University of Technology
Building and Construction Engineering Department
Final Exam 2013-2014

Subject: Reinforced Concrete I
Division: Structural Engineering
Examiner:

Year: Third Year
Time: 3 Hours
Date: 8 / 6 / 2014

Answer **Four** Questions Only

Apply the ACI 318M-05 Specification, $f_c' = 23 \quad N/mm^2$, $f_y = 420 \quad N/mm^2$

Q1) The continuous beam ABC shown in the figure (1) carries a uniformly distributed service dead load of (22.5 kN/m) (including beam self weight) and a uniformly distributed service live load of (25.5 kN/m). By using the Ultimate Strength Design Method, determine the longitudinal steel reinforcement required at section B and section E (draw sections with full details).

Q2) The beam shown in the figure (2) is subjected to an ultimate torsional moment of (T_U=13 kN.m). Design the necessary torsional reinforcement required for this beam. (use Φ14mm closed stirrups and a clear cover of 50mm) (draw full section details).

Q3) The simply supported beam shown in the figure (3) carries a uniformly distributed service dead load of magnitude (2W kN/m) (without beam self weight). By using the Working Stress Design Method, find the maximum value of (W) that the beam can carry.

Q4) The cantilever beam shown in the figure (4) carries a uniformly distributed service dead load of W_D (without beam self weight), and a concentrated service live load of (12 kN) at the free end, calculate:

a) Maximum value of W_D that the beam can carry if the effective moment of inertia equals 1.14 of moment of inertia of the cracked section (I_e=1.14 I_{cr}), and the immediate deflection caused by dead and live loads is (6 mm).
b) Total deflection suppose that 30% of the live load is sustained.

Q5) The reinforced concrete Two-Way slab shown in the figure (5) is supported by reinforced concrete beams of 600^{mm}x300^{mm}. The slab supports a service distributed live load of (7 kN/m^2) and a service distributed dead load of (6 kN/m^2) (including slab self weight). Slab thickness is 140^{mm}. All columns are square of 300^{mm}. Design and detail the middle strip required bars A&B (using Φ12^{mm} bars for the slab).

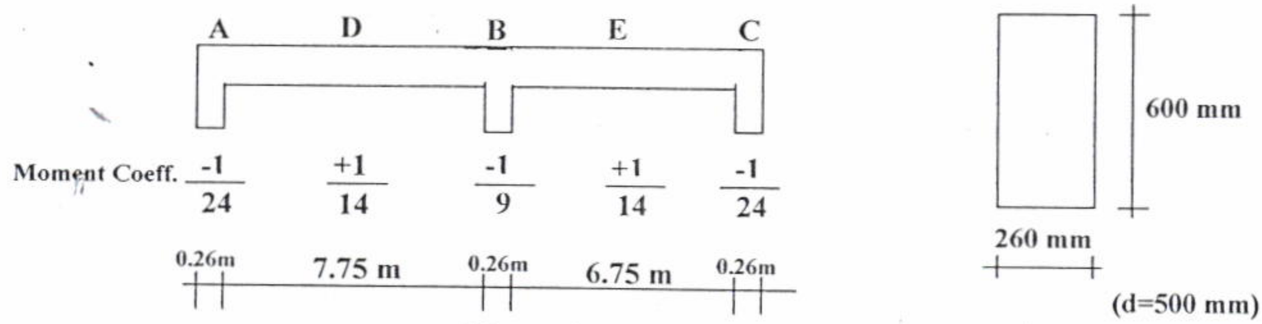


Figure (1)

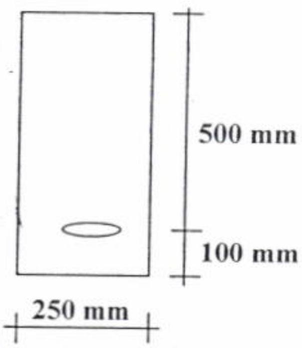


Figure (2)

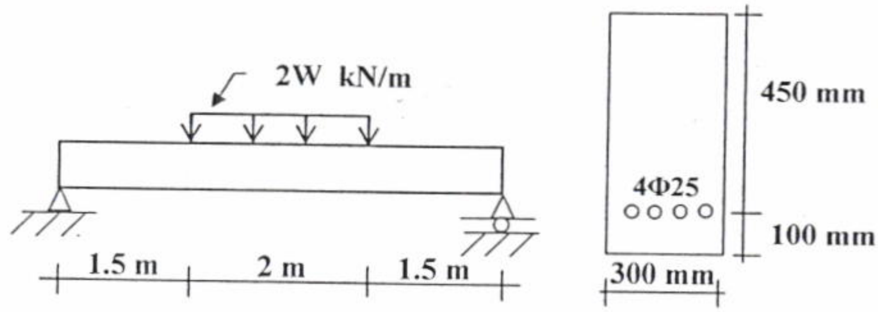


Figure (3)

