University of Technology

Building and Construction Eng. Dept.

Answer - Set (3) 2013

Subject : Engineering Analysis **Class: 3rd Year**

Branch : All Branches **Time: 3 hrs.**

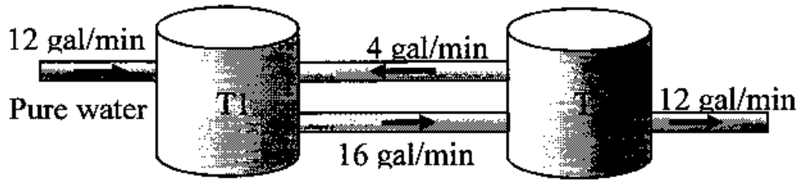
ملاحظة : دقة الحسابات اربع مراتب بعد الفارزة ولكافة الاسئلة

Part one <u>Engineering Analysis</u> Including Q1, Q2, Q3 and Q4

Answer <u>only three</u>

Q1---

In figure shown below, each of the two tanks contains 200 gal of water, in which initially 100 lb (Tank T1) and 200 lb (Tank T2) of salt are dissolved. The inflow, circulation and outflow are shown in figure. The mixture is kept uniform by stirring. Derive the equations which describe the amount of salt at any time in tank (T1) and (T2) and put it in matrix form without solving and write the initial conditions for each tank.



Solution:

Assume x : The amount of salt in tank T1 (lb)

Assume y : The amount of salt in tank T2 (lb)

General Equation:

$$\frac{dw}{dt} = \sum Q_{in}\, C_{in} - \sum Q_{out}\, C_{out}$$

$$\frac{dx}{dt} = \left(12 * 0 + 4 * \frac{y}{200}\right) - \left(16 * \frac{x}{200}\right)$$

$$\frac{dy}{dt} = \left(16 * \frac{x}{200}\right) - \left(4 * \frac{y}{200} + 12 * \frac{y}{200}\right)$$

$$D\,x + \frac{16x}{200} - \frac{4}{200}\; y = 0.0$$

$$\frac{-16x}{200} + \left(Dy + \frac{16}{200}\, y\right) = 0.0$$

Rearrange the above equations:

$$\left(D + \frac{16}{200}\right)x - \frac{4}{200}\, y = 0.0 \text{ --------- } (1)$$

$$\frac{-16}{200}\; x + \left(D + \frac{16}{200}\right)y = 0.0 \text{ ------ } (2)$$

Now put the equations in Matrix form

$$\begin{bmatrix} D+\frac{16}{200} & \frac{-4}{200} \\ -\frac{16}{200} & D+\frac{16}{200} \end{bmatrix} \begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

Or

$$\begin{bmatrix} D+0.08 & -0.02 \\ -0.08 & D+0.08 \end{bmatrix} \begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

Initial condition:

For tank T1 ⟶ at time t = 0, x= 100 lb
For tank T2 ⟶ at time t = 0, y= 200 lb

(16 marks)

Q2---

Solve the following equation using Euler - Cauchy method:

$$(x^2 - 6x + 9)y'' + (2x-6)y' + 6y = 0$$

(16 marks)

Solution:

$$(x-3)^2 y'' + 2(x-3)y' + 6y = 0$$

Solution:

Let y= $(x-3)^m$

y' = $m(x-3)^{m-1}$

y'' = $m(m-1)(x-3)^{m-2}$

$$\therefore \quad (x-3)^2(m(m-1)(x-3)^{m-2} + 2(x-3)(m)(x-3)^{m-1} + 6(x-3)^m = 0$$

$$m^2 + m + 6 = 0$$

$$m = \frac{-1 \pm \sqrt{1-4*6*1}}{2*1}$$

$$m = -\frac{1}{2} \pm \frac{\sqrt{23}}{2} i$$

$$y = (x-3)^{-0.5}\left[A\cos\frac{\sqrt{23}}{2}\ln|x-3| + B\sin\frac{\sqrt{23}}{2}\ln|x-3|\right]$$

Q3- ---

Solve the following partial differential equation using Separation of variables method:

$$U_x + U_y = 2(x+y)U$$

Solution :

