University Of Technology
Building and Construction Eng. Dept.
Final Exam – First Attempt – 2013/2014
Subject :structure II — Class: Four
Branch :Structural Eng. — Time : 3 Hours
Examiner :Asst. Prof. Dr. Alaa Kamal — Date : 2 / 6 / 2014

Note: Answer FOUR questions only.

Q1/ By using the stiffness displacement method, analyze the beam shown in Fig. (1), then draw SFD and BMD.

$$[k]^{e} = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

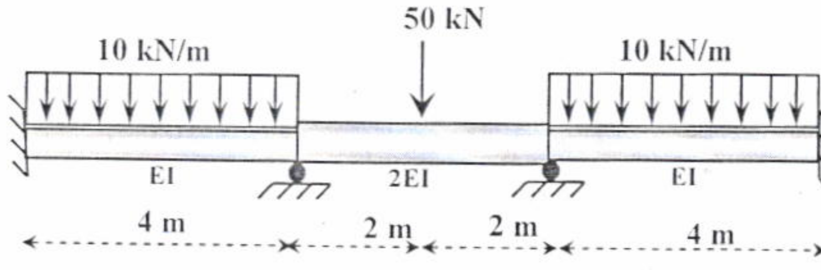

Fig. (1)

Q2/ For the frame shown in Fig. (2), Find the plastic moment (Mp).

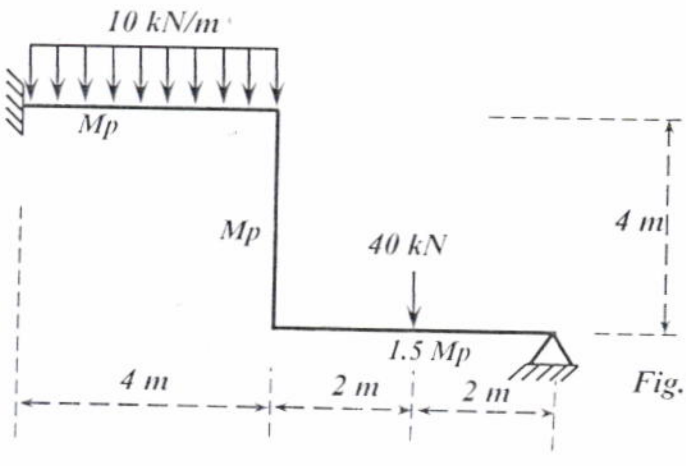

Fig.(2)

Q3/ Answer the following:

(A) For the beam shown in Fig. (3-A), determine the rotation at the point (a) , using stiffness displacement method.

$$[k]^{e} = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

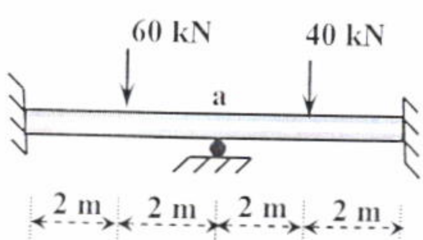

Fig. (3-A)

(B) The composite beam and column cross-sections as shown in Fig. (3-B). Find the relative plastic moment (Mp_{beam} / $Mp_{col.}$).

Given:

$\sigma_{y\ steel} = 210\ N/mm^2$

$\sigma_{y\ Aluminum} = 170\ N/mm^2$

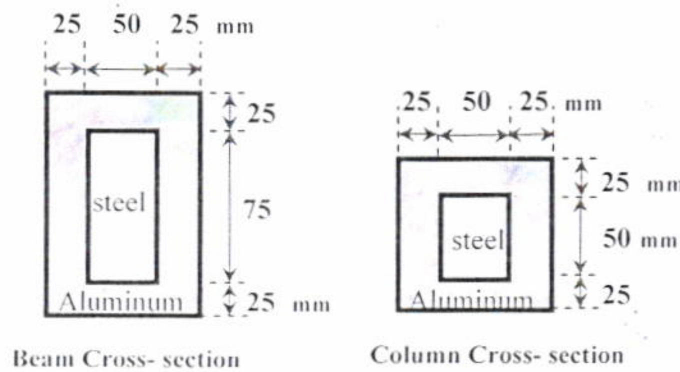

Fig.(3-B)