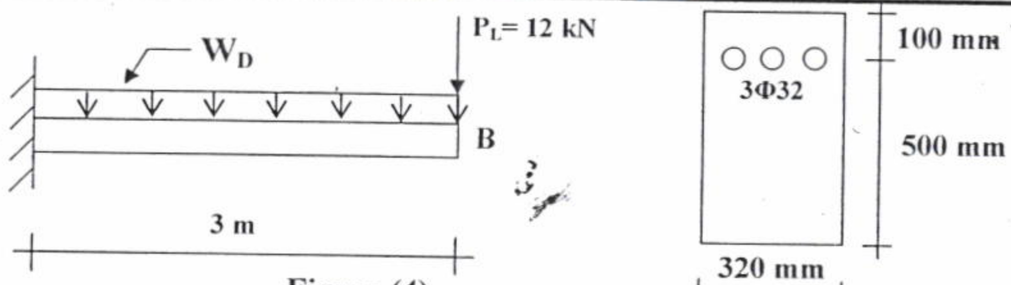


Figure (4)

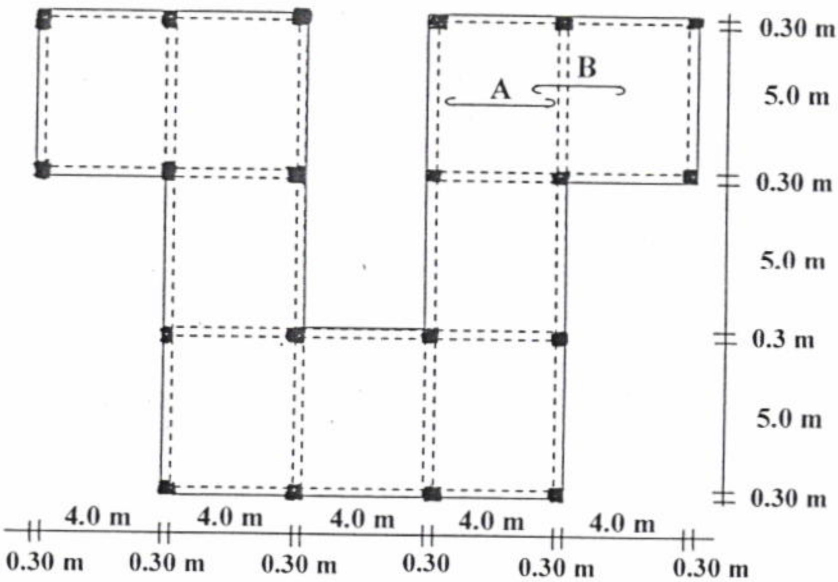


Figure (5)

Q1).

$W_U = 1.2(22.5) + 1.6(25.5) = 67.8 \text{ KN/m}$

$M_U)_B = \frac{1}{9} * 67.8 * (\frac{7.75 + 6.75}{2})^2 = 395.97 \text{ KN.m}$

$\rho_{max} = \frac{51 * 0.85 * 23}{140 * 420} = 0.0169$

$A_{s_1} = 0.0169 * 260 * 500 = 2197 \text{ mm}^2$

$a = \frac{2197 * 420}{0.85 * 23 * 260} = 181.5 \text{ mm}$

$\max M_U = 0.817 * 2197 * 420 (500 - \frac{181.5}{2}) * 10^{-6}$

$\max M_U = 308.51 \text{ KN.m} < M_U)_B = 395.97 \text{ KN.m}$

$\therefore$ Doubly Reinf. Conc. section

$M_{U_2} = 395.97 - 308.51 = 87.46 \text{ KN.m}$

$c = \frac{181.5}{0.85} = 213.5 \text{ mm}$

$\epsilon_s' = 0.003(\frac{213.5 - 60}{213.5}) = 0.00215 > \epsilon_y = \frac{420}{200000} = 0.0021$