Assume $U(x,y) = G(x).F(y)$

$$U_x = \frac{\partial u}{\partial x} = G'.F$$

$$U_y = \frac{\partial u}{\partial y} = G.F'$$

$$\therefore \; G'.F + G.F' = 2\,(x+y)G.F \qquad \div \quad G.F$$

$$\frac{G'}{G} + \frac{F'}{F} = 2\,(x+y)$$

$$\frac{G'}{G} - 2x = 2y - \frac{F'}{F} = \lambda$$

$$\frac{G'}{G} - 2x = \lambda$$

$$\underline{G = A\,e^{\lambda x + x^2}}$$

$$2y - \frac{F'}{F} = \lambda$$

$$\frac{F'}{F} = 2y - \lambda$$

$$\underline{F = B\,e^{y^2 - \lambda y}}$$

Assume $k = A.B$

$$\therefore U = k\,e^{\lambda(x-y)+x^2+y^2}$$

(16 marks)

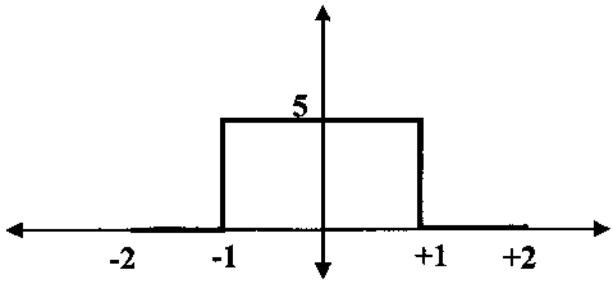
Q4---

For the given function:

$$f(x) = \begin{cases} 0 & if & -2 < x < -1 \\ 5 & if & -1 < x < 1 \\ 0 & if & 1 < x < 2 \end{cases}$$

1- **Draw the function.**
2- **State whether the function is even or odd.**
3- **Find the Fourier series for the function.**

1-



2- The function is <u>even</u>

3- $F(x) = \frac{a_0}{2} + \sum_1^n a_n \cos\frac{n\pi x}{L} + \sum_1^n b_n \sin\frac{n\pi x}{L}$

$\because$ *the function is even* , $\therefore\ b_n = 0$

$$L = \frac{2-(-2)}{2} = 2$$

$$a_0 = \frac{2}{L}\int_0^2 F(x)dx = \frac{2}{L}\left(\int_0^1 5\,dx + \int_1^2 0\,dx\right)$$

$$a_0 = \frac{2}{L}\int_0^1 5\,dx = 5 \qquad \therefore\ a_0 = 5$$

$$a_n = \frac{2}{L}\int_0^2 F(x)\cos\frac{n\pi x}{L}dx$$

$$a_n = \frac{2}{2}\int_0^1 f(x)\cos\frac{n\pi x}{L}dx = \frac{10}{n\pi}\sin\frac{n\pi}{2}$$

$$a_n = \frac{10}{n\pi} \qquad if\ n = 1, 5, 9, ----$$

$$a_n = \frac{-10}{n\pi} \qquad if\ n = 3, 7, 11, ----$$

$$a_n = 0 \qquad if\ n = even\ (2, 4, ------).$$

$$F(x) = \frac{5}{2} + \frac{10}{\pi}\left(\cos\frac{\pi}{2}x - \frac{1}{3}\cos\frac{3\pi}{2}x + \frac{1}{5}\cos\frac{5\pi}{2}x ----\right)$$

(16 marks)

اقلب الصفحة لطفا

Part two <u>Numerical Analysis</u> / include Q5, Q6 ,Q7 and Q8 Answer only <u>three</u> questions <u>including Q8</u>.

