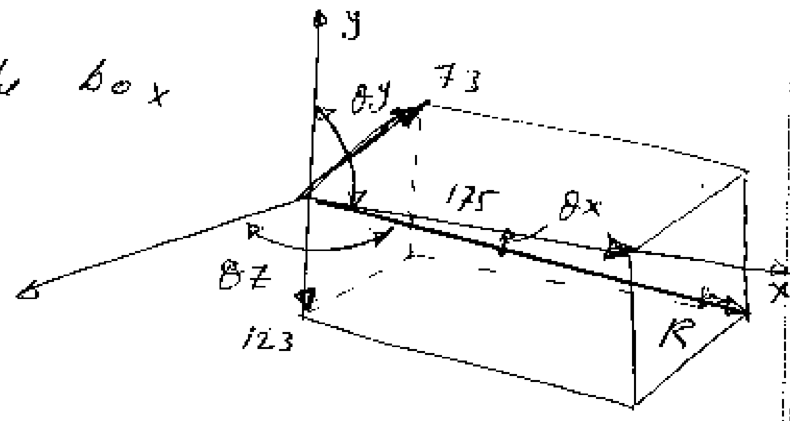
$R_y = 123\ N \downarrow$

$R_z = 73\ N$

$$\therefore R = \sqrt{(175)^2 + (123)^2 + (73)^2}$$

$= 226\ N$ from $(0, 0, 0)$ to $(1.75, -1.23, -.73)$

The diagonal of the box from the point of concurrence will represent the resultant (R).



Another method of specifying the action line of (R) in space is to give its direction cosines :-

$\cos\theta_x = \frac{R_x}{R} = \frac{175}{226}$, $\cos\theta_y = \frac{R_y}{R} = \frac{-123}{226}$

$\cos\theta_x = 0.774$ $\qquad \cos\theta_y = -0.544$

$\cos\theta_z = \frac{R_z}{R} = \frac{-73}{226} = -.323$

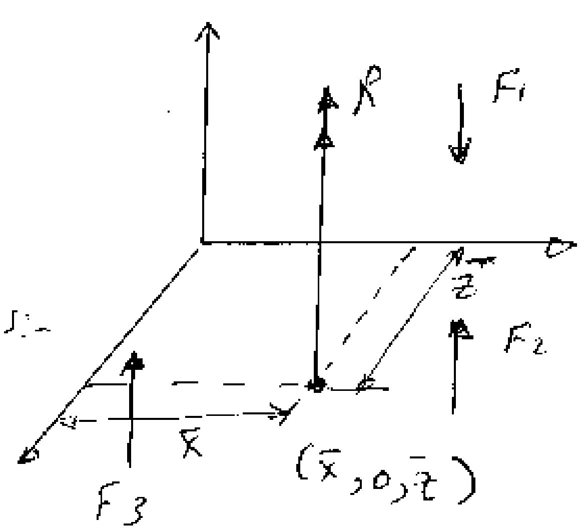
$\therefore \theta_x = 39.28^\circ \qquad \theta_y = 122.95^\circ \qquad \theta_z = 108.8^\circ$

2-4 Resultant of a parallel, NonCoplanar force System:

The resultant of parallel, NonCoplanar force system is <u>either</u> a <u>single force or a Couple</u>.

- If the resultant is a single force, its magnitude is equal to the algebraic sum of the forces of the system, and its position in space can be determined by the Coordinates of the intersection of its action line with a plane perpendicular to the forces of the system.

as an example In Fig (3), The resultant of forces is parallel to the y-axis and it is Completely determined by the equations:-



$R = \Sigma F_y$

$R\bar{x} = \Sigma M_z$

$R\bar{z} = \Sigma M_x$

- If the sum of forces is zero and the sum of the moments about one or both of two rectangular axes in a plane perpendicular to the forces, is not zero, the resultant is a Couple.

[For forces // to y-axis, the resultant is a Couple if $\Sigma F_y = 0$ and $\Sigma M_x \neq 0$ or $\Sigma M_z \neq 0$ or $\Sigma M_x \neq 0$ and $\Sigma M_z \neq 0$].

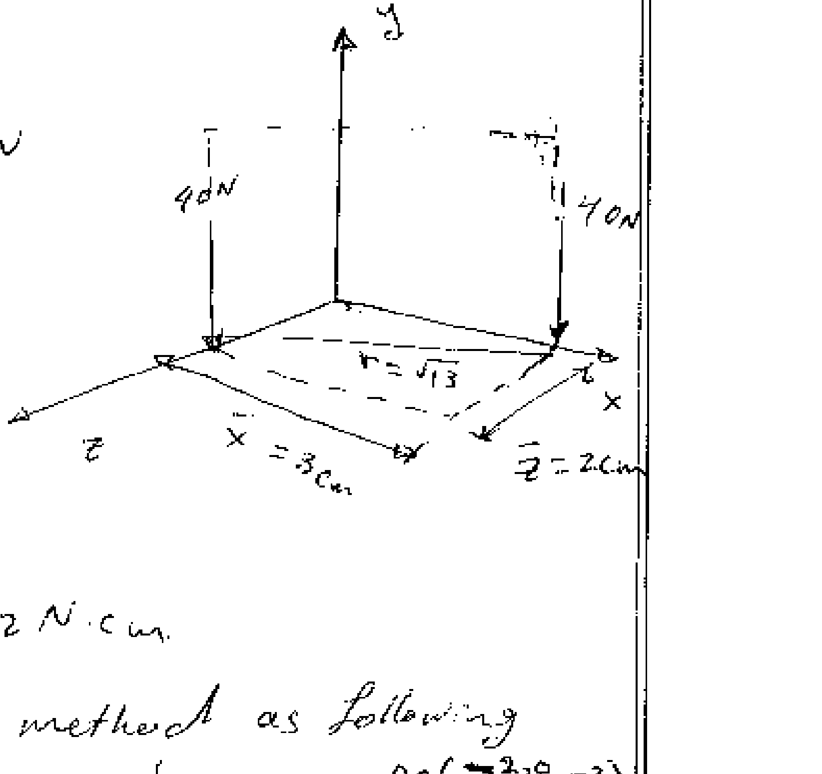
The resultant couple can be shown on a sketch by placing an upward force on one moment axis and an equal downward force on the other moment axis, so spaced as to produce the proper resultant moments.

For example:-

$\Sigma F_y = 0$, $\Sigma M_z = 120\ N\text{-}cm$

$\Sigma M_x = 80\ N.cm$

assume the magnitude of the forces of the couple $= 40N$



since $\Sigma M_x = 80\ N.cm$

$\therefore\ \bar{z} = \frac{80}{40} = 2\ cm$

since $\Sigma M_z = 120\ N.cm$

$\bar{x} = \frac{120}{40} = 3\ cm$

$r = \sqrt{2^2 + 3^2} = \sqrt{13}$

$M_c = 40\sqrt{13} = 144.2\ N.cm$

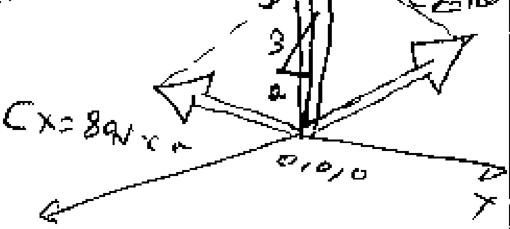
Or using open vector method as following

$\Sigma F_y = 0 \quad \Sigma M_z = 120\ N.cm$

$C_z = 120\ N.cm$

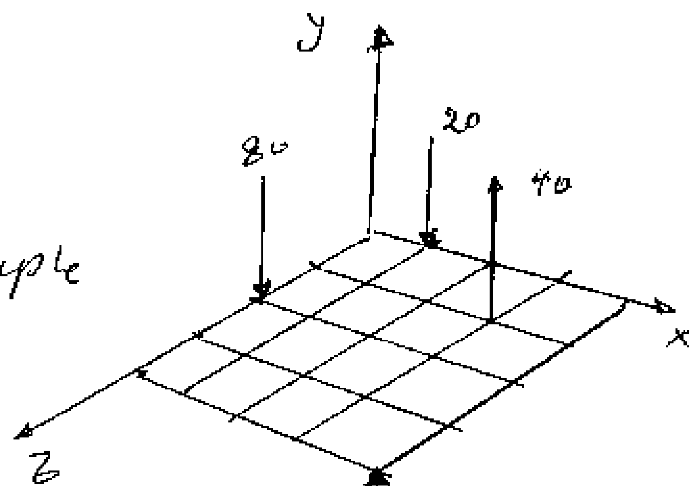
$\Sigma M_x = 80\ N.cm$

$C_x = 80\ N.cm$



$R_c = \sqrt{120^2 + 80^2} = 144.2\ N.cm$ from $(0,0,0)$ through $(-2,0,-3)$

Ex. Determine the resultant of the force system

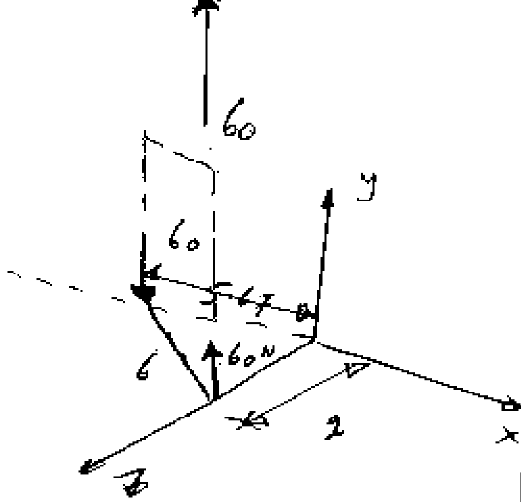


(+↑) $R = \Sigma F_y$

$= 60 + 40 - 80 - 20$

$R = 0$

∴ the resultant is a Couple



$M_x = 60(4) + 40(1) - 80(2)$

$= +120 \ N.cm$

∴ $\Sigma M_x = 120 \ N.cm$

$\Sigma M_z = -40(3) - 60(4) + 20(1)$

$= -340 \ N.cm$

∴ $\Sigma M_z = 340 \ N.cm$

$\bar{x} = \frac{\Sigma M_z}{F}$, $\bar{z} = \frac{\Sigma M_x}{F}$

assume the forces of the Couple = 60 N $\Rightarrow \bar{x} = \frac{340}{60} = 5.67$ cm