Q4/ Using the stiffness displacement method to find displacement at node 1 for the truss shown in Fig. (4), then find the support reactions.

$$[k]^e = \frac{EA}{L}\begin{bmatrix} C^2 & CS & -C^2 & -CS \\ CS & S^2 & -CS & -S^2 \\ -C^2 & -CS & C^2 & CS \\ -CS & -S^2 & CS & S^2 \end{bmatrix}$$

$C = \cos\theta \qquad S = \sin\theta$

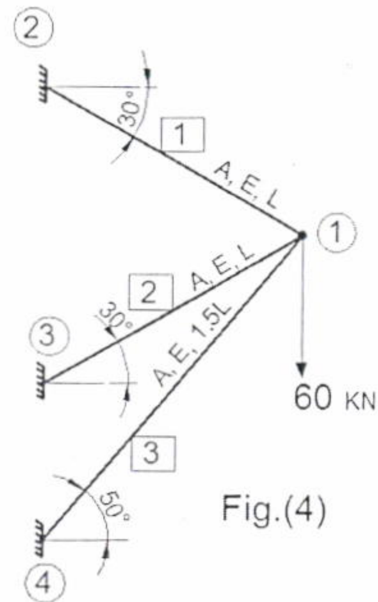

Fig.(4)

Q5/ For the frame shown in the Fig. (5), calculate the critical load (P_{cr}). E, I are constant, show one trail only.

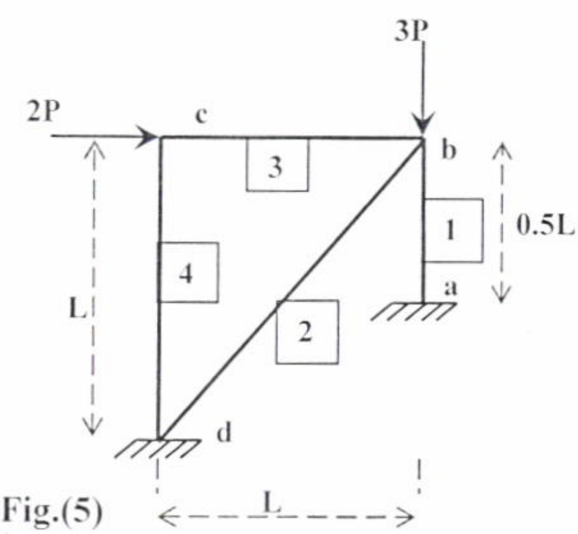

Fig.(5)