Q5--

Find the solution of the system of equations using <u>Gauss –Seidel</u> method: (<u>Note only use 3 iteration</u>).

$$10x_1 + 2x_2 - x_3 = 7$$
$$x_1 + 8x_2 + 3x_3 = -4$$
$$-2x_1 - x_2 + 10x_3 = 9$$

$$x_1 = \frac{7 - (2x_2 - x_3)}{10}$$
$$x_2 = \frac{-4 - (x_1 + 3x_3)}{8}$$
$$x_3 = \frac{9 - (-2x_1 - x_2)}{10}$$

Assume initial value :

$$x_2 = x_3 = 0$$

$$x_1 = \frac{7 - (2 * 0 - 0)}{10} = 0.7$$

$$x_2 = \frac{-4 - (0.7 + 3 * 0)}{8} = -0.5875$$
$$x_3 = \frac{9 - (-2 * 0.7 - (-0.5875))}{10} = 0.9813$$

Second iteration:

$$x_1 = 0.9156$$
$$x_2 = -0.9824$$
$$x_3 = 0.9849$$

Third iteration and the final answer will be as follows:

$$x_1 = 0.9949$$
$$x_2 = -0.9937$$
$$x_3 = 0.9996$$

(16 marks)

Q6---

(a)-The depth x in meter to which the ball submerged under water is given by the following equation:

$f(x) = x^3 - 0.165\,x^2 + 3.993 * 10^{-4}$

Use Newton Raphson method to find the roots of equation (Note: use only 3 iterations, Start With x= 0.05 as initial value).

Solution:

$F(x) = x^3 - 0.165\,x^2 + 3.993 * 10^{-4}$

$F'(x) = 3x^2 - 0.33\,x$

I	X	F(x)	F'(x)	$\frac{F(x)}{F'(x)}$	$x_{i+1=x_i} - \frac{F(x)}{F'(x)}$
1	0.05	0.0001	-0.009	-0.0122	0.0622
2	0.0622	0.0000	-0.0089	0	0.0622
X		= 0.0622			

(b)- Use Simpson's $\frac{3}{8}$ rule to find the following integration

$\int_{0.3}^{0.6} x\, e^{x^2}\, dx$, **Note: use n= 3.**

Solution:

$$h = \frac{0.6 - 0.3}{3} = 0.1$$

X	F(x)= xe^{x^2}
0.3	0.3283
0.4	0.4694
0.5	0.6420
0.6	0.8600

$$\int_{0.3}^{0.6} xe^{x^2}\, dx = \frac{3}{8} * 0.1\,[0.3283 + 3(0.4694 + 0.6420) + 0.8600] = 0.1696$$

(16 marks)

Q7---

For the differential equation $y' = \cos x + y$ **, if y (0) = 2 and h = 0.1. Find y(0.2) by <u>one</u> of the following method:**

a)- modified Euler method.

b)- 4th order Range- kutta method.

Solution :

a- Using modified Euler method

I	x_i	y_i	y' =cos x+y	$y_{i+1} = y_i + hy'_i$	$x_{i+1} = x_i + hy'_i$	$y'_{i+1} = f(x_{i+1}, y_{i+1})$	y'_{av}	$y_{i+1} = y_i + hy'_{ave}$
0	0	2	3	2.3	0.1	3.2950 3.3098 3.3105	3.1475 3.1549 3.1553	2.3148 2.3155 2.3155
1	0.1	2.3155	3.3105	2.6466	0.2	3.6267 3.6441 3.6432	3.4686 3.4773 3.4768	2.6641 2.6632 2.6632
2	0.2	2.6632						

F (0.1) = 2.3155
F (0.2) = 2.6632

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b)- using Runge --kutta method

$$y_{i+1} = y_i + \frac{1}{6}[k_1 + 2(k_2 + k_3) + k_4]$$
$$k_1 = F(x_0, y_0) * h$$
$$k_2 = F\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right) * h$$
$$k_3 = F\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right) * h$$
$$k_4 = F(x_0 + h, y_0 + k_3) * h$$

$Stage\ 1 - - - - - - - -\ find\ y(0.1)$ **using x=0 , y=2**

$$k_1 = F(0,2) * 0.1 = (\cos 0 + 2) * 0.1 = 0.3$$
$$k_2 = F\left(0 + \frac{0.1}{2}, 2 + \frac{0.3}{2}\right) * 0.1 = F(0.05, 2.15) * 0.1 = 0.3149$$
$$k_3 = F\left(0 + \frac{0.1}{2}, 2 + \frac{0.3149}{2}\right) * 0.1 = F(0.05, 2.1575) * 0.1 = 0.3156$$
$$k_4 = F(0 + 0.1, 2 + 0.3156) * 0.1 = F(0.1, 2.3156) * 0.1 = 0.3311$$

$$y_{0.1} = y_0 + \frac{1}{6}[k_1 + 2(k_2 + k_3) + k_4]$$
$$y_{0.1} = 2 + \frac{1}{6}[0.3 + 2(0.3149 + 0.3156) + 0.3311] = 2.3154$$