$\bar{z} = \frac{120}{60} = 2$ cm

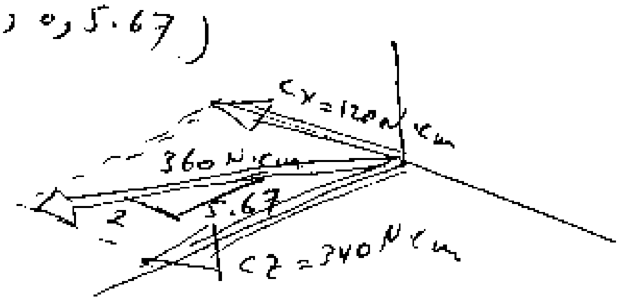
$C = 60 \times 6 = 360 \ N.cm$

or $\Sigma M_x = 120 \ N.cm \longrightarrow C_x = 120 \ N.cm$

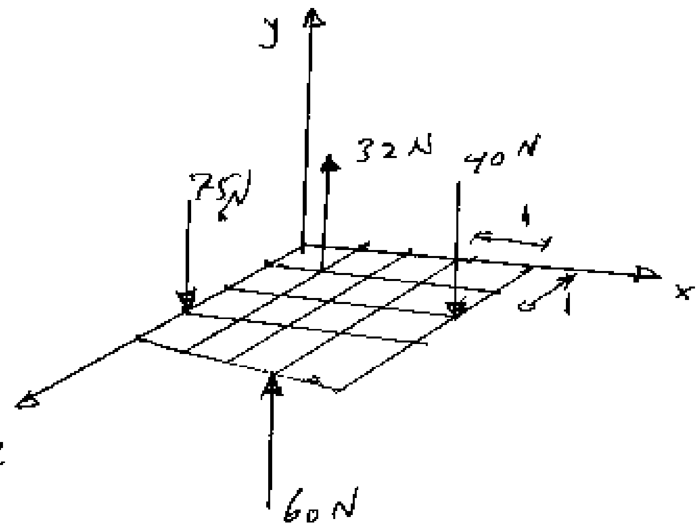
$\Sigma M_z = 340 \ N.cm \qquad C_z = 340 \ N.cm$

∴ $R_c = \sqrt{C_x^2 + C_z^2} = \sqrt{120^2 + 340^2} = 360 \ N.cm$

m (0, 0, 0) through (−2, 0, 5.67)



Ex: Determine the resultant of the four parallel forces and show it on a sketch.



(+↑) $R = \Sigma F_y$

$R = 60 + 32 - 75 - 40$

$R = -23\ N$

$R = 23\ N \downarrow$

The moment of the forces @ x-axis can be calculated as:-

$+\ \Sigma M_x = 60\ (4) - 40(2) - 75(3) + 32(1)$

$\Sigma M_x = 33\ N.cm$

$R . \bar{z} = \Sigma M_x = 33\ N.cm$

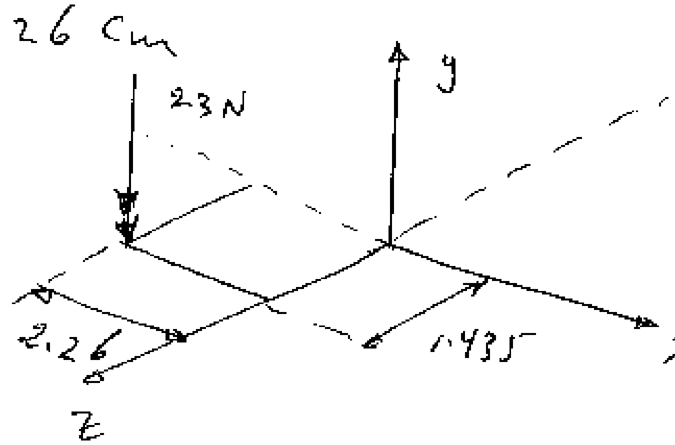
$\therefore\ \bar{z} = \frac{33}{23} = 1.435\ cm$

$\Sigma M_z = -60(3) + 40(4) - 32(1) = 52\ N.cm$

$R . \bar{x} = 52$

$\therefore\ \bar{x} = \frac{52}{23} = 2.26\ cm$

$\therefore\ R = 23\ N \downarrow$



$\therefore\ R = 23\ N \downarrow$

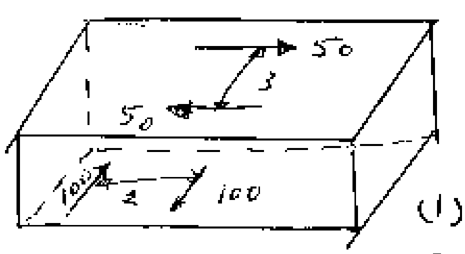
Through $(-2.26, 0, 1.435)$

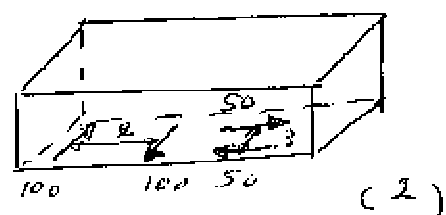
2-5 Resultant of a System of Couples in Space

The resultant of any force system Composed of Couples is a Couple.

a- The resultant of Couples in same plane or in parallel planes is equal to algebraic Sum of the moments of the original Couples.

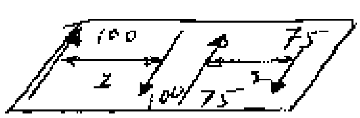
For example:

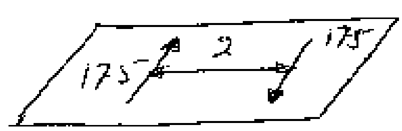




$R_c = 50 * 3 + 100(2)$

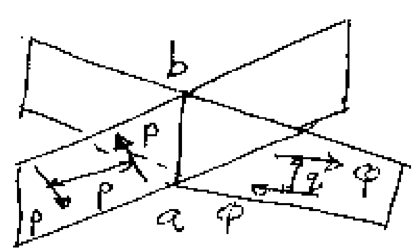
$R_c = 350$ N.cm





$R_c = 175(2) = 350$ N.cm

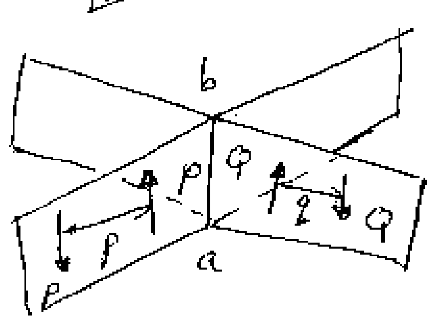
b- The resultant of Couples in non parallel planes is also a Couple.



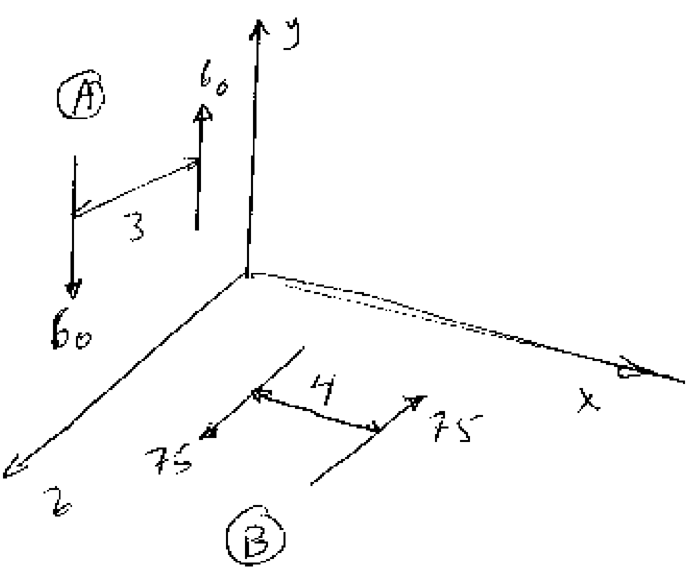
Parallel, non Coplanar forces

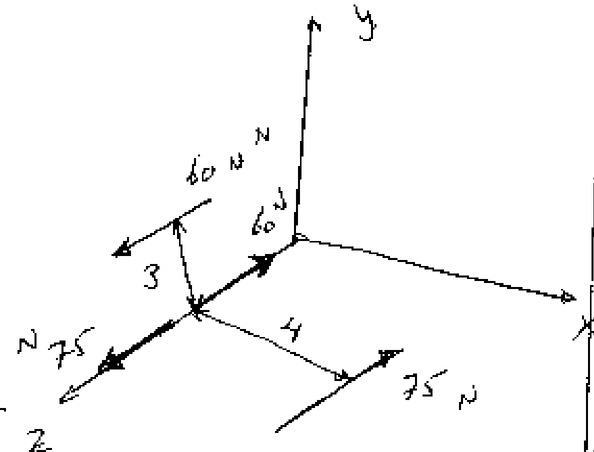
$R = 0$

∴ The resultant is a Couple, in a plane // to ab.