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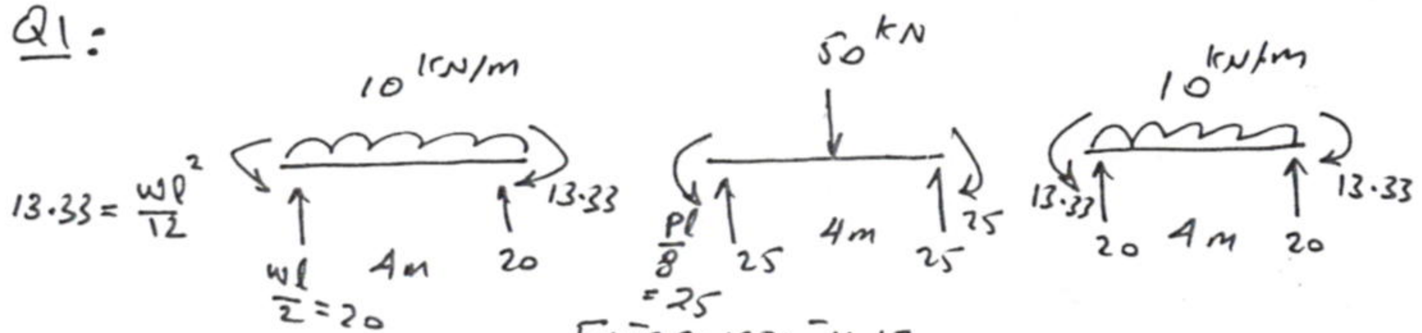
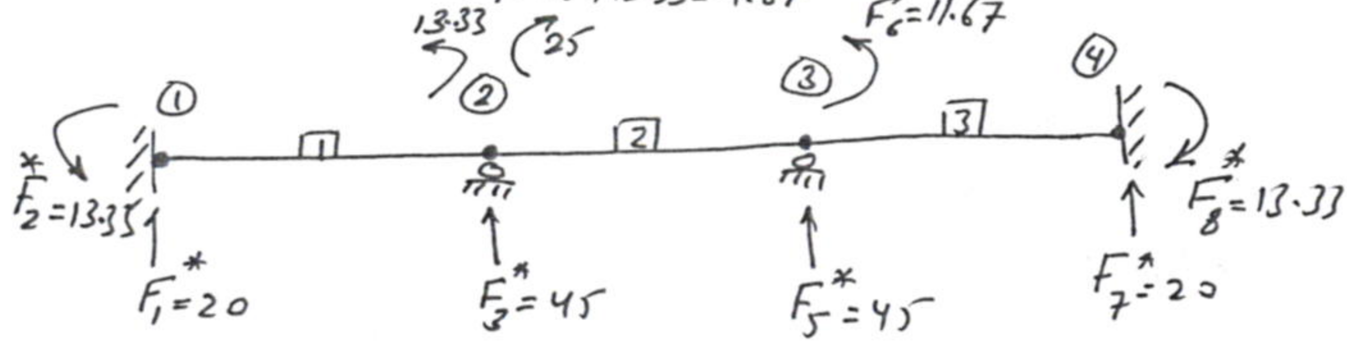
Q1:



$F_4 = 25 + 13.33 = 11.67$



$\Delta_4, \Delta_6 = ?$ $F_4 = -11.67$ kN.m $F_6 = 11.67$

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \\ F_7 \\ F_8 \end{Bmatrix} = \frac{EI}{L^3} \begin{array}{c} \begin{array}{cc} 4 & 6 \end{array} \\ \begin{bmatrix} 24 & 0 \\ 32 & 0 \\ -24+48 & 48 \\ 64+128 & 64 \\ -48 & -48+24 \\ 64 & 128+64 \\ 0 & -24 \\ 0 & 32 \end{bmatrix} \end{array} \begin{Bmatrix} \Delta_4 \\ \Delta_6 \end{Bmatrix}$$

$$\begin{Bmatrix} -11.67 \\ 11.67 \end{Bmatrix} = \frac{EI}{L^3} \begin{bmatrix} 192 & 64 \\ 64 & 192 \end{bmatrix} \begin{Bmatrix} \Delta_4 \\ \Delta_6 \end{Bmatrix}$$

$$-11.67 = 192\,\Delta_4 \frac{L^3}{EI} + 64\,\Delta_6 \frac{L^3}{EI} \quad — ① \quad * 3$$

$$11.67 = 64\,\Delta_4 \frac{L^3}{EI} + 192\,\Delta_6 \frac{L^3}{EI} \quad — ②$$

$$46.68 = -512\,\Delta_4\, L^3/EI$$

$$\therefore \Delta_4 = -0.0911718\, L^3/EI \quad \left(-\frac{L^3}{10.968EI}\right)$$

$$\therefore \Delta_6 = 0.0911718\, L^3/EI$$

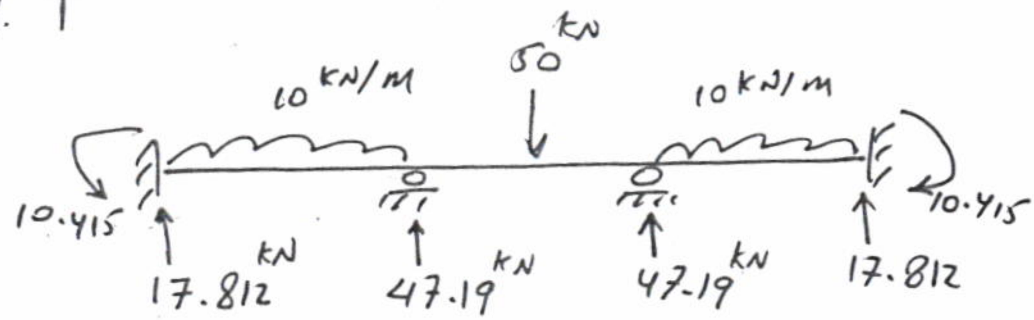
OR from symm. $\Delta_4 = -\Delta_6$

$$\therefore F_{1_{final}} = F_1^* + F_1 = 24\, L^3/EI * \left(-\frac{L^3}{10.968EI}\right) + 20$$