$\therefore f_s' = f_y \; ; A_s' = A_{s_2}$

$A_s' = \frac{87.46 * 10^6}{0.817 * 420 * (500 - 60)} = 580 \text{ mm}^2$

use $3\phi16$

$A_s = 2197 + 580 = 2777$

$6\phi25$

$$M_u)_E = \frac{1}{14} * 67.8 * (6.75)^2 = 220.65 \text{ KN.m}$$

let $\phi = 0.9$; assume $a = 108$ mm

$$A_s = \frac{220.65 * 10^6}{0.9 * 420 \left(500 - \frac{108}{2}\right)} = 1309 \text{ mm}^2$$

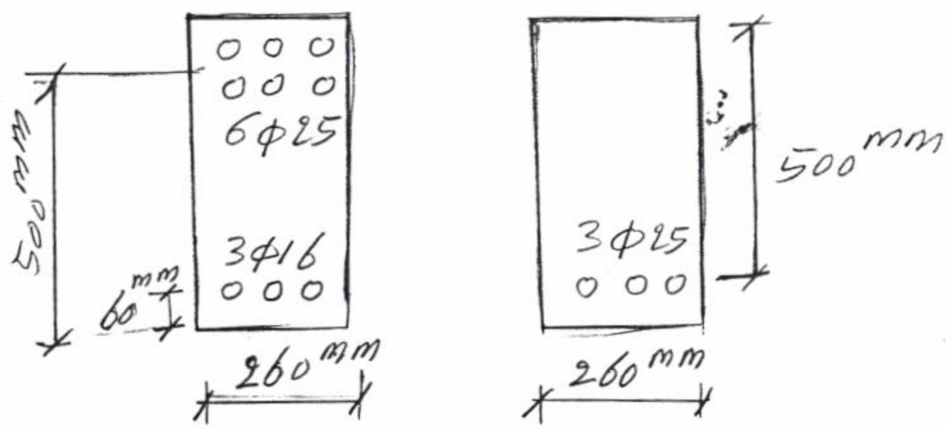
$$a = \frac{1312 * 420}{0.85 * 23 * 260} = 108.1 \text{ mm}$$

$c = 127$ mm

$$\phi = \left[0.65 + 0.25\left(\frac{1}{127/500} - \frac{5}{3}\right)\right] \leq 0.9$$

$\phi = 0.9$ O.K

use $3\phi25$



sec. B

sec. E

Q2) $f'_c = 23$ MPa , $f_y = 420$ MPa.

Solution:

$A_{cp} = 250 \times 600 = 150\,000\ mm^2$, $P_{cp} = (250+600) \times 2 = 1700$ mm.

$T_u = 13\ kN.m > \phi\ 0.083 \sqrt{f'_c}\ (\frac{A_{cp}^2}{P_{cp}}) = 0.75 \times 0.083 \times \sqrt{23} \times \frac{(150\,000)^2}{1700 \times 10^6} = 3.95\ kN.m$

∴ Torsional effect shall be consider.

$y_1 = 600 - 2\times50 - 14 = 486$ mm, $x_1 = 250 - 2\times50 - 14 = 136$ mm.

$A_{oh} = 136 \times 486 = 66096\ mm^2$, $P_h = (136+486)\times 2 = 1244$ mm.

$\frac{T_u . P_h}{1.7 A_{oh}^2} \leq \phi\ 0.83\sqrt{f'_c} \Rightarrow \frac{13\times10^6 \times 1244}{1.7\times(66096)^2} = 2.18 < 0.75\times0.83\times\sqrt{23} = 2.99$ ok

$\frac{A_t}{s} = \frac{T_n}{1.7 A_{oh} f_{yv}} = \frac{(13/0.75)\times10^6}{1.7\times66096\times420} = 0.367$ mm.

$\frac{A_v + 2A_t}{s} = 0 + 2\times0.367 = 0.734$ mm.

$\frac{A_v + 2A_t}{s} \geq 0.062\sqrt{f'_c}\ \frac{b_w}{f_{yv}} \Rightarrow 0.734 > 0.062\sqrt{23}\times\frac{250}{420} = 0.177$ ok

$\frac{A_v + 2A_t}{s} \geq 0.35\ \frac{b_w}{f_{yv}} \Rightarrow 0.734 > 0.35\times\frac{250}{420} = 0.208$ ok

with ϕ 14 mm closed stirrups $\Rightarrow A_t = 153.9\ mm^2$

$\frac{A_t}{s} = 0.367$ mm $\Rightarrow S = \frac{153.9}{0.367} = 419.3$ mm

$S_{max} = \frac{P_h}{8} = \frac{1244}{8} = 155.5$ mm or 300 mm

∴ use ϕ14 mm closed stirrups @150mm c/c.