Ex. Determine the resultant of the two Couples shown in Fig. below.

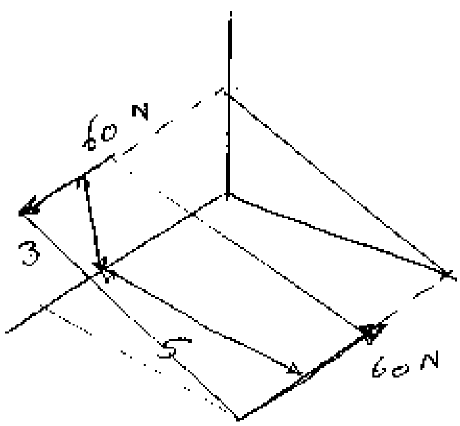




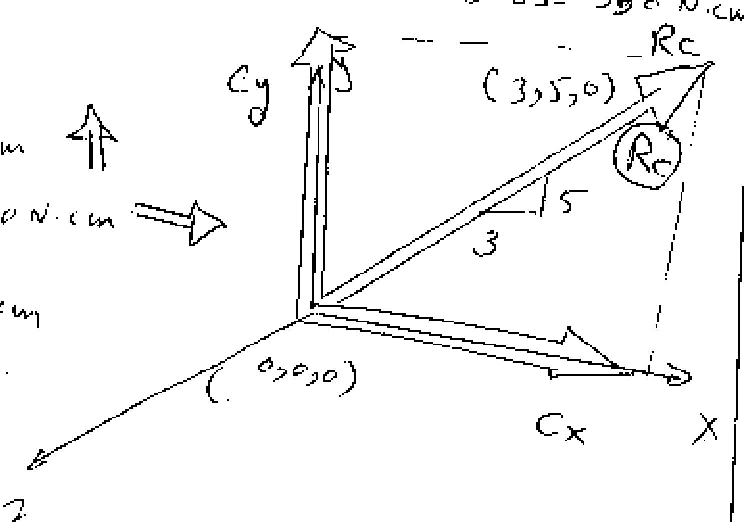
$75 \times 4 = 300$ N.cm

For forces 60 N

$d = \frac{300}{60} = 5$ cm



$R_C = 60 \times 5.83 = 350$ N.cm



or

$C_y = 75 \times 4 = 300$ N.cm

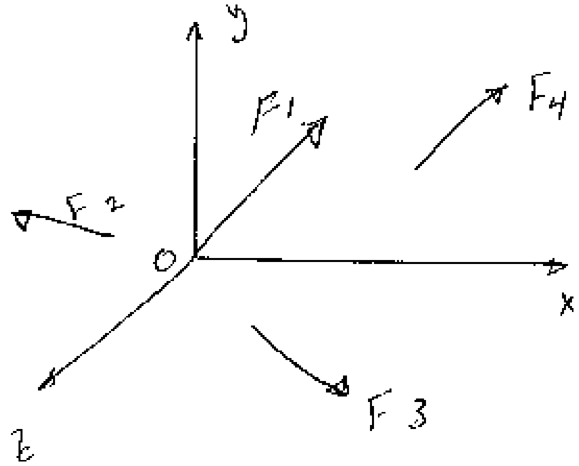
$C_x = 60 \times 3 = 180$ N.cm

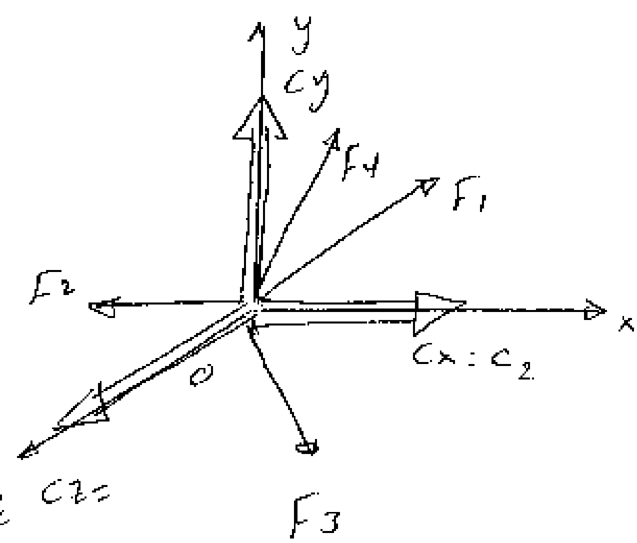
$\therefore R_c = \sqrt{C_x^2 + C_y^2} = 350$ N.cm

from (0,0,0) through (3,5,0)

Resultant of a Non Concurrent, Non Parallel Non Coplanar Force System.

The resultant of this force system can be a single force or a couple, but in general it is a force and a couple.
The resultant of non concurrent, non parallel and non coplanar force system can be obtained by resolving each force into a parallel force through some common point origin and a couple.





Note :-
F_1 is in x-y plane & passing through point o
F_4 is in x-y plane.
F_2 is in y-z plane.
F_3 is in z-x plane.
∴ The resultant force is:

$R = \sqrt{Rx^2 + Ry^2 + Rz^2}$ when $Rx = \Sigma Fx$, $Ry = \Sigma Fy$
$Rz = \Sigma Fz$.
and the resultant couple is:

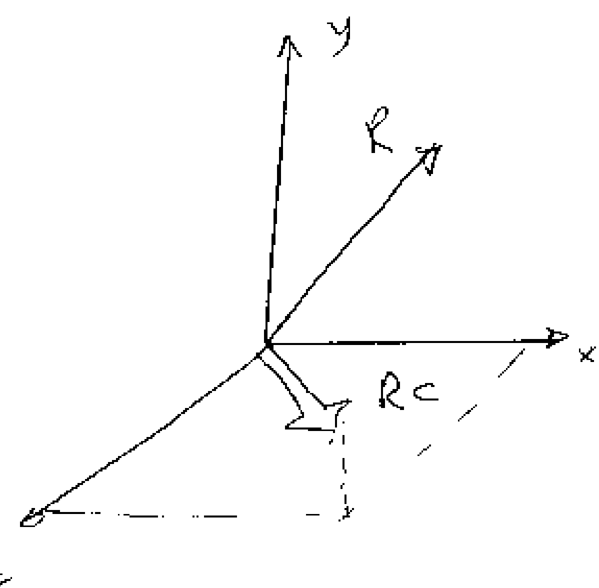
$R_c = \sqrt{Cx^2 + Cy^2 + Cz^2}$

where

$Cx = \Sigma M_x$

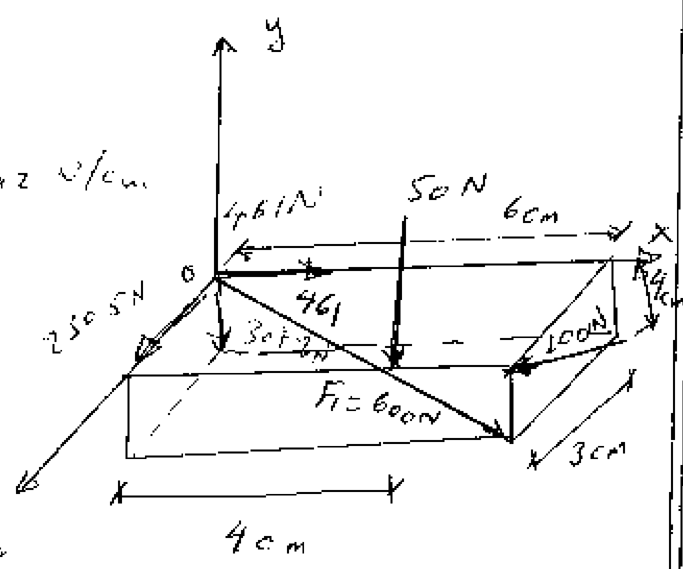
$Cy = \Sigma M_y$

$Cz = \Sigma M_z$



Example :-

Determine the resultant of the force system shown in Fig.