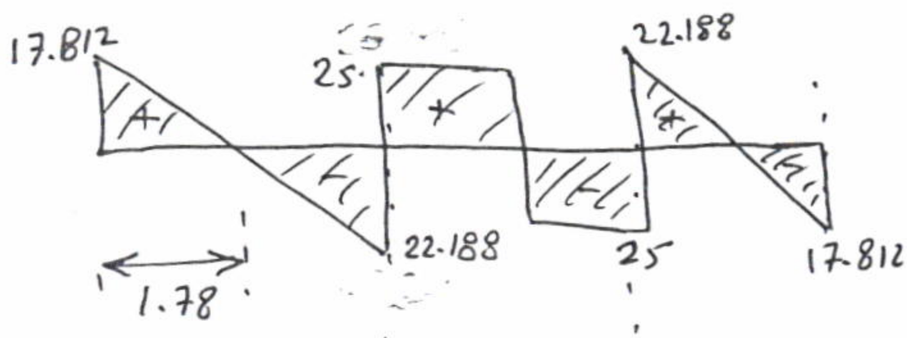
$$= 17.812 \text{ kN} \uparrow$$

$$F_{2_{final}} = 10.415 \text{ kN.m.}$$

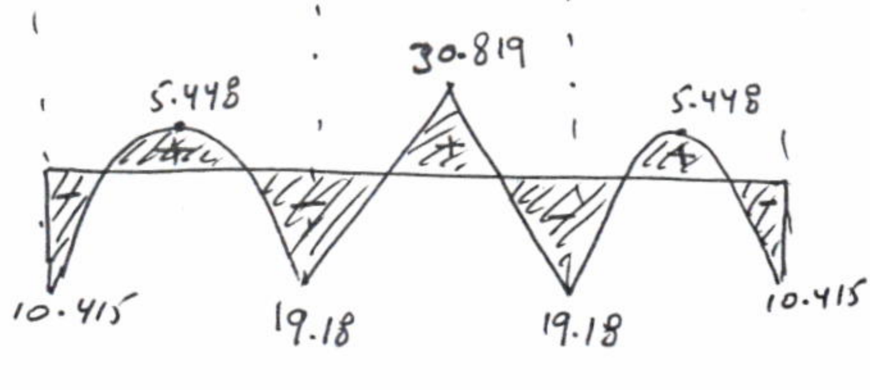
$$F_{3_{final}} = 47.19 \uparrow$$



SFD

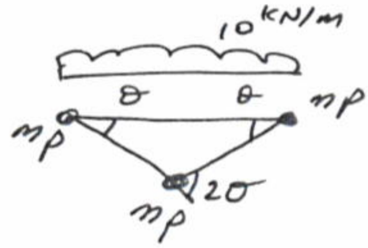


BMD



Q2:

beam mech. (1)

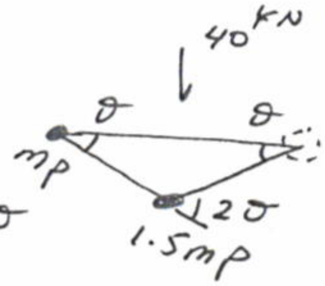


$$\frac{10 \times 4}{2} * 2\theta = mp\theta + mp\theta + mp\,2\theta$$

$$40\theta = 4mp\theta$$

$$\therefore mp = 10 \text{ kN.m}$$

beam mech. (2)

