



Note: Answer **FOUR** questions only.

Q1/ By using the stiffness displacement method, analyze the beam shown in Fig. (1), then draw SFD and BMD.

$$[k] = \frac{EI}{L^3} \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

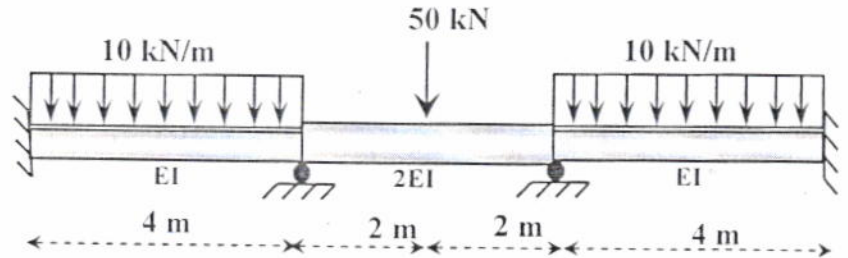


Fig. (1)

Q2/ For the frame shown in Fig. (2), Find the plastic moment (M_p).

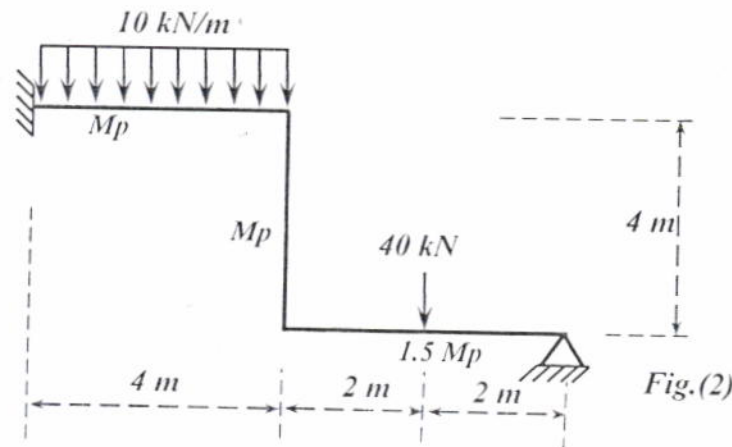


Fig.(2)

Q3/ Answer the following:

(A) For the beam shown in Fig. (3-A), determine the rotation at the point (a), using stiffness displacement method.

$$[k] = \frac{EI}{L^3} \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

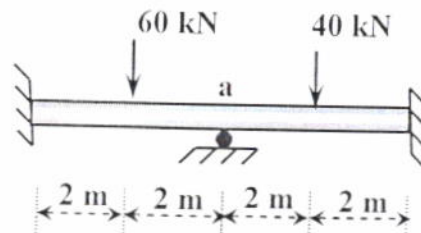


Fig. (3-A)

(B) The composite beam and column cross-sections as shown in Fig. (3-B). Find the relative plastic moment ($M_{p_{beam}} / M_{p_{col.}}$).

Given:

$$\sigma_{y_{steel}} = 210 \text{ N/mm}^2$$

$$\sigma_{y_{Aluminum}} = 170 \text{ N/mm}^2$$

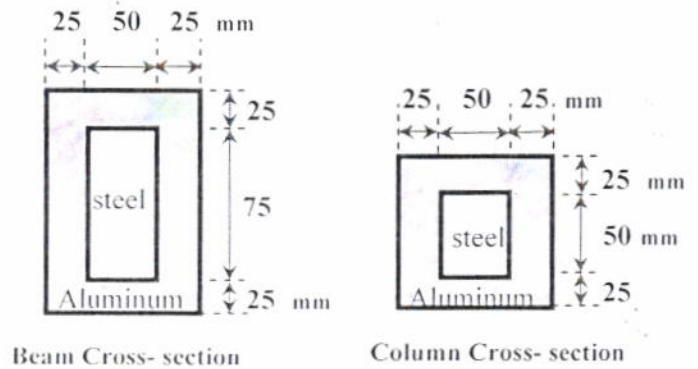


Fig.(3-B)

Q4/ Using the stiffness displacement method to find displacement at node 1 for the truss shown in Fig. (4), then find the support reactions.

