University of Technology
Building and Construction Eng. Dept.
Final Exam –2013/2014

Subject : Mathematics I **Class: 1st stage**
Branch : All Branches **Time : 3 Hours**

ملاحظات 1- الاجابة على خمسة اسئلة فقط 2- توزع الدرجة الكلية على الاسئلة بالتساوي

Q.1 A) Find $\frac{dy}{dx}$: $if\ \pi^y y^\pi = x^{x^2} + \sqrt[4]{tan^{-1}(50) + 4^\pi}$

B) Find $\lim_{x\to 0} \frac{1-(cos3x)^4}{9x^2}$

C) Evaluate: $\int \frac{x^2\ dx}{\sqrt{x^2-4}}$

Q.2 A) Graph and find the domain and range for $y = -2 - 3|x - 1|$

B) Find the value of (H) if: $\begin{vmatrix} H & 2 & 3 \\ 1 & 5 & 7 \\ 2 & 1 & 3 \end{vmatrix} = \lim_{x\to 1} \frac{x^2-1}{x-1}$

C) Evaluate: $\int (x)^{\frac{1}{2}}\ ln\ x\ dx$

Q.3 A) $\int sin^3 x(\ \cos x)^{\frac{1}{3}}\ dx$

B) Find the area bounded by y=ln x, y=1 and x= e^3.

C) Find x and y if: $(3 + 4i)^2 - 2(x - yi) = x + yi$

Q.4 A) Evaluate $\frac{d}{dx}\int_0^{\frac{\pi}{2}} (sin3x)^{100}\ dx$.

B) If A (0, 0), B (1, 0) and C (2, 2)

1. Find the area of triangle ABC by integral method.
2. Find the area of triangle ABC by vector method

C) Find the Domain only for $y = \frac{1}{\sqrt{1-x^2}}$

Q.5 A) Find $\frac{dy}{d\theta}\ if\ y = ln\frac{\sqrt{sin\theta cos\theta}}{1+2ln\theta}$ and simplify the solution.

B) Find $\int_0^1 \frac{x^3}{x^2+2x+1} dx$

C) Find the solution for $\sinh(x^2) - 3\cosh(x^2) = -3$

Q.6 A) Find the radius and center for circle $x^2 + y^2 + 4x - 6y - 3 = 0$

B) Evaluate $\int \frac{lnx^2}{x}$ dx

C) Find x, if $x^{x^x} = x^{x^2}$

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Q.1 A) $\pi^y \cdot y^\pi = e^{x^2 \ln x} + \sqrt[4]{\tan^{-1} 50 + 4\pi}$

$\pi^y * \pi\, y^{\pi-1} + y^\pi * \pi^y * y' * \ln\pi = e^{x^2 \ln x}$ $\left[x^2 \cdot \frac{1}{x} + \ln x \cdot 2x\right]$

Q.1 B) $\lim_{x \to 0} \frac{(1-\cos^2 3x)(1+\cos^2 3x)}{3x \cdot 3x}$ + Zero

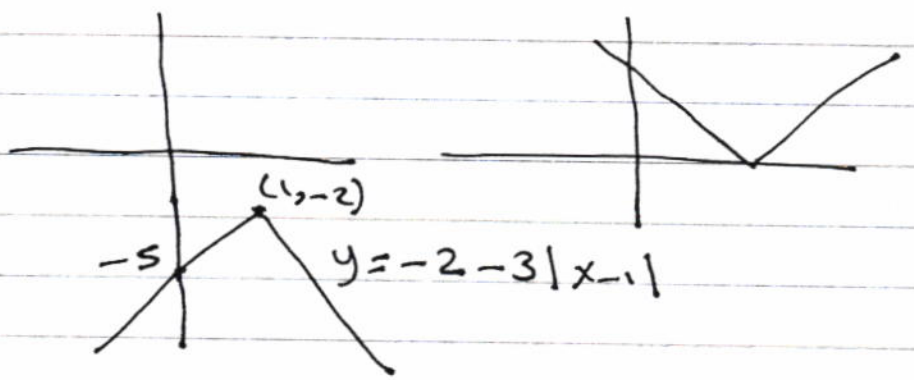
$= \frac{\sin 3x}{3x} \cdot \frac{\sin 3x}{3x}(1+\cos^2 3x) = 1+1 = 2$

Q.1 C) let $x = 2\sec\theta \Rightarrow dx = 2\sec\theta\tan\theta$

$I = \int \frac{4\sec^2\theta * 2\sec\theta\tan\theta\, d\theta}{\sqrt{4\sec^2\theta - 4}} = 4\int \sec^3\theta\, d\theta$ udv by part

Q.2 A) $|x-1| = \begin{cases} x-1 & \text{when } x-1 \geq 0 \\ -(x-1) & \text{when } x-1 < 0 \end{cases}$