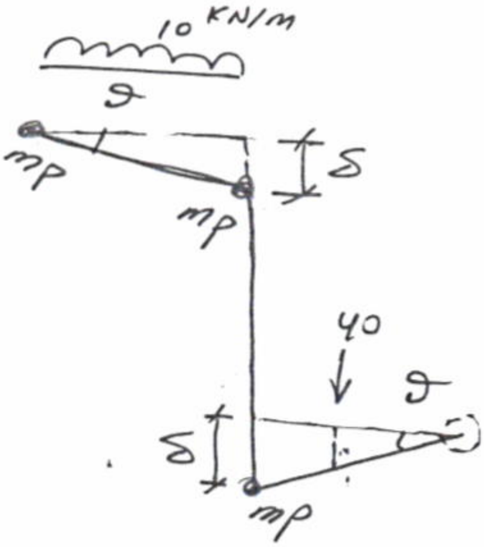


$$40 \times 2\theta = mp\theta + 1.5mp\,2\theta$$

$$80\theta = 4mp\theta$$

$$\therefore mp = 20 \text{ kN.m.}$$

sway mech.



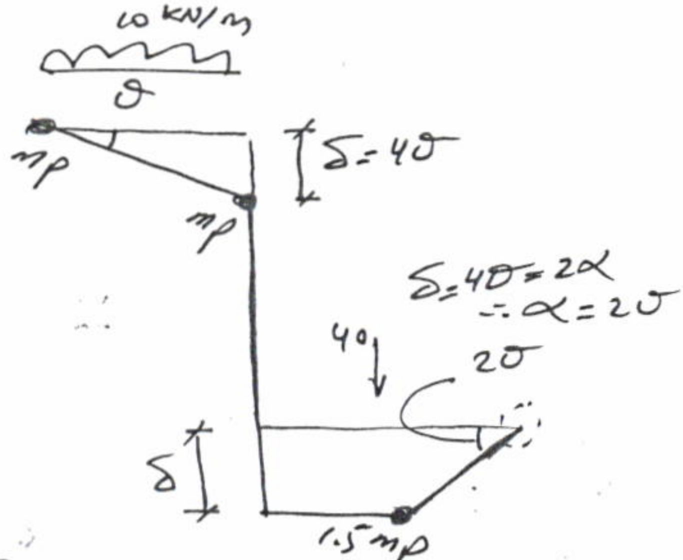
$$\frac{10 \times 4}{2} * 4\theta + 40 * 2\theta$$

$$= mp\theta + mp\theta + mp\theta$$

$$160\theta = 3mp\theta$$

$$\therefore mp = 53.33 \text{ kN.m}$$

comp. mech.



$$\frac{10 \times 4}{2} \times 4\theta + 40 * 4\theta$$

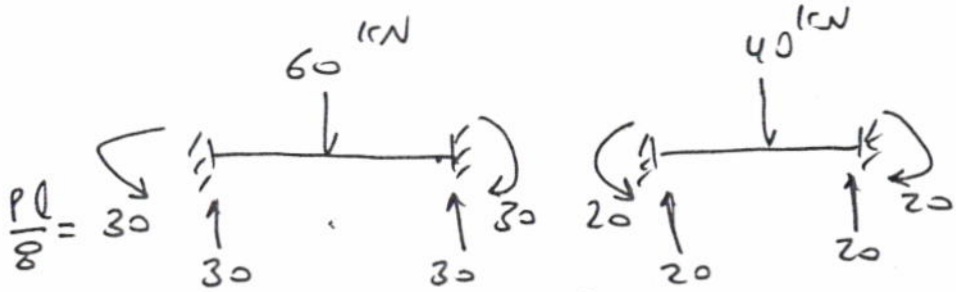
$$= mp\theta + mp\theta + 1.5mp\,2\theta$$