$$[k]^c = \frac{EA}{L} \begin{bmatrix} C^2 & CS & -C^2 & -CS \\ CS & S^2 & -CS & -S^2 \\ -C^2 & -CS & C^2 & CS \\ -CS & -S^2 & CS & S^2 \end{bmatrix}$$

$C = \cos \theta$ $S = \sin \theta$

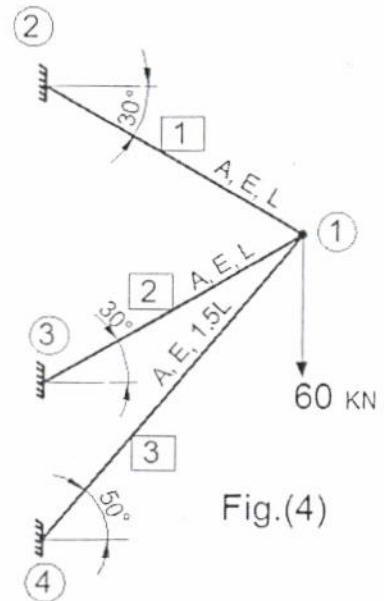


Fig.(4)

Q5/ For the frame shown in the Fig. (5), calculate the critical load (P_{cr}). E, I are constant, show one trail only.

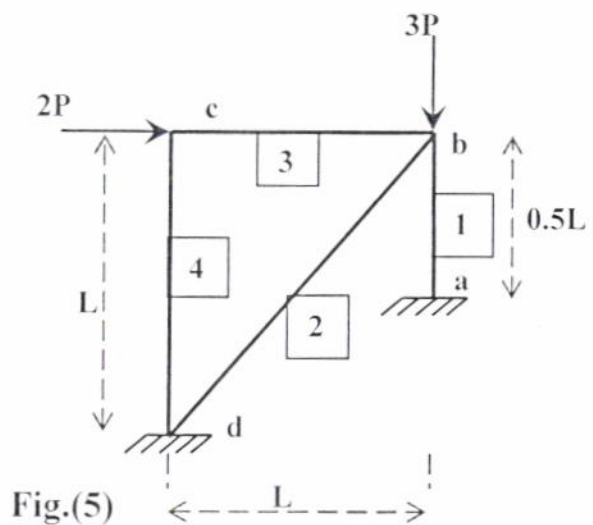
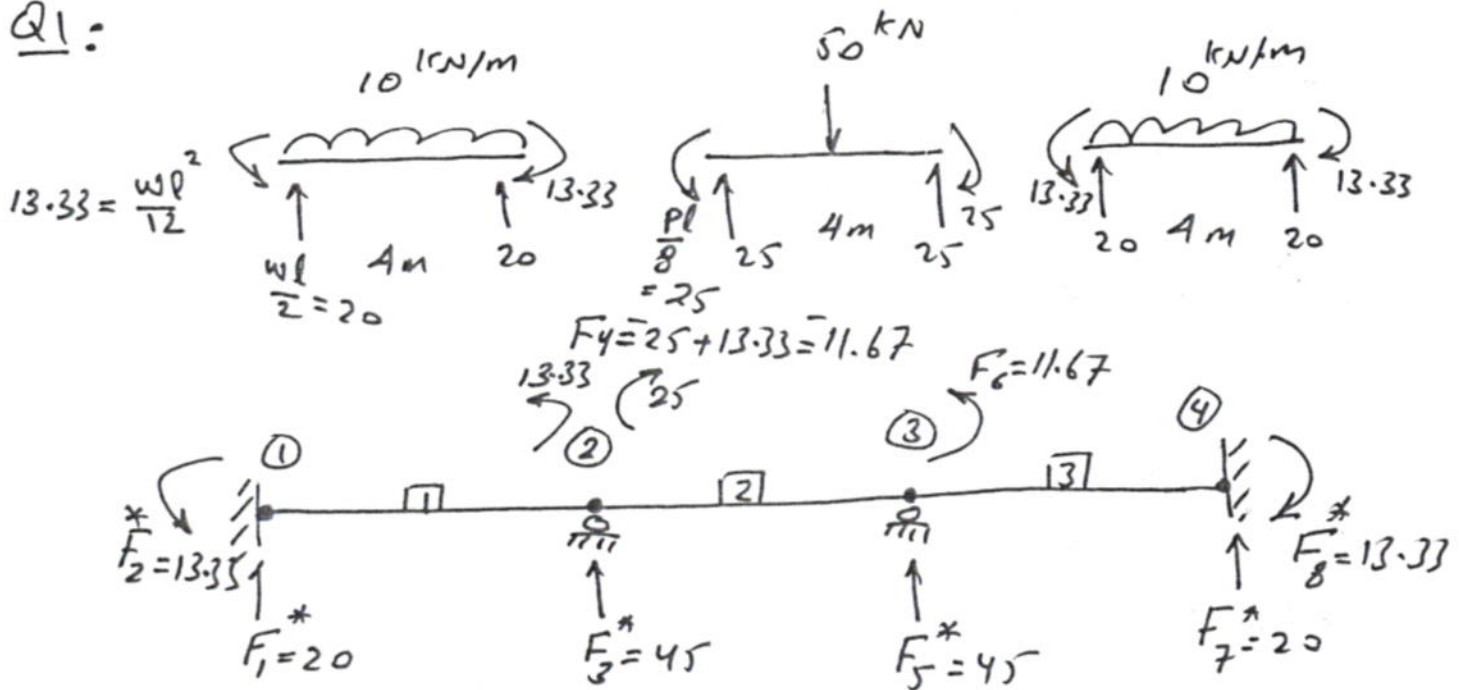