$= \begin{cases} x-1 & \text{when } x \geq 1 \\ 1-x & \text{when } x < 1 \end{cases}$



B) $H(15-7) - 2(3-14) + 3(1-10) = 2$

$8H + 22 - 27 = 2$

$8H = 7 \Rightarrow H = \frac{7}{8}$

$\lim_{x \to 1} \frac{(x-1)(x+1)}{(x-1)} = 2$

Q.2 C) $\int \underbrace{x^{1/2}}_{dv} \underbrace{\ln x}_{u} dx = \ln x \cdot \frac{x^{3/2}}{3/2} - \int \frac{x^{3/2}}{3/2} \cdot \frac{1}{x} dx$

$= \frac{2}{3} \ln x \; x^{3/2} - \frac{2}{3} \int x^{1/2} dx$

$= \frac{2}{3} \ln x \; x^{3/2} - \frac{2}{3} \frac{x^{3/2}}{3/2} + C$

Q.3 A) $\int (1 - \cos^2 x) \sin x (\cos x)^{1/3} dx$

$= \int \sin x (\cos x)^{1/3} dx - \int (\cos x)^{7/3} \times \sin x \, dx$

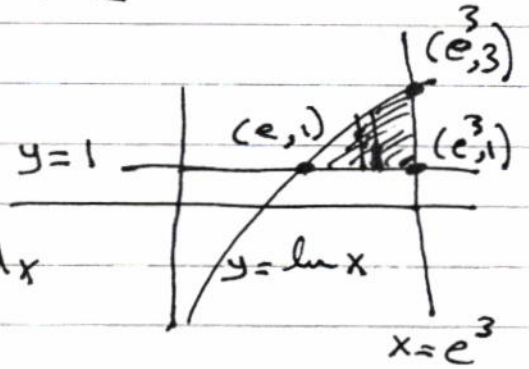
$= -\frac{(\cos x)^{4/3}}{4/3} + \frac{(\cos x)^{10/3}}{10/3} + C$

B) $A = \int_e^{e^3} (\ln x - 1) \, dx$

كيفية الحل

$\int \ln x dx = \ln x \cdot x - \int x \cdot \frac{1}{x} dx$

$= x \ln x - x$



$A = x \ln x - x - x \Big|_e^{e^3} = x \ln x - 2x \Big|_e^{e^3}$

$= e^3 \ln e^3 - 2e^3 - (e \ln e - 2e)$

$= 3e^3 - 2e^3 - (e - 2e) = \boxed{e^3 + e}$

كيفية الحل الثانية

$$\int_1^3 (x_2 - x_1) \, dy = \int_1^3 (e^3 - e^y) \, dy$$

$= e^3 y - e^y \Big|_1^3 = (3e^3 - e^3) - (e^3 - e)$

$= 2e^3 - e^3 + e$

$= \boxed{e^3 + e}$

Q.3 c) $9 + 24i + 16i^2 - 2x + 2yi = x + yi$

-16

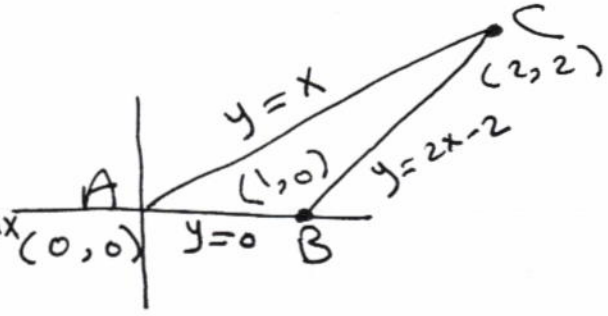
$-7 - 2x + (24 + 2y)i = x + yi$

$-7 - 2x = x \Rightarrow 3x = -7 \Rightarrow x = -\frac{7}{3}$

$24 + 2y = y \Rightarrow y = -24$

Q.4 A) = Zero

Q.4 B)



$A = \int_0^1 (y_2 - y_1)\,dx + \int_1^2 (y_2 - y_1)\,dx$

$= \int_0^1 (x - 0)\,dx + \int_1^2 x - (2x - 2)\,dx$

$= \int_0^1 x\,dx + \int_1^2 (x - 2x + 2)\,dx$

$= \int_0^1 x\,dx + \int_1^2 (-x + 2)\,dx$

$= \frac{x^2}{2}\Big|_0^1 + \frac{-x^2}{2} + 2x\Big|_1^2$

$= \frac{1}{2} + (-2 + 4) - \left(-\frac{1}{2} + 2\right)$

$2 \quad -1\frac{1}{2}$

$= \frac{1}{2} + \frac{1}{2} = 1$

$m = \frac{2-0}{2-0} = 1$

$y - y_0 = m(x - x_0)$

$y - 0 = 1(x - 0)$

$y = x$

$m = \frac{2-0}{2-1} = \frac{2}{1} = 2$

$y - 0 = 2(x - 1)$

$y = 2x - 2$

using vectors $AB = i$, $AC = 2i + 2j$

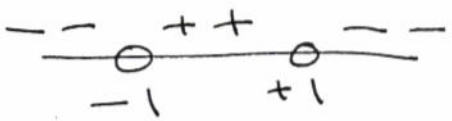
$\text{Area} = \frac{|AB \times AC|}{2} = \frac{2}{2} = 1$