Solution

...ale $= \dfrac{600}{\sqrt{6^2 + 3^2 + 4^2}} = 76.22$ N/cm

$= 76.22(6) = 460.92$

$= 461$ N

$76.22(4) = 307.3$ N

$= 76.22(3) = 230.5$ N

$= 100\left(\frac{3}{5}\right) = 60$ N

$= 100\left(\frac{4}{5}\right) = 80$ N

$= \Sigma F_x = 461$ N

$= \Sigma F_y = 80 - 307.3 - 50 = 277.3$ N

$R_z = \Sigma F_z = 230.5 + 60 = 290.5$ N

$R = \sqrt{(461)^2 + (277.3)^2 + (290.5)^2} = 611.4$ N from $(0,0,0)$ through $(4.61, -2.77, 2.9)$

$\Sigma M_x = C_x$

$\Sigma M_x = 60(4) - 50(3) = 90\ N.cm$

$C_x = 90\ N.cm$

$C_y = \Sigma M_y$

$\Sigma M_y = 60(6) = 360\ N.cm$

$C_z = \Sigma M_z$

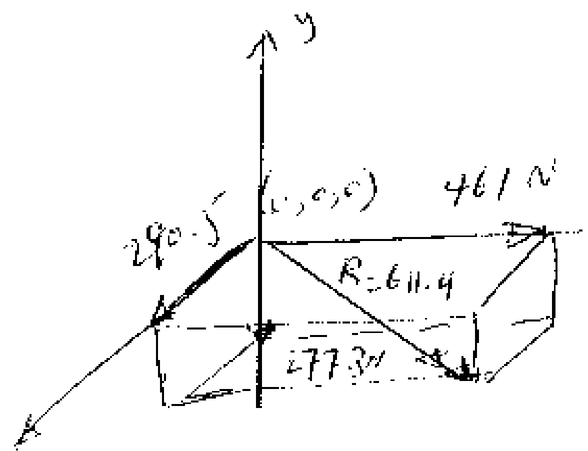
$\Sigma M_z = -80(6) + 50(4) = 280\ N.cm$

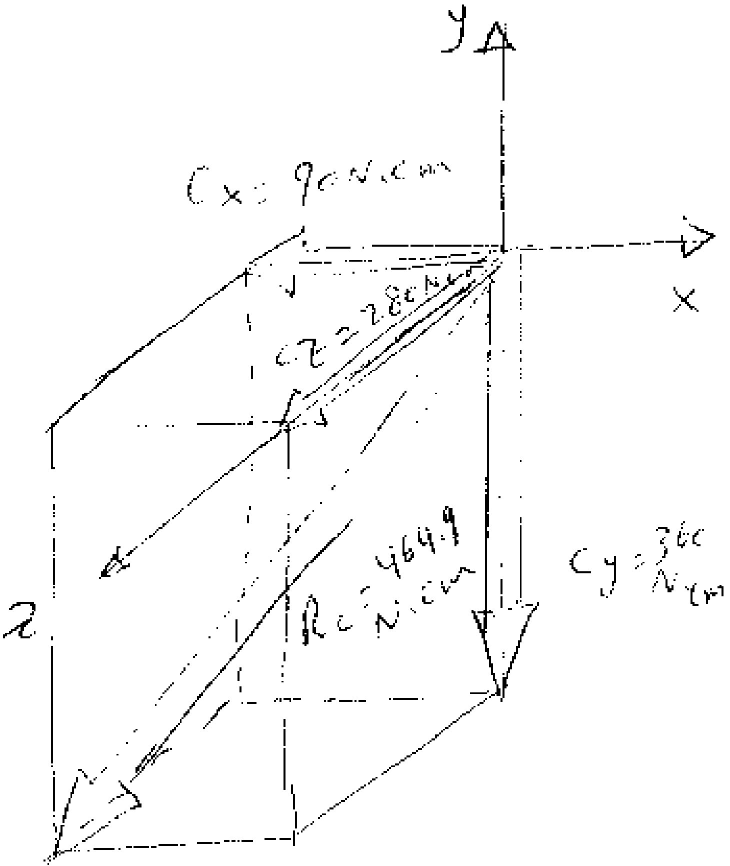
$C_z = 280\ N.cm$

$$R_c = \sqrt{C_x^2 + C_y^2 + C_z^2} = \sqrt{90^2 + 360^2 + 280^2}$$

$= 464.9\ N.cm$

From $(0,0,0)$ through $(-9, -36, 28)$





(Home work)

Q.1 Determine the resultant of the general force system shown in Fig.1 & Fig. 2

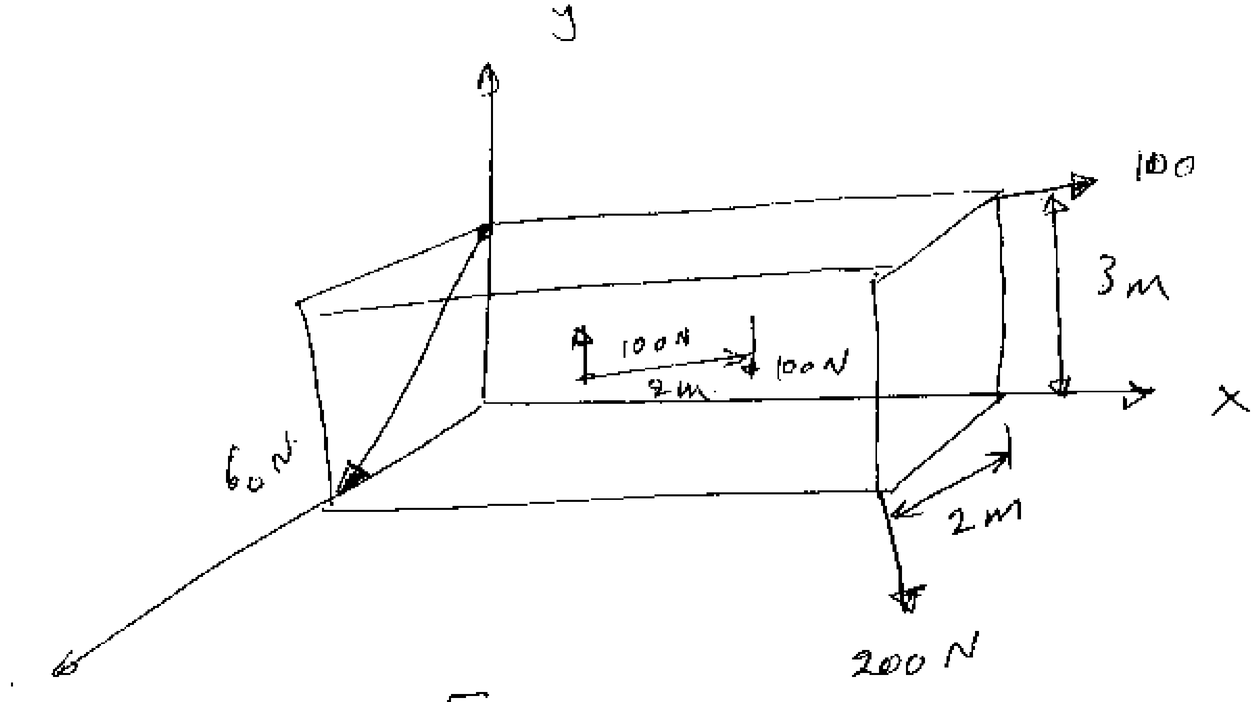


Fig. 1

Example 2

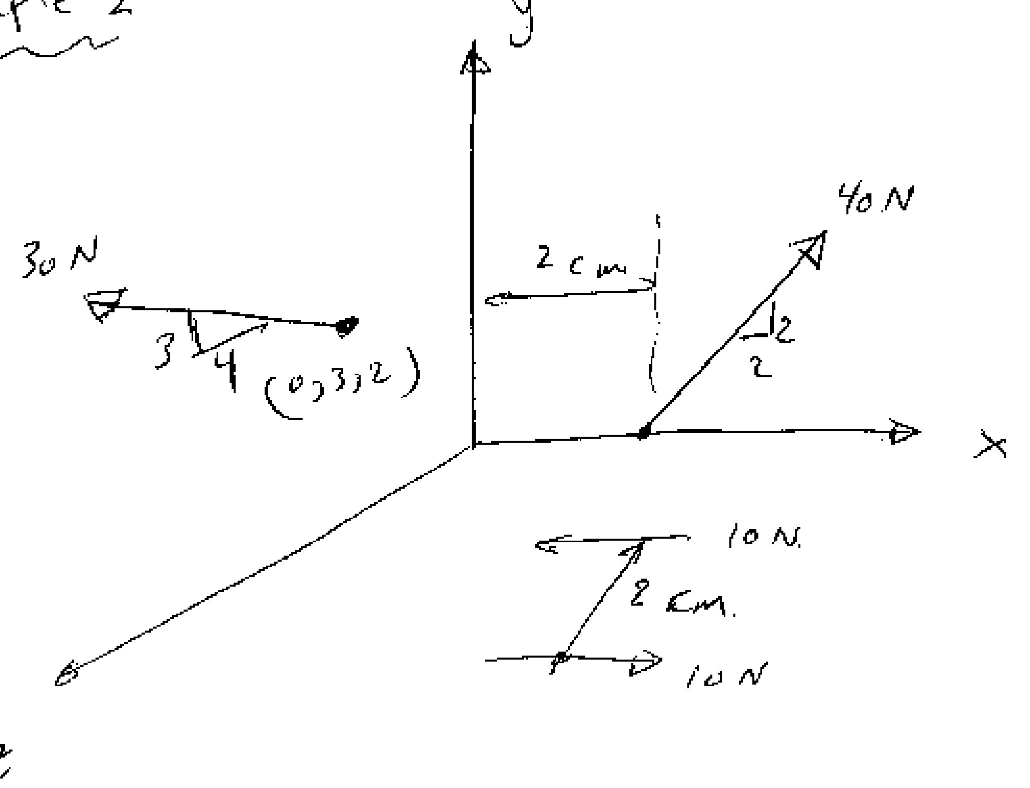


Fig. 2

Chapter Four
Equilibrium

The body is in equilibrium when a system of forces acting on it has no resultant, (equal to zero).