Fig.(5)

Q1:



$\Delta_4, \Delta_6 = ?$ $F_4 = -11.67 \text{ kN}\cdot\text{m}$ $F_6 = 11.67$

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \\ F_7 \\ F_8 \end{Bmatrix} = \frac{EI}{L^3} \begin{bmatrix} 24 & 0 \\ 32 & 0 \\ -24+48 & 48 \\ 64+128 & 64 \\ -48 & -48+24 \\ 64 & 128+64 \\ 0 & -24 \\ 0 & 32 \end{bmatrix} \begin{Bmatrix} \Delta_4 \\ \Delta_6 \end{Bmatrix}$$

$$\begin{Bmatrix} -11.67 \\ 11.67 \end{Bmatrix} = \frac{EI}{L^3} \begin{bmatrix} 192 & 64 \\ 64 & 192 \end{bmatrix} \begin{Bmatrix} \Delta_4 \\ \Delta_6 \end{Bmatrix}$$

$$-11.67 = 192 \Delta_4 \frac{L^3}{EI} + 64 \Delta_6 \frac{L^3}{EI} \quad \text{--- (1) } \times 3$$

$$11.67 = 64 \Delta_4 \frac{L^3}{EI} + 192 \Delta_6 \frac{L^3}{EI} \quad \text{--- (2)}$$

$$46.68 = -512 \Delta_4 \frac{L^3}{EI}$$

$$\therefore \Delta_4 = -0.0911718 \frac{L^3}{EI} \left(-\frac{L^3}{10.968EI} \right)$$