$A_\ell = \frac{A_t}{s} . P_h = 0.367\times1244 = 456.5\ mm^2$

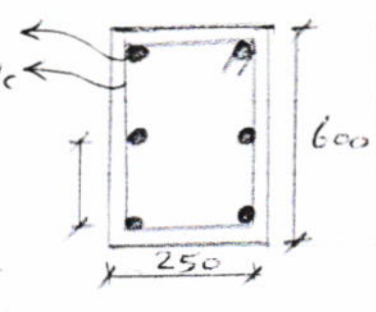
$A_{\ell_{min.}} = \frac{0.42\sqrt{f'_c}.A_{cp}}{f_y} - \frac{A_t}{s} P_h \frac{f_{yv}}{f_{y\ell}}$, where $\frac{A_t}{s} = 0.367 > 0.175\frac{b_w}{f_{yv}}$

$= 0.175\times\frac{250}{420} = 0.104$ ok.

$A_{\ell_{min.}} = \frac{0.42\sqrt{23}\times150\,000}{420} - 456.5 = 719.4 - 456.5 = 262.9\ mm^2$.

bar dia. $\geq 0.042\ S = 0.042\times150 = 6.3$ mm or 10 mm

∴ use 6ϕ10 mm bars ($A_\ell = 471\ mm^2 > 456.5\ mm^2$) ok



$\frac{600 - 2\times50 - 2\times14 - 10}{2}$

$= 231$ mm < 300 mm ok

Q3) $f_c' = 23$ MPa, $f_y = 420$ MPa.

Solution:

$A_s = 1963 \text{ mm}^2$.

$E_c = 4700\sqrt{f_c'} = 4700\sqrt{23} = 22540$ MPa.

$n = E_s / E_c = 200\,000 / 22540 = 8.87 \simeq 9.0$

$$\frac{300\,x^2}{2} = 9 \times 1963 \times (450 - x)$$

$$x^2 + 117.8\,x - 53\,001 = 0$$

$$x = \frac{-117.8 \pm \sqrt{(117.8)^2 + 4 \times 53001}}{2} = 178.7 \text{ mm}.$$

$$I_{cr} = \frac{300(178.7)^3}{3} + 9 \times 1963 \times (450 - 178.7)^2 = 1.871 \times 10^9 \text{ mm}^4.$$

$f_{c_{all.}} = 0.45 f_c' = 0.45 \times 23 = 10.35$ MPa.

$f_{s_{all.}} = 170$ MPa.

$$M_c = \frac{f_{c_{all.}} \times I_{cr}}{c} = \frac{10.35 \times 1.871 \times 10^9}{178.7 \times 10^6} = 108.365 \text{ kN·m}.$$

$$M_s = \frac{f_{s_{all.}} \times I_{cr}}{n \times c} = \frac{170 \times 1.871 \times 10^9}{9 \times (450 - 178.7) \times 10^6} = 130.266 \text{ kN·m}.$$

$\therefore$ use $M_{all.} = 108.365$ kN·m.

$W_D = 24 \times 0.3 \times 0.55 = 3.96$ kN/m

$$M_{all.} = \frac{3.96 \times (5)^2}{8} + 2W \times 2.5 - \frac{2W \times (1)^2}{2}$$

$108.365 = 12.375 + 4W \Rightarrow W = 23.998$ kN/m.

Q4) $f'_c = 23$ MPa , $f_y = 420$ MPa.

Solution:

(a) $A_s = 2413 \ mm^2$

$E_c = 4700\sqrt{f'_c} = 4700\sqrt{23} = 22540$ MPa.

$n = E_s / E_c = 200000 / 22540 = 8.87 \simeq 9.0$

$$\frac{320 x^2}{2} = 9 \times 2413 \times (500 - x)$$

$$x^2 + 135.7 x - 67865.6 = 0$$

$$x = \frac{-135.7 \pm \sqrt{(135.7)^2 + 4 \times 67865.6}}{2} = 201.4 \ mm.$$

$$I_{cr} = \frac{320(201.4)^3}{3} + 9 \times 2413 \times (500 - 201.4)^2 = 2.8077 \times 10^9 \ mm^4.$$

$$I_e = 1.14 I_{cr} = 1.14 \times 2.8077 \times 10^9 = 3.201 \times 10^9 \ mm^4.$$

$$(\Delta_i)_{D+L} = \frac{w l^4}{8EI} + \frac{P l^3}{3EI}$$

$$6 = \frac{w (3 \times 1000)^4}{8 \times 22540 \times 3.201 \times 10^9} + \frac{12 \times 1000 \times (3 \times 1000)^3}{3 \times 22540 \times 3.201 \times 10^9}$$

$\therefore W = 32.089$ kN/m.

beam self weight $= 24 \times 0.32 \times 0.6 = 4.608$ kN/m.