To study the force system acting upon any body or any portion of a body, it is first necessary to recognize both the known & unknown forces acting on the body.

4-1 FRee - Body Diagrams

A free - body diagram is a sketch of a body, a portion of a body, completely insolated and free from all other bodies, showing the forces exerted by all other bodies on the one being considered.

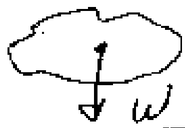
A free - body diagram has three characteristics:-

1- It is a sketch of the body.
2- The body shown is separated (cut free) from all other bodies and from supports.
3- The action on the free body of each removed is shown as a force or forces on the diagram.

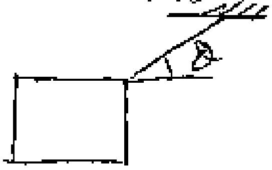
Note:- The sense of unknown forces may be assumed and corrected later if it is incorrect.

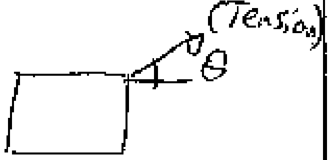
The body to be removed	Sketch of reacting bodies	Action of body removed

Earth.

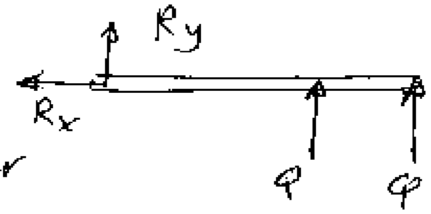


rope, Cable

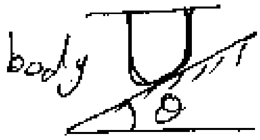


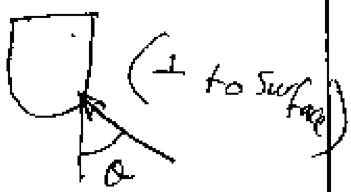


Roller or ball and smooth pin

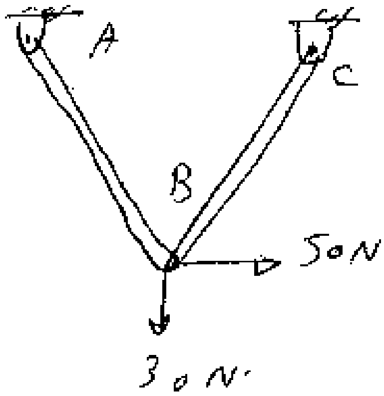


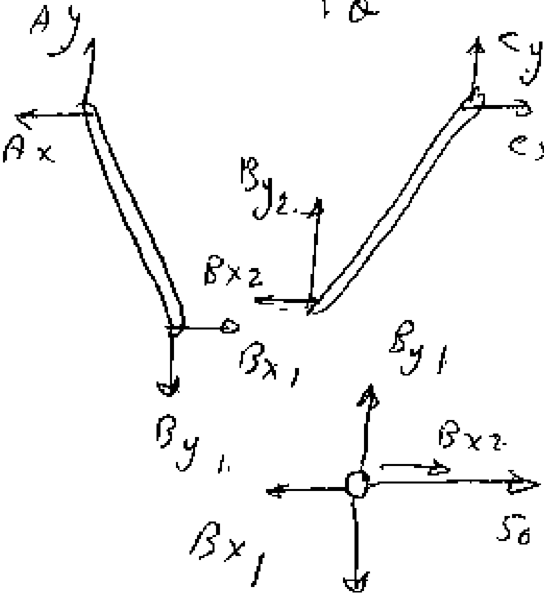
smooth surface



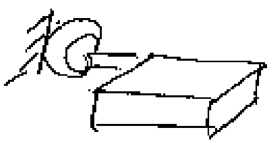


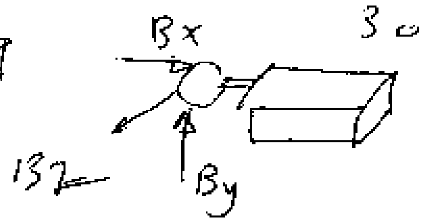
Smooth pin with additional forces on pin



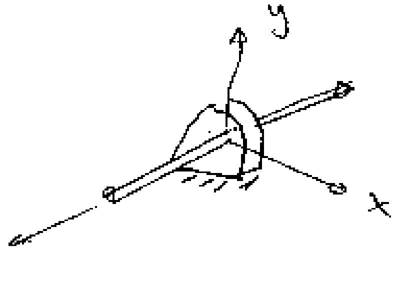


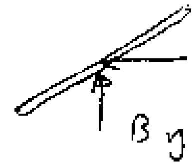
Ball & socket



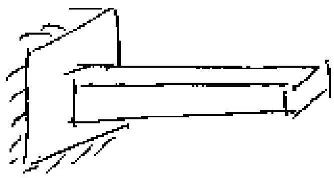


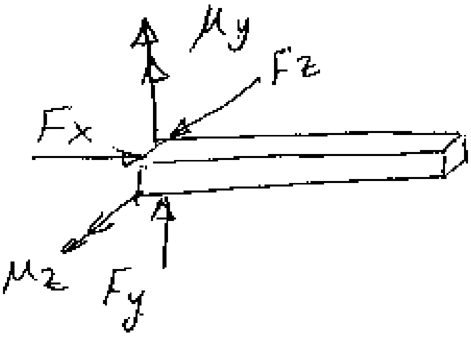
Smooth bearing on a shaft



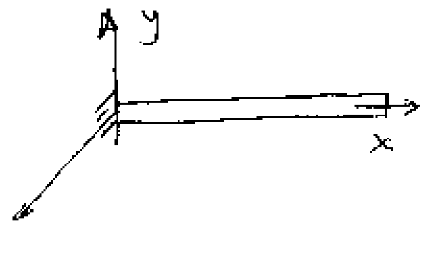


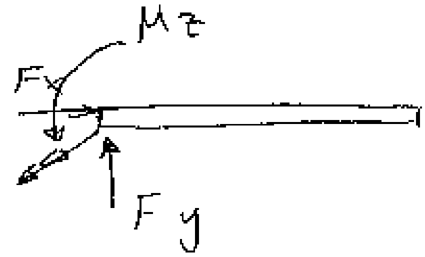
Fixed end beam (three dimensional)



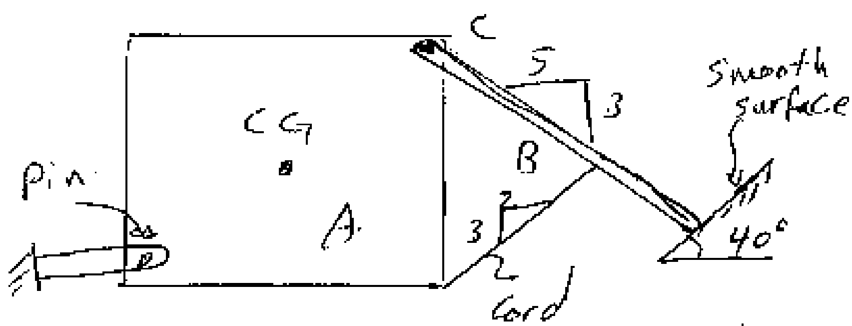


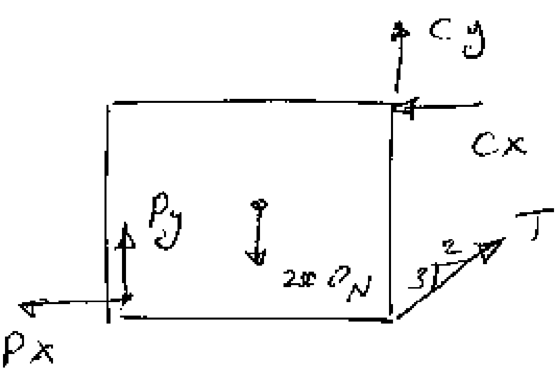
Fixed end beam (Two dimensional)

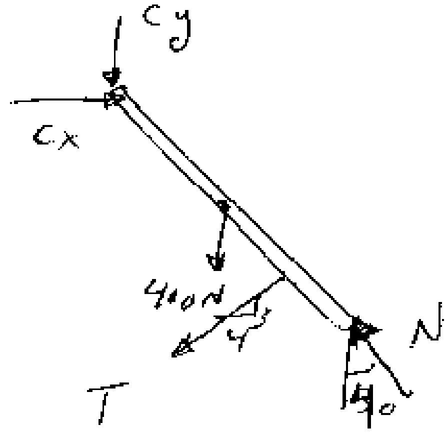




Ex: Draw a free-body diagram of body A weights 200N and B weights 40N.







General procedure for the solution of problems in Equilibrium

1- Determine the given data & what the unknowns results are required.

2- Draw F.B.D for the member on which the unknown forces are acting.

3- Determine the type of force system acting on the F.B.D.

4- Compare the no. of unknowns on the F.B.D. with the no. of independent equations of equilibrium and:-

a) if the no. of equation = the no. of unknowns, then start the solution.

b) if the no. of unknowns > the no. of independent equations, then draw F.B.D for another body and repeat step 3 & 4.