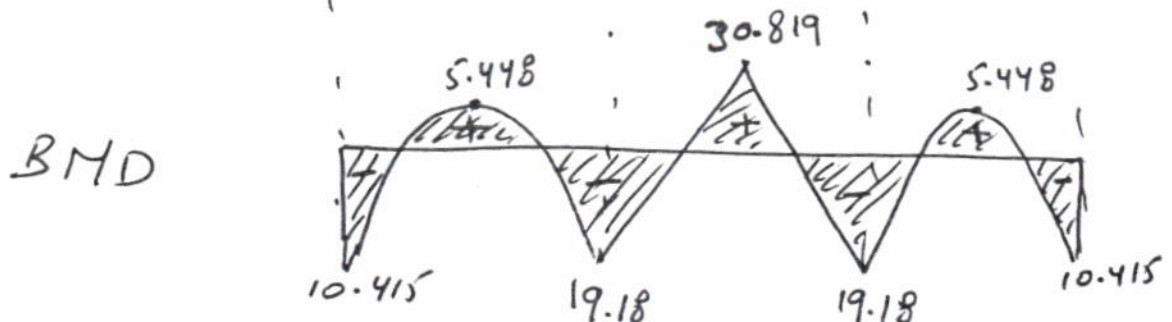
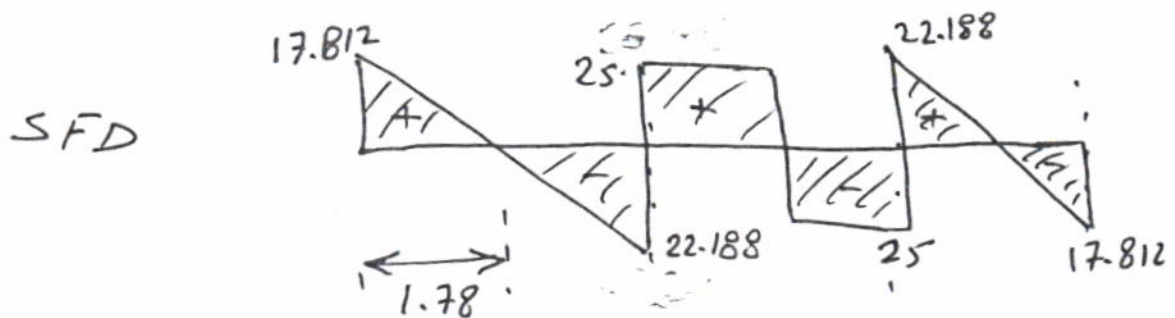
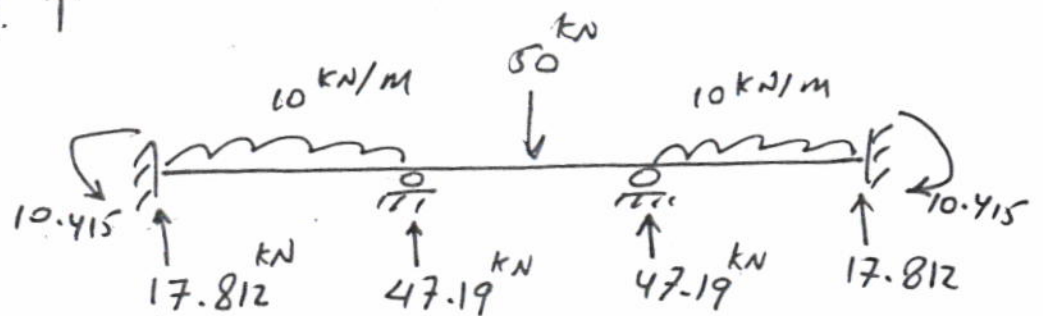
$$\therefore \Delta_6 = 0.0911718 \frac{L^3}{EI}$$

OR from symm. $\Delta_4 = -\Delta_6$

$$\begin{aligned} \therefore \vec{F}_{1 \text{ final}} &= F_1^* + F_1 = 24 \frac{L^3}{EI} \times \left(-\frac{L^3}{10.968EI} \right) + 20 \\ &= 17.812 \text{ kN } \uparrow \end{aligned}$$

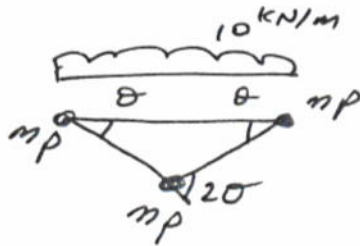
$$F_{2 \text{ final}} = 10.415 \text{ kN.m. } \curvearrowright$$

$$F_{3 \text{ final}} = 47.19 \text{ kN } \uparrow$$



Q2:

beam mech. ①

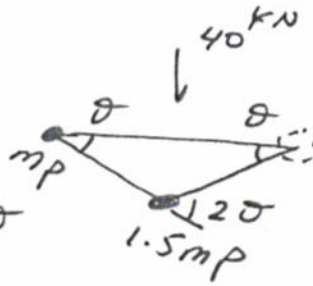


$$\frac{10 \times 4}{2} \times 2\theta = mp\theta + mp\theta + mp \cdot 2\theta$$