$Stage\ 2 - - - - - - - - Find\ y(0.2)$, using x=0.1 , y(0.1)= 2.3154

$$y_{0.2} = y_{0.1} + \frac{1}{6}[k_1 + 2(k_2 + k_3) + k_4]$$
$$k_1 = F(0.1, 2.3154) * 0.1 = 0.3310$$
$$k_2 = F\left(0.1 + 0.05, 2.3154 + \frac{0.3310}{2}\right) * 0.1 = 0.3470$$
$$k_3 = F\left(0.1 + 0.05, 2.3154 + \frac{0.3470}{2}\right) * 0.1 = 0.3478$$
$$k_4 = F(0.1 + 0.1, 2.3154 + 0.3478) * 0.1 = 2.6629$$

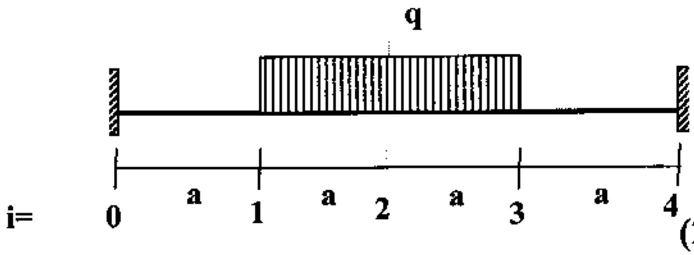
y(0.1)= 2.3154
y(0.2)=2.6629

(16 marks)

Q8---

Find the deflection at pivotal points shown in figure below <u>using finite difference method</u> and the general equation is given below.(Note: make use of symmetry).

$$EI\frac{d^4y}{dx^4} = w \qquad \text{if} \quad \left.\begin{matrix} y = 0 \\ y' = 0 \end{matrix}\right\} \quad \text{at } x = 0 \quad \text{and} \quad x = L$$



(20 marks)

Solution :

$at\ i = 0,\ y_0 = 0\ and\ \ \frac{dy}{dx} = 0$

$at\ i = 4,\ \ \ y_4 = 0\ and\ \ \frac{dy}{dx} = 0$

$\because \frac{dy}{dx} = \frac{y_{i+1} - y_{i-1}}{2h}$, **apply this equation at nodes i=0 and i=4 we get :**

$y_{-1} = y_1\ \ and\ \ \ y_5 = y_3$ **Respectively**

Now use the general equation : $EI\frac{d^4y}{dx^4} = w$:

$$\frac{-w_i}{EI} = \frac{d^4y}{dx^4} = \frac{y_{i+2} - 4y_{i+1} + 6y_i - 4y_{i-1} + y_{i-2}}{h^4}$$

Apply this equation at nodes i= 1 and 2 , take into consideration the symmetry

$\therefore y_3 = y_1$

At node i =1 , $w_i = \frac{q+0}{2} = \frac{q}{2}$

$$\frac{-q}{2EI} = \frac{d^4y}{dx^4} = \frac{y_3 - 4y_2 + 6y_1 - 4y_0 + y_{-1}}{a^4}$$

$$\frac{-qa^4}{2EI} = 8y_1 - 4y_2 \text{-------- } (1)$$

At node i=2 : $w_i = -q$

$$\frac{-q}{EI} = \frac{d^4y}{dx^4} = \frac{y_4 - 4y_3 + 6y_2 - 4y_1 + y_0}{a^4}$$

$$\frac{-qa^4}{EI} = -8y_1 + 6y_2 \text{-------- } (2)$$

Solving equations (1) and (2) :

$$y_1 = y_3 = \frac{-7}{16}\frac{qa^4}{EI}$$

$$y_2 = \frac{-3}{4}\frac{qa^4}{EI}$$

مع تمنياتنا بالنجاح والتوفيق