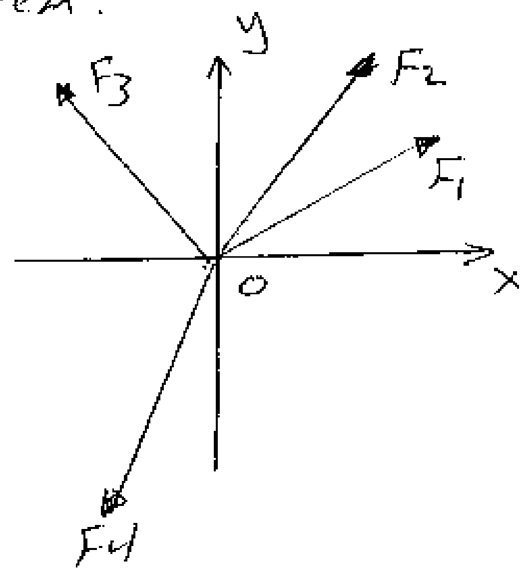
5- If no. of unknowns in the second F.B.D = the no. of equations then solve the problem.
If it is not then repeat step (4-b)

- If there are still too many unknowns after drawing F.B.D for all bodies, then the problem is statically indeterminate -

Equations of Equilibrium for a Concurrent Coplanar force system

The resultant of a concurrent, coplanar force system is a single force, and when this resultant force is zero, the body on which the force system acts is in equilibrium.

The equations necessary to ensure a zero resultant are the equation of equilibrium for this type of force system.



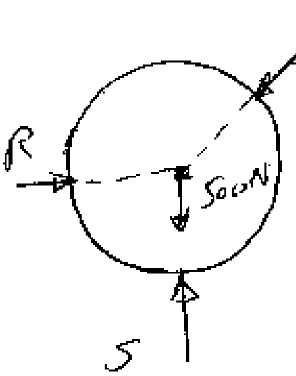
There are three sets of equation of equilibrium for this type of force system.

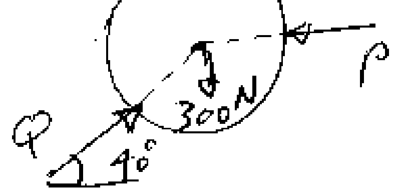
① $\sum F_x = 0$, $\sum F_y = 0$

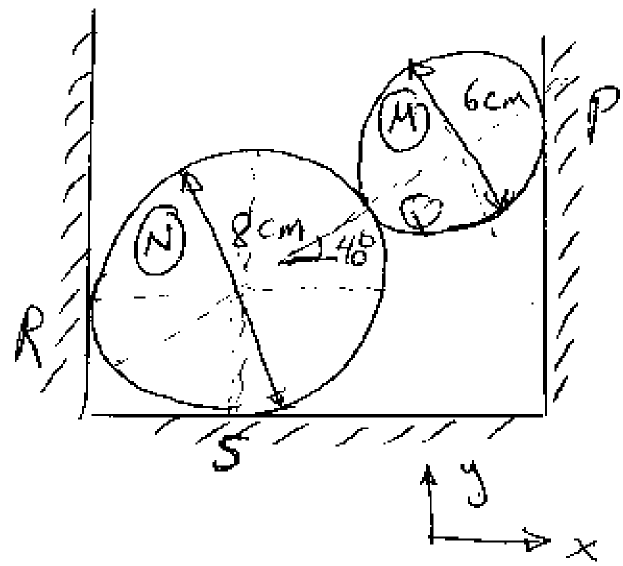
② $\sum F_x = 0$, $\sum M_A = 0$ [A is any point in the plane and not on the y- axis]

③ $\sum M_A = 0$ $\sum M_B = 0$ [where line AB does not pass through the point of concurrence of the forces of the system]

Ex. The 300 N shaft (M) & the 500 N shaft (N) are supported as shown in Fig. Neglecting friction at the contact surfaces determine the reactions at R & S on shaft N.







for M

(+↑) $\Sigma F_y = 0$

$Q \sin 40 - 300 = 0$ $\quad$ ∠40° on M

$\therefore Q = 467\ N$

for N

(+↑) $\Sigma F_y = 0$

$S - 500 - Q \sin 40 = 0$

$S = 500 + 467 \sin 40 = 800\ N$ ↑ on N

(+→) $\Sigma F_x = 0$

$R - Q \cos 40 = 0$ $\quad$ $\therefore R = 358\ N$ → on N

Equilibrium of Bodies Acted on by Non-Concurrent, Coplanar force systems

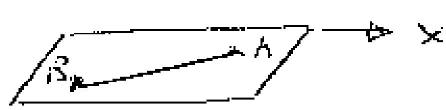
The resultant of this force system is either a single force or a couple. The equations which eliminate all possible resultant are the equations of equilibrium. For this type of force system, there are only three independent equations of equilibrium

There are three sets of equation of equilibrium

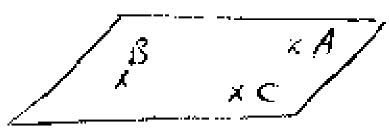
1- $\Sigma M_A = 0$ [A is any point in the plane of forces or any axis $\perp$ to the plane]

$\Sigma F_x = 0$
$\Sigma F_y = 0$

2- $\Sigma M_A = 0$ [A is any point in the plane]
$\Sigma M_B = 0$ [B is any another point in the plane]
$\Sigma F_x = 0$ [x-axis is in the plane of forces and not $\perp$ to the line AB

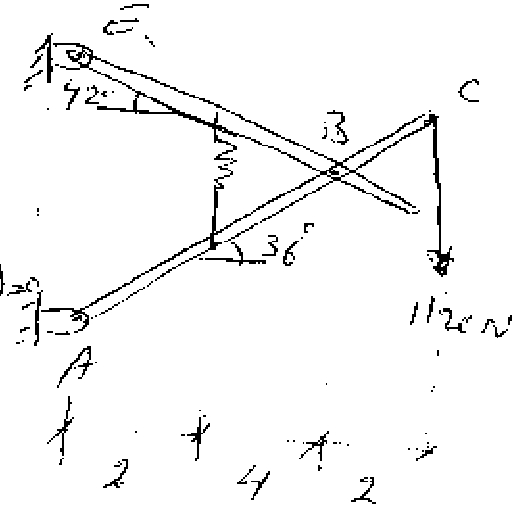


3- $\Sigma M_A = 0$
$\Sigma M_B = 0$ [point A, B and C are in the plane and are not collinear]
$\Sigma M_C = 0$



Ex.1 The tension in the spring is 540N the weights of members and friction can be neglected. Determine the horizontal and vertical components of the pin reaction at (B) on member EB.

Solution



From F.B D (2)

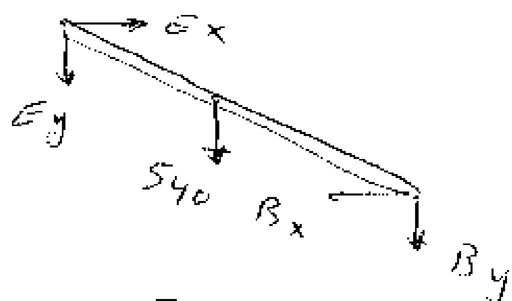
(+) $\Sigma M_A = 0$

$E_x(6\tan 42 + 6\tan 36) + 1120(8) = 0$

$9.76\,E_x + 8960 = 0$

$E_x = -918\,N$

$\therefore E_x = 918\,N \leftarrow$ on EB



F.B D (1)

F.B.D (1)

$\Sigma F_x = 0$

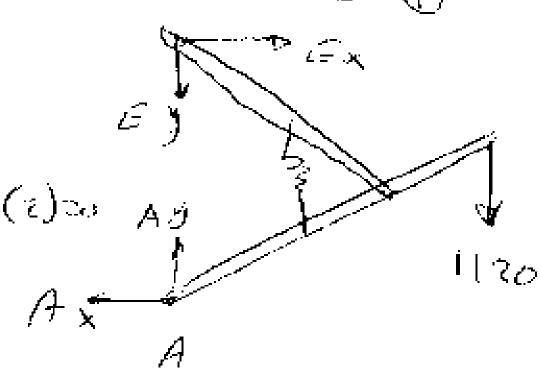
$B_x = E_x = -918\,N$

$\therefore B_x = 918\,N \rightarrow$ on EB

(+) $\Sigma M_E = 0$

$B_x(6\tan 42) + B_y(6) + 540(2) = 0$

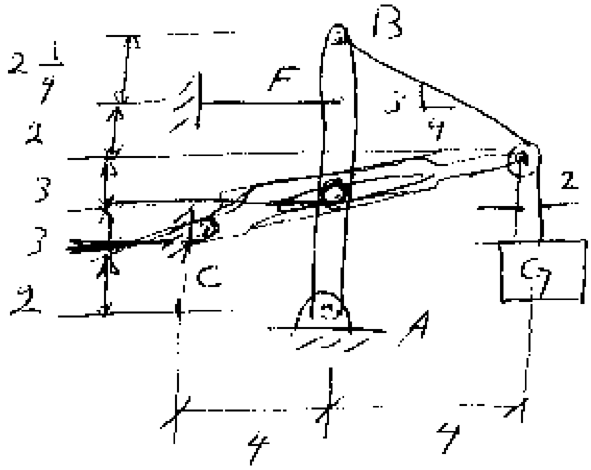
$B_y = 647\,N \downarrow$ on EB.



F.B.D (2)

Ex. 2 Body (G) weights 1500 N. Determine the horizontal and vertical components of force at (A) on AB.