$$40\theta = 4mp\theta$$

$$\therefore mp = 10 \text{ kN.m}$$

beam mech. ②

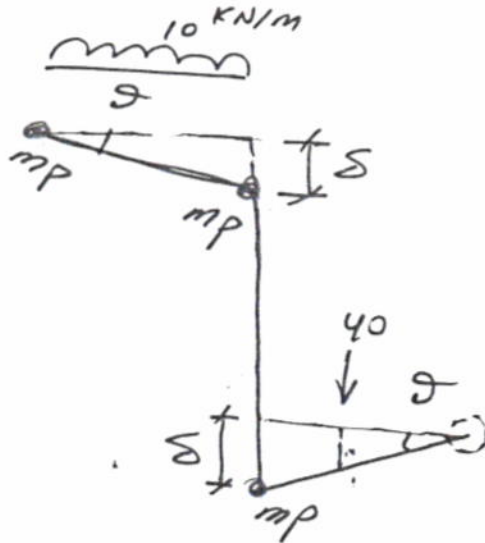


$$40 \times 2\theta = mp\theta + 1.5mp \cdot 2\theta$$

$$80\theta = 4mp\theta$$

$$\therefore mp = 20 \text{ kN.m}$$

sway mech.



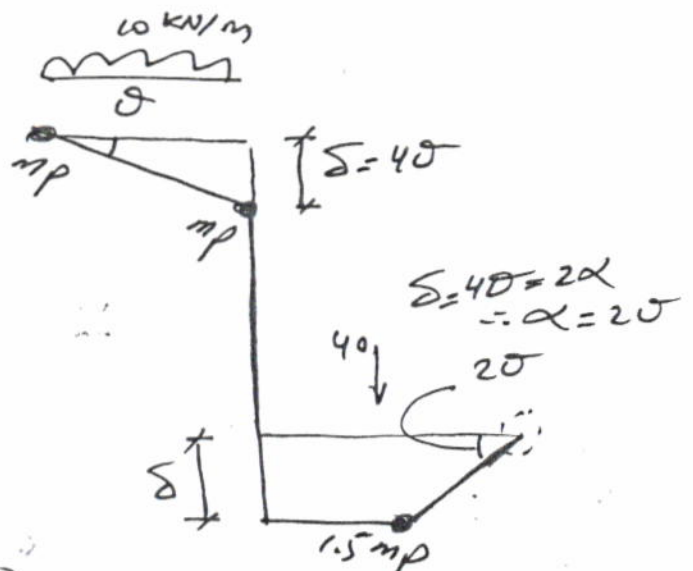
$$\frac{10 \times 4}{2} \times 4\theta + 40 \times 2\theta$$

$$= mp\theta + mp\theta + mp\theta$$

$$160\theta = 3mp\theta$$

$$\therefore mp = 53.33 \text{ kN.m}$$

comp. mech.



$$\frac{10 \times 4}{2} \times 4\theta + 40 \times 4\theta$$

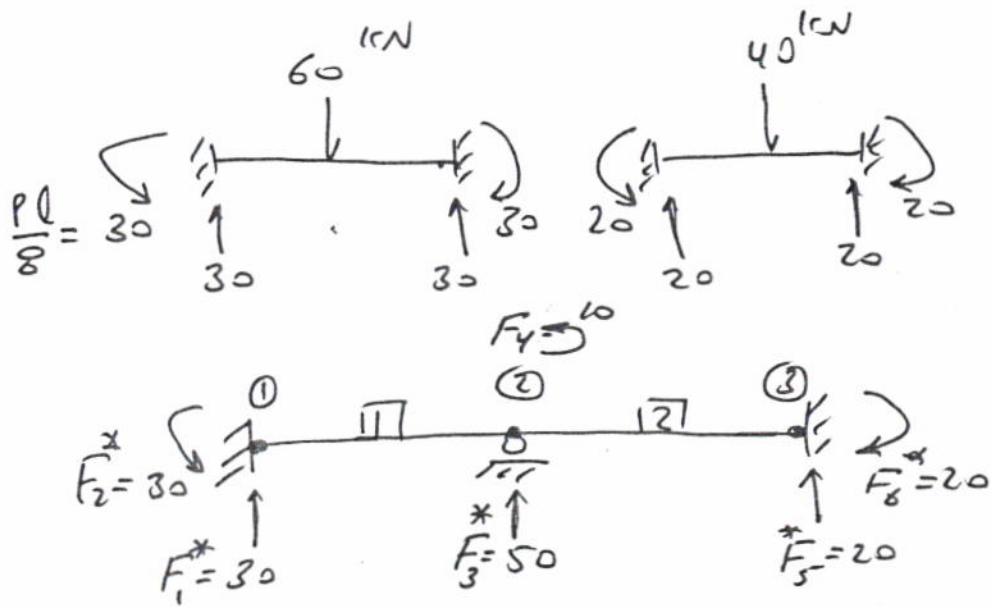
$$= mp\theta + mp\theta + 1.5mp \cdot 2\theta$$

$$240\theta = 5mp\theta$$

$$\therefore mp = 48 \text{ kN.m}$$

$\therefore M_p = 53.33 \text{ kN.m}$

Q3: (a)