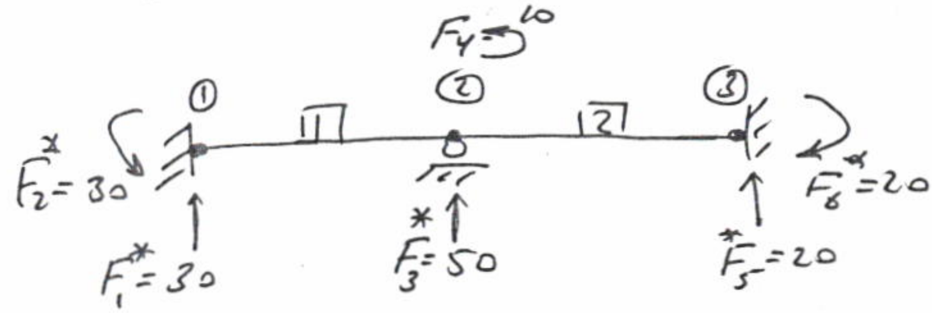
$$240\theta = 5mp\theta$$

$$\therefore mp = 48 \text{ kN.m.}$$

$$\therefore M_p = 53.33 \text{ kN.m}$$

Q3: (a)





$\Delta_4 = ?$

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \end{Bmatrix} = \frac{EI}{L^3} \begin{array}{c} (4) \\ \begin{bmatrix} 24 \\ 32 \\ -24+24 \\ 64+64 \\ -24 \\ 32 \end{bmatrix} \end{array} \{\Delta_4\}$$

$$\{10\} = \frac{EI}{L^3}[128]\{\Delta_4\}$$

$$\boxed{\therefore \Delta_4 = 0.07812\,\frac{L^3}{EI}}$$

Ans.

Q3: (b)

$$M_{p_{beam}} = 170000\left[\frac{0.1*0.125^2}{4} - \frac{0.05*0.075^2}{4}\right] + 210000\left[\frac{0.05*0.075^2}{4}\right] = 54.45 + 14.76 = 69.215$$

$$M_{p_{col.}} = 170000\left[\frac{0.1^3}{4} - \frac{0.05^3}{4}\right] + 210000*\frac{0.05^3}{4} = 37.187 + 6.56 = 43.75$$

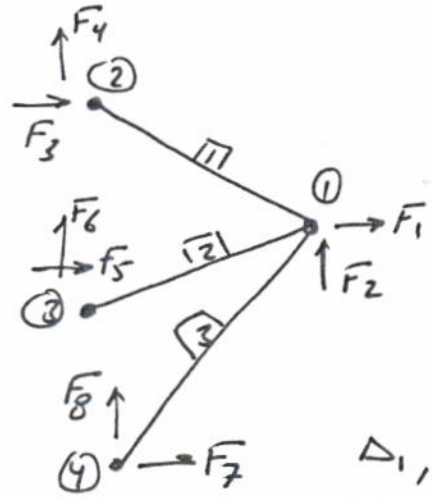
$$\frac{M_{p_{beam}}}{M_{p_{col.}}} = \frac{69.215}{43.75} = 1.582$$

Ans.

Q4:

member	node	θ	$\cos^2$	$\sin^2$	CS	AE/L
1	1–2	150°	0.75	0.25	−0.433	AE/L
2	1–3	210°	0.75	0.25	0.433	AE/L
3	1–4	230°	0.413	0.586	0.492	$\frac{AE}{1.5L}$

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \\ F_7 \\ F_8 \end{Bmatrix} = \frac{AE}{L} \begin{array}{cc} ① & ② \\ \left[\begin{array}{c|c} 0.75+0.75+0.413/1.5 & -0.433+0.433+0.492/1.5 \\ \hline -0.433+0.433+0.492/1.5 & 0.25+0.25+0.586/1.5 \\ \hline -0.75 & 0.433 \\ \hline 0.433 & -0.25 \\ \hline -0.75 & -0.433 \\ \hline -0.433 & -0.25 \\ \hline -\frac{0.413}{1.5} & -\frac{0.492}{1.5} \\ \hline -\frac{0.492}{1.5} & -\frac{0.586}{1.5} \end{array}\right] \end{array} \begin{Bmatrix} \Delta_1 \\ \Delta_2 \end{Bmatrix}$$



$\Delta_1, \Delta_2 = ?$

$F_1 = 0$

$F_2 = -60^{KN}$.

$$\begin{Bmatrix} 0 \\ -60 \end{Bmatrix} = \frac{AE}{L} \begin{pmatrix} 1.775 & 0.328 \\ 0.328 & 0.89 \end{pmatrix} \begin{Bmatrix} \Delta_1 \\ \Delta_2 \end{Bmatrix}$$