$\therefore W_D = 32.089 - 4.608 = 27.481$ kN/m

(b)

$$(\Delta_i)_{sust.} = \frac{32.089 \times (3 \times 1000)^4}{8 \times 22540 \times 3.201 \times 10^9} + \frac{0.3 \times 12 \times 1000 \times (3 \times 1000)^3}{3 \times 22540 \times 3.201 \times 10^9}$$

$$= 4.952 \ mm.$$

$$\lambda = \frac{\xi}{1 + 50\rho'} = \frac{2}{1+0} = 2$$

$\Delta_{sust.} = 2 \times 4.952 = 9.904$ mm.

$\Delta_{total} = 6 + 9.904 = 15.904$ mm.

Q5)

$W_D = 1.2(6) = 7.2\ KN/m^2$

$W_L = 1.6(7) = 11.2\ KN/m^2$

$W_U = 7.2 + 11.2 = 18.4\ KN/m^2$

$d = 140 - 20 - \frac{12}{2} = 114^{mm}$

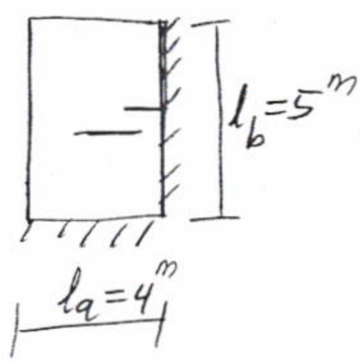
$\rho_{max} = \frac{51 * 0.85 * 23}{140 * 420} = 0.0169$

$As_{max} = 0.0169 * 1000 * 114 = 1927^{mm^2}$

$As'_{min} = 0.002 * 1000 * 140 = 280\ mm^2$

$S_{max} = \min(2h, 450) = \min(280, 450) = 280^{mm}$

case 4: $m = \frac{4}{5} = 0.8$



$Ma_{+ve)_D} = 0.039 * 7.2 * (4)^2 = 4.49\ KN.m/m$

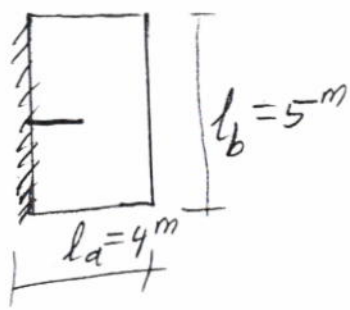
$Ma_{+ve)_L} = 0.048 * 11.2 * (4)^2 = 8.6\ KN.m/m$

$\Sigma = 13.09\ KN.m/m$

$Ma_{-ve)_{D+L}} = 0.071 * 18.4 * (4)^2 = 20.9\ KN.m/m$

Case 6: $m = 0.8$

$Ma_{-ve)_{D+L}} = 0.086 * 18.4 * (4)^2 = 25.31\ KN.m/m$



$\frac{20.9}{25.31} = 0.82 > 0.8$

M_U for bar A = 13.09 KN.m/m

M_U for bar B = 25.31 KN.m/m

For $M_u = 25.31$ KN.m/m

let $\phi = 0.9$; assume $a = 14^{mm}$

$$A_s = \frac{25.31 * 10^6}{0.9 * 420 * (114 - \frac{14}{2})} = 626^{mm}$$

$$a = \frac{626 * 420}{0.85 * 23 * 1000} = 13.4\ mm$$

$c = 16.4^{mm}$

$$\phi = \left[0.65 + 0.25 \left(\frac{1}{16.4/114} - \frac{5}{3}\right)\right] \leq 0.9$$

$\phi = 0.9$

$$S = \frac{1000 * 113}{626} = 180.5^{mm}$$

Use $\phi 12^{mm}$ @ 180^{mm} c/c

For $M_u = 13.09$ KN.m/m

$$A_s = \frac{13.09}{25.31}(626) = 324\ mm^2$$

$$S = \frac{1000 * 113}{324} = 348^{mm} > S_{max} = 280^{mm}$$

$\therefore$ Use $\phi 12^{mm}$ @ 280^{mm} c/c