$\Delta_4 = ?$

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \end{Bmatrix} = \frac{EI}{L^3} \begin{Bmatrix} 24 \\ 32 \\ -24+24 \\ 64+64 \\ -24 \\ 32 \end{Bmatrix} \Delta_4$$

$$\{10\} = \frac{EI}{L^3} [128] \{\Delta_4\}$$

$$\Delta_4 = 0.07812 \frac{L^3}{EI}$$

Ans.

Q3: (b)

$$M_{P_{beam}} = 170000 \left[\frac{0.1 \times 0.125^2}{4} - \frac{0.05 \times 0.075^2}{4} \right] + 210000 \left[\frac{0.05 \times 0.075^2}{4} \right] = 54.45 + 14.76 = 69.215$$

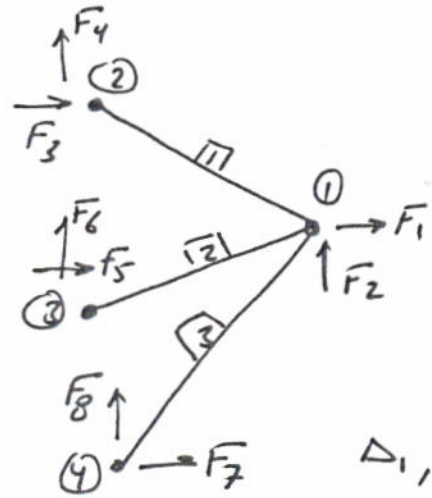
$$M_{P_{col.}} = 170000 \left[\frac{0.1^3}{4} - \frac{0.05^3}{4} \right] + 210000 \times \frac{0.05}{4} = 37.187 + 6.56 = 43.75$$

$$\frac{M_{P_{beam}}}{M_{P_{col.}}} = \frac{69.215}{43.75} = 1.582 \quad \underline{\underline{Ans.}}$$

Q4:

member	node	θ	\cos^2	\sin^2	CS	$\frac{AE}{L}$
1	1-2	150°	0.75	0.25	-0.433	$\frac{AE}{L}$
2	1-3	210°	0.75	0.25	0.433	$\frac{AE}{L}$
3	1-4	230°	0.413	0.586	0.492	$\frac{AE}{1.5L}$

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \\ F_7 \\ F_8 \end{Bmatrix} = \frac{AE}{L} \begin{bmatrix} 0.75+0.75 & -0.433 \\ +0.413/1.5 & +0.433 \\ -0.433 & 0.25 \\ +0.433 & +0.25 \\ -0.75 & 0.433 \\ 0.433 & -0.25 \\ -0.75 & -0.433 \\ -0.433 & -0.25 \\ -0.413 & -0.492 \\ -0.492 & -0.586 \end{bmatrix} \begin{Bmatrix} \Delta_1 \\ \Delta_2 \end{Bmatrix}$$



$\Delta_1, \Delta_2 = ?$
 $F_1 = 0$
 $F_2 = -60 \text{ KN}$

$$\begin{Bmatrix} 0 \\ -60 \end{Bmatrix} = \frac{AE}{L} \begin{bmatrix} 1.775 & 0.328 \\ 0.328 & 0.89 \end{bmatrix} \begin{Bmatrix} \Delta_1 \\ \Delta_2 \end{Bmatrix}$$

$$\Delta_1 = 13.368 L / AE$$

$$\Delta_2 = -72.342 L / AE$$

$$F_3 = -41.32 \text{ KN} \leftarrow$$

$$F_4 = 23.87 \text{ KN} \uparrow$$

$$F_5 = 21.32 \text{ KN} \rightarrow$$

$$F_6 = 12.29 \text{ KN} \uparrow$$

$$F_7 = 20 \text{ KN} \rightarrow$$

$$F_8 = 24.16 \text{ KN} \uparrow$$