Solution



From F.B.D (3)

$\Sigma M_D = 0$

$T = 1500\ N$ as shown

$\Sigma F_x = 0$

$D_x = T\frac{4}{5} = 1200\ N \rightarrow$ on pully

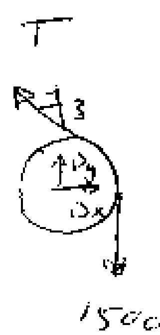
$D_x = 1200\ N \leftarrow$ on CD

$\Sigma F_y = 0$

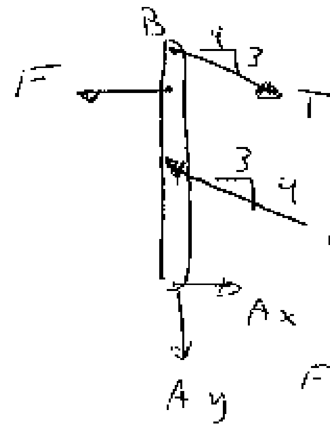
$D_y + \frac{3}{5}T - 1500 = 0$

$D_y = 600\ N \uparrow$ on pully

$= 600\ N \downarrow$ on CD



F.B D(3)



F.B D(1)

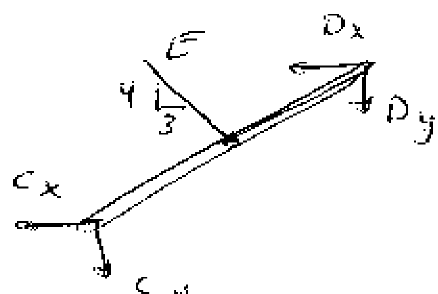
From F.B.D 2

$\Sigma M_c = 0$

$5E + 8D_y - 6D_x = 0$

$E = 480\ N$ on CD

$E = 480\ N$ on AB



F.B D(2)

From F.B.D (1)

$\Sigma F_y = 0$

$-A_y - T\frac{3}{5} + E\frac{4}{5} = 0$

$A_y = -516$

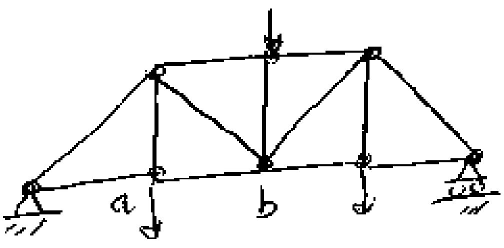
$\boxed{\therefore A_y = 516\ N \uparrow \text{ on } AB}$

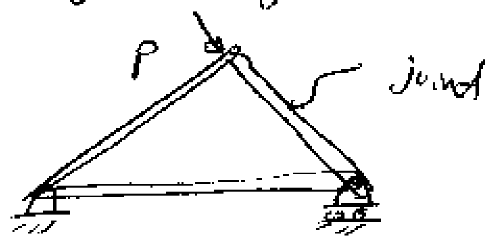
$\Sigma M_F = 0 \Rightarrow T(\frac{4}{5})(2\frac{1}{4}) - A_x(10) + E(\frac{3}{5})5 = 0$

$\boxed{A_x = 414\ N \rightarrow \text{ on } AB}$

" TRUSSES "

A truss is a structure made up of a number of members fastened together at their ends in such a manner as to form a rigid body.





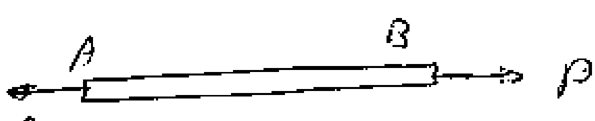
The calculation for the internal forces in the members of a truss are based on the following assumptions.

1- The members of the truss are joined together by smooth pins

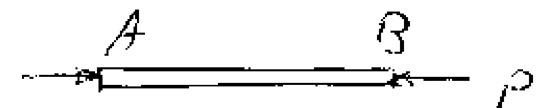
2- The loads and reaction act only at the joints.

3- The weights of the members can be neglected.

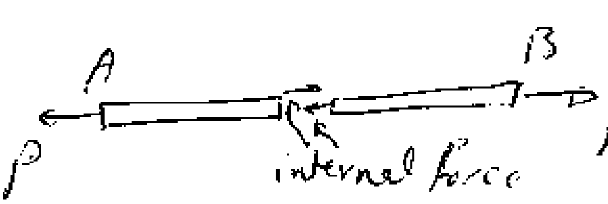
Each member of the truss in a two-force member

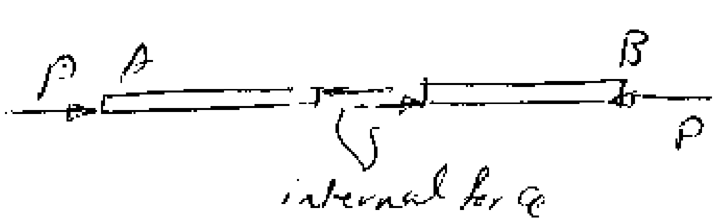


Tensile force



Compressive force

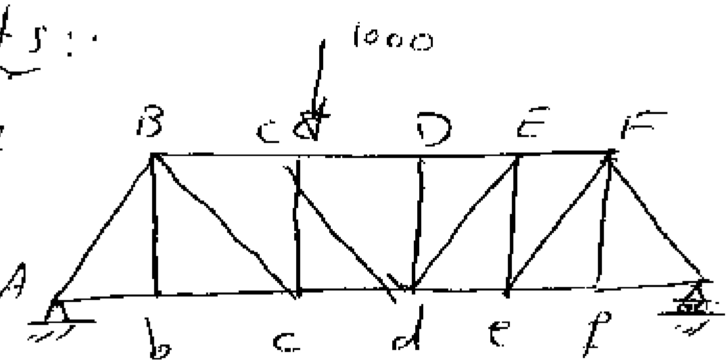




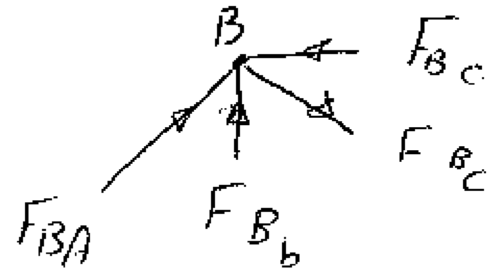
Analysis of Trusses

The forces in the members of a truss can be determined by two methods:

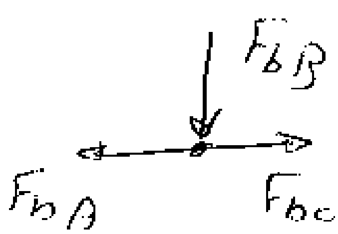
1- **method of joints:**



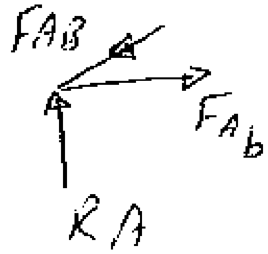
a single joint in the truss is isolated as a free body and then applying equation of equilibrium



$\Sigma F_x = 0$
$\Sigma F_y = 0$



Note: The forces applied at each joint represent a concurrent coplanar force system.

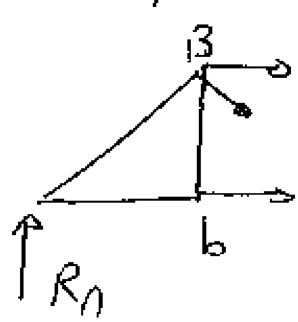


2 - **Method of section:** Two or more non-concurrent members are cut to obtain a free body and then applying equations of equilibrium

$\Sigma F_x = 0$

$\Sigma F_y = 0$

$\Sigma M_B = 0$

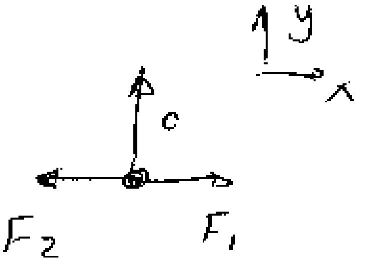


Note ①:- The forces applied at any section represent <u>non concurrent coplanar force system.</u>

Note ②. If all but one of the members at a joint are Collinear the force in the non collinear member Can be determined by summing forces perpendicular to the collinear members

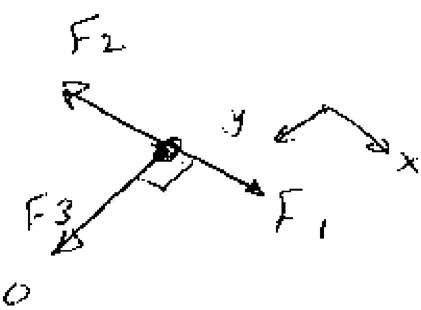
$\rightarrow \sum F_x = 0 \longrightarrow F_1 = F_2$

$\uparrow \sum F_y = 0 \longrightarrow F_y = 0$



$\searrow \sum F_y = 0 \longrightarrow F_3 = 0$

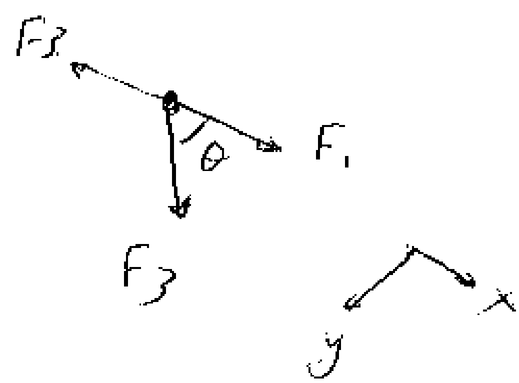
$\nearrow \sum F_x = 0 \rightarrow F_1 = F_2$