$\Delta_1 = 13.368 L/AE$

$\Delta_2 = -72.342 L/AE$

$F_3 = -41.32^{KN} \leftarrow$

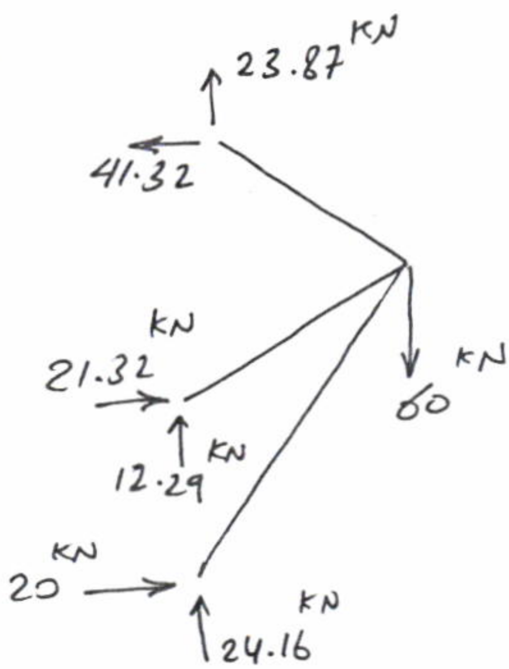
$F_4 = 23.87 \uparrow$

$F_5 = 21.32 \rightarrow$

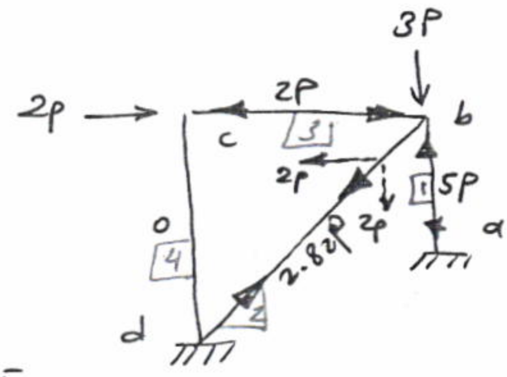
$F_6 = 12.29 \uparrow$

$F_7 = 20 \rightarrow$

$F_8 = 24.16 \uparrow$



DS:



mem.	length	$K = \frac{EI}{L}$	P	$P_e = \frac{EI}{L^2}$	$\frac{P}{P_e} = \rho$	$\rho_{rel.}$	
1	0.5	2	5	4	1.25	0.625	
2	$\sqrt{2}$	$\frac{1}{\sqrt{2}}$	−2.828	$\frac{1}{2}$	−5.656	−2.828	
3	1	1	2	1	2	1	← critical
4	1	1	0	1	0	0	

$$M_{bc} = S_3 K_3 \theta_b + SC_3 K_3 \theta_c$$

$$M_{ba} = S_1 K_1 \theta_b$$

$$M_{bd} = S_2 K_2 \theta_b$$

$$M_{cb} = S_3 K_3 \theta_c + SC_3 K_3 \theta_b$$

$$M_{cd} = 4 K_4 \theta_c$$

$\Sigma M_b = 0$

$\Sigma M_c = 0$

$$\left(2S_1 + S_3 + \frac{S_2}{\sqrt{2}}\right)\theta_b + SC_3\,\theta_c = 0 \quad \text{— ①}$$

$$SC_3\,\theta_b + (4 + S_3)\,\theta_c = 0 \quad \text{— ②}$$

$$[k] = \begin{bmatrix} 2S_1 + S_3 + \frac{S_2}{\sqrt{2}} & SC_3 \\ SC_3 & 4 + S_3 \end{bmatrix} \begin{bmatrix} \theta_b \\ \theta_c \end{bmatrix} = 0$$

$1 < \rho < 4$

Try ①: $\rho_3 = 1.5 \quad \therefore \rho_1 = 1.5 * 0.625 = 0.9375$

$\rho_2 = -2.828 \times 1.5 = -4.242$

↓ from tables find S_3, S_1, S_2 ...
due to the values of ρ.