$\begin{vmatrix} i & j & k \\ 1 & 0 & 0 \\ 2 & 2 & 0 \end{vmatrix}$

$= -1(-2k) = 2k$

Q.4 c) $1 - x^2 > 0$

$(1-x)(1+x) > 0$



Domain $(-1, +1)$
or $-1 < x < +1$

Q.5 A) $y = \ln(\sqrt{\sin\theta \cos\theta}) - \ln(1 + 2\ln\theta)$

$= \frac{1}{2}\ln(\sin\theta\cos\theta) - \ln(1 + \ln\theta^2)$

$= \frac{1}{2}[\ln\sin\theta + \ln\cos\theta] - \ln(1 + \ln\theta^2]$

$\frac{dy}{d\theta} = \frac{1}{2}\left[\frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta}\right] - \frac{\frac{2\theta}{\theta^2}}{1+\ln\theta^2}$

$= \frac{1}{2}(\cot\theta - \tan\theta) - \frac{2}{\theta(1+2\ln\theta)}$

Q.5 B) $I = \int x - 2 + \int \frac{3x+2}{x^2+2x+1}\,dx$

$\frac{A}{x+1} + \frac{B}{(x+1)^2}$

$\frac{3x+2}{(x+1)^2} = \frac{A(x+1)+B}{(x+1)^2}$

$3x + 2 = Ax + A + B$

$3 = A$
$2 = A + B$ $\Rightarrow B = -1$

$\int (x-2)\,dx + \int \frac{3}{x+1} + \int \frac{-1}{(x+1)^2}\,dx$

$= \left[\frac{x^2}{2} - 2x + 3\ln|x+1| + \frac{1}{x+1}\right]_0^1$

$= \left[\frac{1}{2} - 2 + 3\ln 2 + \frac{1}{2}\right] - [1] = 3\ln 2 - 2$

$$
\begin{array}{r|l}
 & x - 2 \\
\hline
x^2+2x+1 & x^3 \\
 & x^3 + 2x^2 + x \\
\hline
 & -2x^2 - x \\
 & -2x^2 - 4x - 2 \\
\hline
 & 3x + 2
\end{array}
$$

Q.5 c) $\left(\frac{1}{2}(e^{x^2}-e^{-x^2})-\frac{3}{2}(e^{x^2}+e^{-x^2})=-3\right)*2$

$e^{x^2}-e^{-x^2}-3e^{x^2}-3e^{-x^2}=-6$

$-2e^{x^2}-4e^{-x^2}=-6$

$\left(-2e^{x^2}-\frac{4}{e^{x^2}}=-6\right)*e^{x^2}$

$-2(e^{x^2})^2-4=-6e^{x^2}$

$-2(e^{x^2})^2+6e^{x^2}-4=0$

$(-2e^{x^2}+4)(1e^{x^2}-1)=0$

either $e^{x^2}=2 \Rightarrow x^2=\ln 2 \Rightarrow x=\pm\sqrt{\ln 2}$

or $e^{x^2}=1 \Rightarrow x^2=\ln 1=0 \Rightarrow x=0$

Q.6 A) $x^2+4x+4+y^2-6y+9=3+4+9$

$(x+2)^2+(y-3)^2=16$

$h=-2, k=3, r=4$

B) $2\int \frac{\ln x}{x} = 2\frac{(\ln x)^2}{2} = \ln^2 x + C$

or $\frac{1}{2}\int \frac{2}{x}\ln(x^2)\,dx = \frac{1}{2}\frac{(\ln x^2)^2}{2}+C$

$=\frac{1}{4}(\ln x^2)^2 = \frac{1}{4}(2\ln x)^2 = (\ln x)^2 + C$

c) $\ln x^{x^x} = \ln x^{x^2} \Rightarrow x^x \ln x = x^2 \ln x$

$(x^x - x^2)(\ln x) = 0$

either $\ln x = 0 \Rightarrow x = e^0 = 1$

or $x^x - x^2 = 0 \Rightarrow x^x = x^2 \Rightarrow x = 2$