$\sum F_x = 0 \longrightarrow F_1 = F_2$

$\sum F_y = 0 \longrightarrow F_3 \sin\theta = 0$

$\therefore F_3 = 0$



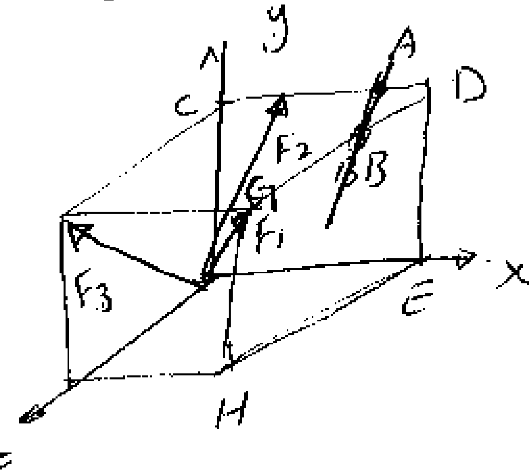
See example 4-6 page 171

Equilibrium of Bodies Acted on by Concurrent, Non Coplanar Force Systems

The resultant of a Concurrent force System in space is a single force through the point of concurrence. So A complete set of equations of equilibrium for this force System is:

(1) $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma F_z = 0$

one or more of the above force equations Can be replaced by the same number of moment equations, for example:-



(2) $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma M_{CD} = 0$

[CD is not // to Z axis and it does not intersect the Z axis]

Note: Line CD or DE might be more convenient.

(3) $\Sigma F_x = 0$ 6 $\Sigma M_{HE} = 0$, $\Sigma M_{CD} = 0$

(4) $\Sigma M_{HG} = 0$, $\Sigma M_{HE} = 0$, $\Sigma M_{CD} = 0$

See Ex. 4-9 page 192.

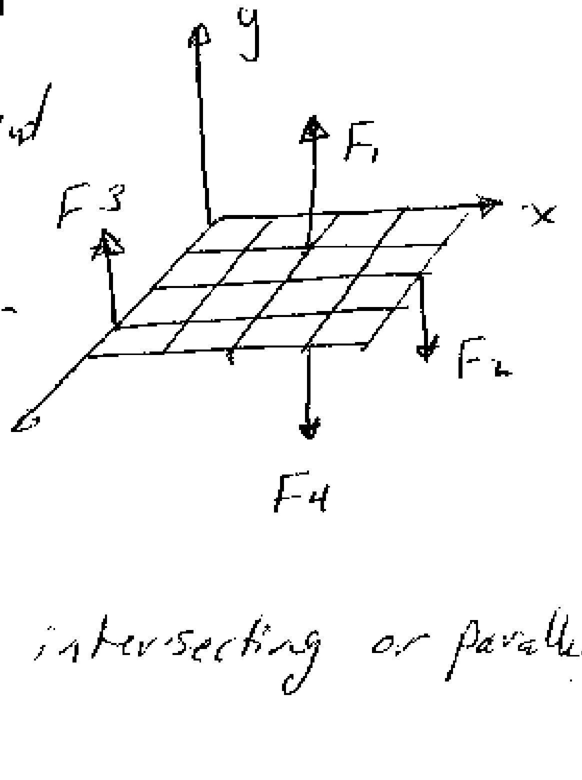
Equilibrium of Bodies Acted on by Parallel, NonCoplanar Force System:

The resultant of a parallel force system in space is either a single force or a couple.

A complete set of independent equations of equilibrium for this system is:-

$\Sigma F_y = 0$, $\Sigma M_x = 0$, $\Sigma M_z = 0$ [y axis is parallel to the forces]

Another set of independent equations of equilibrium for this force system is:-



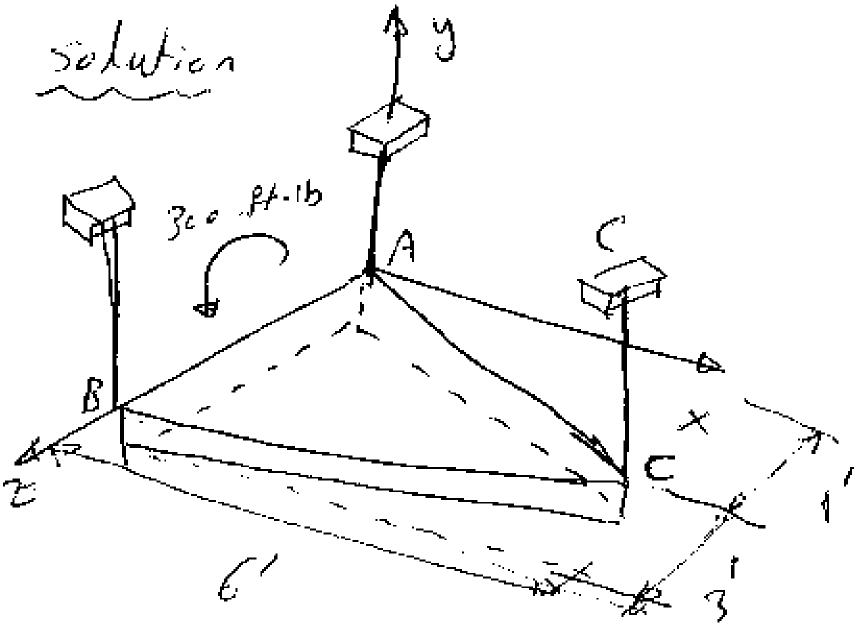
$\Sigma M_A = 0$, $\Sigma M_x = 0$, $\Sigma M_z = 0$

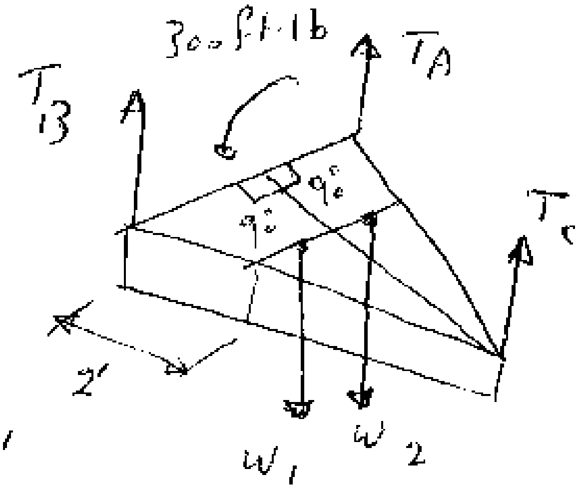
[A is an axis not intersecting or parallel to the y-axis]

Example Page 198

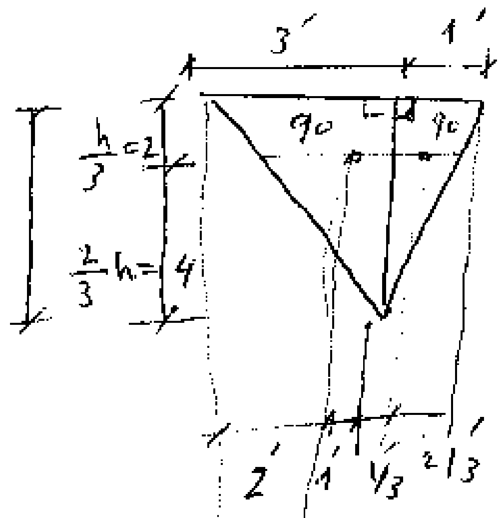
A homogeneous triangular steel plate, 6 in, thick, is supported in a horizontal position by vertical cables at the vertices as shown in Fig. and is acted on by a 300 ft-lb Couple in the yz plane. Determine the tension in each Cable. The specific weight of steel is 490 lb/ft3

Solution





thickness = 6" = 0.5 ft



h = 6'

$W_1 = \frac{3 * 6}{2} * 0.5 * 490 = 2205$ lb

$W_2 = \frac{1 * 6}{2} * 0.5 * 490 = 735$ lb

$\Sigma M_z = 0$

$6T_C - 2W_2 - 2W_1 = 0$

$T_C = 980$ lb (tension)

$\Sigma M_x = 0 \Rightarrow 300 - 4T_B - T_C(1) + 0.667(W_2) + 2W_1 = 0$

$T_B = 1055$ lb (tension)

$\Sigma F_y = 0 \Rightarrow T_A + 1055 + 980 - 2205 - 735 = 0$

$\therefore T_A = 905$ lb (tension).

Equilibrium of Bodies Acted on by NonConcurrent, Nonparallel, NonCoplanar Force Systems

The resultant of this force system is a single force, a Couple, or a force and a Couple.

A set of equations of equilibrium for this general force system is :–

$$\left.\begin{array}{lll} \Sigma F_x = 0 & \Sigma F_y = 0 & \Sigma F_z = 0 \\ \Sigma M_x = 0 & \Sigma M_y = 0 & \Sigma M_z = 0 \end{array}\right\} \begin{array}{l} \text{------ } 1 \\ \text{--- } 2 \end{array}$$

One or all of the force equations (1), Can be replaced by additional moment equations, provided the moment axes so selected that six independent equations are obtained.

Note :– only six unknowns (magnitudes, distances or slopes) Can be determined from one free-body diagram acted on by a general force system.

See Ex. 4-11 page 201